TDT4171: Methods in AI

Assignment 1

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1 Task 1

1.1

a) The probability that more than 2 bananas are eaten is given as $P(X > 2) = P(X = 3) + P(X = 4) + P(X \geq 5) = 0.28 + 0.10 + 0.17 = 0.55$.

b) $P(X \leq 4) = 0.03 + 0.18 + 0.24 + 0.28 + 0.10 = 0.83$.

c) $P(X \geq 4) = 0.10 + 0.17 = 0.27$.

1.2

I first define some probabilities that are needed later on:

- P(Two) = The probability that two apples are randomly selected and inspected, and neither is rotten.
- P(Two|ZeroRotten) = The probability that two randomly picked apples are not rotten, given that there is one rotten apple in the batch. P(Two|ZeroRotten) = 1.
- P(Two|OneRotten) = The probability that two randomly picked apples are not rotten, given that there is one rotten apple in the batch. P(Two|OneRotten) = $\frac{19}{20} \cdot \frac{18}{19} = 0.9$.
- P(Two|TwoRotten) = The probability that two randomly picked apples are not rotten, given that there are two rotten apples in the batch. P(Two|TwoRotten) = $\frac{18}{20} \cdot \frac{17}{19} = 0.805$.

Now P(Two) = P(Two|ZeroRotten) · P(ZeroRotten) + P(Two|OneRotten) · P(OneRotten) + P(Two|TwoRotten) · P(TwoRotten) = $1 \cdot 0.6 + 0.9 \cdot 0.3 + 0.805 \cdot 0.1 = 0.9505$. Using Bayes' Theorem we can now find the probability of the existence of a certain number of rotten apples in the batch we select two good ones from.

a) P(ZeroRotten|Two) = $\frac{P(Two|ZeroRotten) \cdot P(ZeroRotten)}{P(Two)} = \frac{1 \cdot 0.6}{0.9505} = 0.6312$.

b) P(OneRotten|Two) = $\frac{P(Two|OneRotten) \cdot P(OneRotten)}{P(Two)} = \frac{0.9 \cdot 0.3}{0.9505} = 0.2841$.

c) P(TwoRotten|Two) = $\frac{P(Two|TwoRotten) \cdot P(TwoRotten)}{P(Two)} = \frac{0.805 \cdot 0.1}{0.9505} = 0.0847$.

1.3

Want to find P(Sick|Negative). In order to use Bayes' Rule we need P(Negative|Sick) = 0.1, P(Sick) = 0.07 and P(Negative). Since $P(\overline{Negative}|\overline{Sick}) = 0.05$, $P(Negative) = 0.1 \cdot 0.07 + 0.95 \cdot 0.93 = 0.8905$. Thus $P(Sick|Negative) = \frac{P(Negative|Sick) \cdot P(Sick)}{P(Negative)} = \frac{0.1 \cdot 0.07}{0.8905} = 0.00786075239 \approx 0.00786$.

1.4

In total there are

$$\binom{12}{5} = 792$$

ways to purchase 5 sets. From this we need to subtract the number of selections where

1. there are no defective sets
2. there is only one defective set.

