

CS174A: Introduction to Computer Graphics

Kinsey 1240 MW 4-6pm

Scott Friedman, Ph.D
UCLA Institute for Digital Research and Education



Term Project

- Details will be introduced next Monday.
- Things to start considering now are:
 - Teams will be a minimum of *three* people.
 - Use the class forum to find partners if you need to.
 - Teams can have up to *five* people in your group.
 - Your team will submit a one page project proposal.
 - "we are going to write a game", will not do for a proposal.
 - The proposals are due 2/10/2017 by midnight.
 - We will review proposals and make comments back to your group.



- When the depth buffer is enabled
 - A z value is written into buffer for every pixel.
 - If an incoming pixel has a z value less than the value already in the buffer
 - The pixel is written into the frame (color) buffer.
 - The z value is updated in the depth buffer.
 - Else
 - The pixel is rejected and not written to the frame buffer.
 - The depth buffer is not modified.

Lecture 6 CS174A – Winter 2017



- What happens without it?
 - Last write to the frame buffer "wins".
 - The order that objects are drawn then *matters*.
 - Objects have to be rendered back to front.
 - Why?
 - Potentially problematic cases
 - For inter-penetrating objects
 - Moving objects

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- There is very little to do in order to use it
 - Enable the depth buffer.

```
function init()
{
    ...
    gl.enable( gl.DEPTH_TEST );
    ...
}
```

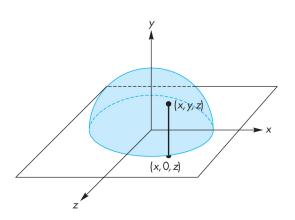


- Why do we need to enable?
 - OpenGL is a state machine, remember.
 - Also have to *clear (or reset)* the depth buffer when starting a new image.

```
function display()
{
    gl.viewport( ... );
    gl.clear( gl.COLOR_BUFFER_BIT | gl.DEPTH_BUFFER_BIT );
    ...
}
```

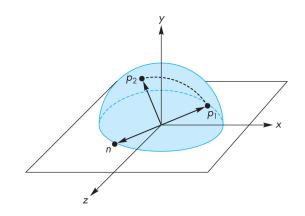
- Last time we talked about rotation...
 - A useful/intuitive interaction technique.
 - Plant a unit hemisphere into the *x-z* plane.
 - Using mouse positions we can reconstruct *y*

- Because,
$$x^2 + y^2 + z^2 = 1$$
, $y = \sqrt{1 - x^2 - z^2}$



- That's nice, we know y...
 - Now we can track, as the mouse moves, a position p_1 to p_2 on the surface of the hemisphere.
 - What we really want to do is rotate in the direction of the arc swept out from p_1 to p_2 .
 - That axis of rotation can be found via the cross product of p_1 to p_2 , the *normal*.

$$n = p_1 \times p_2$$





- That's nice, we know *n*...
 - Conveniently, n can tell us the angle between p_1 to p_2 as well because we are using a *unit* hemisphere.

$$|\sin\theta| = |n|$$

- Now we know an angle and a vector around which the rotation is supposed to occur.
- The book mentions a nice trick when animating the rotation in small increments and that is to recognize that for small
 - and you can avoid the inverse sine operation.

$$\theta \approx \sin \theta$$

- A side note...
 - When animating a rotation in small increments
 - A lot of trigonometry is involved = slow
 - Helpful to recognize that for small $\theta \approx \sin \theta$
 - and then you can avoid the inverse sine operation.
 - This implies that, for **small** angles we can use
 - Another graphics "trick"

$$R = R_{z}(\psi)R_{y}(\phi)R_{x}(\theta) \approx \begin{bmatrix} 1 & -\psi & \phi & 0 \\ \psi & 1 & -\theta & 0 \\ -\phi & \theta & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

