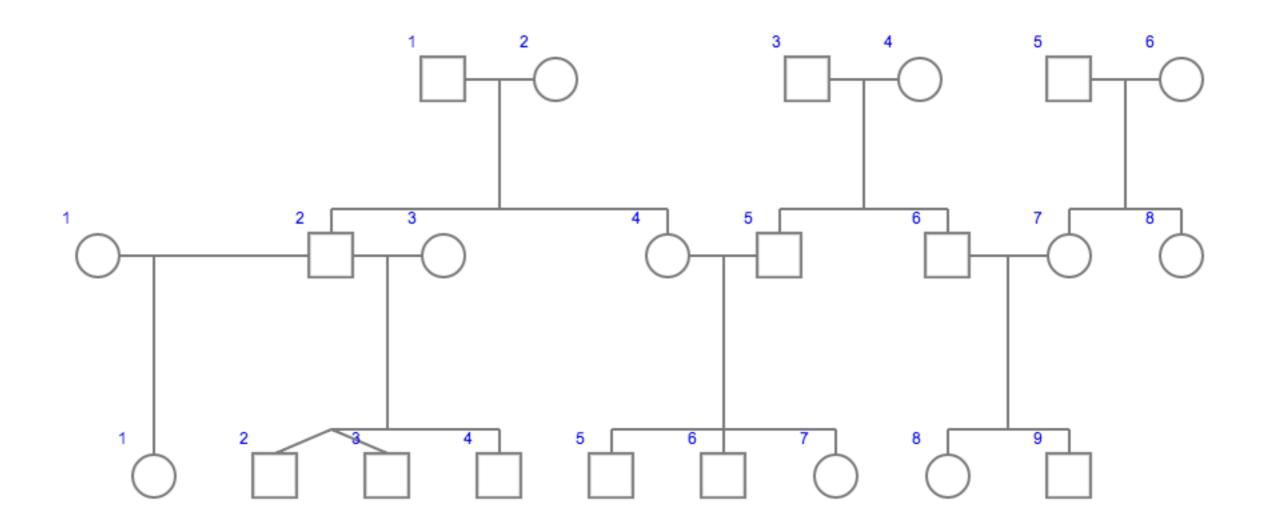
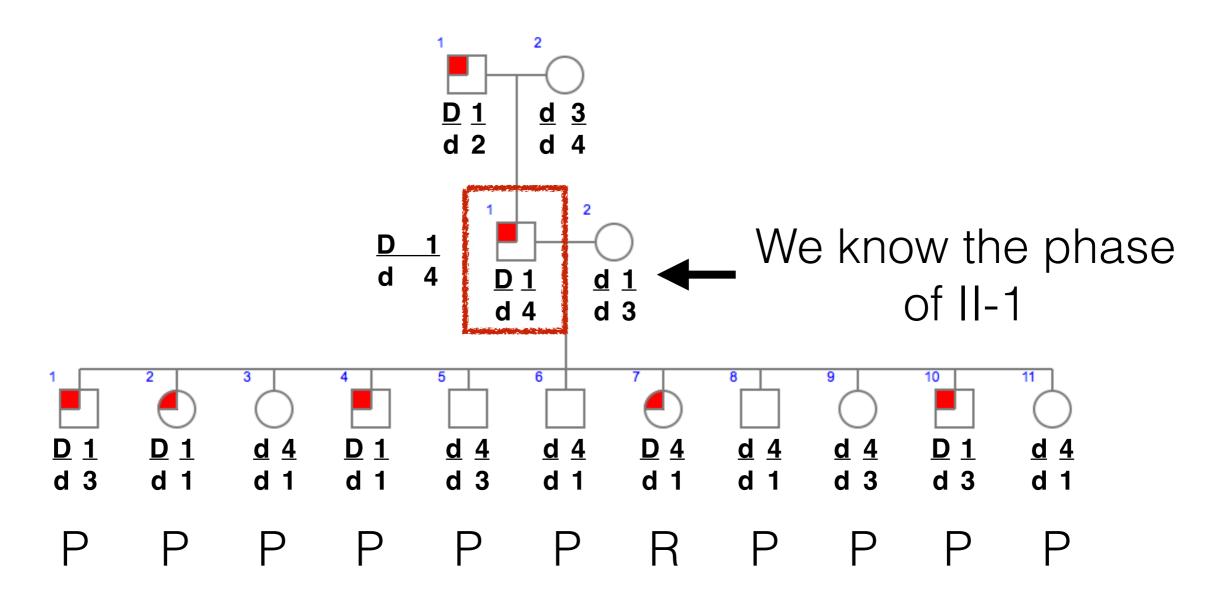
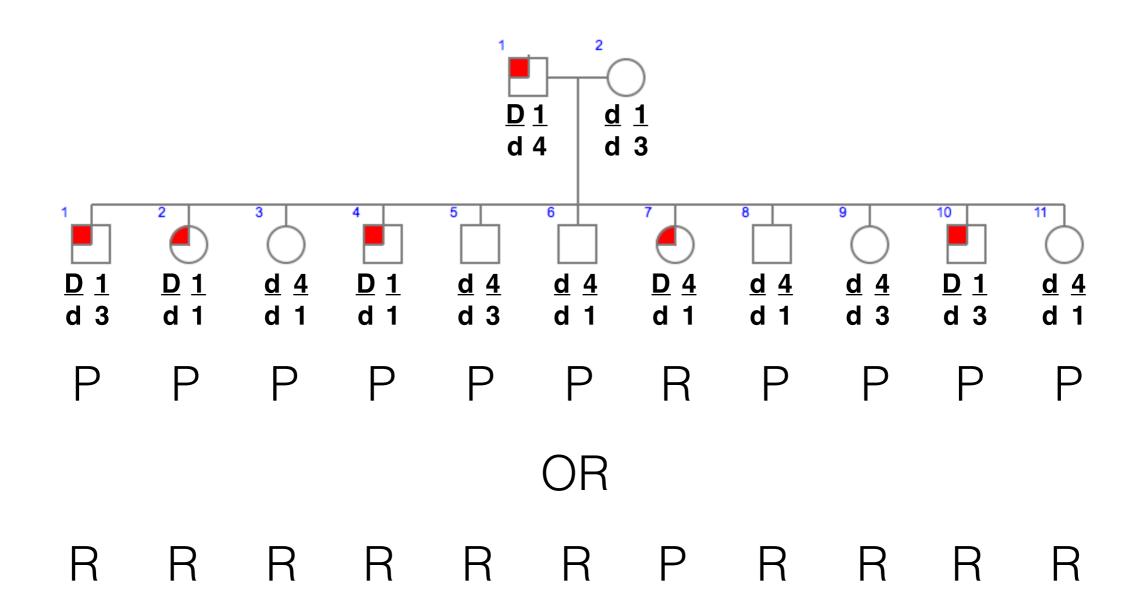
### Linkage mapping in families



