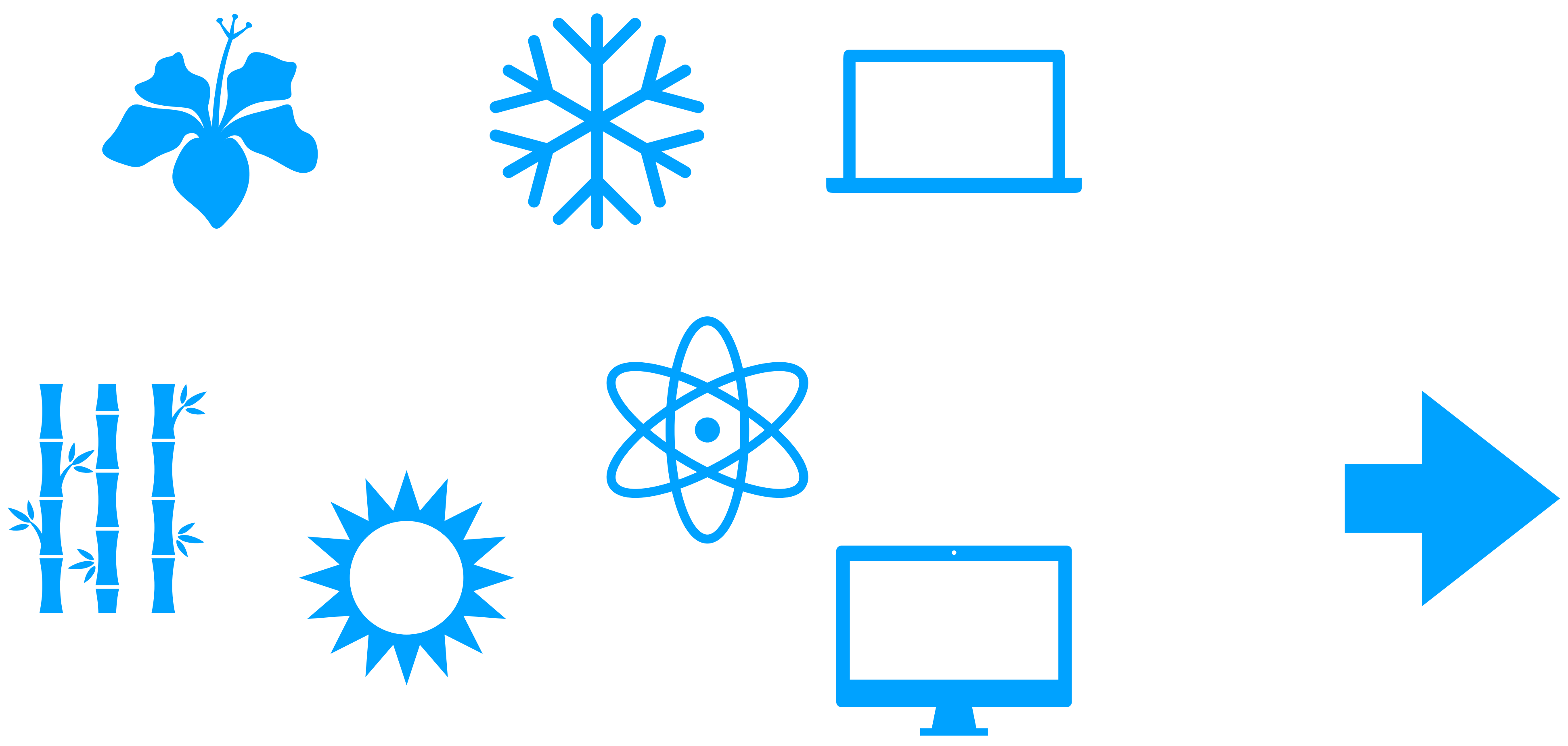
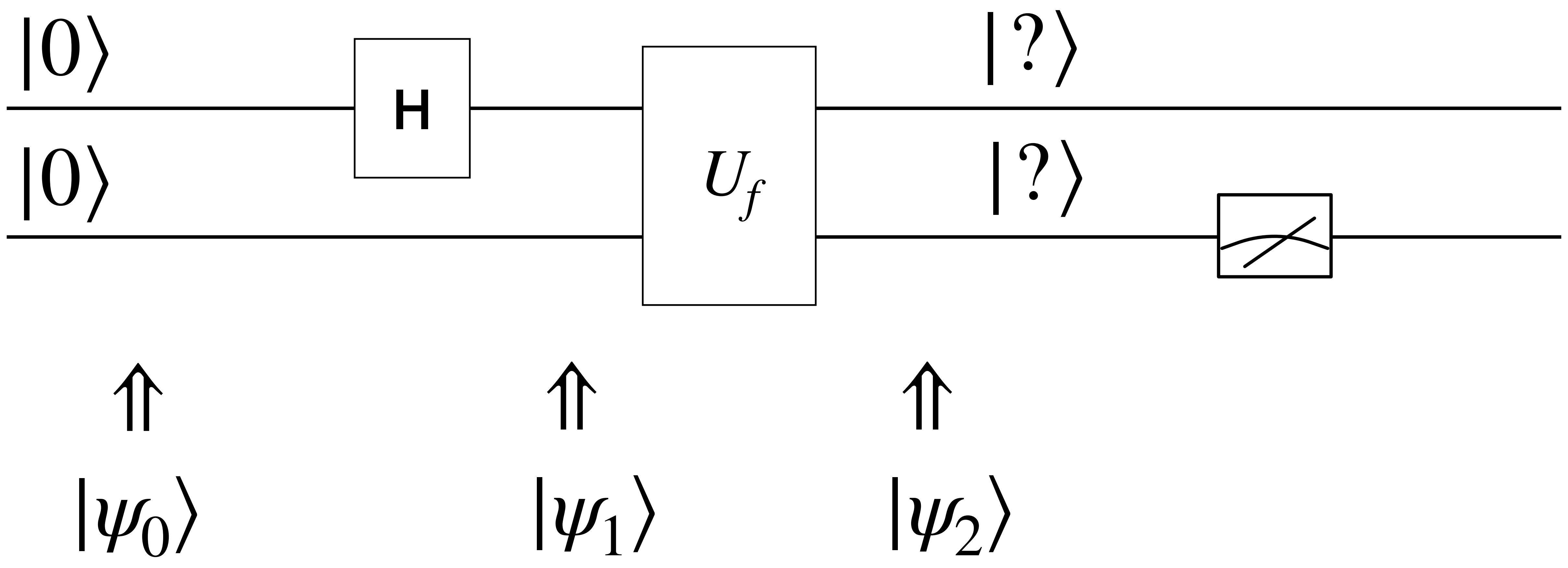
Simulación de sistema clásicos y probabilisticos

Luis Daniel Benavides Navarro, Ph.D

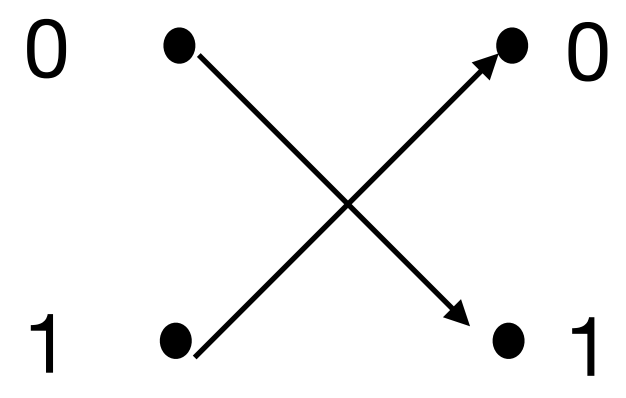
Simulación





$$|\psi_0\rangle = |00\rangle$$

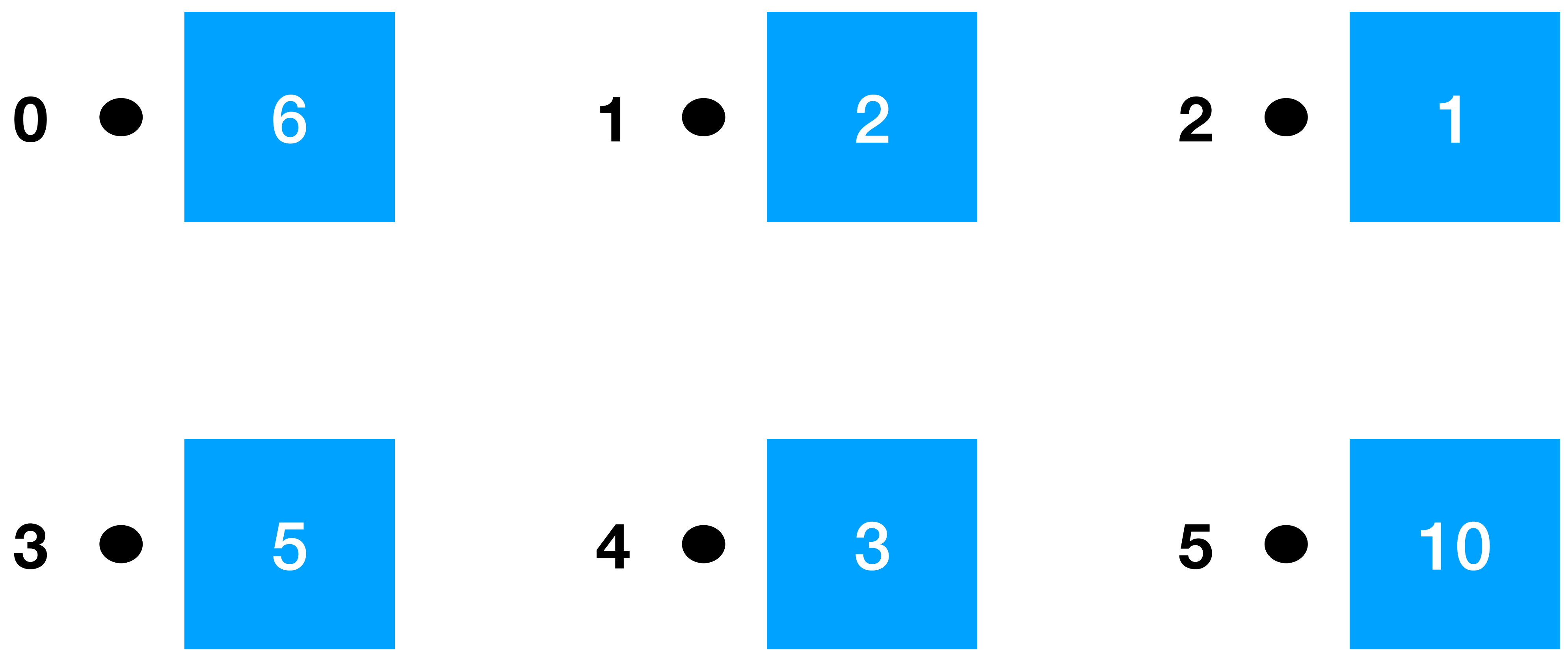
$$|\psi_1\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}}$$



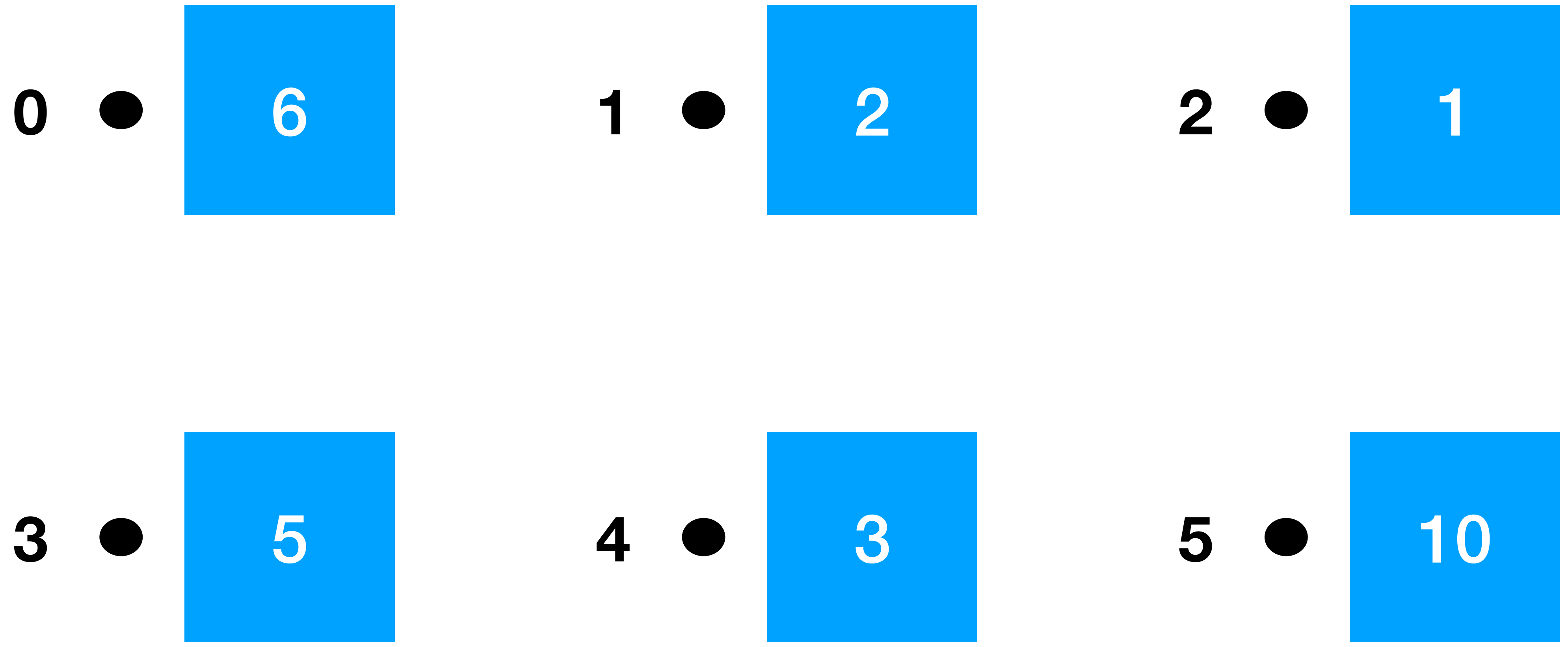
	00	01	10	11
00	0	1	0	0
01	1	0	0	0
10	0	0	1	0
11	0	0	0	1

$$|\psi_2\rangle = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} (1/\sqrt{2} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}) = 1/\sqrt{2} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Sistemas determinísticos clásicos