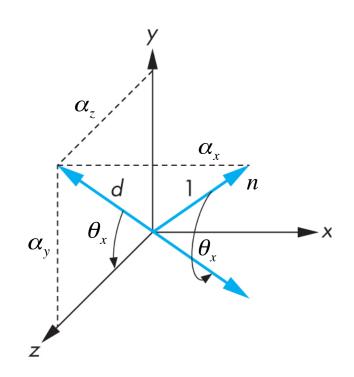
- That's nice, we know the angle...
 - But, we just know how to rotate around x, y and z!
 - Yes, but if we align the rotation vector with, say, the z-axis we perform our rotation. Simple!

$$R = R_x(-\theta_x)R_y(-\theta_y)R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)$$

- Ugh, we don't know $R_x(\theta_x)$ or $R_y(\theta_y)$ even if we decided that $R_z(\theta_z)$ was the rotation we wanted.
- Yes, but let's understand what is going on first.

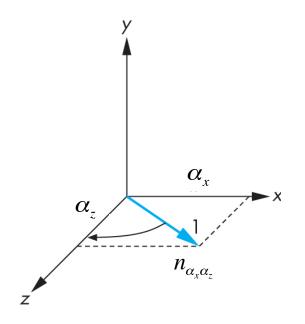
- How can we find the x and y rotations?
 - First we need to rotate around the x-axis onto the x-z plane.
 - Recall that $\cos \theta_x = \alpha_x$
 - Then,

$$R_{x}(\theta_{x}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha_{z}/d & -\alpha_{y}/d & 0 \\ 0 & \alpha_{y}/d & \alpha_{z}/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \alpha_{y} \qquad \theta_{x}$$



- Now we need the *y* rotation?
 - We can follow the same process
 - Again, recall that $\cos \theta_y = \alpha_y$
 - Then,

$$R_{y}(\theta_{y}) = \begin{bmatrix} \alpha_{z} & 0 & -\alpha_{x} & 0 \\ 0 & 1 & 0 & 0 \\ \alpha_{x} & 0 & \alpha_{z} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





- Actually, we can now rotate about *any* vector *v* fixed at a point *p*.
 - Concatenating into M

$$M = T(p)R_x(-\theta_x)R_y(-\theta_y)R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)T(-p)$$

- Now we can rotate our trackball vector!
- Our point p is the **origin** and our rotation vector is n.
- This is a lot of work is there a better way?



Quaternions

- Same result with less computation.
 - A quaternion has the form q=w+xi+yj+zk.
 - The terms i, j and k are imaginary.
 - Fortunately, we can ignore this fact in this class.
 - But, they are what ultimately make quaternions work.
 - Lets consider them this way q(w,x,y,z)
 - Lets make w the rotation angle
 - Lets make x, y and z be the rotation vector.

Quaternions

- Same result with less computation.
 - It is *critical* that q be *normalized*, i.e. $q=|q^2|$.
 - Or this does. not. work.
 - The resulting rotation matrix is

$$Q = \begin{bmatrix} 1 - 2(y^2 + z^2) & 2(xy - wz) & 2(xz + wy) & 0 \\ 2(xy + wz) & 1 - 2(x^2 + z^2) & 2(yz - wx) & 0 \\ 2(xz - wy) & 2(yz + wx) & 1 - 2(x^2 + y^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Quaternions

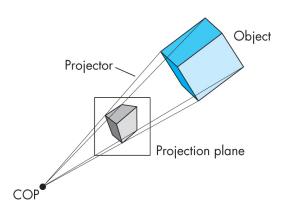
- Same result with less computation.
 - A very nice feature of quaternions is that they allow for straightforward smooth interpolation.
 - You do this with a current rotation R and an increment R_I
 - R starts with some initial/or no rotation and rotation vector
 - $-R_I$ has a small rotation and the same rotation vector.
 - Need to renormalize R occasionally to keep computation stable.