## Linkage to genetic markers tells us where disease genes are



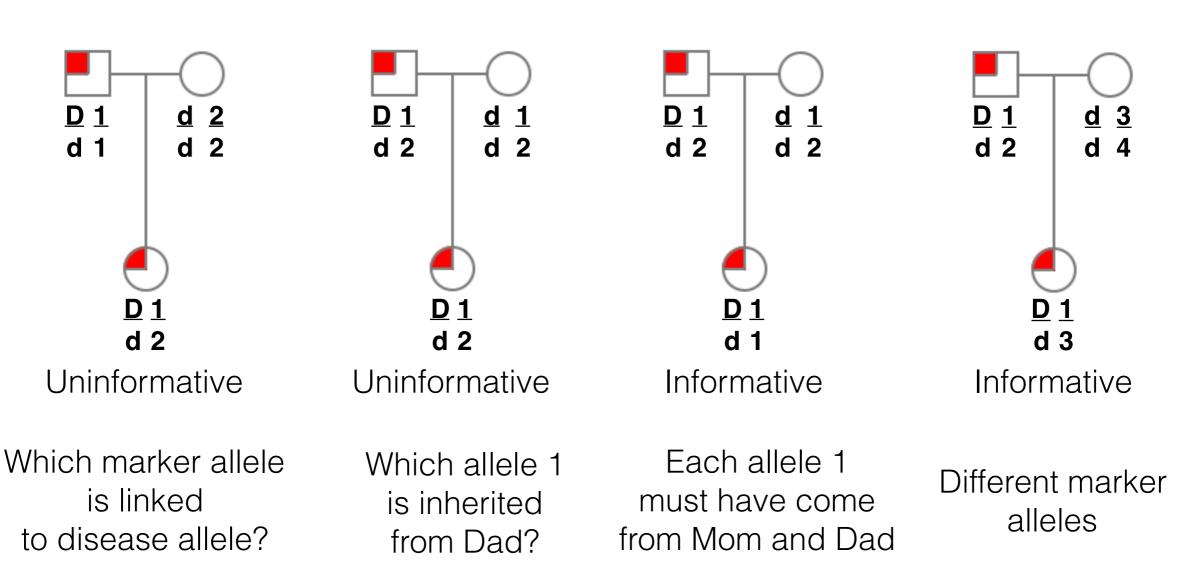
## Linkage to genetic markers tells us where disease genes are



Sometimes, we don't know the phase of the parent, and both possibilities of phase are equally likely

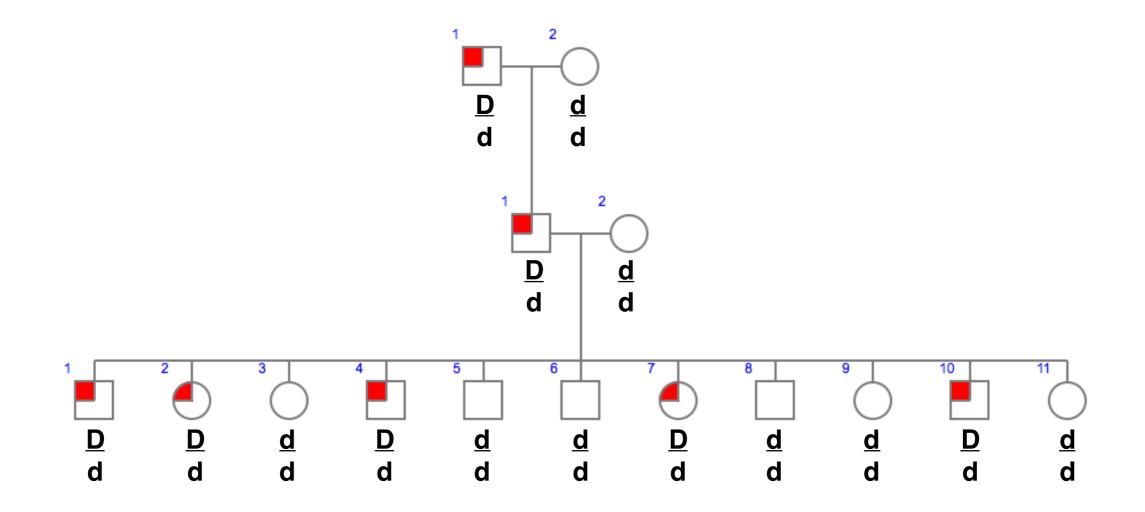
# Some allelic combinations are non-informative and can not be included in mappings

Consider a dominant trait and a variant marker:



