```
| | | }
| | | else if (dfn[y]<dfn[x]&&y!=fa) chkmin(low[x],dfn[y]);
| | }
| }
| int check(int s,int t,int n) { // return topo
| | if (n==1) return topo[1]=1,1;
| | if (s==t) return 0;
| | cc=flg=0; dfs(s,s,s,t);
| | if (flg) return 0;
| | sgn[s]=0;
| | static int pre[sz],suf[sz];
| | suf[0]=s,pre[s]=0,suf[s]=t;
| | pre[t]=s,suf[t]=n+1,pre[n+1]=t;
| | rep(i,3,n) {
| | | int v=inv[i];
| | | if (!sgn[inv[low[v]]]) {
| | | | int P=pre[p[v]];
| | | | pre[v]=P,suf[v]=p[v];
| | | | suf[P]=pre[p[v]]=v;
| | | }
| | | else {
| | | | int S=suf[p[v]];
| | | | pre[v]=p[v],suf[v]=S;
| | | | suf[p[v]]=pre[S]=v;
| | | }
| | | sgn[p[v]]=!sgn[inv[low[v]]];
| | }
| | for (int x=s,cnt=0;x!=n+1;x=suf[x]) topo[++cnt]=x;
| | return 1;
| }
| void clr(int n) {
| | rep(i,1,n) dfn[i]=low[i]=p[i]=inv[i]=topo[i]=sgn[i]=0,G[i].clear();
| }
}
```

1.17 Tree And Graph

1.17.1 树的计数 Prufer序列

树和其prufer编码一一对应, 一颗 n 个点的树, 其prufer编码长度为 $n-2$, 且度数为 d_i 的点在prufer 编码中出现 $d_i - 1$ 次.

由树得到序列: 总共需要 $n-2$ 步, 第 i 步在当前的树中寻找具有最小标号的叶子节点, 将与其相连的点的标号设为Prufer序列的第 i 个元素 p_i, 并将此叶子节点从树中删除, 直到最后得到一个长度为 $n-2$ 的Prufer 序列和一个只有两个节点的树.

由序列得到树: 先将所有点的度赋初值为 1, 然后加上它的编号在Prufer序列中出现的次数, 得到每个点的度; 执行 $n-2$ 步, 第 i 步选取具有最小标号的度为 1 的点 u 与 $v = p_i$ 相连, 得到树中的一条边, 并将 u 和 v 的度减一. 最后再把剩下的两个度为 1 的点连边, 加入到树中.

相关结论: n 个点完全图, 每个点度数依次为 d_1,d_2,...,d_n, 这样生成树的棵树为: $\frac{(n-2)!}{(d_1-1)!(d_2-1)!...(d_n-1)!}$.

左边有 n_1 个点, 右边有 n_2 个点的完全二分图的生成树棵树为 $n_1^{n_2-1} \times n_2^{n_1-1}$.

m 个连通块, 每个连通块有 c_i 个点, 把他们全部连通的生成树方案数: $(\sum c_i)^{m-2} \prod c_i$

1.17.2 有根树的计数

首先, 令 $S_{n,j} = \sum_{1\le j\le n/j}$; 于是 $n+1$ 个结点的有根树的总数为 $a_{n+1} = \frac{\sum_{j=1}^{n} j a_j S_{n-j}}{n}$. 注: $a_1 = 1, a_2 = 1, a_3 = 2, a_4 = 4, a_5 = 9, a_6 = 20, a_9 = 286, a_{11} = 1842$.

1.17.3 无根树的计数

n 是奇数时, 有 $a_n - \sum_i^{n/2} a_i a_{n-i}$ 种不同的无根树.

n 时偶数时, 有 $a_n - \sum_i^{n/2} a_i a_{n-i} + \frac{1}{2} a_{n/2}(a_{n/2}+1)$ 种不同的无根树.

1.17.4 生成树计数 Kirchhoff's Matrix-Tree Thoerem

Kirchhoff Matrix $T = Deg - A$, Deg 是度数对角阵, A 是邻接矩阵. 无向图度数矩阵是每个点度数; 有向图度数矩阵是每个点入度.

邻接矩阵 $A[u][v]$ 表示 $u \to v$ 边个数, 重边按照边数计算, 自环不计入度数.

无向图生成树计数: $c = |K$ 的任意1个 $n1$ 阶主子式 $|$

有向图外向树计数: $c = |$ 去掉根所在的那阶得到的主子式 $|$

1.17.5 有向图欧拉回路计数 BEST Thoerem

$$\mathrm{ec}(G) = t_w(G) \prod_{v\in V} (\deg(v) - 1)!$$

其中 deg 为入度 (欧拉图中等于出度), $t_w(G)$ 为以 w 为根的外向树的个数. 相关计算参考生成树计数.