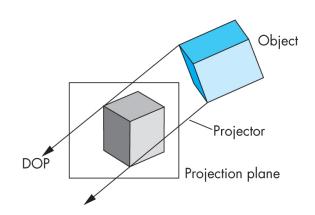
$$R=RR_I$$



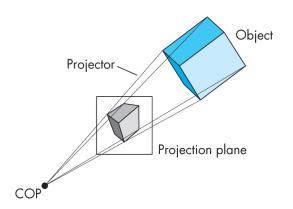
- We are going to concern ourselves with two types of *viewing*.
 - Perspective viewing
 - Parallel viewing

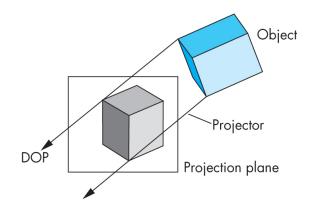
- In both cases we have
 - Objects,
 - Projection lines
 - Projection plane
 - Eye (COP: center of projection or DOP: direction of projection)



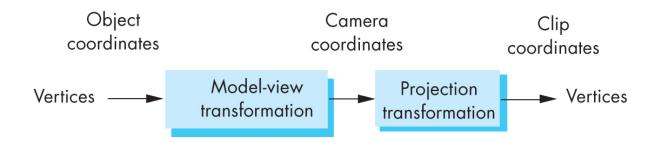


- Our goal is to ultimately
 - Use transformations to project the vertices of objects onto the projection plane.
 - Specifically we will create transformations to go from object space to world space to camera space to clip space.



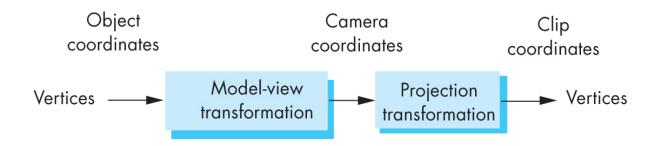


- Previously
 - We used the default canonical view volume.
 - Which exists in clip space, i.e. (-1,1),(-1,1),(-1,1).
 - The 'projections' we implicit
 - Last time we saw how transformations can be combined to bring objects into camera (world) coordinates
 - Collectively, the *model-view transformation*.
 - space == coordinates, terminology is equalivalent





- Model-view transformation
 - Does not take us all the way to clip coordinates.
 - we need a *projection transformation* for that.
 - Model-view gets objects in front of the camera.
 - A Projection defines which and how and where those objects (verticies) will appear on the screen.





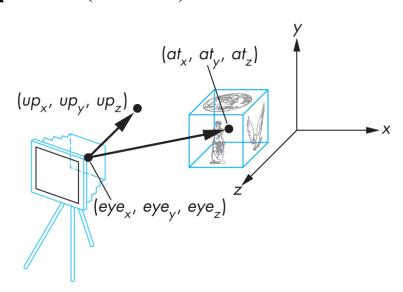
Instancing

- Useful for Assignments and Project:
 - Take a single geometric object.
 - "Clone" it with only transformations (and possibly state).
 - Define the geometry of a cube (once).
 - Instance the cube by setting a transformation and setting some state (e.g. color) and drawing it.
 - Instance another cube by setting another transformation and setting state(e.g. color and scale) and drawing the *same geometry*.



- Positioning the (getting things in from of) "camera"
 - Recall that the default is "looking" down the -z axis at the origin (0,0,0).
 - This is equivalent to model-view set to the identity matrix.
 - Remember, transformations are specified in *reverse*
 - in a post-multiplication world.
 - That means we specify the position of the camera *first*.
 - We are going to look at two methods
 - Look-at
 - Yaw, pitch and roll (euler angles)

- Look-at
 - We define three terms
 - A point describing the location of the *eye*.
 - A point the eye is looking *at* in the scene.
 - An *up* vector (direction) for the camera.