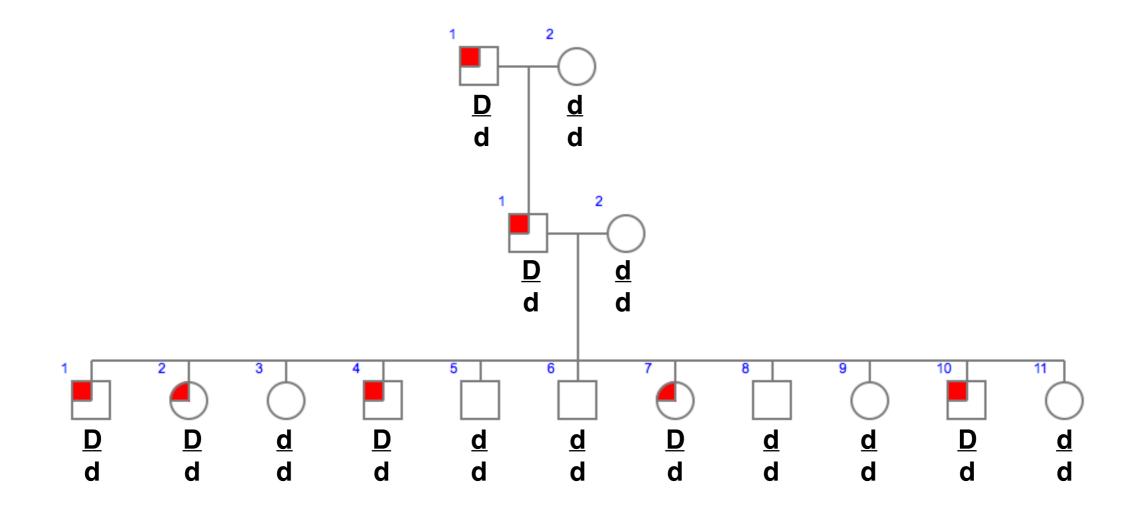
We want to determine if the daughter inherited a recombinant or parental chromosome

## Imagine you could genotype millions of markers in each individual



The goal is to measure linkage of many markers to the disease-causing allele to map the gene

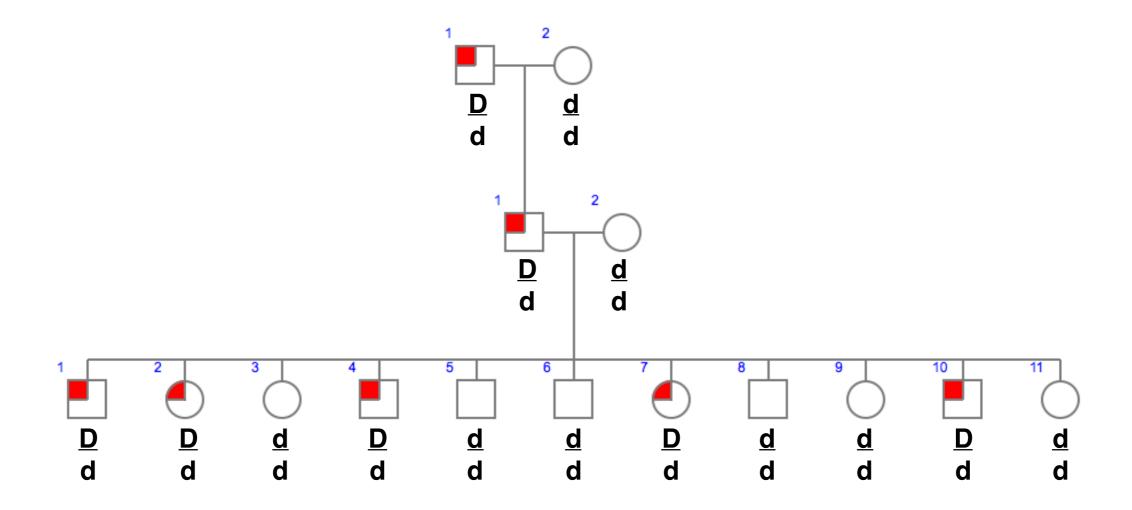
# We want to measure how close each marker is to the disease-causing allele



$$\frac{\text{Number of recombinants}}{\text{Total progeny}} \quad \times \quad 100 = \frac{\text{Recombination}}{\text{frequency}}$$

But we don't know who is a recombinant because we don't have true-breeding strains

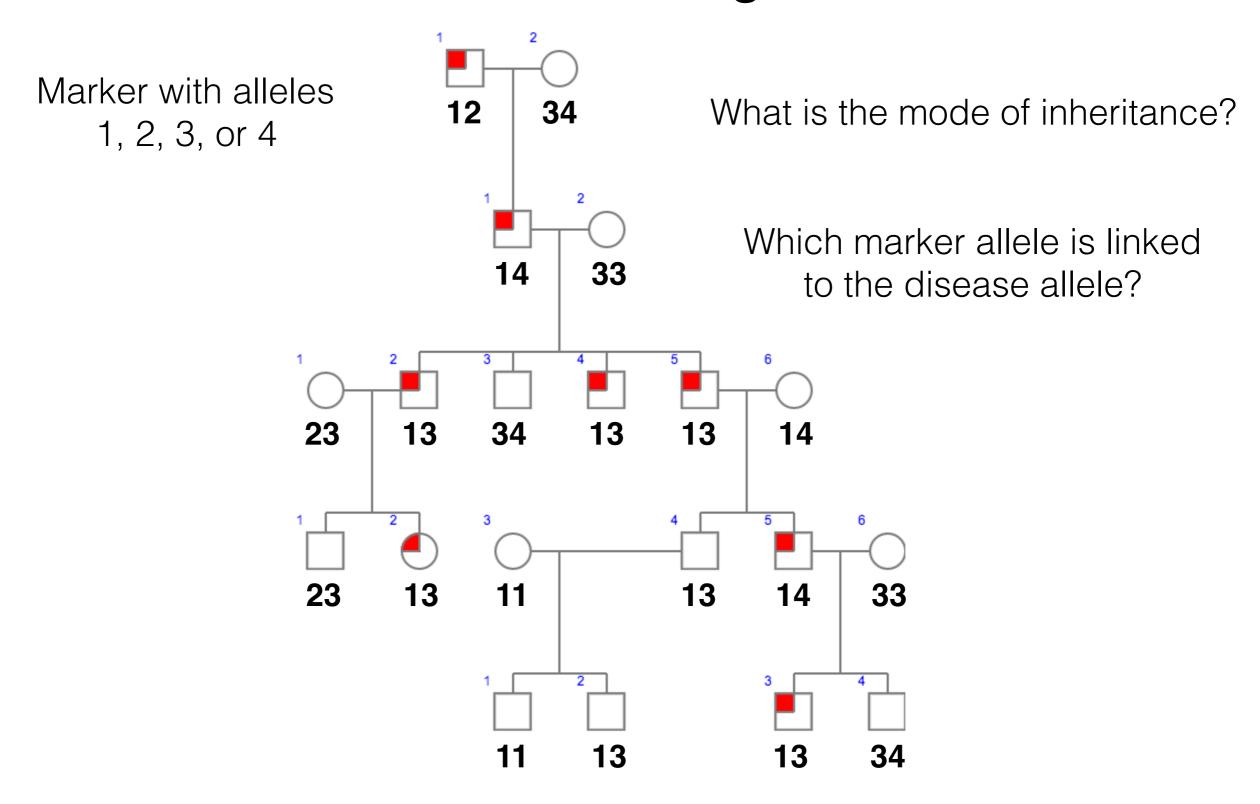
## We want to measure how close each marker is to the disease-causing allele



