TDT4171: Methods in AI

Assignment 2

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January 2022

1 Task 1

a) The set of unobserved variables for a given time-slice t is whether it is raining or not, call it R_t. Thus the set $\boldsymbol{X_t}$ contains only R_t.

b) The set of observable variables for a given time-slice t is whether or not the director carries an umbrella, call it U_t. Then the set $\boldsymbol{E_t}$ just contains U_t.

c) The dynamic model is

$$\boldsymbol{P(X_t|X_{t-1})} = \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}$$

assuming a time-slice t is one day, this model says that it will rain at day t with a probability 0.7 given that it rained on day $t-1$. Likewise the probability of not rain on day t is 0.3 given that it rained on day $t-1$.

d) Similarly the observation model is

$$\boldsymbol{P(E_t|X_t)} = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$$

which says that the probability of the director carrying his umbrella is 0.9 given that it rains that day and 0.2 if it does not rain.

e) We are making several assumptions in this model, namely:

- The Markov assumption which says that the current state only depends on a finite fixed number of previous states. In this model it only depends on one previous state and is therefore a first order Markov model.
- The model is a stationary process.
- The sensor Markov assumption. It says that the probability of the director carrying his umbrella is only dependant on that days weather. This is unlikely, since he would probably bring it anyway if there was a morning of no rain in the middle of a period with a lot of rain.

2 Task 2

Using that

$$\mathbf{f}_{1:t+1} = \alpha \mathbf{O}_{t+1} \mathbf{T}^{\mathsf{T}} \mathbf{f}_{1:t}$$

where $O_1 = \begin{pmatrix} 0.9 & 0.0 \\ 0.0 & 0.2 \end{pmatrix}$ on day one, when we have observed the director with an umbrella and $O_3 = \begin{pmatrix} 0.1 & 0.0 \\ 0.0 & 0.8 \end{pmatrix}$ on day three, when he does not carry his umbrella, the forward messages are found and presented in Table 1. Here I initiate $\mathbf{f}_{0:0} = \begin{pmatrix} 0.5 & 0.5 \end{pmatrix}$. Thus the probability of `rain` at day 5 given the sequence of observations is ≈ 0.867.

Table 1: Forward

Day	**Evidence**	**f**
1	True	[0.81818182, 0.18181818]
2	False	[0.88335704, 0.11664296]
3	True	[0.19066794, 0.80933206]
4	True	[0.730794, 0.269206]
5	True	[0.86733889, 0.13266111]

3 Task 3

By implementing equation `14.13`, the backward algorithm in matrix form, namely that

$$\mathbf{b}_{k+1:t} = \mathbf{T}\mathbf{O}_{k+1}\mathbf{b}_{k+2:t}$$

and starting with $\mathbf{b}_{5:5} = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}$, we get the following normalized backward probabilities

$\mathbf{b}_{4:5} = \begin{pmatrix} 0.62727273 \\ 0.37272727 \end{pmatrix}$

$\mathbf{b}_{3:5} = \begin{pmatrix} 0.65334282 \\ 0.34665718 \end{pmatrix}$

$\mathbf{b}_{2:5} = \begin{pmatrix} 0.37626718 \\ 0.62373282 \end{pmatrix}$

$\mathbf{b}_{1:5} = \begin{pmatrix} 0.5923176 \\ 0.4076824 \end{pmatrix}$

$\mathbf{b}_{0:5} = \begin{pmatrix} 0.64693556 \\ 0.35306444 \end{pmatrix}$

By multiplying the forward probabilities with the reversed backward probabilities we get the smoothed values, or the total probability of being in each state at a given time t: $\mathbf{P}(\mathbf{X_t}|\mathbf{e}_{t:5})$. Using this I find that the probability of rain at day 1, given the same evidence, is
$\mathbf{P}(\mathbf{X_1}|\mathbf{e}_{1:5}) = 0.86733889$.