

Ejercicios Teorema maestro

$$a) T(n) = 2T(n/2) + n^4$$

$$\equiv \langle T_m \# 3.1 \rangle$$

$$n^4 = \Omega(n^{\log_2 2 + \epsilon})$$

$$\equiv \langle \log_2 2 = 1, \epsilon = 3 \rangle$$

$$n^4 = \Omega(n^4)$$

$\equiv \text{true}$

$$T(n) = 2T(n/2) + n^4$$

$$\equiv \langle T_m \# 3.2 \rangle$$

$$2f(n/2) \leq cf(n)$$

$$\equiv \langle f(n) = n^4 \rangle$$

$$2\left(\frac{n}{2}\right)^4 \leq cn^4$$

$$\equiv \langle \text{Potencia} \rangle$$

$$2 \frac{n^4}{2^4} \leq cn^4$$

$$\equiv \langle \quad \rangle$$

$$\frac{n^4}{2^3} \leq cn^4$$

$$\equiv \langle c = \frac{1}{8} \rangle$$

$$n^4 \leq n^4$$

$\equiv \text{true}$

$$a = 2$$

$$b = 2$$

$$f(n) = n^4$$

$$f(n) ? n^{\log_b a}$$

$$n^4 ? n^{\log_2 2}$$

$$n^4 > n^1$$

$$T(n) = \Theta(n^4)$$

Web sockets

Garbage collector

$$b) T(n) = T(7n/10) + n$$

$$\equiv \langle T_m \#3.1 \rangle$$

$$a = 1$$

$$b = \frac{10}{7}$$

$$f(n) = n$$

$$n = \Omega(n^{\log_{10/7} 1 + \epsilon})$$

$$\equiv \langle \log_{10/7} 1 = 0, \epsilon = 1 \rangle$$

$$n = \Omega(n^1)$$

$\equiv \text{true}$

$$f(n) ? \quad n^{\log_{10/7} a}$$

$$n ? \quad n^{\log_{10/7} 1}$$

$$n > 1$$

$$T(n) = T(7n/10) + n$$

$$\equiv \langle T_m \#3.2 \rangle$$

$$f(n/10/7) \leq c f(n)$$

$$\equiv \langle f(n) = n \rangle$$

$$\frac{n}{10/7} \leq c n$$

$$\equiv \langle \text{Designation } a-b < c \leq a-c < b \rangle$$

$$\frac{7n}{10} - cn \leq 0$$

$$\equiv \langle \text{factorization} \rangle$$

$$n \left(\frac{7}{10} - c \right) \leq 0$$

$$\equiv \langle c = \frac{7}{10} \rangle$$

true

$$T(n) = \Theta(n)$$

$$c) T(n) = 16T(n/4) + n^2$$

$$\equiv \langle T_m \# 2 \rangle$$

$$n^2 = \Theta(n^{\log_4 16})$$

$$\equiv \langle \log_4 16 = 2 \rangle$$

$$n^2 = \Theta(n^2)$$

$$\equiv \langle f(n) = \Theta(f(n)) \rangle$$

true

$$\rightarrow T(n) = \Theta(n^{\log_4 16} \lg n) \\ = \Theta(n^2 \lg n)$$

$$a = 16$$

$$b = 4$$

$$f(n) = n^2$$

$$f(n) ? \quad n^{\log_b a}$$

$$n^2 ? \quad n^{\log_4 16}$$

$$n^2 \equiv n^2$$

$$d) T(n) = 7T(n/3) + n^2$$

$$\equiv \langle T_m \# 3.1 \rangle$$

$$n^2 = \Omega(n^{\log_3 7 + \epsilon})$$

$$\equiv \langle \epsilon = 2 - \log_3 7 \rangle$$

$$n^2 = \Omega(n^2)$$

$\equiv \text{true}$

$$T(n) = 7T(n/3) + n^2$$

$$\equiv \langle T_m \# 3.2 \rangle$$

$$7f(n/3) \leq cf(n)$$

$$\equiv \langle f(n) = n^2 \rangle$$

$$7\left(\frac{n}{3}\right)^2 \leq cn^2$$

$$\equiv \langle \text{Potencia} \rangle$$

$$7\frac{n^2}{9} \leq cn^2$$

$$\equiv \langle c = \frac{7}{9} \rangle$$

$$n^2 \leq n^2$$

\equiv

true

$$a = 7$$

$$b = 3$$

$$f(n) = n^2$$

$$f(n) ?$$

$$n^2 ?$$

$$n^2 >$$

$$n^{\log_b a}$$

$$n^{\log_3 7}$$

$$n^{\log_3 7}$$

$$T(n) = \Theta(n^2)$$

$$e) T(n) = 7T(n/2) + n^2$$

$$\equiv \langle T_m \# 1 \rangle$$

$$n^2 = O(n^{\log_2 7 - \epsilon})$$

$$\equiv \langle \epsilon = 2 - \log_2 7 \rangle$$

$$n^2 = O(n^2)$$

$$\equiv \langle f(n) = O(f(n)) \rangle$$

true

$$\rightarrow T(n) = \Theta(n^{\log_2 7})$$

$$a = 7$$

$$b = 2$$

$$f(n) = n^2$$

$$f(n) ? n^{\log_b a}$$

$$n^2 ? n^{\log_2 7}$$

$$n^2 < n^{\log_2 7}$$

$$f) T(n) = 2T(n/4) + \sqrt{n}$$

$$\equiv \langle T_m \# 2 \rangle$$

$$\sqrt{n} = \Theta(n^{\log_4 2})$$

$$\equiv \langle \log_4 2 = \frac{1}{2} \rangle$$

$$\sqrt{n} = \Theta(\sqrt{n})$$

$$\equiv \langle f(n) = \Theta(f(n)) \rangle$$

true

$$\rightarrow T(n) = \Theta(n^{\log_4 2} \lg n) \\ = \Theta(\sqrt{n} \lg n)$$

$$a = 2$$

$$b = 4$$

$$f(n) = \sqrt{n}$$

$$f(n) ? n^{\log_b a}$$

$$\sqrt{n} ? n^{\log_4 2}$$

$$\sqrt{n} \cong \sqrt{n}$$