Likelihood of linkage between marker and disease-causing allele

Likelihood of NO linkage between marker and disease-causing allele

# The goal of linkage analysis is to identify a marker nearby the disease-causing allele



The odds ratio is a statistical method to measure association of some attribute with the presence or absence of another attribute.

Probability of pedigree under linkage versus no linkage

LOD = 
$$log_{10}$$
  $\frac{P(pedigree with linkage)}{P(pedigree with no linkage)}$ 

LOD > 3 is good evidence of linkage 1 in 1,000 chance of data by random assortment

LOD < 0 means no linkage and observed data are more likely by chance

# How linked is our marker to the disease-causing allele? Recombination frequency

Recombination frequency is written as  $\theta$ 

Percentage of recombinant gametes passed down

 $\theta$  = 0 is perfect linkage

 $\theta$  = 0.5 is no linkage

The odds ratio is a statistical method to measure association of some attribute with the presence or absence of another attribute.

Probability of pedigree under linkage versus no linkage

LOD = 
$$log_{10}$$
  $\frac{P(pedigree with linkage)}{P(pedigree with no linkage)}$ 

LOD = 
$$log_{10}$$
 
$$\frac{P(data \mid \boldsymbol{\theta})}{P(data \mid \boldsymbol{\theta} = 0.5)}$$
 P(data) = probability that a particular gamete was inherited

 $P(data | \theta = 0.5)$  is independent assortment

$$LOD = log_{10} \qquad \frac{P(data \mid \boldsymbol{\theta})}{P(data \mid \boldsymbol{\theta} = 0.5)} \qquad P(data) = probability that a}$$

$$P(data | \theta) =$$

Probability that recombination did not occur for parental 1 -  $\theta$ 

Probability that recombination did occur for recombinant  $\theta$ 

Probability of phase of parent

Each individual is independent.
Use product rule to multiply probabilities.

LOD = 
$$log_{10}$$
 
$$\frac{P(data \mid \boldsymbol{\theta})}{P(data \mid \boldsymbol{\theta} = 0.5)}$$
 P(data) = probability that a particular gamete was inherited

$$P(data \mid \theta = 0.5)$$

Equal probability of two loci independently assorting and one gamete being passed down

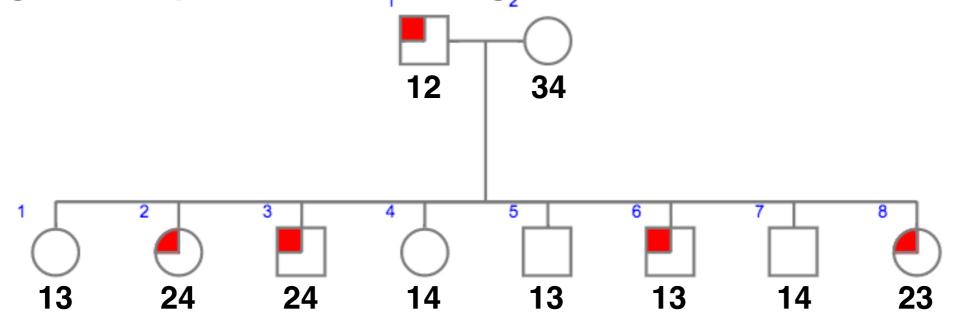
Each individual is independent.
Use product rule to multiply probabilities.

Probability of pedigree under linkage versus no linkage

LOD = 
$$log_{10}$$
  $\frac{P(pedigree with linkage)}{P(pedigree with no linkage)}$ 

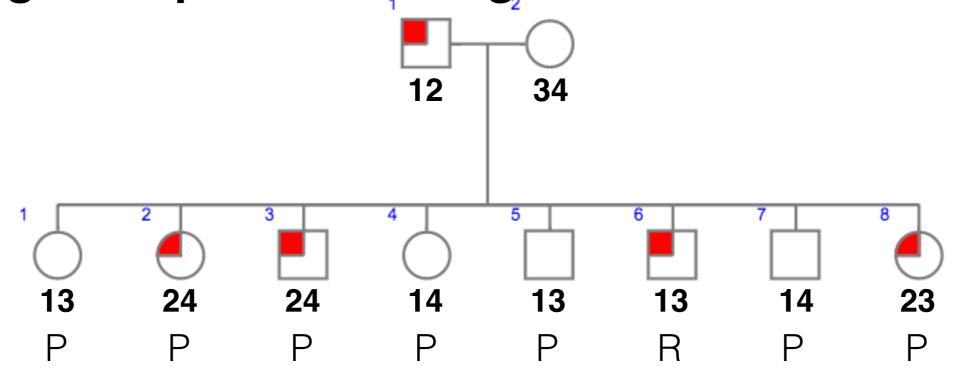
LOD = 
$$log_{10}$$
 
$$\frac{P(data \mid \boldsymbol{\theta})}{P(data \mid \boldsymbol{\theta} = 0.5)}$$
 P(data) = probability that a particular gamete was inherited

LOD = 
$$log_{10}$$
  $\frac{(1 - \theta)^P \times \theta^R}{0.5^{(P + R)}}$  Use this equation when we know phase.