- Look-at
 - The at and eye points give us
 - the *view-plane-normal* or *vpn*
 - a vector perpendicular to the view plane
 - the up vector is usually (0, 1, 0)
 - Or, (0, 1, 0, 0) in homogeneous coordinates!
 - We then calculate the following

$$vpn = at - eye$$
 $u = \frac{up \times n}{|up \times n|}$ $n = \frac{vpn}{|vpn|}$ $v = \frac{n \times u}{|n \times u|}$

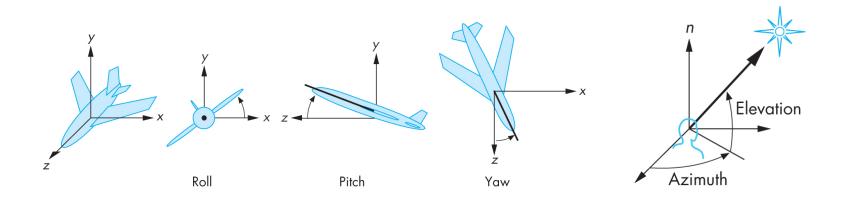
- Look-at
 - Once *u*, *v* and *n* are *normalized*
 - The following will position our camera

$$V = RT = \begin{bmatrix} u_{x} & u_{y} & u_{z} & -eye_{x}u_{x} - eye_{y}u_{y} - eye_{z}u_{z} \\ v_{x} & v_{y} & v_{z} & -eye_{x}v_{x} - eye_{y}v_{y} - eye_{z}v_{z} \\ n_{x} & n_{y} & n_{z} & -eye_{x}n_{x} - eye_{y}n_{y} - eye_{z}n_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Look-at
 - Works reasonably well for positioning.
 - But not so well for moving the camera smoothly.
 - MV.js has a function for setting this up

$$V = RT = \begin{bmatrix} u_{x} & u_{y} & u_{z} & -eye_{x}u_{x} - eye_{y}u_{y} - eye_{z}u_{z} \\ v_{x} & v_{y} & v_{z} & -eye_{x}v_{x} - eye_{y}v_{y} - eye_{z}v_{z} \\ n_{x} & n_{y} & n_{z} & -eye_{x}n_{x} - eye_{y}n_{y} - eye_{z}n_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Yaw, pitch and roll (like an airplane)
 - Here we, essentially, use polar coordinates.
 - A simplified version uses just azimuth and elevation.
 - Rotate camera in the direction we desire.
 - Translate camera to *eye* point.

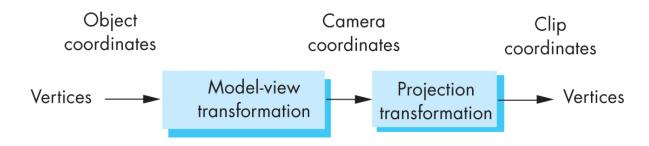




- Yaw, pitch and roll
 - In reality we perform the *inverse* of what we want.
 - We are transforming world coordinates (all objects) into camera coordinates.
 - Move the world to the camera.
 - » That is, if I want to appear to rotate left 20 degrees.
 - » The transformation about the y-axis would be -20.
 - » Similarly, if I want to appear to move forward 5 units.
 - » I would transform everything by -5.
- So far we have only gotten things in front of the camera!
 - We now have both model and view transformations

- Projections Parallel (orthographic)
 - Once in camera coordinates we need a projection transformation to get us to clip coordinates.
 - The transformation matrix that gives us an orthographic projection is:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

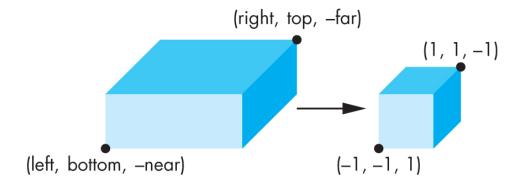


31



- Projections Parallel (orthographic)
 - However, M is applied by the hardware after the vertex shader.
 - Which gets our geometry into clip coordinates
 - How do we "include" or "see" more of our scene?

