Operating research hw0

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1 Problem 1

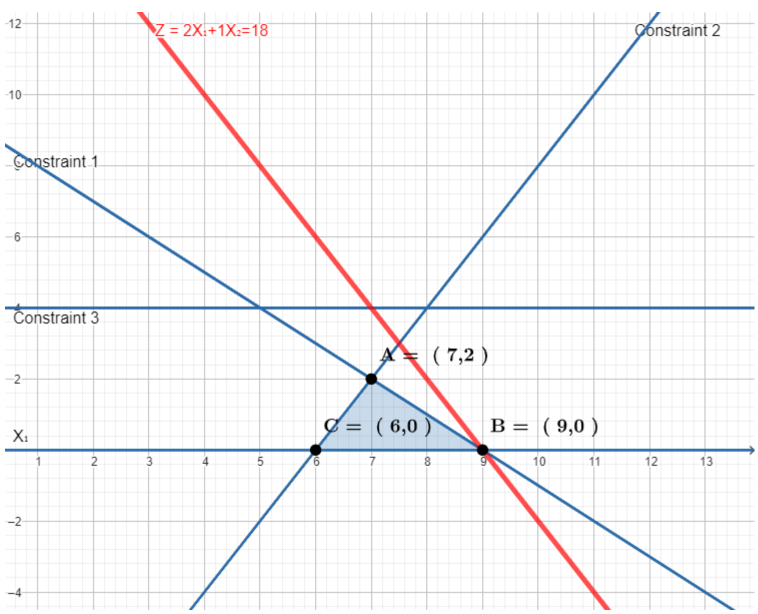
1. To find a in $f(x)$ that maximizes at $x = 2$, we need to find the number of a such that $f'(2) = 0$. $f'(x) = 2ax + 4$ so that we solve $f'(2) = 4a + 4 = 0$. Therefore, $a = -1$.

2. To solve $F(x)$, we first solve $\int f(x)dx = \frac{a}{3}x^3 + 2x^2 + 6x + c$. Then we get $F(x) = (\frac{a}{3}t^3 + 2t^2 + 6t + c) - (0 + c) = \frac{a}{3}t^3 + 2t^2 + 6t$ for all $t > 0$.

3. First name the matrix in the problem A . To let the inverse of the matrix given in the question not exist, $det(A)$ should equal zero. Which is solving the equation $det(A) = a + 0 + 4 - 3 - 0 - 2 = 0$. And we got $a = 1$.