The paternal grandmother had the disease and her genotype was 2,4. Her husband's genotype was 11 and he was not affected

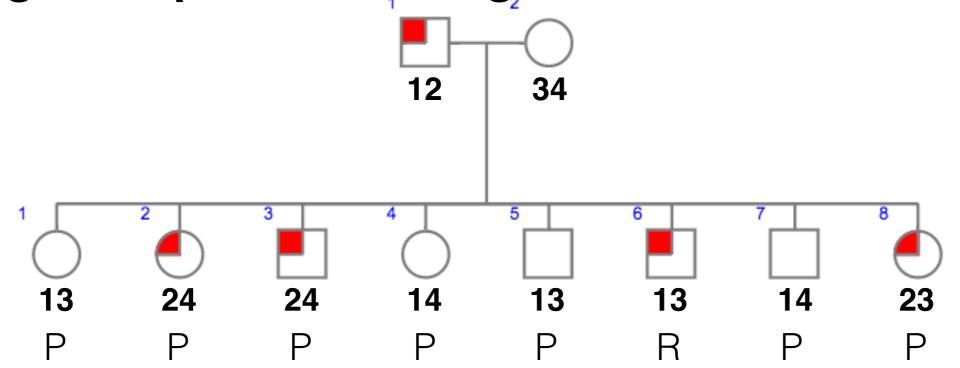
$$LOD = log_{10} \qquad \frac{(1 - \theta)^{P} \times \theta^{R}}{0.5^{(P + R)}}$$



The paternal grandmother had the disease and her genotype was 24. Her husband's genotype was 11 and he was not affected

$$LOD = log_{10} \qquad \frac{(1 - \theta)^{P} \times \theta^{R}}{0.5^{(P + R)}}$$

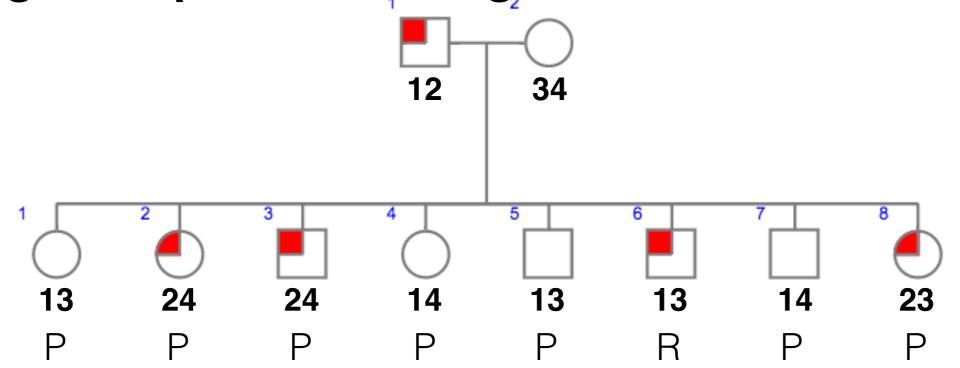
#### 1. Determine who is recombinant or parental



The paternal grandmother had the disease and her genotype was 24. Her husband's genotype was 11 and he was not affected

LOD = 
$$log_{10}$$
 
$$\frac{(1 - \theta)^7 \times \theta^1}{0.5^{(7+1)}}$$

### 2. Put the P and R numbers in the equation

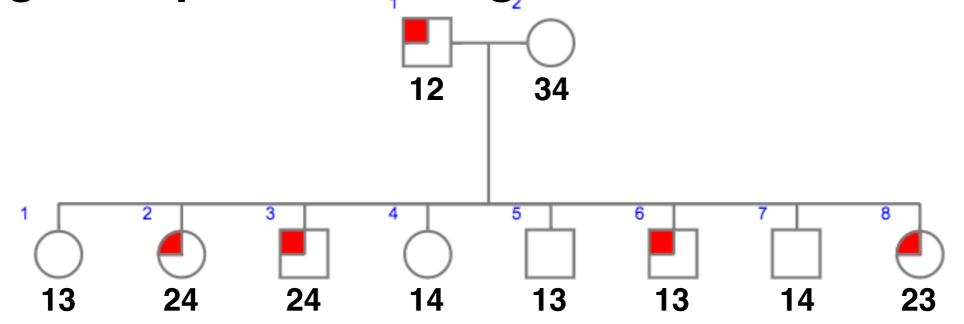


The paternal grandmother had the disease and her genotype was 24. Her husband's genotype was 11 and he was not affected

LOD = 
$$log_{10}$$
 
$$(1 - 0.125)^7 \times 0.125^1$$
$$0.5^{(7+1)}$$

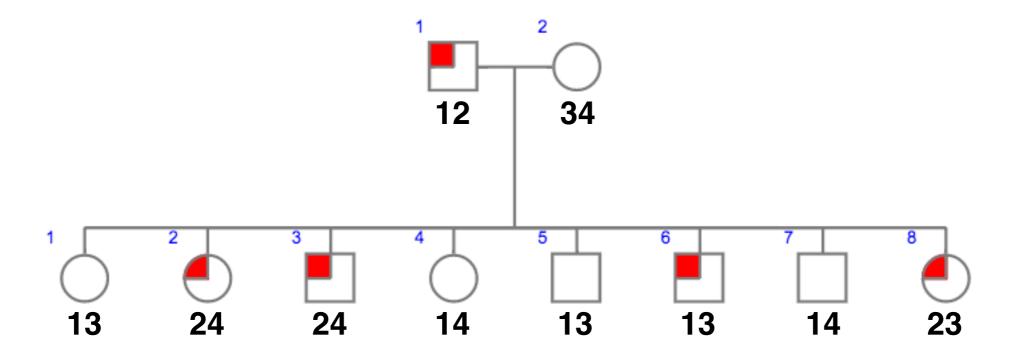
#### 3. Put the recombination fraction in the equation