- Projections Parallel (orthographic)
 - We *scale* what we want to "include" to fit within the canonical view volume. i.e. (-1,1),(-1,1),(-1,1)
 - OpenGL provides a function for this called
 - ortho(left, right, bottom, top, near, far) (ex: MV.js)

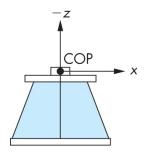


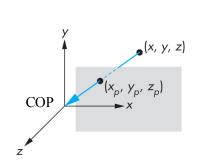
- Projections Parallel (orthographic)
 - If you think about it ortho contains a scale and translation.
 - Here is what the transformation matrix looks like.
 - The translate moves everything relative to the origin (and the canonical view volume)
 - The scale (symmetrically) resizes everything to squeeze or stretch into the canonical view volume.

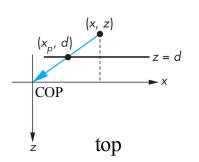
$$N = ST = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{left + right}{right - left} \\ 0 & \frac{2}{top - bottom} & 0 & -\frac{top + bottom}{top - bottom} \\ 0 & 0 & -\frac{2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

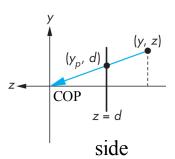
- Projections Perspective
 - Basic symmetrical perspective projection
 - The point (x, y, z) is projected through the projection plane to the eye point (or center of projection COP)
 - We can compute the point of intersection with

$$x_p = \frac{x}{z/d}$$
, $y_p = \frac{y}{z/d}$









- Projections Perspective
 - The simple perspective projection matrix is

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- The important thing to notice here is the position of the -1 term.
- This means our homogeneous coordinate, w, can be modified (will no longer be 1) when a vertex is multiplied by *M*.

- Projections Perspective
 - Uh oh, the homogeneous coordinate is no longer 1?

$$q = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

• Not the end of the world, remember

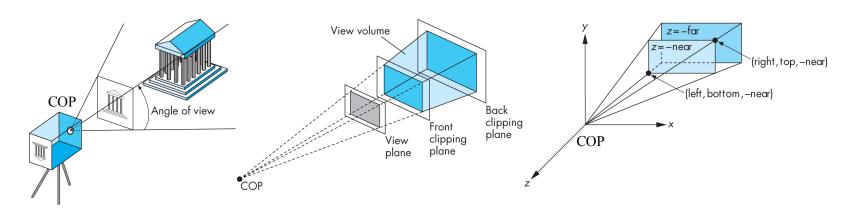
We have to divide by the homogeneous coordinate to get back to 3D space.

37

 $q' = \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ \frac{d}{d} \\ 1 \end{bmatrix}$

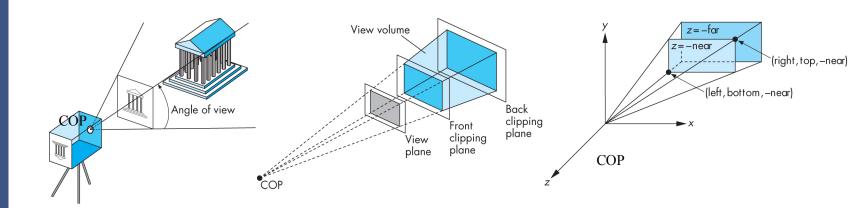


- Projections Perspective
 - That's nice, but only gets our points onto the projection plane.
 - We want a transformation into clip coordinates!
 - That requires not only the specification of a perspective projection, but of a view volume as well.
 - Similar to the box we just defined for orthographic projection



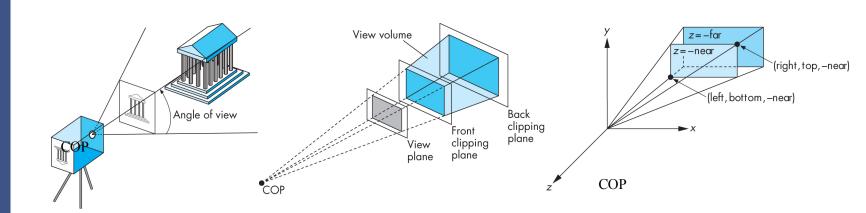
38

- Projections Perspective
 - View volume is a pyramid with apex its at the COP.
 - Top and bottom are the near and far clip planes, respectively.
 - Notice that the view and clip planes do not *necessarily* need to be the same/coincident.
 - In fact, don't worry about the view plane now