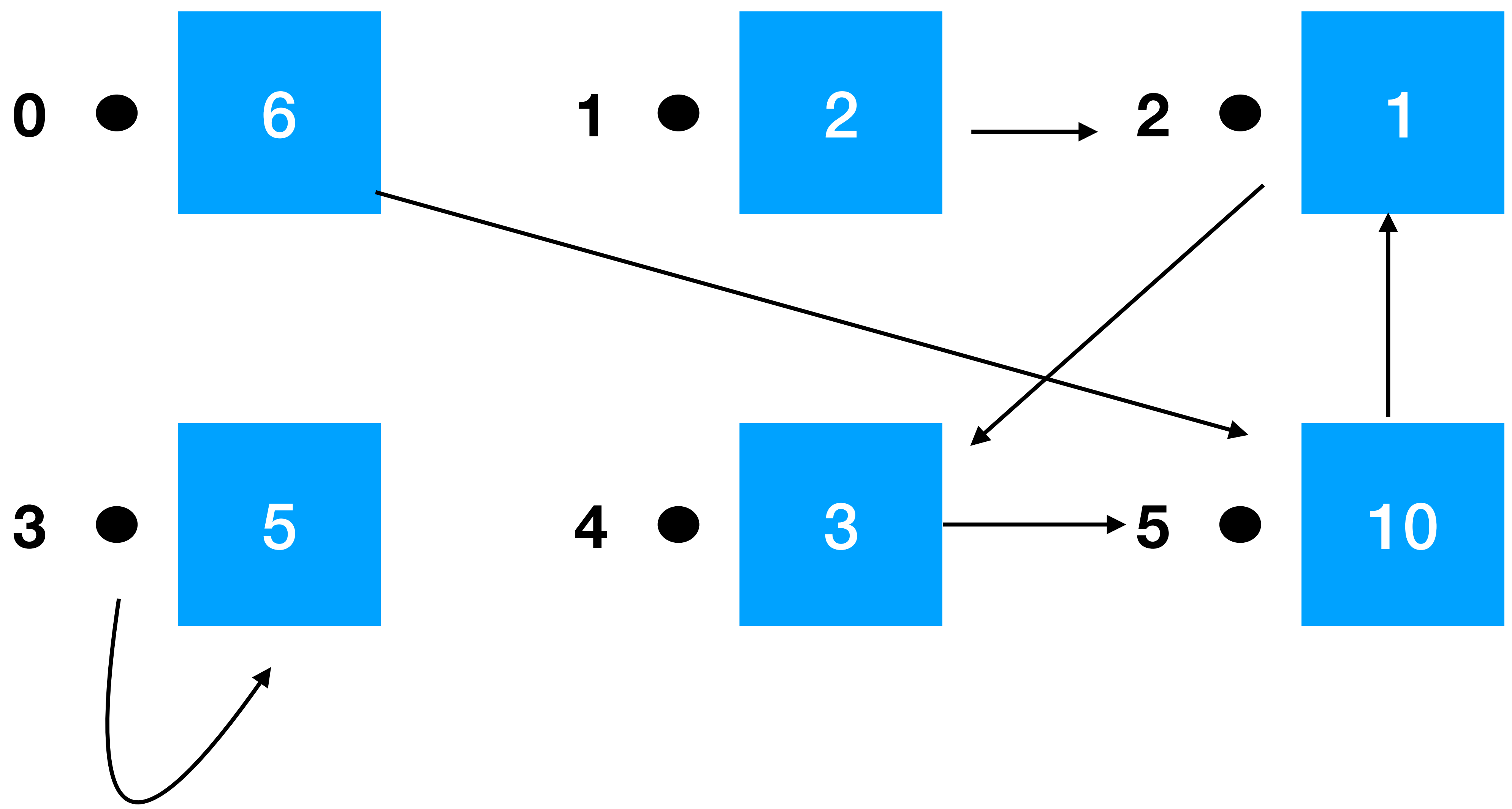
Sistemas discretos



Estado

$$X = [6,2,1,5,3,10]^T$$

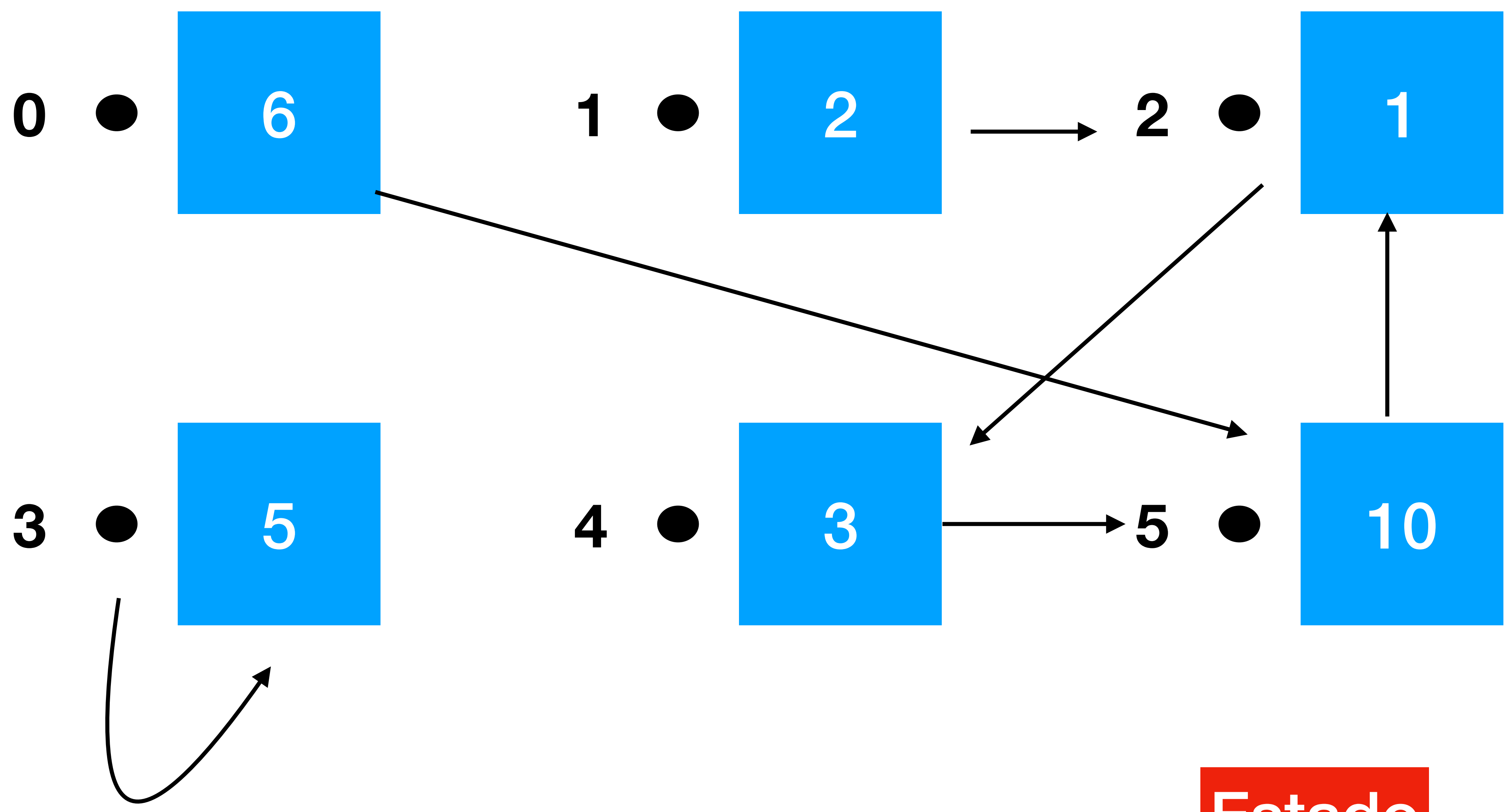
Sistemas discretos



$$X = [6,2,1,5,3,10]^T$$

Restricción para ser determinístico:
Cada vértice tiene una sola flecha de salida

Sistemas discretos



Estado

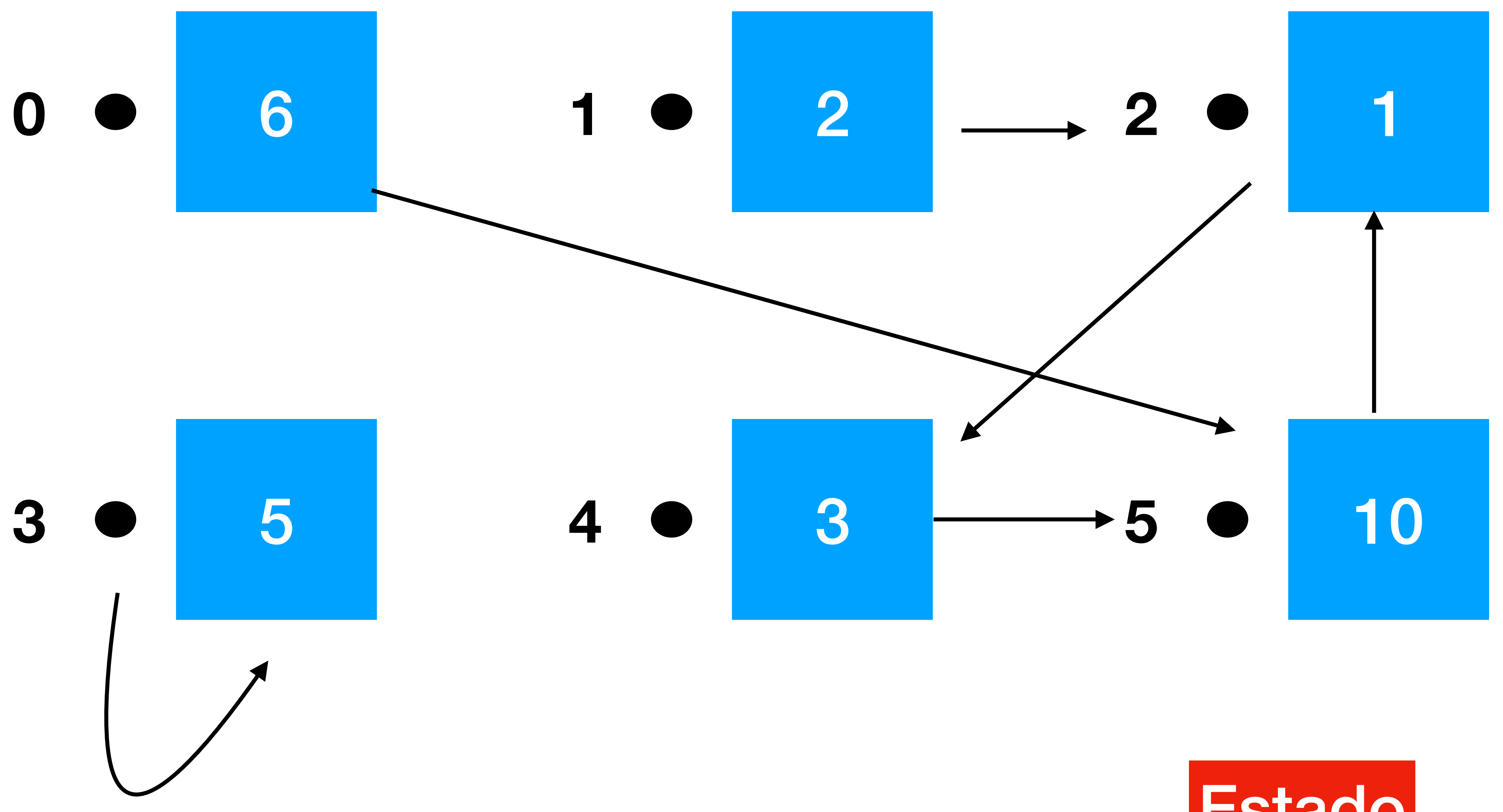
$$X = [6,2,1,5,3,10]^T$$

Restricción para ser determinístico:
Cada vértice tiene una sola flecha de salida