欧拉连通图中任意两点外向树个数相同: $\mathrm{t_v}(G) = \mathrm{t_w}(G)$.

以 1 结尾的欧拉路径计数就是把 deg 视为出度，把 $\deg(1)$ 的贡献改为 $\deg(1)!$.

1.17.6 Tutte Matrix

Tutte matrix A of a graph $G = (V, E)$:

$$A_{ij} = \begin{cases} x_{ij} & \text{if } (i,j) \in E \text{ and } i < j \\ -x_{ij} & \text{if } (i,j) \in E \text{ and } i > j \\ 0 & \text{otherwise} \end{cases}$$

where x_{ij} are indeterminates. The determinant of this skew-symmetric matrix is then a polynomial (in the variables x_{ij}, $i < j$): this coincides with the square of the pfaffian of the matrix A and is non-zero (as a polynomial) if and only if a perfect matching exists.

1.17.7 Edmonds Matrix

Edmonds matrix A of a balanced ($|U| = |V|$) bipartite graph $G = (U, V, E)$:

$$A_{ij} = \begin{cases} x_{ij} & (u_i, v_j) \in E \\ 0 & (u_i, v_j) \notin E \end{cases}$$

where the x_{ij} are indeterminates. G 有完美匹配当且仅当关于 x_{ij} 的多项式 $det(A_{ij})$ 不恒为 0. 完美匹配的个数等于多项式中单项式的个数.

1.18 拟阵交

```
// max size, minimum weight
namespace MatroidIntersection {
    int K;
    ll W[sz]; // weight
    int in[sz]; // ans
    namespace Check { // implementation needed
```

8.4 日期公式

```cpp
// Mon = 0, ... % 7
// days since 1/1/1
int getday(int y, int m, int d) {
    if(m < 3) -- y, m += 12;
    return (365 * y + y / 4 - y / 100 + y / 400 + (153 * (m - 3) + 2) / 5 + d
        - 307);
}
void date(int n, int & y, int & m, int & d) {
    n += 429 + ((4 * n + 1227) / 146097 + 1) * 3 / 4;
    y = (4 * n - 489) / 1461;
    n -= y * 1461 / 4;
    m = (5 * n - 1) / 153;
    d = n - m * 153 / 5;
    if (--m > 12) m -= 12, ++y;
}
```

8.5 Xorshift

```cpp
u64 xorshift(u64 x) { x ^= x << 13; x ^= x >> 7; x ^= x << 17; return x; }
u32 xorshift(u32 x) { x ^= x << 13; x ^= x >> 17; x ^= x << 5; return x; }
```

9 配置

9.1 vimrc

```
set si ci ts=4 sw=4 nu cino=j1 backup undofile
syntax on
map<F9> <ESC>:!make %<<CR>
map<F10> <ESC>:!./%<<CR>
map<F4> <ESC>:!gdb %<<CR>
```

9.2 bashrc

```bash
export CXXFLAGS='-g -Wall -fsanitize=address,undefined -Dzqj -std=gnu++20'
mk() { g++ -O2 -Dzqj -std=gnu++20 $1.cpp -o $1; }
ulimit -s 1048576
ulimit -v 1048576
```

9.3 对拍

需要 chmod +x

```bash
while true; do
    ./gen > 1.in
    ./naive < 1.in > std.out
    ./a < 1.in > 1.out
    if diff 1.out std.out; then
        echo ac
    else
        echo wa
        break
    fi
done
```

9.4 编译参数

-D_GLIBCXX_DEBUG : STL debug mode

-fsanitize=address : 内存错误检查

-fsanitize=undefined : UB 检查

9.5 随机素数

979345007 986854057502126921

935359631 949054338673679153

931936021 989518940305146613

984974633 972090414870546877

984858209 956380060632801307

9.6 常数表

n	$\log_{10} n$	$n!$	$C(n, n/2)$	$\mathrm{LCM}(1 \ldots n)$	P_n
2	0.30102999	2	2	2	2
3	0.47712125	6	3	6	3
4	0.60205999	24	6	12	5
5	0.69897000	120	10	60	7
6	0.77815125	720	20	60	11
7	0.84509804	5040	35	420	15
8	0.90308998	40320	70	840	22
9	0.95424251	362880	126	2520	30
10	1	3628800	252	2520	42
11	1.04139269	39916800	462	27720	56
12	1.07918125	479001600	924	27720	77
15	1.17609126	1.31e12	6435	360360	176
20	1.30103000	2.43e18	184756	232792560	627
25	1.39794001	1.55e25	5200300	26771144400	1958
30	1.47712125	2.65e32	155117520	1.444e14	5604
P_n	37338_{40}	204226_{50}	966467_{60}	190569292_{100}	$1e9_{114}$