$$\theta = 0.125$$



$$LOD = 1.1$$
  $\theta = 0.125$ 

$$\theta = 0.125$$



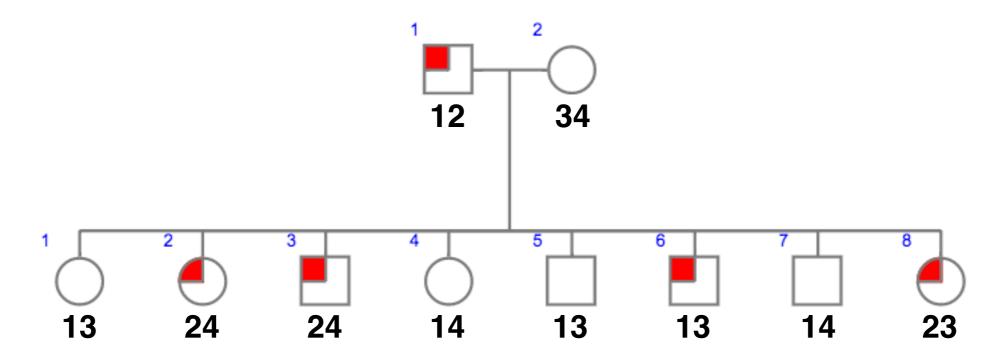
Do we know the phase of I-1?

#### NO

Every child has an equal chance of being a recombinant or parental

#### Difference between being informative and knowing phase

Informative = Parent heterozygous at each of two loci



With unknown phase of parent, the LOD equation is...

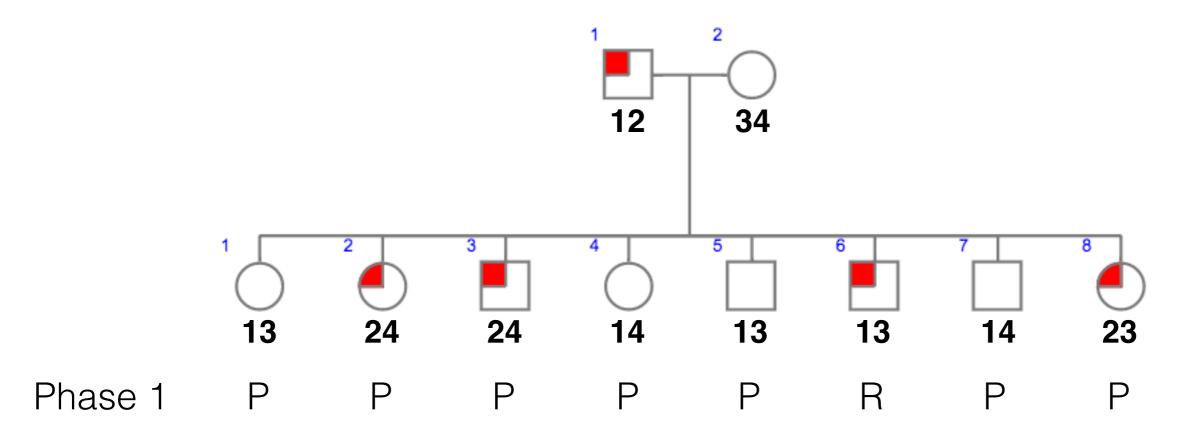
LOD = 
$$log_{10}$$
 
$$\frac{\frac{1}{2}((1 - \theta)^{P} \times \theta^{R}) + \frac{1}{2}((1 - \theta)^{P} \times \theta^{R})}{(0.5^{(P + R)})}$$

½ chance for each phase

Phase 1

$$\frac{D}{d}$$
  $\frac{2}{1}$ 

$$\frac{D}{d}$$
 2



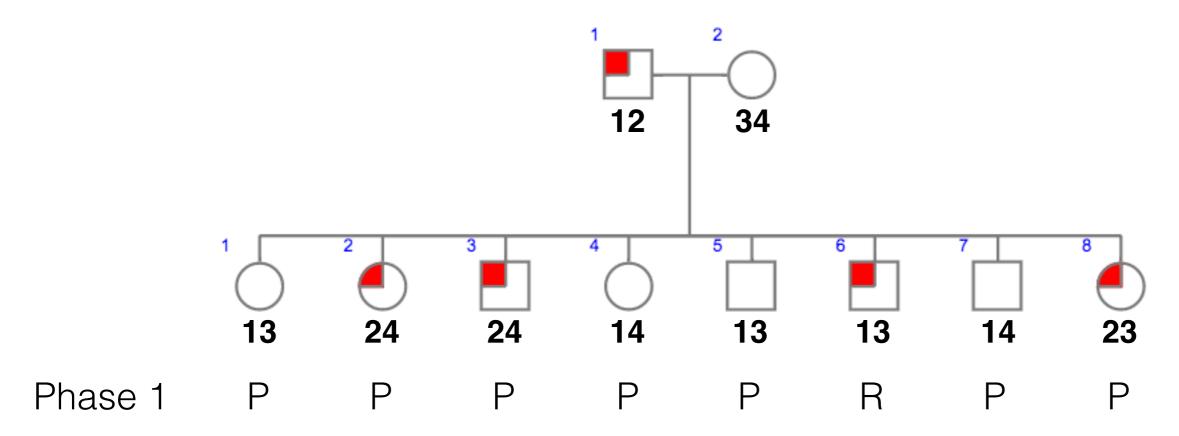
LOD = 
$$log_{10}$$
 
$$\frac{\frac{1}{2}((1 - \theta)^{P} \times \theta^{R}) + \frac{1}{2}((1 - \theta)^{P} \times \theta^{R})}{(0.5^{(P + R)})}$$

