8.4 日期公式

```
// Mon = 0, ... % 7
// days since 1/1/1
int getday(int y, int m, int d) {
  if(m < 3) -- y, m += 12;
  return (365 * y + y / 4 - y / 100 + y / 400 + (153 * (m - 3) + 2) / 5 + d
    ↪ - 307);
}
void date(int n, int & y, int & m, int & d) {
  n += 429 + ((4 * n + 1227) / 146097 + 1) * 3 / 4;
  y = (4 * n - 489) / 1461;
  n -= y * 1461 / 4;
  m = (5 * n - 1) / 153;
  d = n - m * 153 / 5;
  if (--m > 12) m -= 12, ++y;
}
```

8.5 Xorshift

```
u64 xorshift(u64 x) { x ^= x << 13; x ^= x >> 7; x ^= x << 17; return x; }
u32 xorshift(u32 x) { x ^= x << 13; x ^= x >> 17; x ^= x << 5; return x; }
```

9 配置

9.1 vimrc

```
set si ci ts=4 sw=4 nu cino=j1 backup undofile
syntax on
map<F9> <ESC>:!make %<<CR>
map<F10> <ESC>:!./%<<CR>
map<F4> <ESC>:!gdb %<<CR>
```

9.2 bashrc

```
export CXXFLAGS='-g -Wall -fsanitize=address,undefined -Dzqj -std=gnu++20'
mk() { g++ -O2 -Dzqj -std=gnu++20 $1.cpp -o $1; }
ulimit -s 1048576
ulimit -v 1048576
```

9.3 对拍

需要 chmod +x

```
while true; do
  ./gen > 1.in
  ./naive < 1.in > std.out
  ./a < 1.in > 1.out
  if diff 1.out std.out; then
    echo ac
  else
    echo wa
    break
  fi
done
```

9.4 编译参数

```
-D_GLIBCXX_DEBUG : STL debug mode
-fsanitize=address : 内存错误检查
-fsanitize=undefined : UB 检查
```

9.5 随机素数

```
979345007 986854057502126921
935359631 949054338673679153
931936021 989518940305146613
984974633 972090414870546877
984858209 956380060632801307
```

9.6 常数表

n	$\log_{10} n$	$n!$	$C(n, n/2)$	$\mathrm{LCM}(1 \dots n)$	P_n
2	0.30102999	2	2	2	2
3	0.47712125	6	3	6	3
4	0.60205999	24	6	12	5
5	0.69897000	120	10	60	7
6	0.77815125	720	20	60	11
7	0.84509804	5040	35	420	15
8	0.90308998	40320	70	840	22
9	0.95424251	362880	126	2520	30
10	1	3628800	252	2520	42
11	1.04139269	39916800	462	27720	56
12	1.07918125	479001600	924	27720	77
15	1.17609126	1.31e12	6435	360360	176
20	1.30103000	2.43e18	184756	232792560	627
25	1.39794001	1.55e25	5200300	26771144400	1958
30	1.47712125	2.65e32	155117520	1.444e14	5604
P_n	37338_{40}	204226_{50}	966467_{60}	190569292_{100}	$1e9_{114}$

$n \le$	10	100	1e3	1e4	1e5	1e6
$\max \omega(n)$	2	3	4	5	6	7
$\max d(n)$	4	12	32	64	128	240
$\pi(n)$	4	25	168	1229	9592	78498
$n \le$	1e7	1e8	1e9	1e10	1e11	1e12
$\max \omega(n)$	8	8	9	10	10	11
$\max d(n)$	448	768	1344	2304	4032	6720
$\pi(n)$	664579	5761455	5.08e7	4.55e8	4.12e9	3.7e10
$n \le$	1e13	1e14	1e15	1e16	1e17	1e18
$\max \omega(n)$	12	12	13	13	14	15
$\max d(n)$	10752	17280	26880	41472	64512	103680
$\pi(n)$	Prime number theorem: $\pi(x) \sim x/\log(x)$					

10 注意事项

10.1 测试项目

pbds tree, float128, int128, long double submit 命令, printfile, MLE ?= RE, pragma, axv2, python,

10.2 bugs

看数据范围（多测总和）, 变量 shadow, 清空, long long, 数组大小, 模数, MLE?, 对拍记得看输出在不在变, 输出格式，inf 开小，答案初值，STL 重构导致引用失效，极端情况 (n=1)