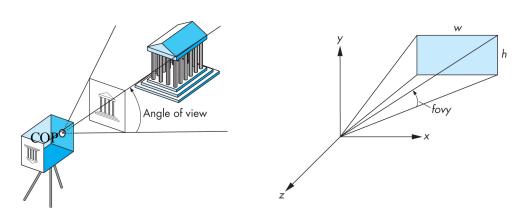




- Projections Perspective
 - The edges specify the near clip plane.
 - The edges implicitly define the *angle of view* of the projection.



- Projections perspective
 - MV.js provides a utility function
 - perspective(fovy, aspect, near, far)
 - This form is sometimes more convenient to specify.
 - *Aspect* is the aspect ratio of the view volume.
 - i.e. width / height



- Projections perspective
 - Once again, what we had earlier is a projection not the full transformation we need to get to clip coordinates.
 - The full matrix we need is defined by:

$$P = NSH = \begin{bmatrix} \frac{right}{near} & 0 & 0 & 0 \\ 0 & \frac{near}{top} & 0 & 0 \\ 0 & 0 & \frac{-far + near}{far - near} & \frac{-2far \cdot near}{far - near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



- Projections perspective
 - Matrices are passed from Javascript to the vertex shader just like we have seen earlier.
 - Matrices are **uniform** variables where all the data parallel processors on the GPU will see the same (uniform) value.

```
attribute vec4 vPosition;
uniform mat4 modelView;
uniform mat4 projection;

void main( )
{
    //
    // The perspective division actually happens to gl_Position immediately
    // after the vertex shader completes. i.e. divided by gl_Position.w
    //
    gl_Position = projection * modelView * vPosition;
}
```





Getting your head around this

• Often there is an initial struggle understanding what is happening

• These next slides will hopefully illuminate what is happening to a vertex, where and when.



Getting your head around this

• First, remember that we are only working with arrays of vertices.

- It is up to us to keep track of what those vertices represent in our code (Javascript).
 - Point, line, triangle, car, dinosaur, kitten, hamburger, etc.



Getting your head around this

- WebGL knows points, lines and triangles
 - That is it
 - They are all made up of vertices

• When we draw an array of vertices we will tell WebGL what to interpret the array of vertices as: points, lines or triangles



- They could come from a modeling program
 - Overkill for the assignments
 - You have to 'load' into your code somehow
- You could define them directly
 - By hand
 - OK for simple geometry, like a cube
 - Algorithmically
 - Using code, can make a sphere this way

• In any case your vertices (geometry) will end up in a Javascript array

• Let's use a regular tetrahedron as an example (all edges the same length)

• We could compute the four vertices very simply using $\left(\pm 1,0,\frac{1}{\sqrt{2}}\right)$ and $\left(0,\pm 1,\frac{1}{\sqrt{2}}\right)$



- (1, 0, 0.707)
- (-1, 0, 0.707)
- (0, 1, 0.707)
- (0, -1, 0.707)

• This is a start but not enough to get a tetrahedron on the screen



- A tetrahedron is four triangles
 - Something WebGL knows about
- We could define an array of four triangles, three vertices each
- We have enough information to that
- But what order?
 - Does it matter?



- The order of the individual triangles does not in this simple example
- The order of the vertices that comprise each triangle *does matter*!
- WebGL uses a 'right handed' system
 - Determines the 'direction' of the triangle's normal (which is the side that will be rasterized/painted)
 - Also known as the 'winding' of the vertices
 - Take your right hand and make a fist with your thumb sticking out.
 - The direction of your fingers will tell you the order of vertices to get the normal facing in the direction of your thumb
 - This is important because the side of the triangle the normal is facing away from is the one that will be rasterized (painted/filled in)
 - The back is not drawn at all!



