1 Why is the pole frequency different than the numerical value pole in S domain?

Say I have a one pole system

$$H(s) = \frac{1}{1 + sRC}$$

The system is often written as

$$H(s) = \frac{1}{1 + \frac{s}{\omega_p}}$$

where ω_p is the -3 dB frequency in rad/s.

The pole is at where the denominator = 0

$$1 + sRC = 0, s = -\frac{1}{RC}$$

but the -3 dB frequency ω_p is $\frac{1}{RC}$ and not $-\frac{1}{RC}$, why?

To find the -3 dB, we're interested in the **magnitude** of this transfer function

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

and using the rule (proof and source : math.stackexchange, Youtube video)

$$\left|\frac{A}{B}\right| = \frac{|A|}{|B|}$$

we can find the frequency at where the magnitude drop by 3 dB $\left(\frac{1}{\sqrt{2}} = 10^{\frac{-3}{20}}\right)$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1^2 + (\omega RC)^2}} = \frac{1}{\sqrt{2}}$$

$$\omega = \frac{1}{RC} = \omega_p$$

This is why we have the numerical number of the pole isn't quite the same as the -3 dB frequency. You can say that in 1 pole system

$$s = -\omega_p$$

2 Another Example with 2 Pole System

Let

$$H(s) = \frac{1}{(1 + 5s)(1 + 7s)}$$

Poles are

$$s = -5, +7$$

and the pole frequencies are

$$s_1 = -\omega_{p1} = 5 \text{ rad/sec}$$
$$s_2 = -\omega_{p2} = 7 \text{ rad/sec}$$

Remember that conversion between Hz and rad/sec is

$$2\pi * f \text{ Hz} = \omega \text{ rad/sec}$$

3 Additional Tip: Angle of a complex number ratio

source : youtube video

$$\angle\left(\frac{A}{B}\right) = \angle A - \angle B$$