11 tables

11.1 导数积分

$$(\frac{u}{v})' = \frac{u'v-uv'}{v^2}$$
$$(a^x)' = (\ln a)a^x$$
$$(\tan x)' = \sec^2 x$$
$$(\cot x)' = \csc^2 x$$
$$(\sec x)' = \tan x \sec x$$
$$(\csc x)' = -\cot x \csc x$$
$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$
$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$
$$(\arctan x)' = \frac{1}{1+x^2}$$
$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$
$$(\operatorname{arccsc} x)' = -\frac{1}{x\sqrt{1-x^2}}$$
$$(\operatorname{arcsec} x)' = \frac{1}{x\sqrt{1-x^2}}$$
$$(\tanh x)' = \operatorname{sech}^2 x$$
$$(\coth x)' = -\operatorname{csch}^2 x$$
$$(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$$
$$(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$$
$$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{1+x^2}}$$
$$(\operatorname{arccosh} x)' = \frac{1}{\sqrt{x^2-1}}$$
$$(\operatorname{arctanh} x)' = \frac{1}{1-x^2}$$
$$(\operatorname{arccoth} x)' = \frac{1}{x^2-1}$$
$$(\operatorname{arccsch} x)' = -\frac{1}{|x|\sqrt{1+x^2}}$$
$$(\operatorname{arcsech} x)' = -\frac{1}{x\sqrt{1-x^2}}$$

$ax^2+bx+c (a>0)$

1. $$\int \frac{\mathrm{d}x}{ax^2+bx+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}} + C & (b^2 < 4ac) \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| + C & (b^2 > 4ac) \end{cases}$$
2. $$\int \frac{x}{ax^2+bx+c}\mathrm{d}x = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{2a}\int \frac{\mathrm{d}x}{ax^2+bx+c}$$

$\sqrt{ax^2+bx+c} (a>0)$

1. $$\int \frac{\mathrm{d}x}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}} \ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}| + C$$
2. $$\int \sqrt{ax^2+bx+c}\mathrm{d}x = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8\sqrt{a^3}} \ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}| + C$$
3. $$\int \frac{x}{\sqrt{ax^2+bx+c}}\mathrm{d}x = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2\sqrt{a^3}} \ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}| + C$$
4. $$\int \frac{\mathrm{d}x}{\sqrt{c+bx-ax^2}} = -\frac{1}{\sqrt{a}} \arcsin \frac{2ax-b}{\sqrt{b^2+4ac}} + C$$
5. $$\int \sqrt{c+bx-ax^2}\mathrm{d}x = \frac{2ax-b}{4a}\sqrt{c+bx-ax^2} + \frac{b^2+4ac}{8\sqrt{a^3}} \arcsin \frac{2ax-b}{\sqrt{b^2+4ac}} + C$$
6. $$\int \frac{x}{\sqrt{c+bx-ax^2}}\mathrm{d}x = -\frac{1}{a}\sqrt{c+bx-ax^2} + \frac{b}{2\sqrt{a^3}} \arcsin \frac{2ax-b}{\sqrt{b^2+4ac}} + C$$

$\sqrt{\pm\frac{x-a}{x-b}}$ 或 $\sqrt{(x-a)(x-b)}$

1. $$\int \frac{\mathrm{d}x}{\sqrt{(x-a)(b-x)}} = 2\arcsin\sqrt{\frac{x-a}{b-x}} + C\,(a<b)$$
2. $$\int \sqrt{(x-a)(b-x)}\mathrm{d}x = \frac{2x-a-b}{4}\sqrt{(x-a)(b-x)} + \frac{(b-a)^2}{4}\arcsin\sqrt{\frac{x-a}{b-x}} + C,\,(a<b)$$