For our teraheadron we have the following

```
-(1, 0, 0.707), (0, 1, 0.707), (-1, 0, 0.707)
```

- -(1, 0, 0.707), (0, -1, 0.707), (0, 1, 0.707)
- -(1, 0, 0.707), (-1, 0, 0.707), (0, -1, 0.707)
- -(0, 1, 0.707), (0, -1, 0.707), (-1, 0, 0.707)
- We now have four triangles
- Triangle vertices are in correct order



- We could put each triangle's data into it's own array and draw each one individually
- We could also put all four into a single array
- You can also just store each vertex once in an array and use another array of indexes into the the first.
- We'll use the second approach



- Here is one array with all the data
 - (1, 0, 0.707), (0, 1, 0.707), (-1, 0, 0.707), (1, 0, 0.707), (0, -1, 0.707), (0, -1, 0.707), (0, 1, 0.707), (1, 0, 0.707), (-1, 0, 0.707), (0, -1, 0.707), (0, -1, 0.707), (0, -1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707), (-1, 0.707),
- This would still be an array in CPU memory
- This data is also centered around the origin
 - In 'model space'
- we can now define a WebGL buffer and assign this data to it
 - We will receive a reference back to a copy of the data stored in GPU memory



• If we wanted the object to be somewhere else we need to transform it into 'world' space

• How do we do that?

 We set up a transformation that is part of the model-view transformation



- Let's assume we want to draw the tetrahedron twice
 - Once, centered at (1,0,0)
 - And again, centered at (-1,0,0)

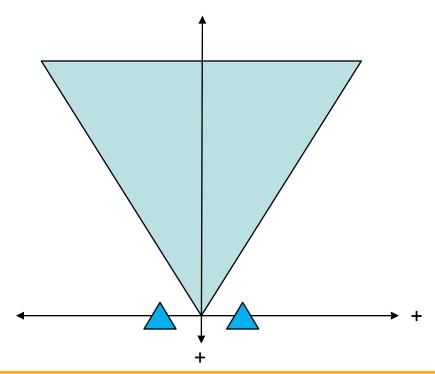
- The implies two transformations
 - We will apply them one at a time while we draw the geometry



- Before we draw anything let's set up the view
- The default perspective view has a COP at the origin
- If we do not reposition the view we will not see anything, even after applying our model transformations.
 - WHY?

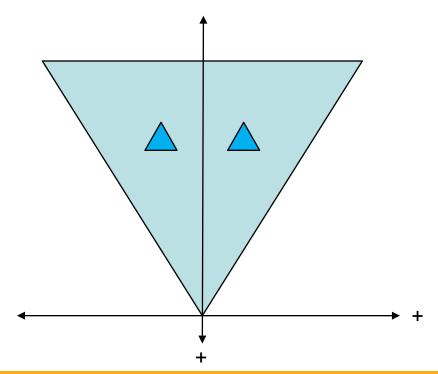


• Because our default view is looking away from where the geometry is in **world** space



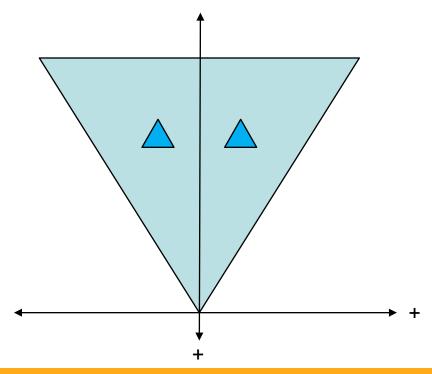


• Our view transformation needs to position the objects in the world in such as way to match what we want to see





• Let's transform the objects by (0,0,-6)





- We now have
 - Two model transformations
 - One view transformation
- We also need (have) a projection transformation
 - This would be the light blue area on the previous slide



Almost ready to draw

- Four transformation matrices
- Two objects

• What do we 'send' to WebGL to draw?