½ chance for each phase

Phase 1

$$\frac{D}{d}$$
  $\frac{2}{1}$ 

$$\frac{D}{d}$$
 2



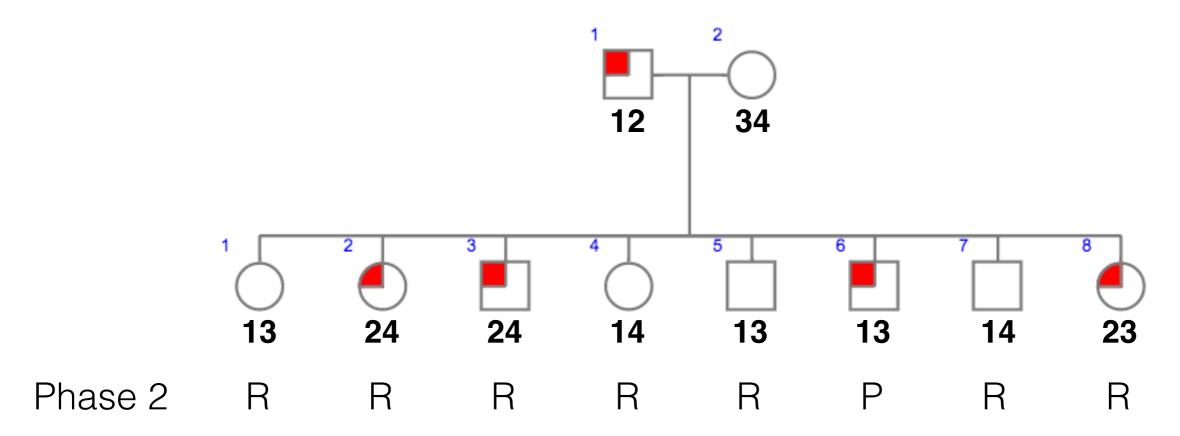
LOD = 
$$log_{10}$$
 
$$\frac{\frac{1}{2}((1 - \theta)^7 \times \theta^1) + \frac{1}{2}((1 - \theta)^{NR} \times \theta^R)}{(0.5^{(8)})}$$

½ chance for each phase

Phase 1

$$\frac{D}{d}$$
 1

$$\frac{D}{d}$$
 2



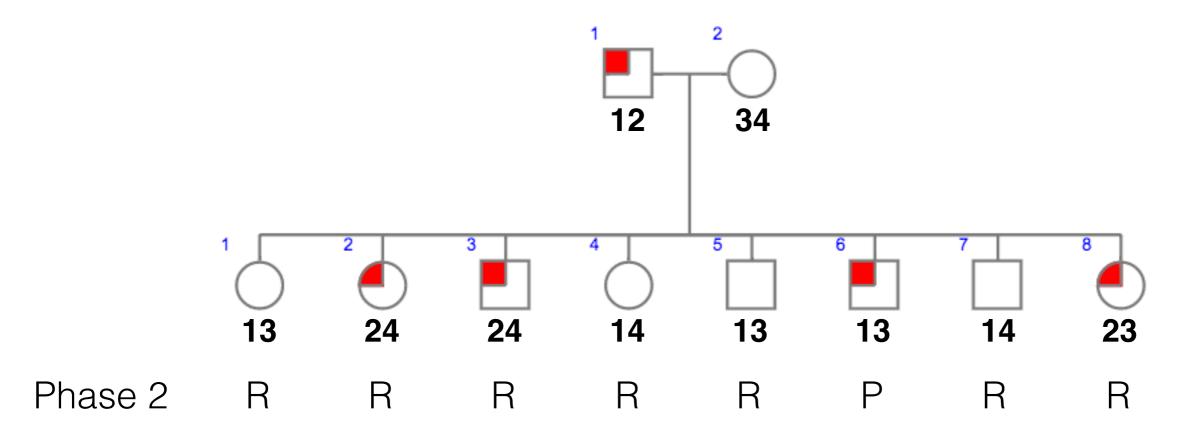
LOD = 
$$\log_{10} \frac{\frac{1}{2}((1-\theta)^{7} \times \theta^{1}) + \frac{1}{2}((1-\theta)^{1} \times \theta^{7})}{(0.5^{(8)})}$$

½ chance for each phase

Phase 1

$$\frac{D}{d}$$
 1

$$\frac{D}{d}$$
 2



$$LOD = log_{10} \frac{1}{2}((1 - 0.125)^7 \times 0.125^1) + \frac{1}{2}((1 - 0.125)^1 \times 0.125^7)$$