三角函数的积分

1. $$\int \tan x\mathrm{d}x = -\ln|\cos x| + C$$
2. $$\int \cot x\mathrm{d}x = \ln|\sin x| + C$$
3. $$\int \sec x\mathrm{d}x = \ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C = \ln|\sec x+\tan x| + C$$
4. $$\int \csc x\mathrm{d}x = \ln\left|\tan\frac{x}{2}\right| + C = \ln|\csc x-\cot x| + C$$
5. $$\int \sec^2 x\mathrm{d}x = \tan x + C$$
6. $$\int \csc^2 x\mathrm{d}x = -\cot x + C$$
7. $$\int \sec x\tan x\mathrm{d}x = \sec x + C$$
8. $$\int \csc x\cot x\mathrm{d}x = -\csc x + C$$
9. $$\int \sin^2 x\mathrm{d}x = \frac{x}{2} - \frac{1}{4}\sin 2x + C$$
10. $$\int \cos^2 x\mathrm{d}x = \frac{x}{2} + \frac{1}{4}\sin 2x + C$$
11. $$\int \sin^n x\mathrm{d}x = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\mathrm{d}x$$
12. $$\int \cos^n x\mathrm{d}x = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\mathrm{d}x$$
13. $$\int \frac{\mathrm{d}x}{\sin^n x} = -\frac{1}{n-1}\frac{\cos x}{\sin^{n-1}x} + \frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\sin^{n-2}x}$$
14. $$\int \frac{\mathrm{d}x}{\cos^n x} = \frac{1}{n-1}\frac{\sin x}{\cos^{n-1}x} + \frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\cos^{n-2}x}$$
15. $$\begin{aligned} &\int \cos^m x\sin^n x\mathrm{d}x \\ =&\frac{1}{m+n}\cos^{m-1}x\sin^{n+1}x + \frac{m-1}{m+n}\int \cos^{m-2}x\sin^n x\mathrm{d}x \\ =&-\frac{1}{m+n}\cos^{m+1}x\sin^{n-1}x + \frac{n-1}{m+1}\int \cos^m x\sin^{n-2}x\mathrm{d}x \end{aligned}$$
16. $$\int \frac{\mathrm{d}x}{a+b\sin x} = \begin{cases} \frac{2}{\sqrt{a^2-b^2}}\arctan\frac{a\tan\frac{x}{2}+b}{\sqrt{a^2-b^2}} + C & (a^2>b^2) \\ \frac{1}{\sqrt{b^2-a^2}}\ln\left|\frac{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan\frac{x}{2}+b+\sqrt{b^2-a^2}}\right| + C & (a^2<b^2) \end{cases}$$
17. $$\int \frac{\mathrm{d}x}{a+b\cos x} = \begin{cases} \frac{2}{a+b}\sqrt{\frac{a+b}{a-b}}\arctan\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) + C & (a^2>b^2) \\ \frac{1}{a+b}\sqrt{\frac{a+b}{a-b}}\ln\left|\frac{\tan\frac{x}{2}+\sqrt{\frac{a+b}{b-a}}}{\tan\frac{x}{2}-\sqrt{\frac{a+b}{b-a}}}\right| + C & (a^2<b^2) \end{cases}$$
18. $$\int \frac{\mathrm{d}x}{a^2\cos^2 x+b^2\sin^2 x} = \frac{1}{ab}\arctan\left(\frac{b}{a}\tan x\right) + C$$
19. $$\int \frac{\mathrm{d}x}{a^2\cos^2 x-b^2\sin^2 x} = \frac{1}{2ab}\ln\left|\frac{b\tan x+a}{b\tan x-a}\right| + C$$
20. $$\int x\sin ax\mathrm{d}x = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax + C$$
21. $$\int x^2\sin ax\mathrm{d}x = -\frac{1}{a}x^2\cos ax + \frac{2}{a^2}x\sin ax + \frac{2}{a^3}\cos ax + C$$
22. $$\int x\cos ax\mathrm{d}x = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax + C$$
23. $$\int x^2\cos ax\mathrm{d}x = \frac{1}{a}x^2\sin ax + \frac{2}{a^2}x\cos ax - \frac{2}{a^3}\sin ax + C$$

反三角函数的积分 (其中 $a>0$)

1. $$\int \arcsin\frac{x}{a}\mathrm{d}x = x\arcsin\frac{x}{a} + \sqrt{a^2-x^2} + C$$
2. $$\int x\arcsin\frac{x}{a}\mathrm{d}x = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\arcsin\frac{x}{a} + \frac{x}{4}\sqrt{x^2-x^2} + C$$
3. $$\int x^2\arcsin\frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arcsin\frac{x}{a} + \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2} + C$$
4. $$\int \arccos\frac{x}{a}\mathrm{d}x = x\,arccos\frac{x}{a} - \sqrt{a^2-x^2} + C$$
5. $$\int x\arccos\frac{x}{a}\mathrm{d}x = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\arccos\frac{x}{a} - \frac{x}{4}\sqrt{a^2-x^2} + C$$
6. $$\int x^2\arccos\frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arccos\frac{x}{a} - \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2} + C$$
7. $$\int \arctan\frac{x}{a}\mathrm{d}x = x\arctan\frac{x}{a} - \frac{a}{2}\ln(a^2+x^2) + C$$
8. $$\int x\arctan\frac{x}{a}\mathrm{d}x = \frac{1}{2}(a^2+x^2)\arctan\frac{x}{a} - \frac{a}{2}x + C$$
9. $$\int x^2\arctan\frac{x}{a}\mathrm{d}x = \frac{x^3}{3}\arctan\frac{x}{a} - \frac{a}{6}x^2 + \frac{a^3}{6}\ln(a^2+x^2) + C$$