	0	1	2	3	4	5
0						
1						
2		1				1
3				1		
4			1			
5	1				1	

Dinámica

Sistemas discretos



Estado

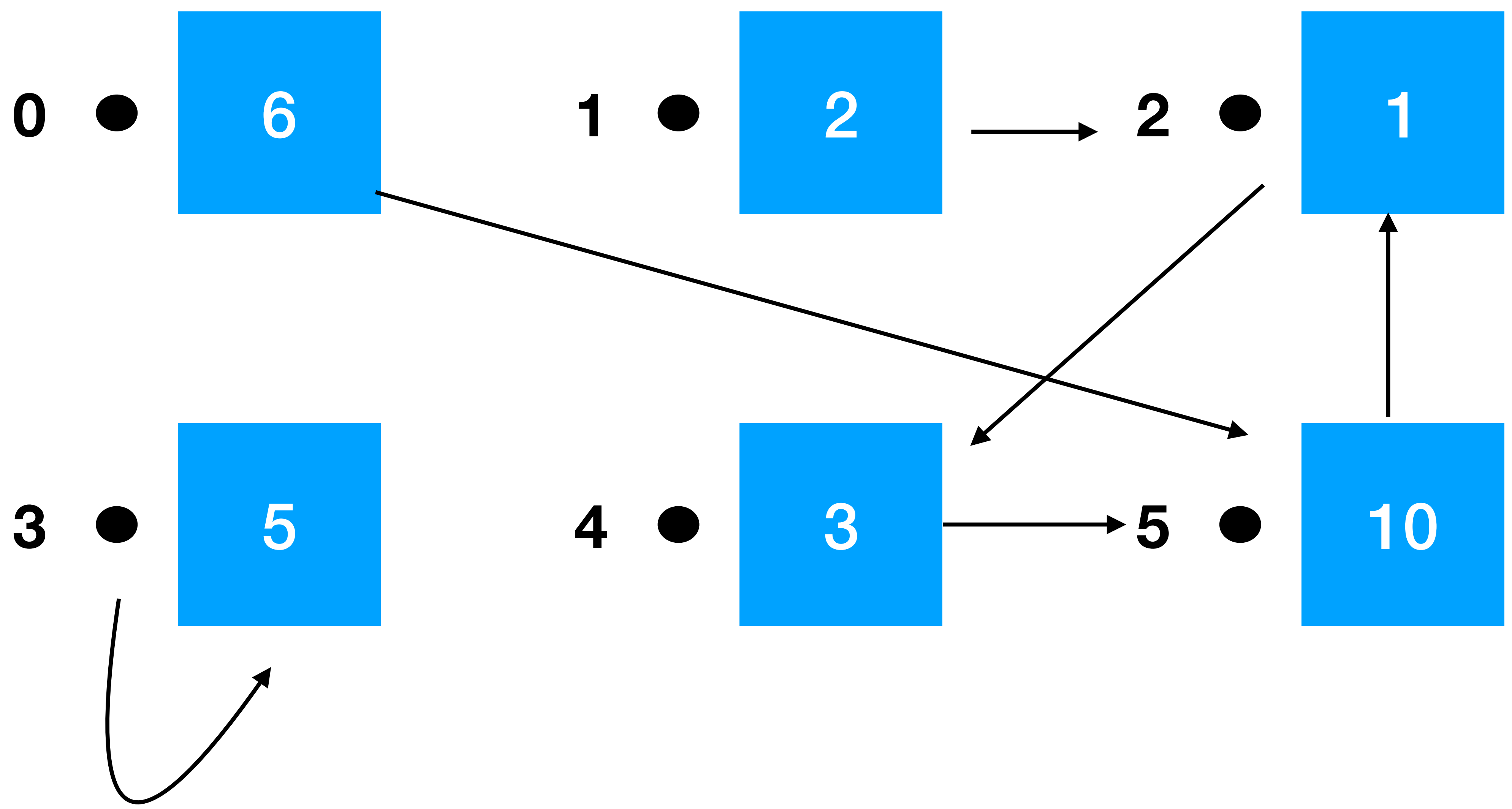
$$X = [6,2,1,5,3,10]^T$$

Restricción para ser determinístico:
Cada vértice tiene una sola flecha de salida

	0	1	2	3	4	5
0						
1						
2		1				1
3				1		
4			1			
5	1				1	

Dinámica

Sistemas discretos



$X = [6,2,1,5,3,10]^T$

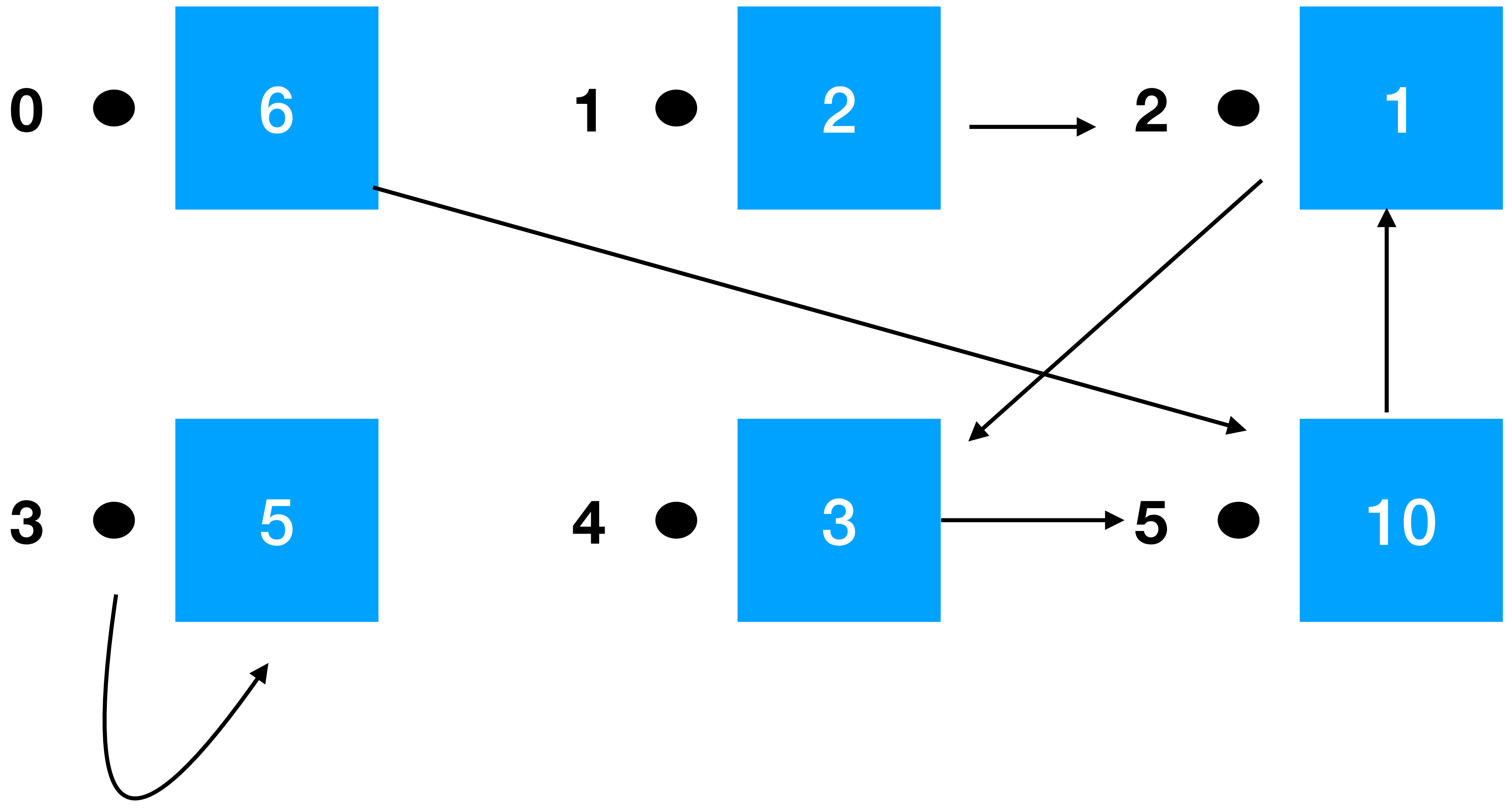
Restricción para ser determinístico:
Cada vértice tiene una sola flecha de salida

$$
M = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{array}
$$

Estado después de un click de tiempo:

$$Y = MX$$

Sistemas discretos



Restricción para ser determinístico:
Cada vértice tiene una sola flecha de salida

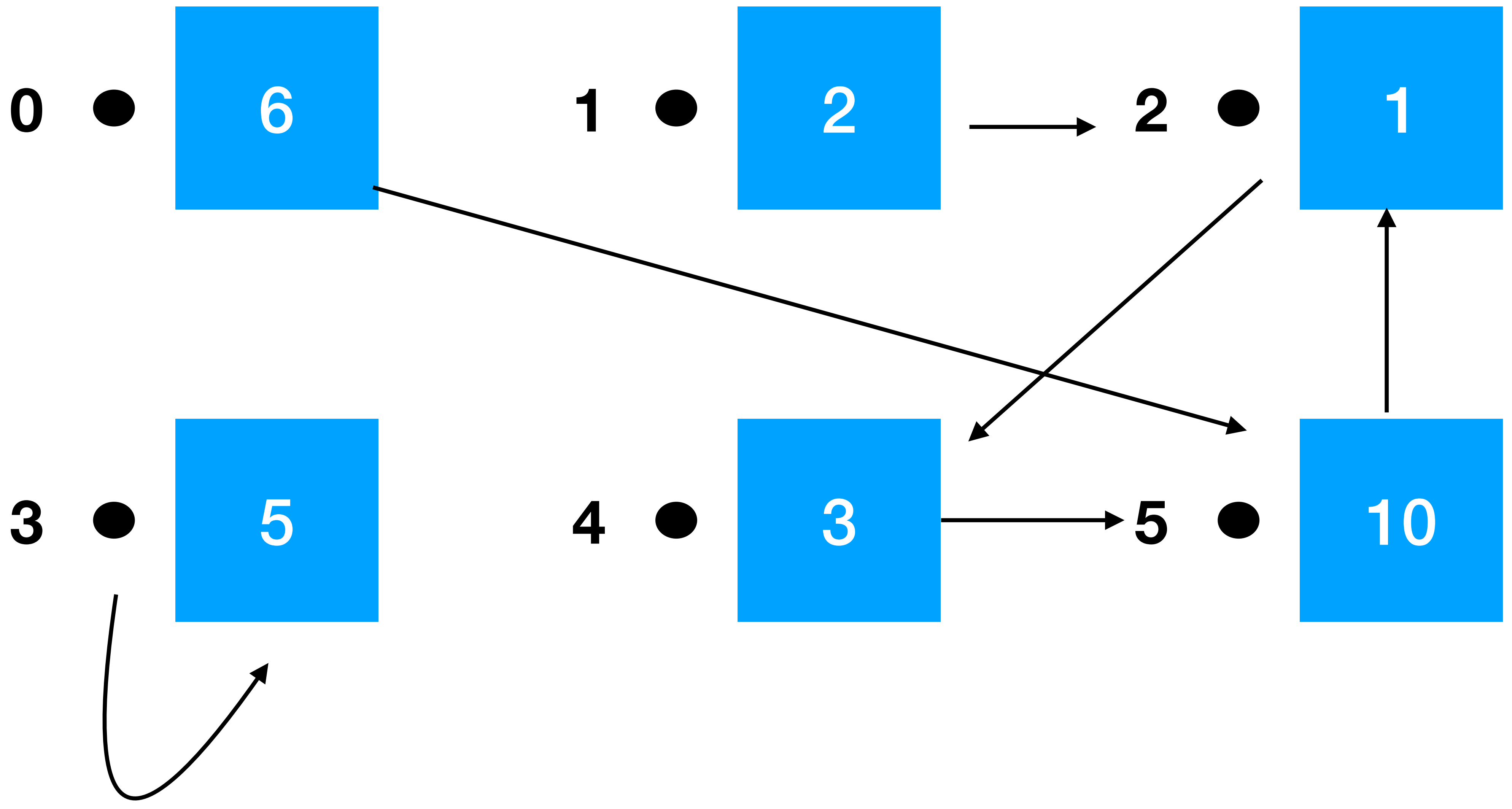
Estado después de un click de tiempo:

$$X = [6,2,1,5,3,10]^T$$

$$Y = MX$$

$$\begin{array}{c|cccccc}
 & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \\
\hline
\mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{2} & 0 & 1 & 0 & 0 & 0 & 1 \\
\mathbf{3} & 0 & 0 & 0 & 1 & 0 & 0 \\
\mathbf{4} & 0 & 0 & 1 & 0 & 0 & 0 \\
\mathbf{5} & 1 & 0 & 0 & 0 & 1 & 0
\end{array}
\begin{bmatrix} 6 \\ 2 \\ 1 \\ 5 \\ 3 \\ 10 \end{bmatrix}
=
\begin{bmatrix} \; \\ \; \\ \; \\ \; \\ \; \\ \; \end{bmatrix}$$