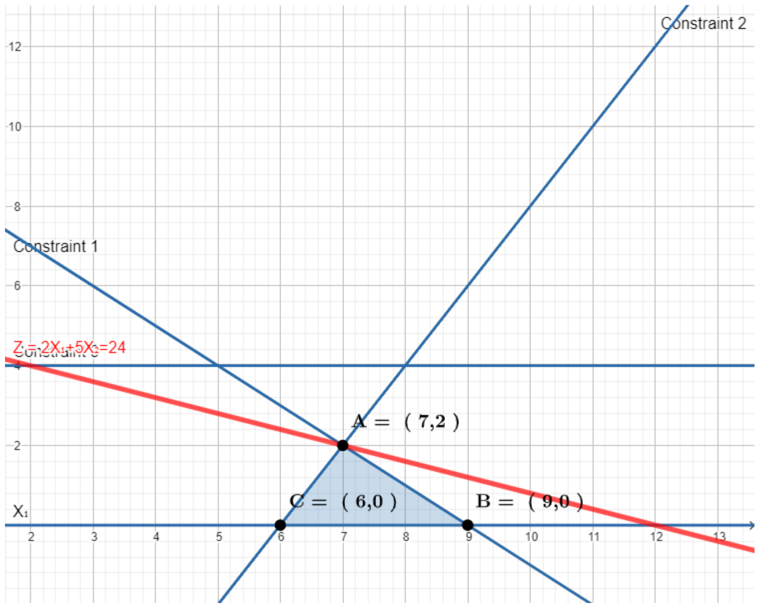
2 Problem 2

1. When $A = 1, B = 4$, the graph will be



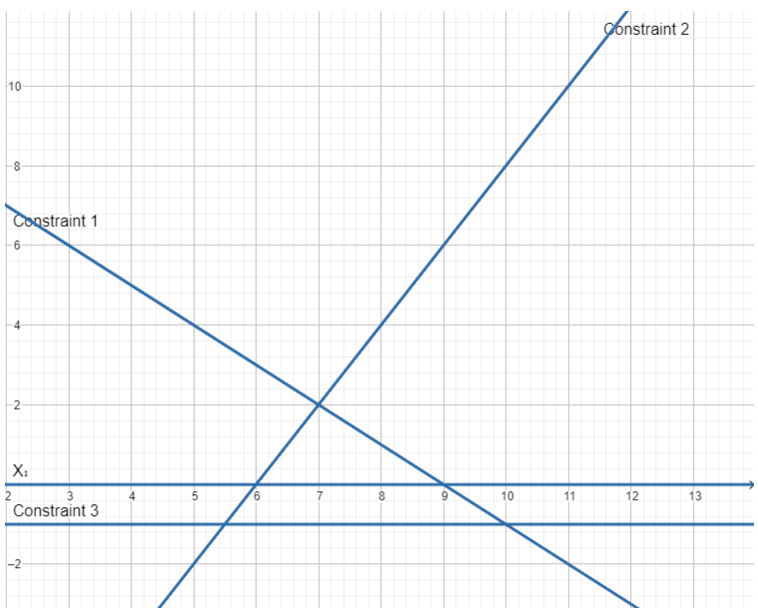
   The optimal solution is at point $B(9, 0)$, $2x_1 + x_2$ will be maximized at 18.

2. When $A = 5, B = 4$, the graph will be



The optimal solution is at point $A(7, 2)$, $2x_1 + 5x_2$ will be maximized at 24.

3. When $A = 1, B = -1$, the graph will be



There is no optimal solution since $x_2 \leq -1$ violates $x2 \geq 0$.

3 Problem 3

1. We want to find the optimal profit by finding the ideal number of barrels to be distributed. We let X_{11}, X_{12}, X_{21}, X_{22} be the number of barrels that have the same starting point and ending point as P_{11}, P_{12},P_{21}, P_{22} respectively. The following linear program can represent the problem.

$$
\begin{aligned}
\max \quad & X_{11}P_{11} + X_{12}P_{12} + X_{21}P_{21} + X_{22}P_{22} \\
\text{s.t.} \quad & X_{11} + X_{12} \leq K_1 \\
& X_{21} + X_{22} \leq K_2 \\
& X_{11} + X_{21} \leq D \\
& X_{12} + X_{22} \leq D \\
& X_{11}, X_{12}, X_{21}, X_{22} \leq 0
\end{aligned} \tag{1}
$$

2. We want to find the optimal profit by finding the ideal number of barrels to be distributed. We let X_{ij} be the number of barrels that have the same starting point and ending point as P_{ij}. The following linear program can represent the problem.

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij}P_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{m} X_{ij} \leq K_i, \forall\{i \ni 1 \leq i \leq n\} \\
& \sum_{i=1}^{n} X_{ij} \leq D, \forall\{j \ni 1 \leq j \leq m\} \\
& X_{ij} \leq 0, \forall\{i \ni 1 \leq i \leq n\}, \forall\{j \ni 1 \leq j \leq m\}
\end{aligned} \tag{2}
$$

3. Continue from 2. We want to find the optimal profit in T years by finding the ideal number of barrels to be distributed. We let X_{ij} be the number of barrels that have the same starting point and ending point as P_{ij}. Then we let N_i be the amount added after expending in city i. The following linear program can represent the problem.

$$
\begin{aligned}
\max \quad & T \times \sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij}P_{ij} - \sum_{i=1}^{n} C_i N_i \\
\text{s.t.} \quad & \sum_{j=1}^{m} X_{ij} \leq K_i + N_i, \forall\{i \ni 1 \leq i \leq n\} \\
& \sum_{i=1}^{n} X_{ij} \leq D, \forall\{j \ni 1 \leq j \leq m\} \\
& X_{ij} \leq 0, \forall\{i \ni 1 \leq i \leq n\}, \forall\{j \ni 1 \leq j \leq m\} \\
& N_i \leq 0, \forall\{i \ni 1 \leq i \leq n\}
\end{aligned} \tag{3}
$$

4. Continue from 3. We want to find the optimal profit in T years by finding the ideal number of barrels to be distributed. We let X_{ij} be the number of barrels that have the same starting point and ending point as P_{ij}. Then we let N_i be the amount added after expending in city i. The following linear program can represent the problem.

$$
\begin{aligned}
\max \quad & T \times \sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij}P_{ij} - \sum_{i=1}^{n} C_i N_i \\
\text{s.t.} \quad & \sum_{j=1}^{m} X_{ij} \leq K_i + N_i, \forall\{i \ni 1 \leq i \leq n\} \\
& \sum_{i=1}^{n} X_{ij} \leq D_{jt}, \forall\{j \ni 1 \leq j \leq m\}, \forall\{t \ni 1 \leq t \leq T\} \\
& X_{ij} \leq 0, \forall\{i \ni 1 \leq i \leq n\}, \forall\{j \ni 1 \leq j \leq m\} \\
& N_i \leq 0, \forall\{i \ni 1 \leq i \leq n\}
\end{aligned} \tag{4}
$$