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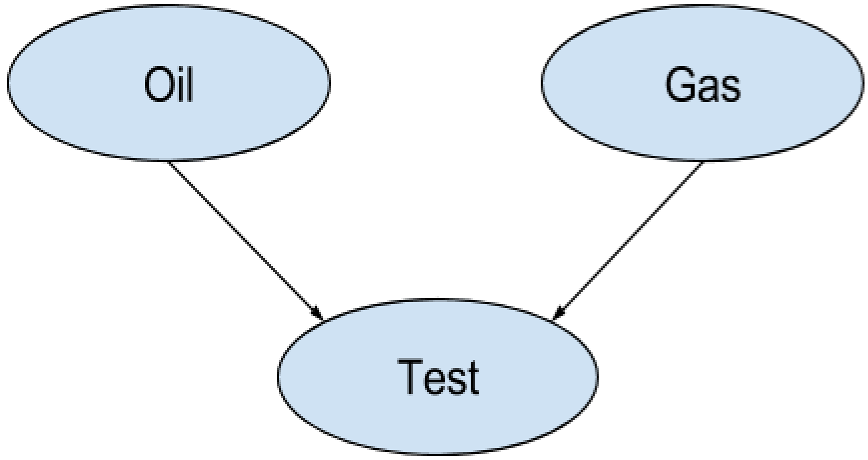
CS161

Homework #6

1. (a)

Oil	Pr(Oil)
0	0.5
1	0.5

Gas	Pr(Gas)
0	0.8
1	0.2



Test	Oil	Gas	Pr(Test | Oil, Gas)
1	0	0	0.1
1	0	1	0.3
1	1	0	0.9
1	1	1	0

(b) We seek Pr(Oil | Test). To calculate this probability, we use Bayes' Theorem.

$$Pr(Oil \mid Test) = \frac{Pr(Test \mid Oil) * Pr(Oil)}{Pr(Test)}$$

From the conditional probability tables (CPTs) above,

Pr(Test | Oil) = 0.9 (disregard Gas, even as Test is doubly dependent on Oil and Gas, because they are mutually exclusive)

Pr(Oil) = 0.5

Pr(Test) = ???

Since Pr(Test) is unknown, use the Law of Total Probability to solve for it.

$$Pr(x) = \sum_i Pr(x, y_i)$$

Choose x = Test, and y = {Oil, ¬Oil}

$Pr(x) = Pr(Test) = Pr(Test, Oil) + Pr(Test, \neg Oil)$

Conditioning,

$Pr(Test) = Pr(Test \mid Oil) * Pr(Oil) + Pr(Test \mid \neg Oil) * Pr(\neg Oil)$
$Pr(Test) = 0.9 * 0.5 + 0.4 * 0.5$
$Pr(Test) = 0.5(0.9 + 0.4)$
$Pr(Test) = 0.65$

With all necessary values,

$Pr(Oil \mid Test) = (0.9 * 0.5) / 0.65 = 0.45/0.65 =$ **0.69**

2. (a) $Pr(A, B, C, D, E, F, G, H)$
$= Pr(A) * Pr(B) * [Pr(C \mid A)] * [Pr(D \mid A,B)] * [Pr(E \mid B)] * [Pr(F \mid C, D)] * [Pr(G \mid F)] * [Pr(H \mid F, E)]$

(b) $Pr(E, F, G, H) = Pr(E \mid B) * Pr(F \mid C, D) * Pr(G \mid F) * Pr(H \mid F, E)$

(c) $Pr(a, \neg b, c, d, \neg e, f, \neg g, h) = 0.2 * 0.3 * [Pr(c \mid a)] * 0.6 * 0.1 * [Pr(f \mid c, d)] * [Pr(\neg g \mid f)] * [Pr(h \mid f, \neg e)]$
$= \mathbf{0.0036 * Pr(c \mid a) * Pr(f \mid c, d) * Pr(\neg g \mid f) * Pr(h \mid f, \neg e)}$

(d) $Pr(a, \neg b) = Pr(a) * Pr(\neg b)$ (independence)
$= 0.2 * 0.3$
$=$ **0.06**

$Pr(\neg e \mid a) = Pr(\neg e, a) / Pr(a)$
$= Pr(\neg e) * Pr(a) / (Pr(a)$ (e is independent of a given its parent, b)
$= Pr(\neg e)$

E depends on B, however.