Sistemas discretos



Restricción para ser determinístico:
Cada vértice tiene una sola flecha de salida

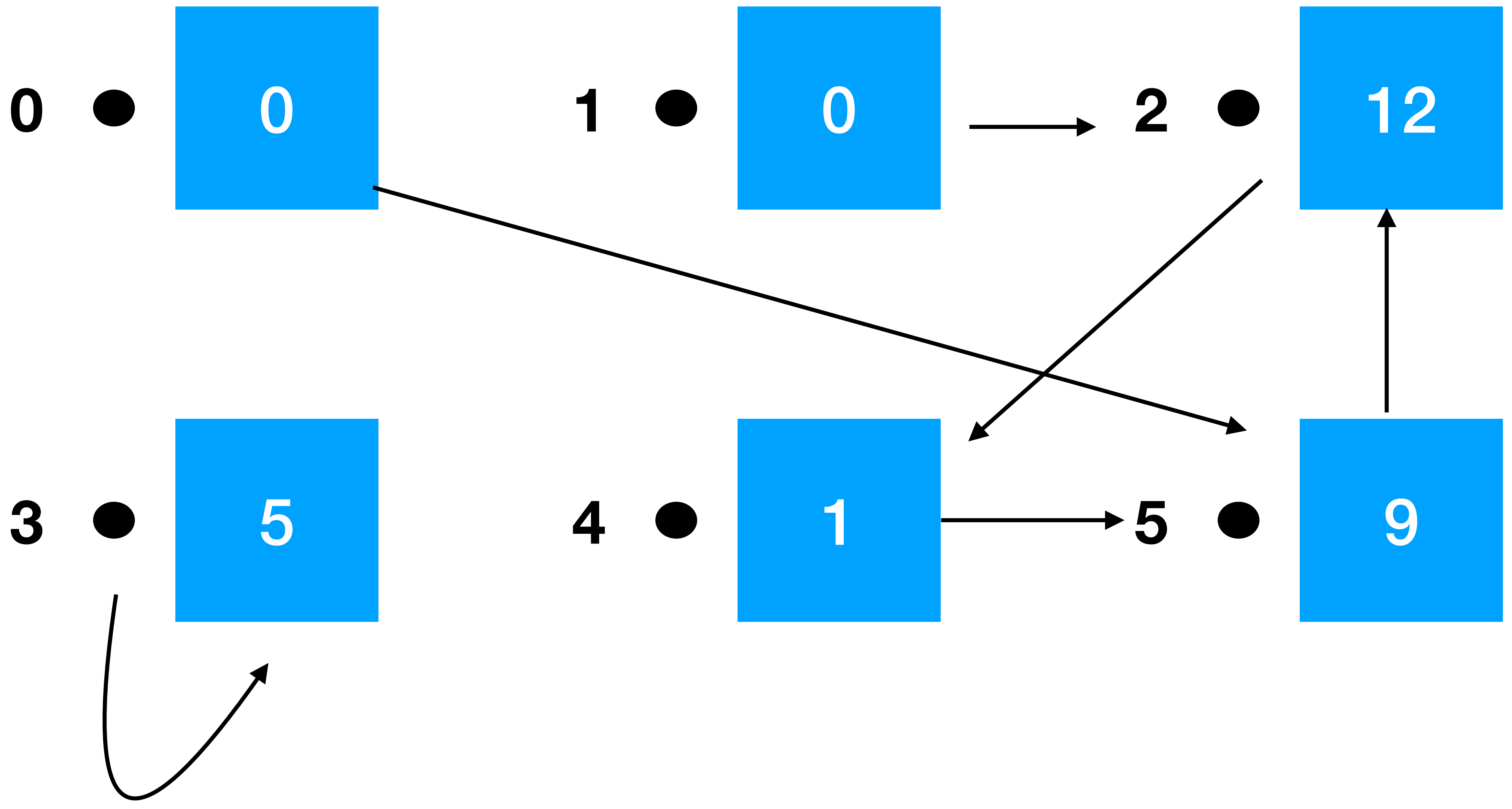
Estado después de un click de tiempo:

$$X = [6,2,1,5,3,10]^T$$

$$Y = MX$$

$$\begin{array}{c|cccccc|}
 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & 0 \\
5 & 1 & 0 & 0 & 0 & 1 & 0
\end{array}
\begin{bmatrix} 6 \\ 2 \\ 1 \\ 5 \\ 3 \\ 10 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 12 \\ 5 \\ 1 \\ 9 \end{bmatrix}$$

Sistemas discretos



Restricción para ser determinístico:
Cada vértice tiene una sola flecha de salida

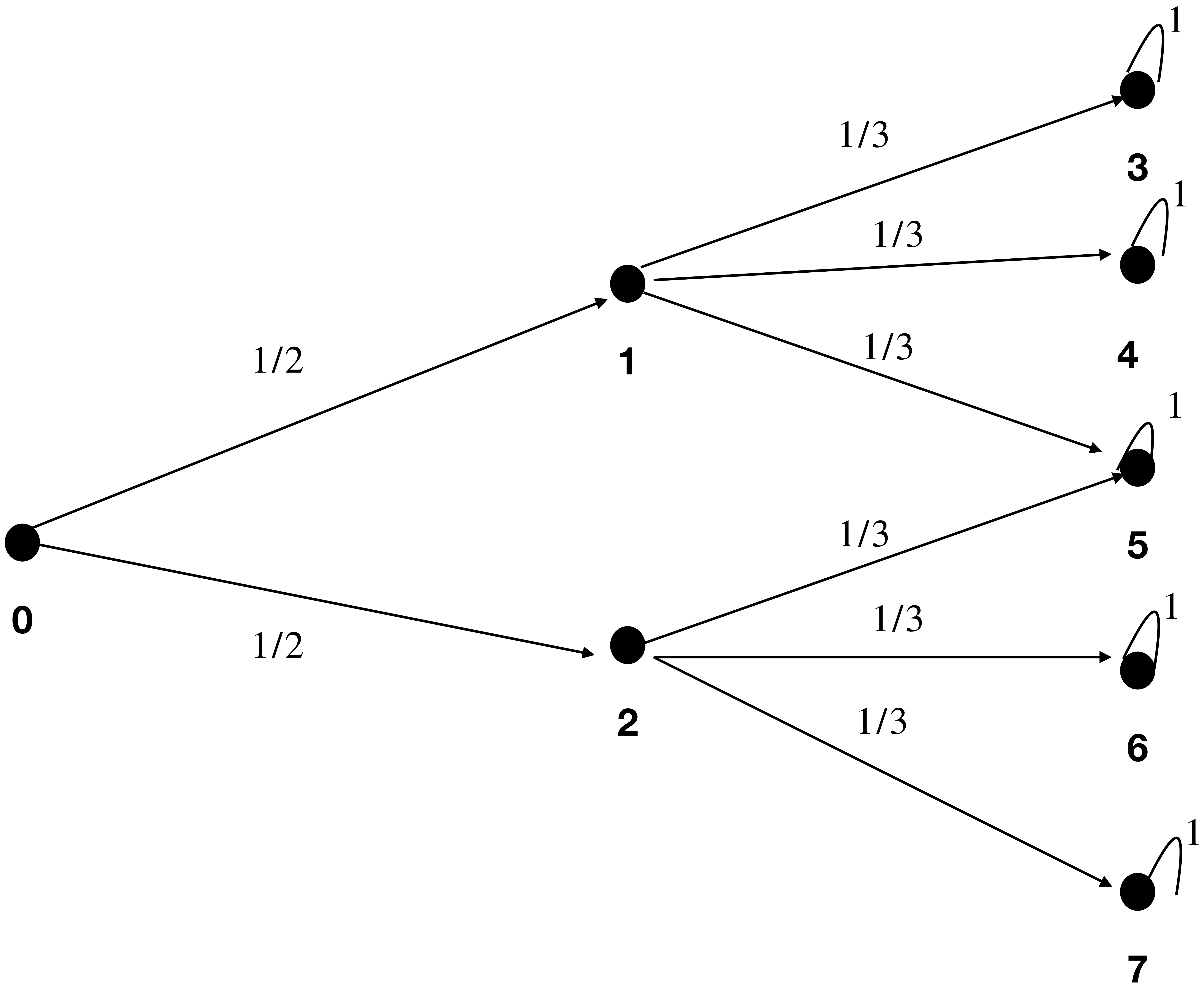
Estado después de dos click de tiempo:

$$Y = [0,0,12,5,1,9]^T$$

$$Y' = MY = MMX$$

$$\begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 & 1 & 0 \end{array} \begin{bmatrix} 0 \\ 0 \\ 12 \\ 5 \\ 1 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 9 \\ 5 \\ 12 \\ 1 \end{bmatrix}$$

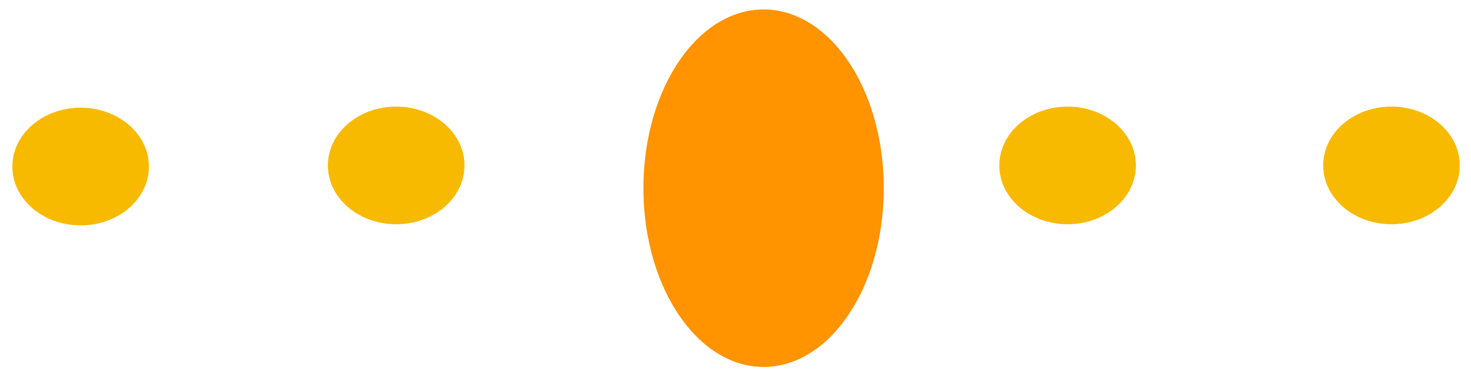
Doble rendija probabilístico



$$X = [1,0,0,0,0,0,0,0]$$

$$X' = [0,\frac{1}{2},\frac{1}{2},0,0,0,0,0]$$

$$X'' = [0,0,0,\frac{1}{6},\frac{1}{6},\frac{1}{3},\frac{1}{6},\frac{1}{6}]$$



	0	1	2	3	4	5	6	7
0								
1	1/2							
2	1/2							
3		1/3		1				
4		1/3			1			
5		1/3	1/3			1		
6			1/3				1	
7			1/3					1

Exercise 3.2.2 Let M be any n-by-n doubly stochastic matrix. Let X be an n-by-1 column vector. Let the result of $MX = Y$.

a) If the sum of the entries of X is 1, prove that the sum of the entries of Y is 1.

b) More generally, prove that if the sum of the entries of X is x, then the sum of the entries of Y is also x, i.e., M preserves the sum of the entries of a column vector multiplied at the right of M.

$\blacksquare$

$$\begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} a00 & a01 & a02 \\ a10 & a11 & a12 \\ a20 & a21 & a22 \end{bmatrix} \end{array} \begin{bmatrix} X0 \\ X1 \\ X2 \end{bmatrix} = \begin{bmatrix} Y0 \\ Y1 \\ Y2 \end{bmatrix}$$

x0+x1+x2=1

Y0= a00 . X0 + a01 . x1 + a02 .x2

Y1= a10 . X0 + a11 . x1 + a12 .x2

Y2= a20 . X0 + a21 . x1 + a22 .x2

Y0+Y1+Y2 = (a00 +10+a20) .X0 + (a01+a11+a21) . X1+ (a02+a12+a22).X2

Y0+Y1+Y2 = X0 + X1+ X2

$\blacksquare$

Exercise 3.2.2 Let M be any n-by-n doubly stochastic matrix. Let X be an n-by-1 column vector. Let the result of $MX = Y$.

a) If the sum of the entries of X is 1, prove that the sum of the entries of Y is 1.

