Astro 425 Cosmology: Problem Set 3

Due Wednesday 26th October 2016

1. [6 pts] We used three equations when deriving the Friedman equations:

$$\frac{\ddot{a(t)}}{a(t)} = -\frac{4\pi G}{3c^2}\left[\varepsilon(t) + 3P(t)\right]$$

$$\left(\frac{\dot{a(t)}}{a(t)}\right)^2 = \frac{8\pi G\varepsilon(t)}{3c^2} - \frac{kc^2}{a(t)^2}$$

and

$$\dot{\varepsilon(t)} + \frac{3\dot{a(t)}}{a(t)}\left(\varepsilon + P\right) = 0$$

In fact only two of these are independent. Show how we get the 3rd of these equations from the first two.

2. [10 pts] Using your understanding of how to integrate (from the second homework) write a program that integrates the Friedman equation,

$$\left(\frac{H(z)}{H_0}\right)^2 = \left(\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0} + (1-\Omega_0)(1+z)^2\right)$$

Using this program calculate and plot (a) the comoving distance from z=0 to 10 (b) the age of the universe (at a given redshift) from z=0 to z=10. Calculate these results for the two universes given below

(a) a flat universe with $\Omega_{m,0} = 0.3$, $\Omega_{r,0}=0.$, $\Omega_{\Lambda,0}=0.7$
(b) a closed universe with $\Omega_{m,0} = 10.$, $\Omega_{r,0}=0.0$, and $\Omega_\Lambda = 0$

Note: to calculate the age of the universe remember that H(z) is directly related to dz/dt . Use H_0=70 km s^{-1} Mpc^{-1} in your calculations and plot the age in Gyr

3. [4 pts] Use the program to find the value of Ω_r for a radiation only open universe (no cosmological constant) that would give the same age of the universe (for the current day) as our benchmark flat universe with $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0}=0.7$