• Some of what we need we have already we just need to get the WebGL state correct



Almost ready to draw

- Need to be sure vertex buffer is 'bound' to the WebGL state
- Need to be sure our shaders active (useProgram)



 We first compute our model-view matrix for object #1 – in Javascript

• MV = V * M1



• We then set vertex shader uniform variables for our **model-view** and **projection** matrices

• This is so the shader has access to these transformation matrices when operating on the vertices



• We then tell WebGL to draw our vertex data

- We do this by passing the reference to the vertex array buffer and telling WebGL to interpret this data as a series of *n* triangles.
 - -n would be 4 in this case



We then tell WebGL to draw our vertex data

- We do this by passing the reference to the vertex array buffer and telling WebGL to interpret this data as a series of *n* triangles.
 - -n would be 4 in this case



```
gl.viewport(0,0, canvas.width, canvas.height);
gl.clearColor(1.0, 1.0, 1.0, 1.0);
var program = initShaders( gl, "vertex-shader", "fragment-shader" );
gl.useProgram( program ); // setting state of which shaders to use
// Load the data into the GPU
var bufferId = gl.createBuffer();
gl.bindBuffer(gl.ARRAY BUFFER, bufferId);
gl.bufferData(gl.ARRAY BUFFER, flatten(points), gl.STATIC DRAW);
var vPosition = gl.getAttribLocation( program, "vPosition" );
gl.vertexAttribPointer(vPosition, 3, gl.FLOAT, false, 0, 0);
gl.enableVertexAttribArray(vPosition););
var modelView= gl.getUniformLocation(program, "modelView");
var projection = gl.getUniformLocation(program, "projection");
// call when we want to update the value passed to the shader
gl.uniformMatrix4fv(projection, false, new flatten(P));
gl.clear(gl.COLOR BUFFER BIT);
gl.uniformMatrix4fv(modelView, false, (V * M1));
gl.drawArrays(gl.TRIANGLES, 0, vertices.length);
gl.uniformMatrix4fv(modelView, false, (V * M2));
gl.drawArrays(gl.TRIANGLES, 0, vertices.length);
```



- When gl.drawArrays is called the GPU comes into the action
- This is when the vertex shader gets executed by the GPU
- Remember, the GPU is highly parallel
 - It first operates on vertices in parallel
 - 'attributes' are different for each execution
 - 'uniforms' are the same for each execution



- Then, **modelView** will be the same everywhere and so will **projection** in the vertex shader
- vPosition will be each of the vertices in our buffer.
- We let the GPU do the heavy lifting of multiplying the modelView and projection matrices to each and every vertex



- Recall we only have one copy of the data for a tetrahedron centered at the origin loaded into the GPU
- Notice we reset the modelView matrix every time we want to draw a new object
- We do not include the projection matrix here because it stays the same for all objects we'll let the GPU multiply it
- There are other ways to accomplish this we could have sent the view matrix once and just kept adjusting the model matrix.

```
gl.uniformMatrix4fv(modelView, false, (V * M1));
gl.drawArrays( gl.TRIANGLES, 0, vertices.length );
gl.uniformMatrix4fv(modelView, false, (V * M2));
gl.drawArrays( gl.TRIANGLES, 0, vertices.length );
...
```



What about color?

- Color, in this example, can be handled the same was as the gasket example
- Pass a uniform variable to the fragment shader
- Once the WebGL pipeline gets to the fragment shader its parallel operations are working on...fragments, which are assigned color values.
- We'll talk more about fragment shaders when we discuss lighting

```
gl.uniformMatrix4fv(modelView, false, (V * M1));
gl.drawArrays( gl.TRIANGLES, 0, vertices.length );
gl.uniformMatrix4fv(modelView, false, (V * M2));
gl.drawArrays( gl.TRIANGLES, 0, vertices.length );
...
```