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■

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & a00 & a01 & a02 \\ 1 & a10 & a11 & a12 \\ 2 & a20 & a21 & a22 \end{array} \begin{bmatrix} X0 \\ X1 \\ X2 \end{bmatrix} = \begin{bmatrix} Y0 \\ Y1 \\ Y2 \end{bmatrix}$$

x0+x1+x2=x

Y0= a00 . X0 + a01 . x1 + a02 .x2

Y1= a10 . X0 + a11 . x1 + a12 .x2

Y2= a20 . X0 + a21 . x1 + a22 .x2

Y0+Y1+Y2 = (a00 +10+a20) .X0 + (a01+a11+a21) . X1+ (a02+a12+a22).X2

Y0+Y1+Y2 = X0 + X1+ X2 = x

■

Sistemas cuánticos

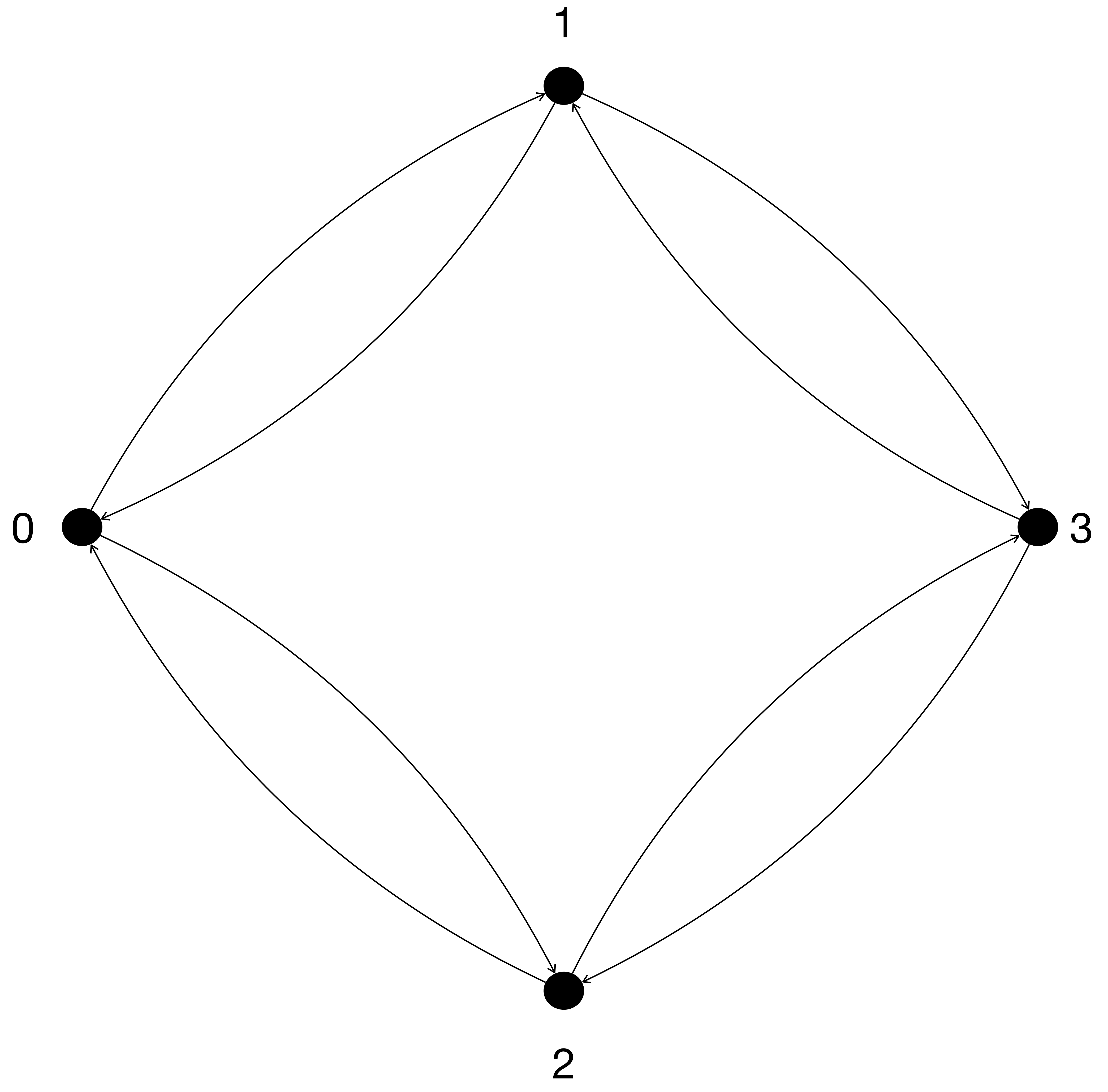
Generalidades del modelo matemático

- Los pesos de las arcos están dados por números complejos
- La probabilidad de una transición está dada por $|c|^2$
- La matriz que se forma en el sistema debe ser unitaria. Una matriz $n \times n$ se denomina unitaria si $U \star U^\dagger = U^\dagger \star U = I_n$.
- Si tomamos el módulo cuadrado de cada elemento de una matriz unitaria formamos una matriz doblemente estocástica.
- Los número complejos pueden interferir y cancelarse unos con otros

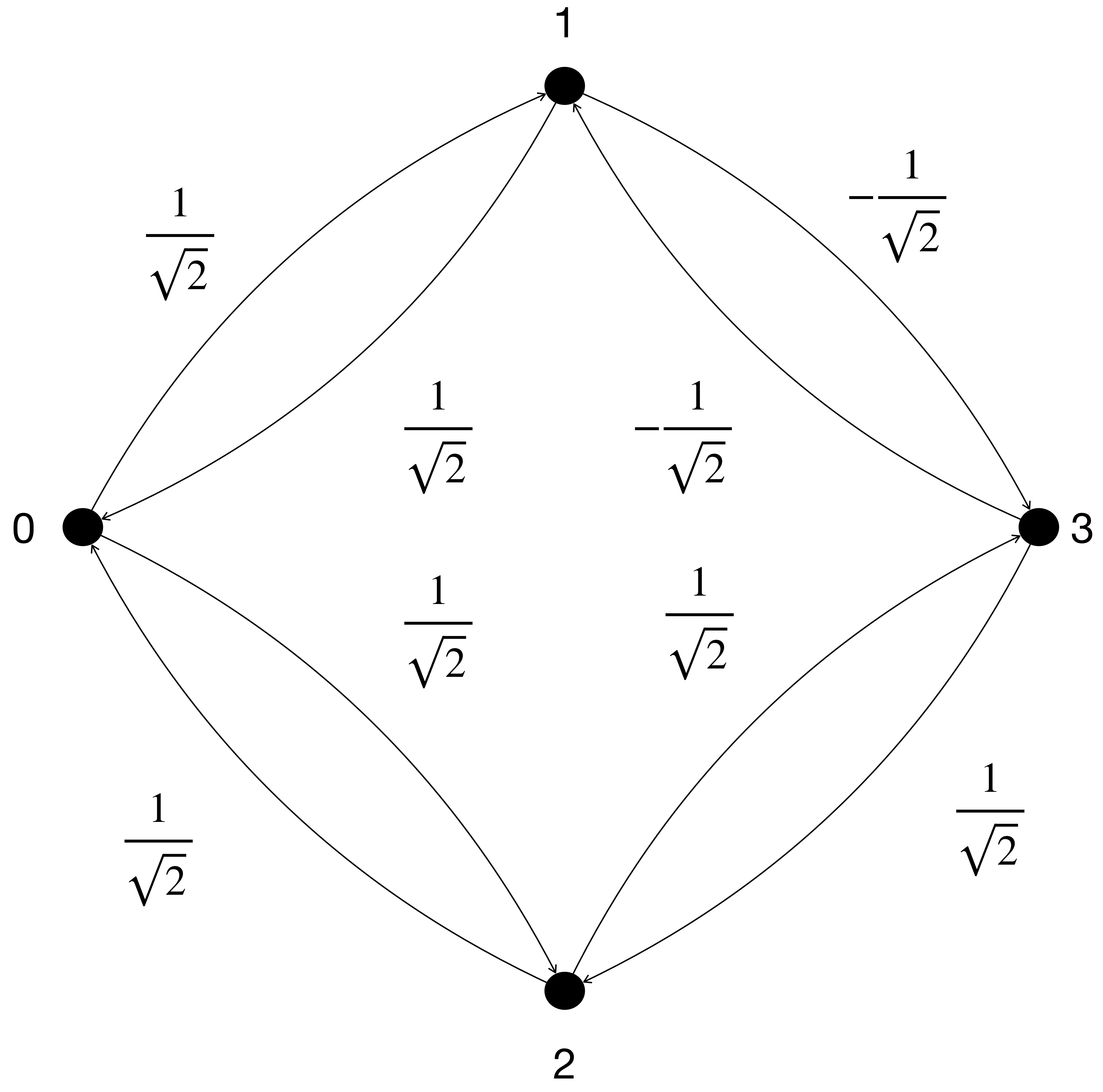
Tres conceptos fundamentales

- **Superposición.** El estado de un sistema puede contener múltiples historias.
- **Interferencia.** Las historias de los sistemas pueden interactuar. (La suma de complejos puede cancelarse)
- **Entrelazamiento.** Un estado de un sistema no se puede representar como la suma de estados individuales.