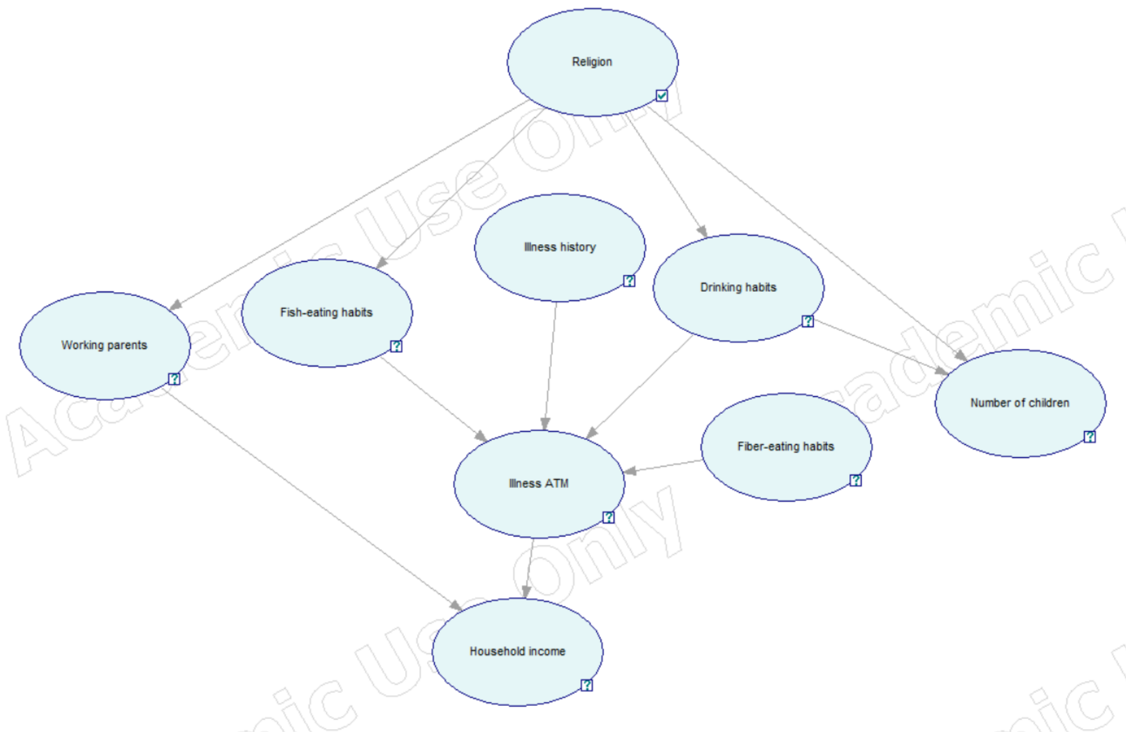

Figure 1: Bayesian network

Thus there are

$$\binom{12}{5} - \binom{9}{5} - \binom{9}{4} \cdot \binom{3}{1} = 792 - 126 - 126 \cdot 3 = 288$$

ways to purchase 5 sets and receive at least 2 defective ones.

2 Task 2

To create the Bayesian network in Figure 1 I have used the following assumptions:

1. Religion:
   - Some religions forbid drinking.
   - Fertility rates are lower in secular countries [1]
   - According to certain religions, women should be homemakers rather than working.
   - Some religions, such as Buddhism, forbid eating fish.
2. Fish-eating habits:
   - Fish is healthy and thus influences someones current illness.
3. Drinking habits:
   - Alcohol is toxic and therefore has a negative influence on ones current illness.
4. Fiber-eating habits:
   - Fiber is healthy and has a positive effect on ones immediate illness.

[1] https://news.cornell.edu/stories/2021/07/religious-have-fewer-children-secular-countries

5. Illness history:
    - Having a chronic disease will affect ones current illness.
6. Working parents:
    - Having both parents working will certainly increase a households income, as compared to having only one.
7. Illness at the moment:
    - Being sick might affect ones ability to perform their job and by extension the income of the household.

Taking a look at some of the conditional independencies, Working parents and Fish-eating habits are conditionally independent given religion, which seems reasonable. Moreover, Number of children and Current Illness are conditionally independent given Drinking habits which also sounds reasonable. The household income is conditionally independent of Fish-eating habits and Fiber-eating habits given the current illness, also reasonable.