$$(0.5^{(8)})$$

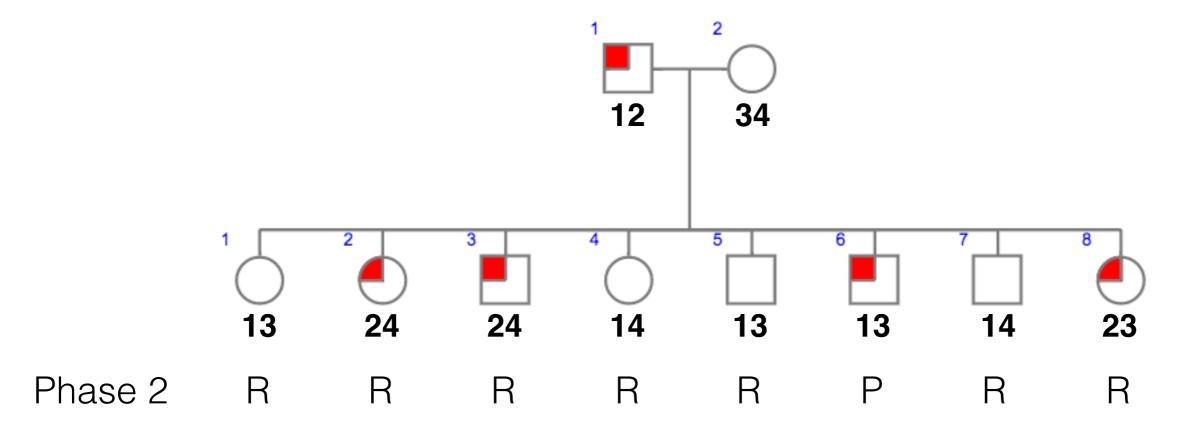
½ chance for each phase

Phase 1

$$\theta = 0.125$$

$$\frac{D}{d}$$
 1

$$\frac{D}{d}$$



$$LOD = 0.79$$
  $\theta = 0.125$ 

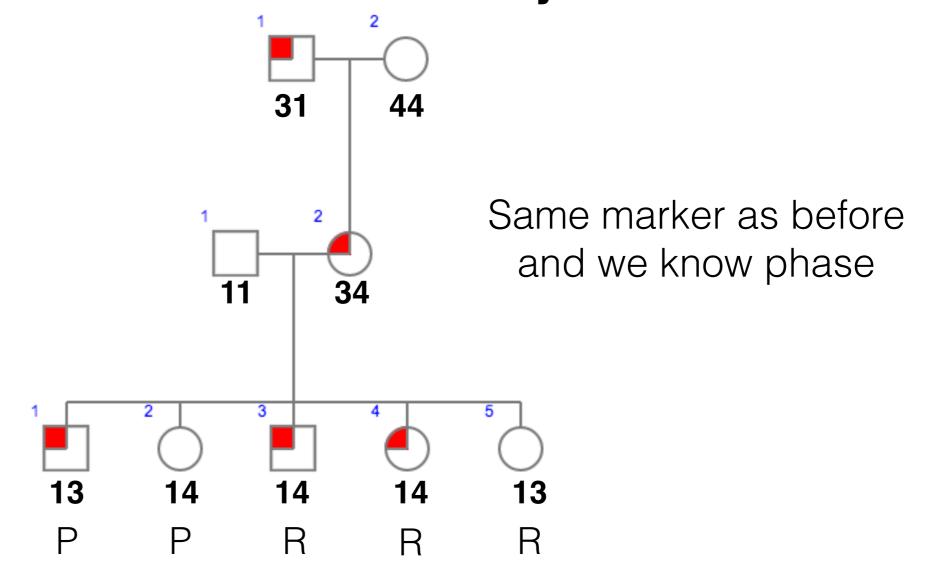
Without phase information, LOD scores decrease

## Some properties of LOD scores

LOD scores from independent families can be added (product rule with logarithms)

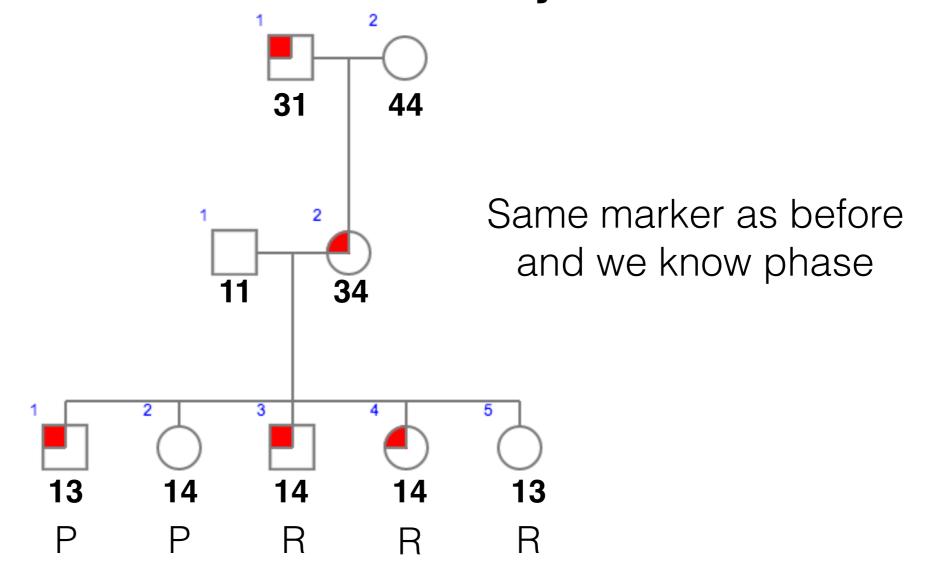
Determining phase increases the LOD score

### Let's add another family



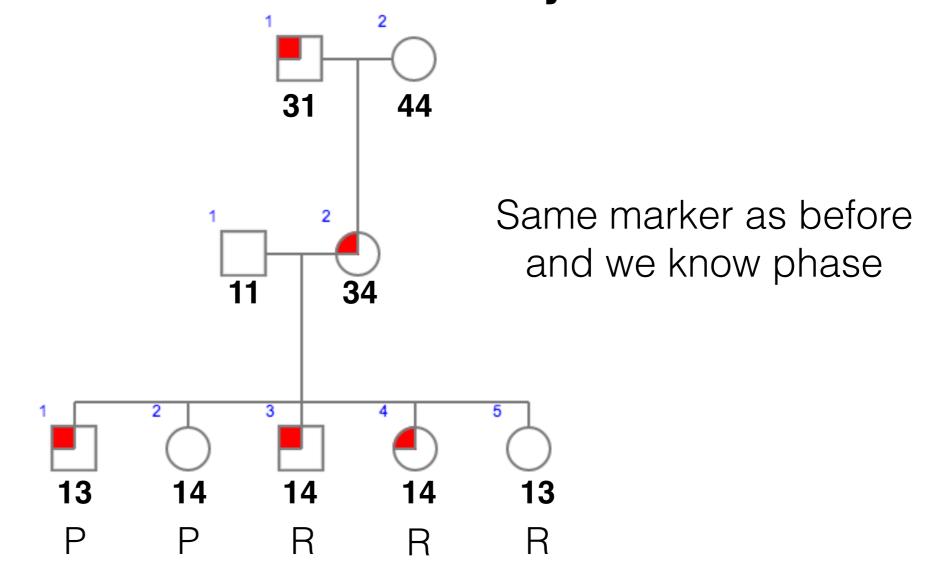
$$LOD = log_{10} \qquad \frac{(1 - \theta)^{P} \times \theta^{R}}{0.5^{(P + R)}}$$

### Let's add another family



LOD = 
$$log_{10}$$
 
$$\frac{(1 - 0.125)^2 \times 0.125}{0.5^{(2+3)}}$$

#### Let's add another family



$$LOD = -0.1159$$

$$\theta = 0.125$$

Change theta to 0.6? No, 0.5 is unlinked