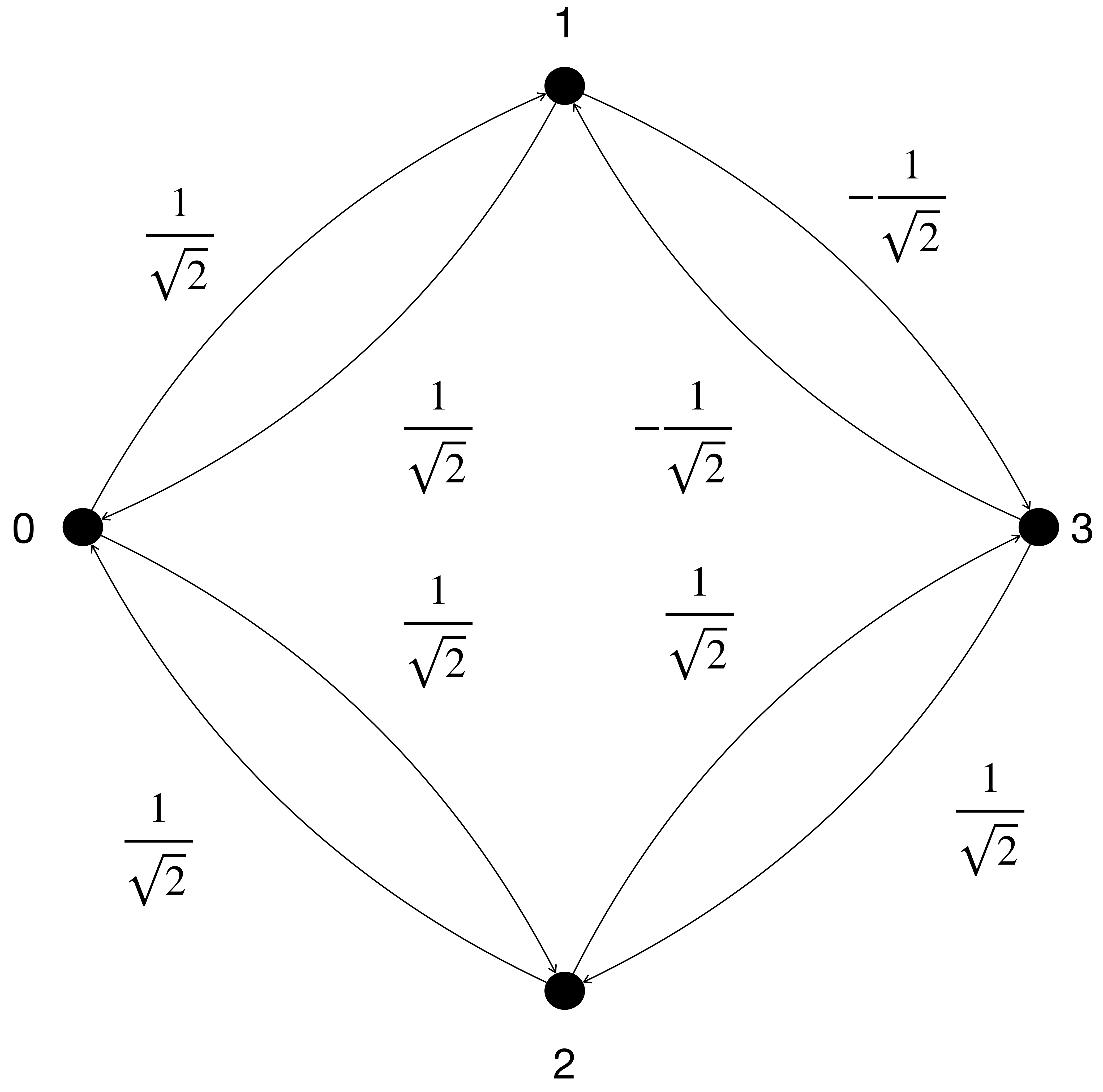
El billar cuántico



El billar cuántico: 1 Click

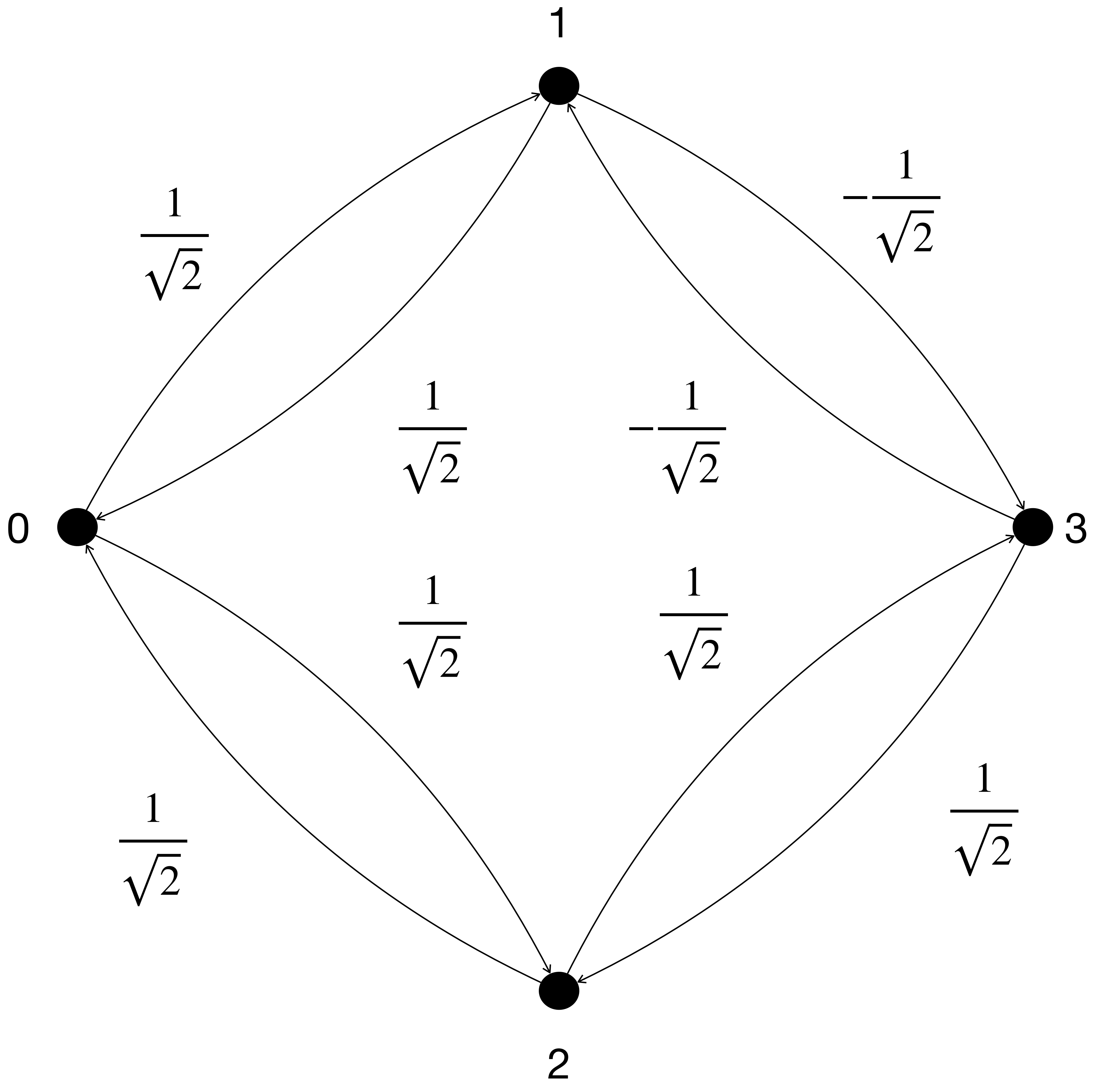


El billar cuántico: 1 Click



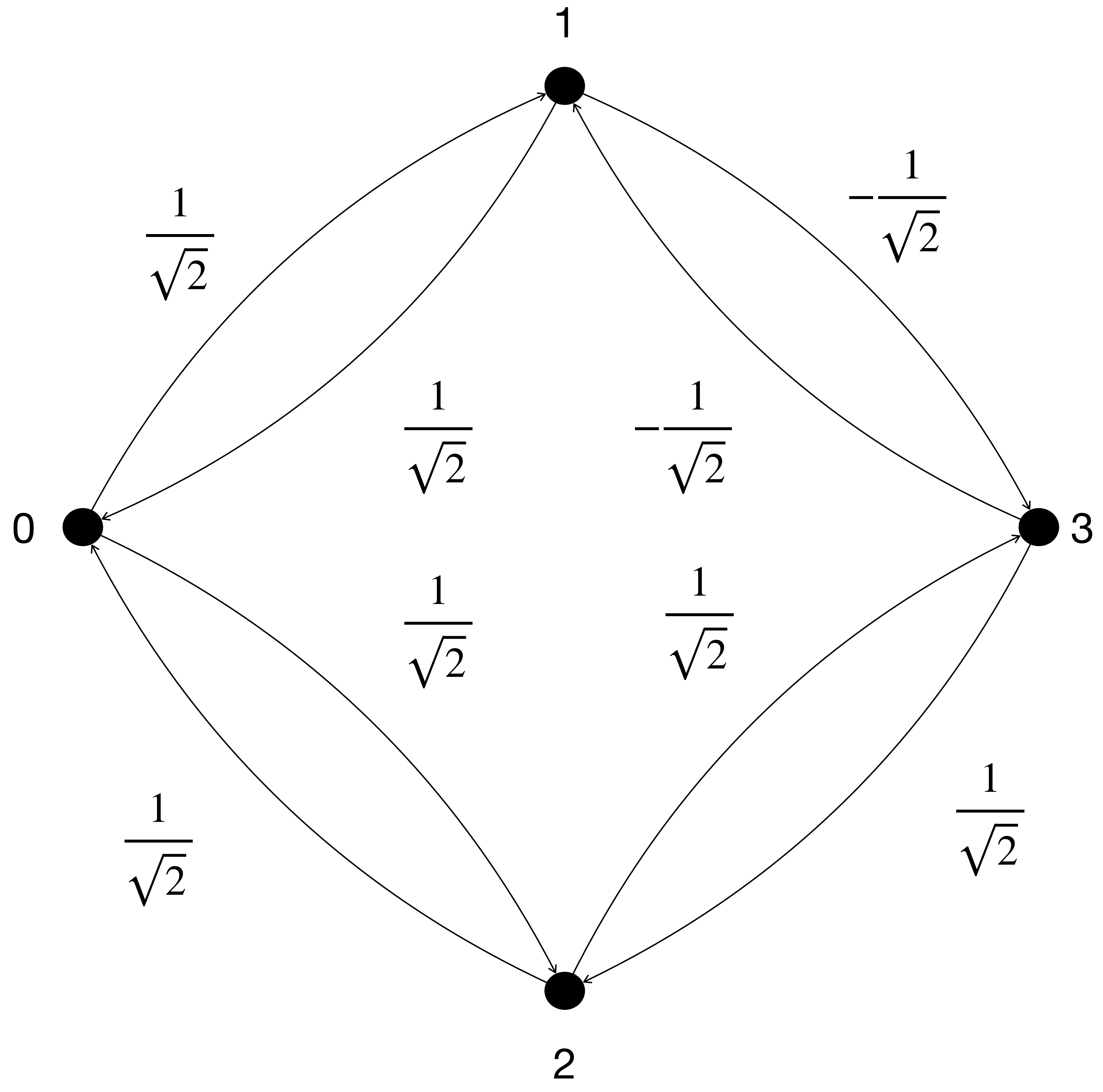
$$M = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

El billar cuántico: 1 Click



$$M = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \qquad X = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

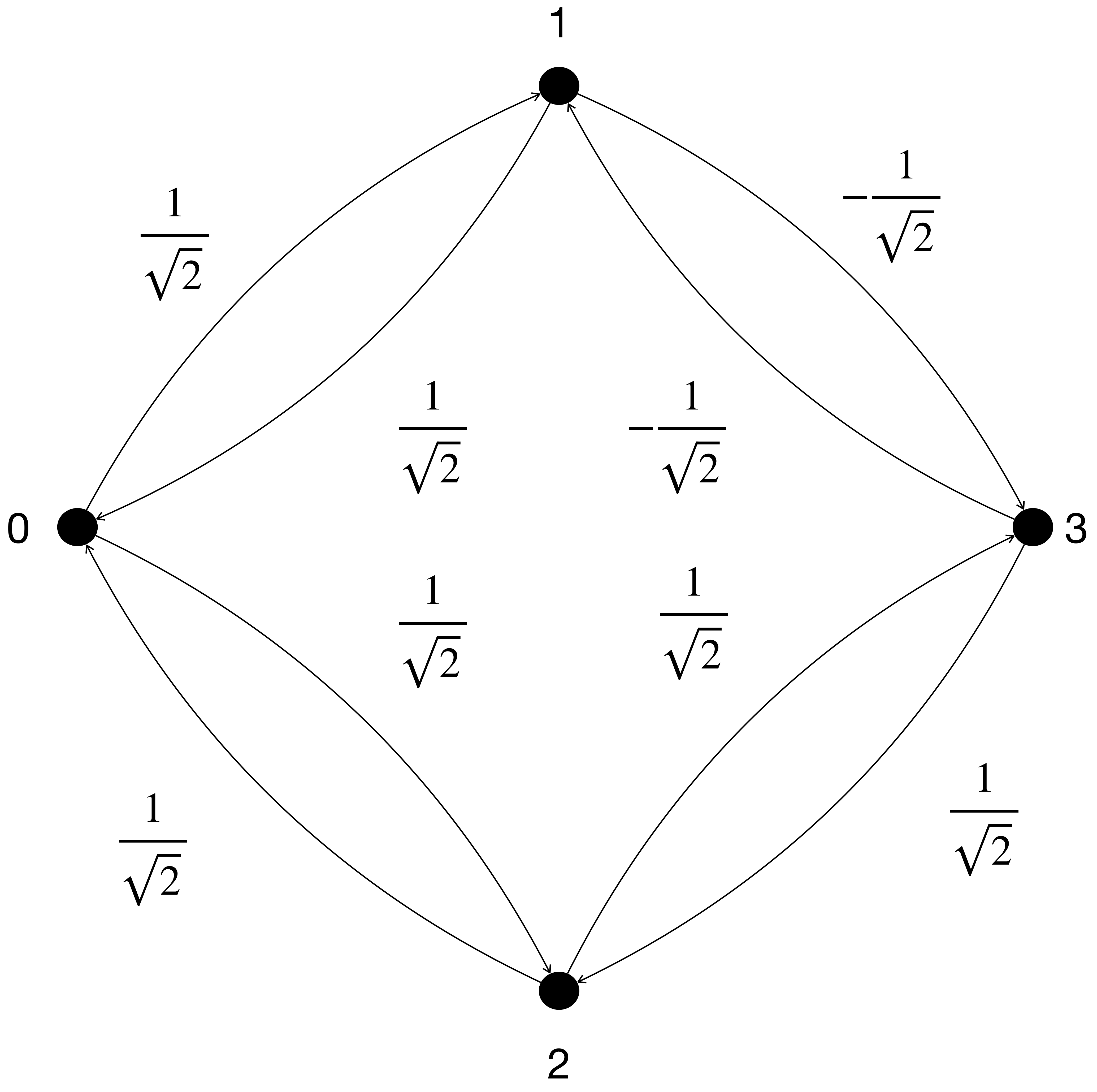
El billar cuántico: 1 Click



$$M = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \qquad X = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