指数函数的积分

1. $$\int a^x\mathrm{d}x = \frac{1}{\ln a}a^x + C$$
2. $$\int \mathrm{e}^{ax}\mathrm{d}x = \frac{1}{a}a^{ax} + C$$
3. $$\int x\mathrm{e}^{ax}\mathrm{d}x = \frac{1}{a^2}(ax-1)a^{ax} + C$$
4. $$\int x^n\mathrm{e}^{ax}\mathrm{d}x = \frac{1}{a}x^n\mathrm{e}^{ax} - \frac{n}{a}\int x^{n-1}\mathrm{e}^{ax}\mathrm{d}x$$
5. $$\int xa^x\mathrm{d}x = \frac{x}{\ln a}a^x - \frac{1}{(\ln a)^2}a^x + C$$
6. $$\int x^n a^x\mathrm{d}x = \frac{1}{\ln a}x^n a^x - \frac{n}{\ln a}\int x^{n-1}a^x\mathrm{d}x$$
7. $$\int \mathrm{e}^{ax}\sin bx\mathrm{d}x = \frac{1}{a^2+b^2}\mathrm{e}^{ax}(a\sin bx - b\cos bx) + C$$
8. $$\int \mathrm{e}^{ax}\cos bx\mathrm{d}x = \frac{1}{a^2+b^2}\mathrm{e}^{ax}(b\sin bx + a\cos bx) + C$$
9. $$\int \mathrm{e}^{ax}\sin^n bx\mathrm{d}x = \frac{1}{a^2+b^2n^2}\mathrm{e}^{ax}\sin^{n-1}bx(a\sin bx - nb\cos bx) + \frac{n(n-1)b^2}{a^2+b^2n^2}\int \mathrm{e}^{ax}\sin^{n-2}bx\mathrm{d}x$$
10. $$\int \mathrm{e}^{ax}\cos^n bx\mathrm{d}x = \frac{1}{a^2+b^2n^2}\mathrm{e}^{ax}\cos^{n-1}bx(a\cos bx + nb\sin bx) + \frac{n(n-1)b^2}{a^2+b^2n^2}\int \mathrm{e}^{ax}\cos^{n-2}bx\mathrm{d}x$$

对数函数的积分

1. $$\int \ln x\mathrm{d}x = x\ln x - x + C$$
2. $$\int \frac{\mathrm{d}x}{x\ln x} = \ln|\ln x| + C$$
3. $$\int x^n\ln x\mathrm{d}x = \frac{1}{n+1}x^{n+1}(\ln x - \frac{1}{n+1}) + C$$
4. $$\int (\ln x)^n\mathrm{d}x = x(\ln x)^n - n\int (\ln x)^{n-1}\mathrm{d}x$$
5. $$\int x^m(\ln x)^n\mathrm{d}x = \frac{1}{m+1}x^{m+1}(\ln x)^n - \frac{n}{m+1}\int x^m(\ln x)^{n-1}\mathrm{d}x$$

STL 积分/求和 (need std::)

1. $\int_0^1 t^{x-1}(1-t)^{y-1}dt = beta(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$
2. $\int_0^{+\infty} t^{num-1}e^{-t}dt = tgamma(num) = e^{lgamma(num)} = \Gamma(num)$
3. $\int_0^{phi} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} = ellint\_1(k,phi)$
4. $\int_0^{phi} \sqrt{1-k^2\sin^2\theta}d\theta = ellint\_2(k,phi)$
5. $\int_{num}^{+\infty} \frac{e^{-t}}{t}dt = -expint(-num)$
6. $\sum_{n=1}^{+\infty} n^{-num} = riemann\_zeta(num)$
7. $\frac{2}{\sqrt{\pi}}\int_0^{arg} e^{-t^2}dt = erf(arg)$