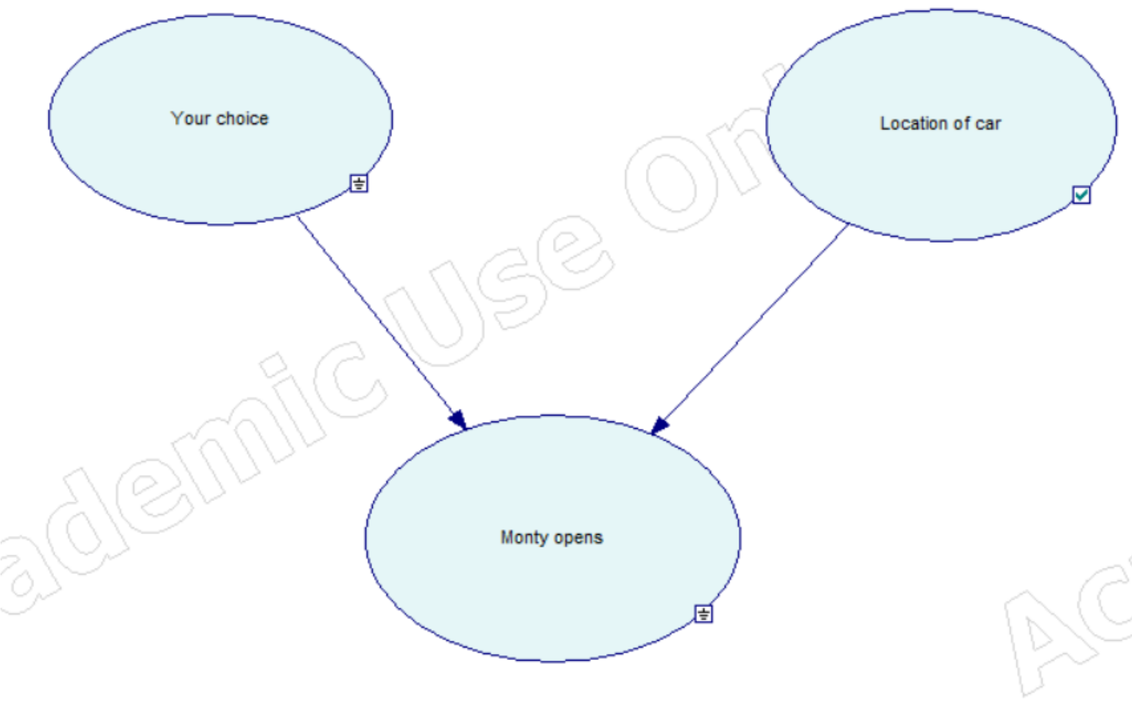

Figure 2: Bayesian network for Monty Hall

3 Task 3

I implemented the Monty Hall Problem as a Bayesian network using GeNIe where it can be described with three nodes, namely `Your choice`, `Location of car` and `Monty opens`. The network is shown in Figure 2. Each of the nodes have three states, `Door 1`, `Door 2` and `Door 3`. Figure 4 shows the Conditional Probability Table for the `Monty opens`, where we can read from the first column that if you choose `Door 1` and the car is in `Door 1`, Monty will open either `Door 2` or `Door 3` with equal probability.
Using this, we can `set evidence`. Let `Door 1` be our choice. Then according to the rules of the game, Monty can open either `Door 2` or `Door 3` depending on where the car is. Let us say he opens `Door 2`. Then by using the `Update`-tool in GeNIe, the result in Figure 3 is obtained. This shows that the probability of the car being in `Door 1`, our original guess is still 0.33, while the probability of it being in `Door 3` now has increased to .67, given that Monty opens `Door 2`. Had he opened `Door 3`, the probabilities of `Door 2` and `Door 3` would have been swapped. There is a symmetry about this, which is that by always swapping to the remaining door when prompted, you will win the game 67% of the time, only losing when your initial guess is correct.

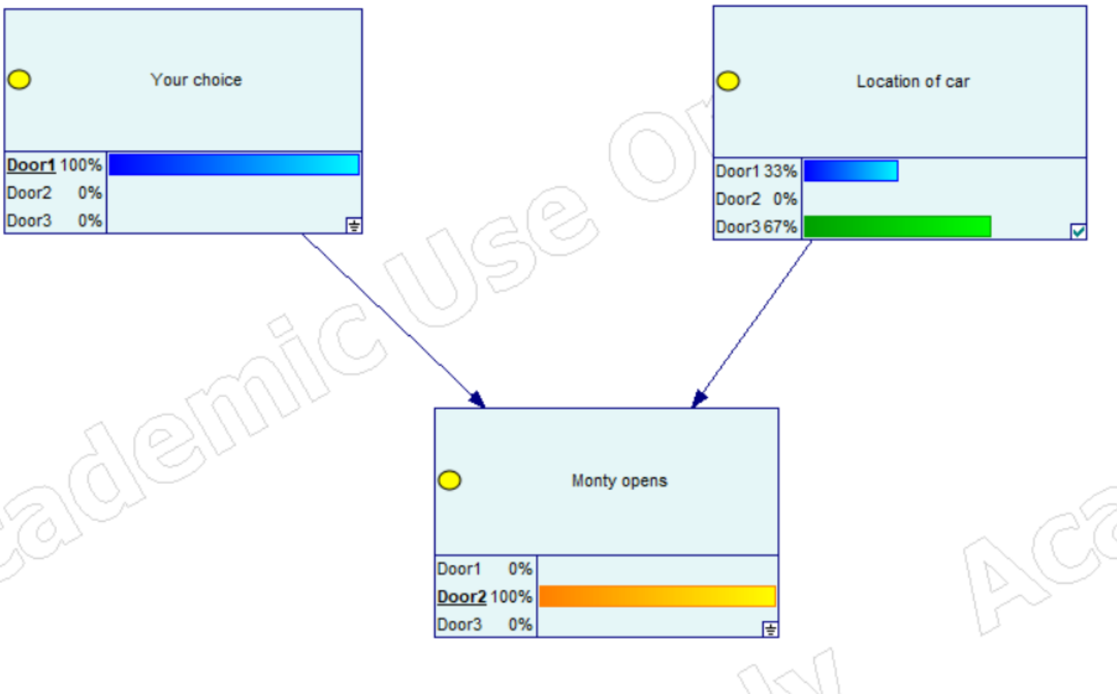

Figure 3: The Monty Hall problem

Your choice	Door1			Door2			Door3		
Location of car	Door1	Door2	Door3	Door1	Door2	Door3	Door1	Door2	Door3
Door1	0	0	0	0	0.5	1	0	1	0.5
Door2	0.5	0	1	0	0	0	1	0	0.5
Door3	0.5	1	0	1	0.5	0	0	0	0

Figure 4: Conditional probability table