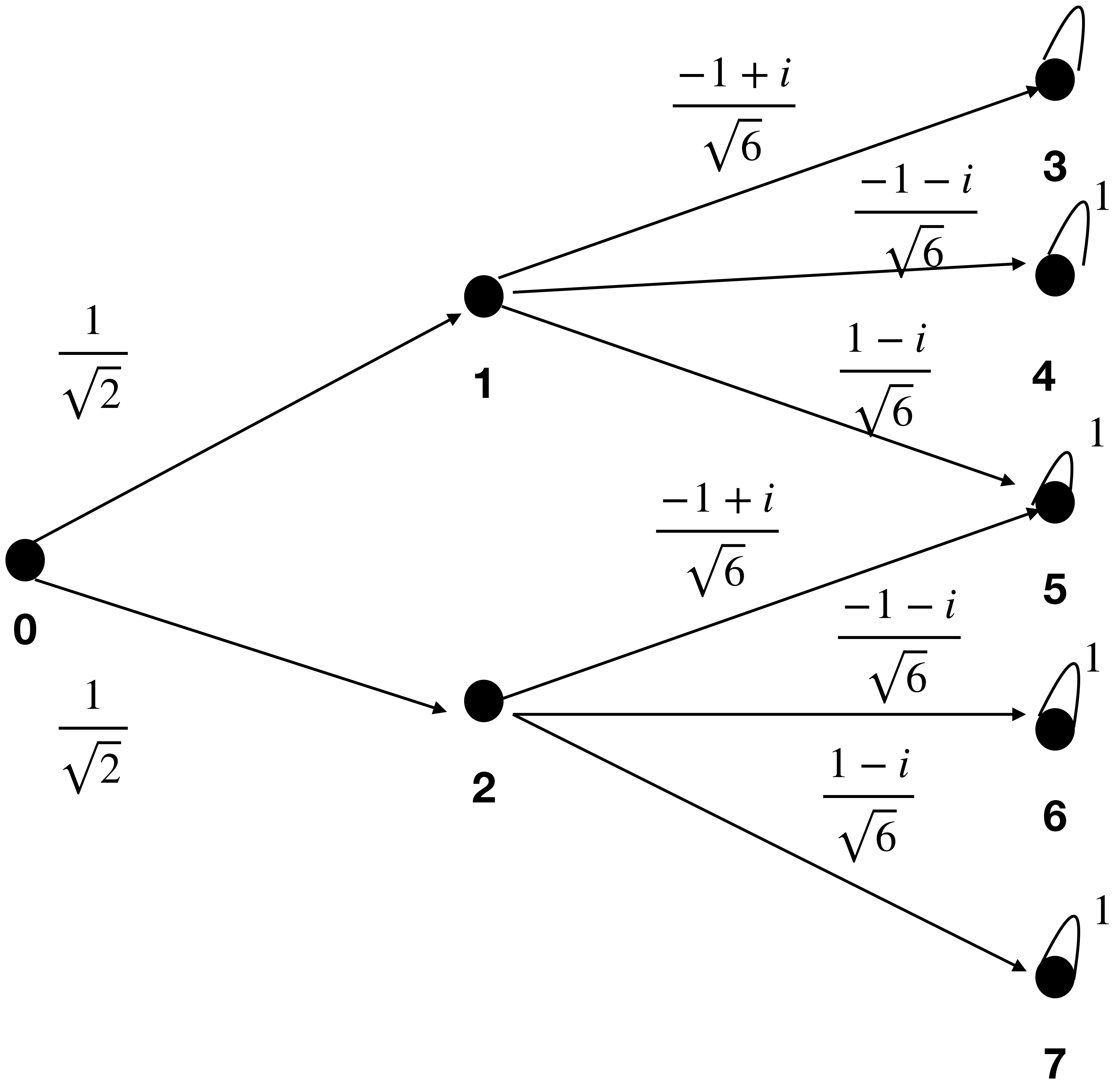
$$Y = MX = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

El billar cuántico: 2 Clicks



$$Y' = MMX = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Experimento de la doble rendija

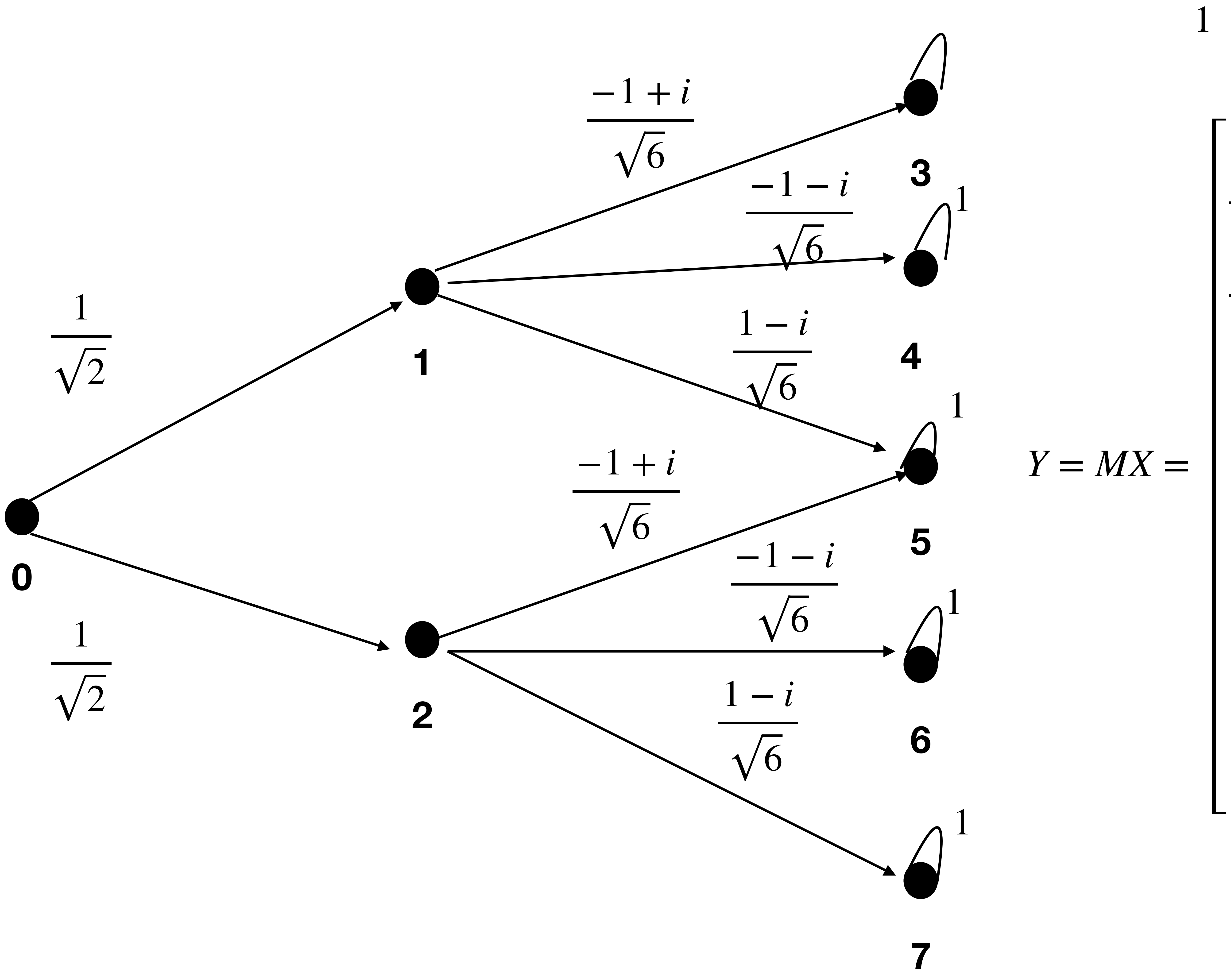


1

$$X = [1,0,0,0,0,0,0,0]^T$$

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

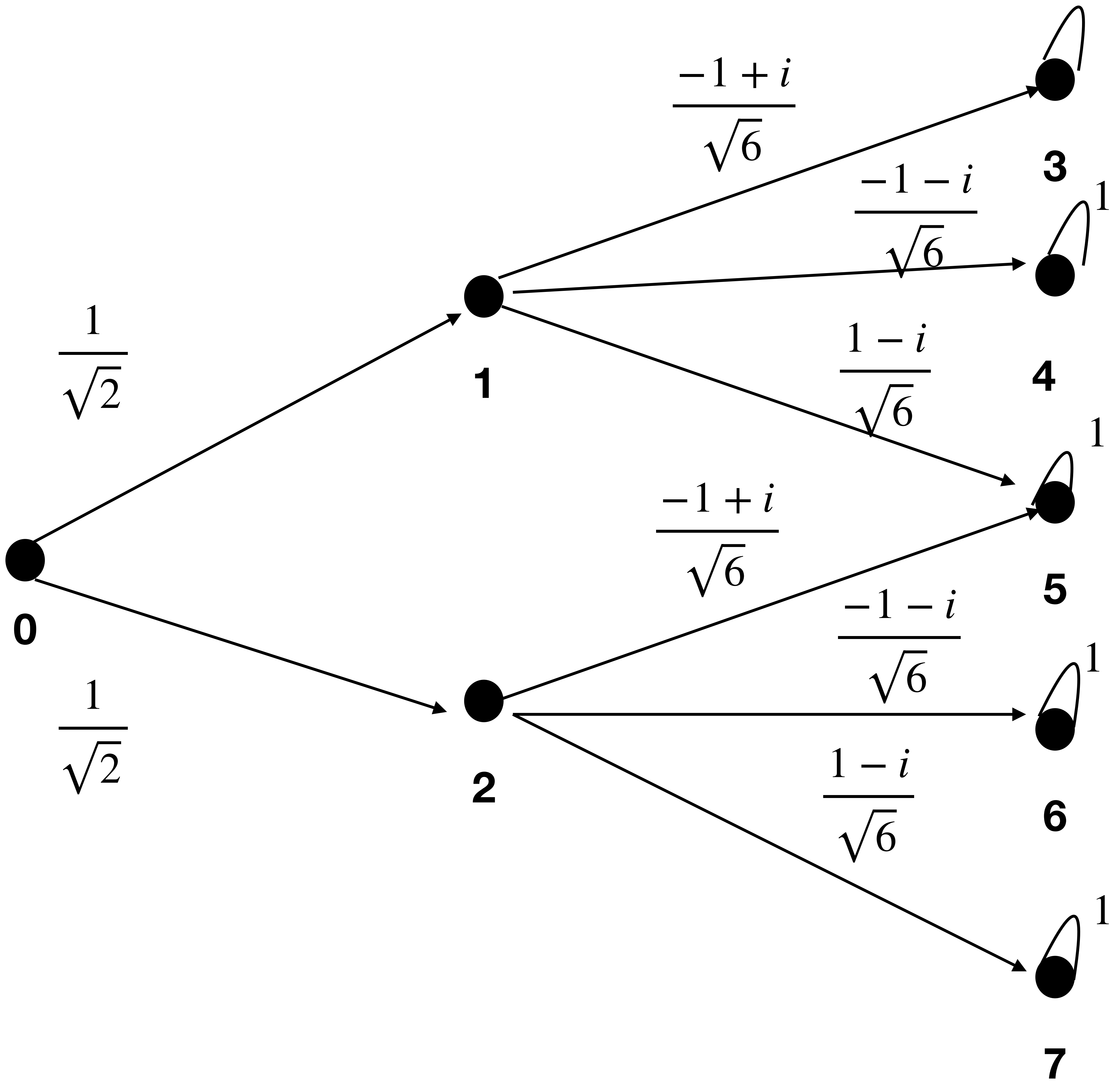
Experimento de la doble rendija



$$X = [1,0,0,0,0,0,0,0]^T$$

$$Y = MX = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Experimento de la doble rendija



1

$$X = [1,0,0,0,0,0,0,0]^T$$

$$Y'' = MY = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-1+i}{2\sqrt{3}} \\ \frac{-1+i}{2\sqrt{3}} \\ 0 \\ \frac{-1-i}{2\sqrt{3}} \\ \frac{1-i}{2\sqrt{3}} \end{bmatrix}$$

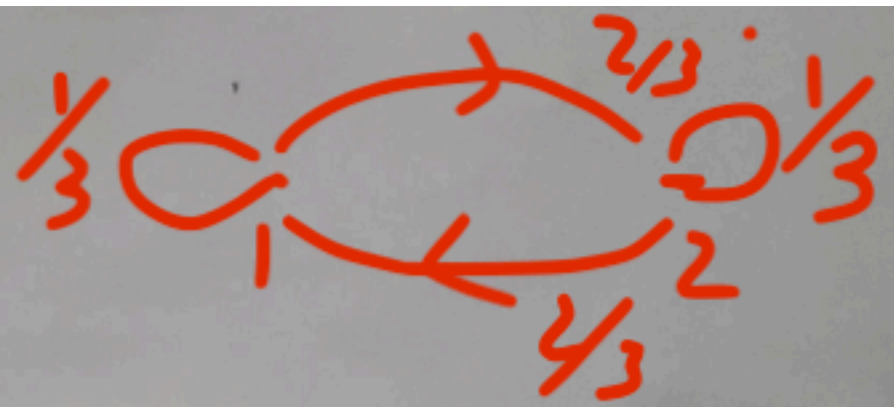
$$\frac{1-i}{2\sqrt{3}} + \frac{-1+i}{2\sqrt{3}} = 0$$

Ensamblar sistemas

Ensamblar Sistemas

3.4.2

$$N = \begin{array}{cc} & \begin{array}{cc}1 & 2\end{array} \\ \begin{array}{c}1\\2\end{array} & \begin{bmatrix}\frac{1}{3} & \frac{2}{3}\\ \frac{2}{3} & \frac{1}{3}\end{bmatrix}\end{array}$$