$Pr(\neg e) = Pr(\neg e,b) + Pr(\neg e,\neg b)$ (Law of Total Probabilities, partition on b)
$Pr(\neg e) = Pr(\neg e,b) * Pr(b) + Pr(\neg e,\neg b) * Pr(\neg b)$ (Conditioning)
$Pr(\neg e) = 0.9 * 0.7 + 0.1 * 0.3$
$Pr(\neg e) = 0.63 + 0.03$
$\mathbf{Pr(\neg e) = Pr(\neg e \mid a) = 0.66}$

(e) The **Markovian assumption** states that every node is independent of its non-descendants given its parents. For a node X symbolically,

$$X \perp \text{Non-Descendants}(X) \mid \text{Parents}(X)$$

Using this notation,

$A \perp B$
$A \perp E$

$B \perp C$

$C \perp D \mid A$
$C \perp E \mid A$

$D \perp C \mid A, B$
$D \perp E \mid A, B$

$E \perp C \mid B$
$E \perp D \mid B$
$E \perp F \mid B$
$E \perp G \mid B$

$F \perp A \mid C, D$
$F \perp B \mid C, D$
$F \perp E \mid C, D$

$G \perp A \mid F$
$G \perp B \mid F$
$G \perp C \mid F$
$G \perp D \mid F$
$G \perp E \mid F$
$G \perp H \mid F$

$H \perp A \mid F, E$
$H \perp B \mid F, E$
$H \perp C \mid F, E$
$H \perp D \mid F, E$
$H \perp G \mid F, E$

(f) In a Bayesian network, the Markov blanket of a node includes its parents, children, and the other parents of all its children.

Markov Blanket(D) = {A, B, C, F}

(g)

A	B	D	Pr(D|AB)	B	E	Pr(E|B)	A	B	D	E	**Pr(D|AB) x Pr(E|B)**
0	0	0	0.5	0	0	0.1	0	0	0	0	**0.5 * 0.1 = 0.05**
0	0	1	0.5	0	1	0.9	0	0	0	1	**0.5 * 0.9 = 0.45**
0	1	0	0.6	1	0	0.9	0	0	1	0	**0.5 * 0.1 = 0.05**
0	1	1	0.4	1	1	0.1	0	0	1	1	**0.5 * 0.9 = 0.45**
1	0	0	0.1				0	1	0	0	**0.6 * 0.9 = 0.54**
1	0	1	0.9				0	1	0	1	**0.6 * 0.1 = 0.06**
1	1	0	0.8				0	1	1	0	**0.4 * 0.9 = 0.36**
1	1	1	0.2				0	1	1	1	**0.4 * 0.1 = 0.04**
							1	0	0	0	**0.1 * 0.1 = 0.01**
							1	0	0	1	**0.1 * 0.9 = 0.09**
							1	0	1	0	**0.9 * 0.1 = 0.09**
							1	0	1	1	**0.9 * 0.9 = 0.81**
							1	1	0	0	**0.8 * 0.9 = 0.72**
							1	1	0	1	**0.8 * 0.1 = 0.08**
							1	1	1	0	**0.2 * 0.9 = 0.18**
							1	1	1	1	**0.2 * 0.1 = 0.02**

(h) To sum out the variable D,

Let f(A, B, D, E) = Pr(D | A, B) x Pr(E | B)

$$f(A, B, E) = \sum_{d} f(A, B, D, E) = f(A, B, d, E) + f(A, B, \neg d, E)$$

We are **marginalizing** on the variable D. To do this, look for all rows where A, B, E agree and sum over all possible values of D.

A	B	E	$\sum_{d} \mathbf{f(A, B, D, E) = f(A, B, E)}$
0	0	0	**0.05 + 0.05 = 0.10**
0	0	1	**0.45 + 0.45 = 0.90**
0	1	0	**0.54 + 0.36 = 0.90**
0	1	1	**0.06 + 0.04 = 0.10**
1	0	0	**0.01 + 0.09 = 0.10**
1	0	1	**0.09 + 0.81 = 0.90**
1	1	0	**0.72 + 0.18 = 0.90**
1	1	1	**0.08 + 0.02 = 0.10**