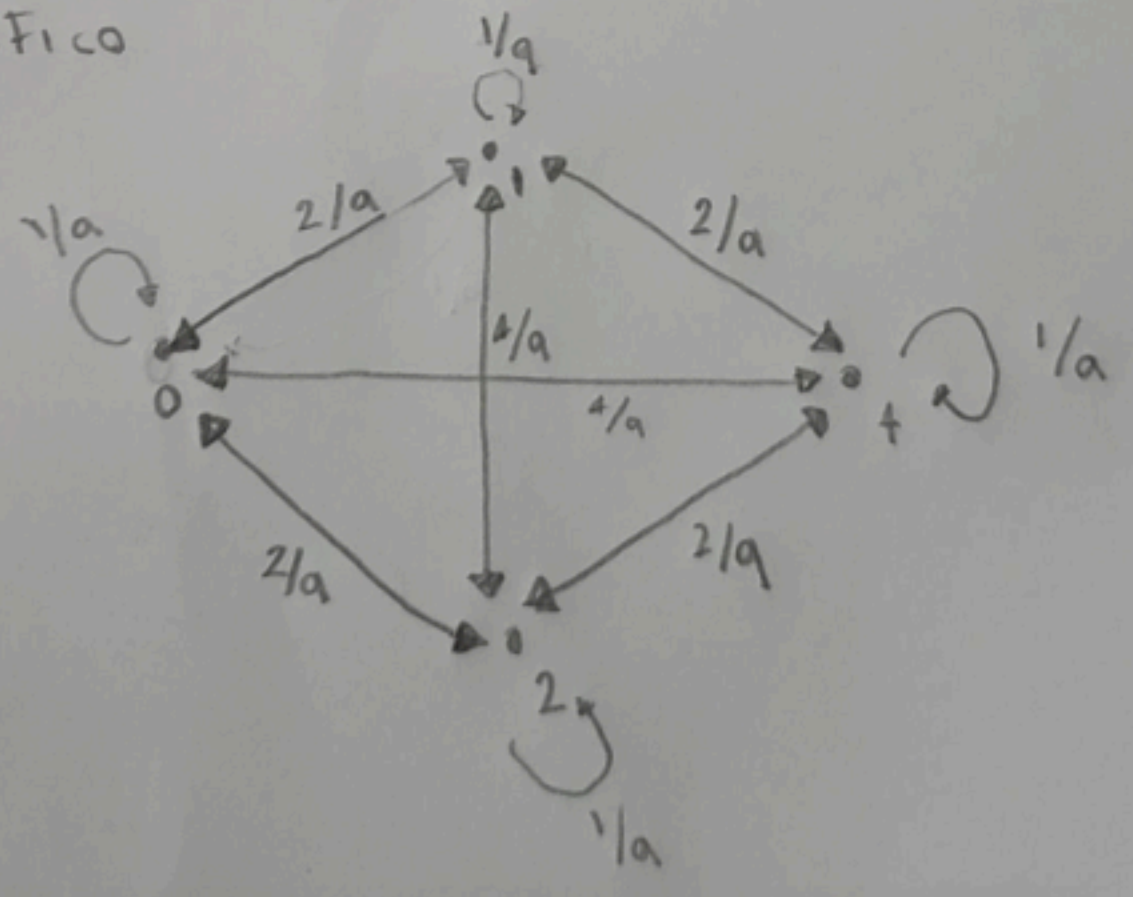
Encuentre $N \otimes N$ y su Grafico

a b

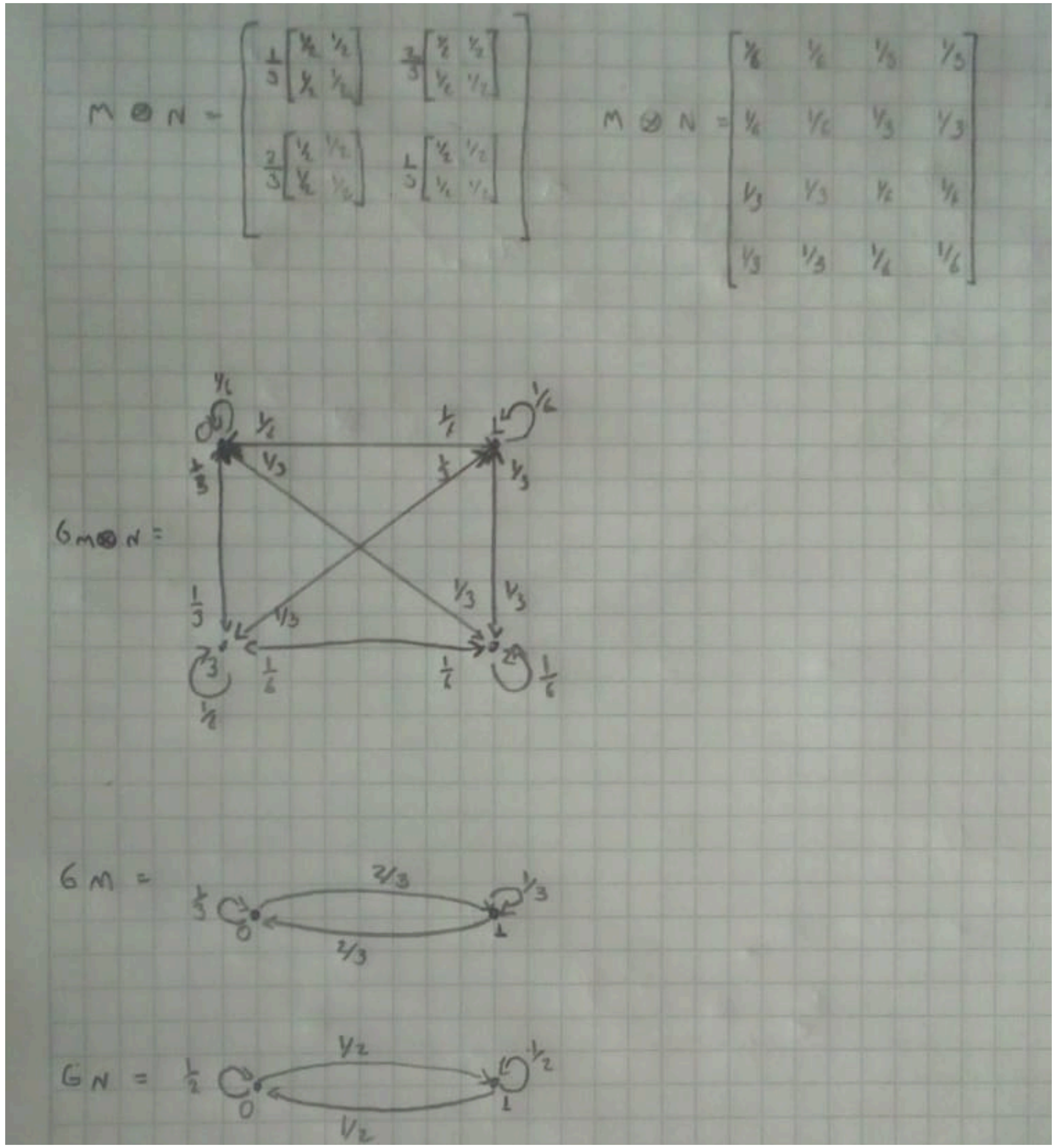
$$\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$$

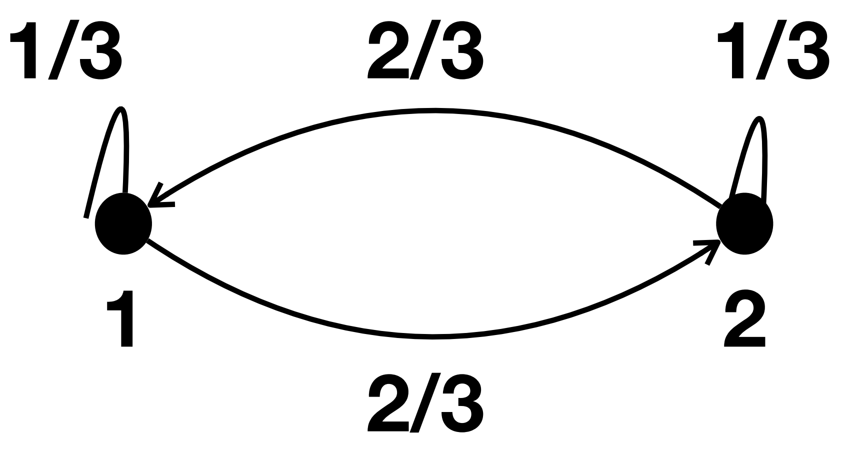
$$* \; N \otimes N \quad \begin{array}{cc} & \begin{array}{cccc}1a & 1b & 2a & 2b\end{array} \\ \begin{array}{c}1a\\1b\\2a\\2b\end{array} & \begin{bmatrix}\frac{1}{9} & \frac{2}{9} & \frac{2}{9} & \frac{4}{9}\\ \frac{2}{9} & \frac{1}{9} & \frac{4}{9} & \frac{2}{9}\\ \frac{2}{9} & \frac{4}{9} & \frac{1}{9} & \frac{2}{9}\\ \frac{4}{9} & \frac{2}{9} & \frac{2}{9} & \frac{1}{9}\end{bmatrix}\end{array}$$

* Grafico

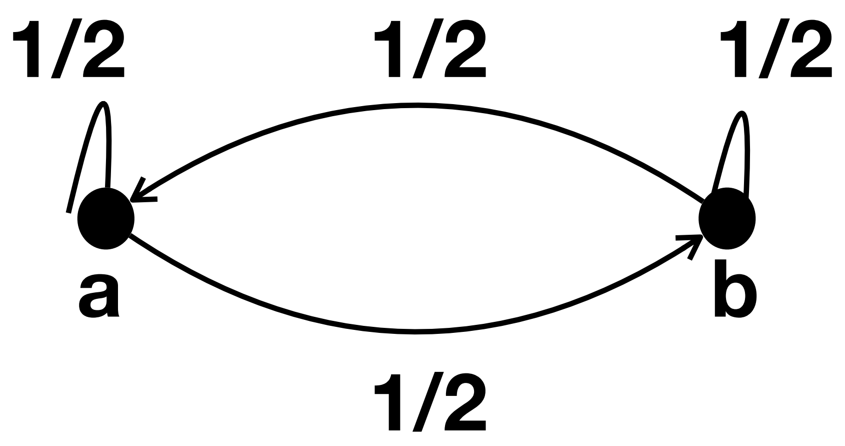


Ensamblar sistemas

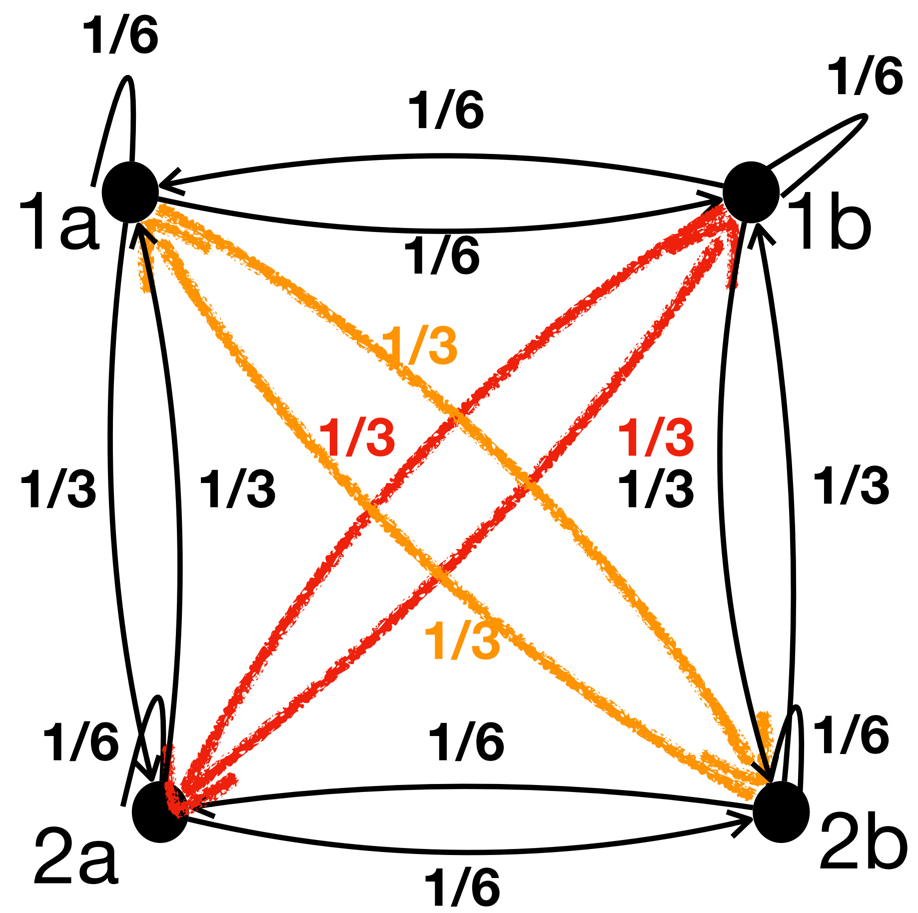




	1	2
1	1/3	2/3
2	2/3	1/3



	a	b
a	1/2	1/2
b	1/2	1/2



	1a	1b	2a	2b
1a	1/6	1/6	1/3	1/3
1b	1/6	1/6	1/3	1/3
2a	1/3	1/3	1/6	1/6
2b	1/3	1/3	1/6	1/6

Fin