$n \le$	10	100	1e3	1e4	1e5	1e6
$\max \omega(n)$	2	3	4	5	6	7
$\max d(n)$	4	12	32	64	128	240
$\pi(n)$	4	25	168	1229	9592	78498
$n \le$	1e7	1e8	1e9	1e10	1e11	1e12
$\max \omega(n)$	8	8	9	10	10	11
$\max d(n)$	448	768	1344	2304	4032	6720
$\pi(n)$	664579	5761455	5.08e7	4.55e8	4.12e9	3.7e10
$n \le$	1e13	1e14	1e15	1e16	1e17	1e18
$\max \omega(n)$	12	12	13	13	14	15
$\max d(n)$	10752	17280	26880	41472	64512	103680
$\pi(n)$	Prime number theorem: $\pi(x) \sim x/\log(x)$					

10 注意事项

10.1 测试项目

pbds tree, float128, int128, long double submit 命令, printfile, MLE ?= RE, pragma, axv2, python,

10.2 bugs

看数据范围（多测总和），变量 shadow, 清空, long long, 数组大小, 模数, MLE?, 对拍记得看输出在不在变, 输出格式，inf 开小，答案初值，STL 重构导致引用失效，极端情况 (n=1)

11 tables

11.1 导数积分

$(\frac{u}{v})' = \frac{u'v-uv'}{v^2}$
$(a^x)' = (\ln a)a^x$
$(\tan x)' = \sec^2 x$
$(\cot x)' = \csc^2 x$
$(\sec x)' = \tan x \ \sec x$
$(\csc x)' = -\cot x \ \csc x$
$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

$(\arctan x)' = \frac{1}{1+x^2}$
$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$
$(\operatorname{arccsc} x)' = -\frac{1}{x\sqrt{1-x^2}}$
$(\operatorname{arcsec} x)' = \frac{1}{x\sqrt{1-x^2}}$
$(\tanh x)' = \operatorname{sech}^2 x$
$(\coth x)' = -\operatorname{csch}^2 x$
$(\operatorname{sech} x)' = -\operatorname{sech} x \ \tanh x$
$(\operatorname{csch} x)' = -\operatorname{csch} x \ \coth x$

$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{1+x^2}}$
$(\operatorname{arccosh} x)' = \frac{1}{\sqrt{x^2-1}}$
$(\operatorname{arctanh} x)' = \frac{1}{1-x^2}$
$(\operatorname{arccoth} x)' = \frac{1}{x^2-1}$
$(\operatorname{arccsch} x)' = -\frac{1}{|x|\sqrt{1+x^2}}$
$(\operatorname{arcsech} x)' = -\frac{1}{x\sqrt{1-x^2}}$

$ax^2+bx+c(a>0)$

1. $\int \frac{dx}{ax^2+bx+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}} + C & (b^2 < 4ac) \\ \frac{1}{\sqrt{b^2-4ac}} \ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| + C & (b^2 > 4ac) \end{cases}$
2. $\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c}$

$\sqrt{\pm ax^2+bx+c}(a>0)$

1. $\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}|+C$
2. $\int \sqrt{ax^2+bx+c}dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8\sqrt{a^3}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}|+C$
3. $\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2\sqrt{a^3}}\ln|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}|+C$
4. $\int \frac{dx}{\sqrt{c+bx-ax^2}} = -\frac{1}{\sqrt{a}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C$
5. $\int \sqrt{c+bx-ax^2}dx = \frac{2ax-b}{4a}\sqrt{c+bx-ax^2} + \frac{b^2+4ac}{8\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C$
6. $\int \frac{x}{\sqrt{c+bx-ax^2}}dx = -\frac{1}{a}\sqrt{c+bx-ax^2} + \frac{b}{2\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C$

$\sqrt{\pm\frac{x-a}{x-b}}$ 或 $\sqrt{(x-a)(x-b)}$

1. $\int \frac{dx}{\sqrt{(x-a)(b-x)}} = 2\arcsin\sqrt{\frac{x-a}{b-x}}+C\ (a<b)$
2. $\int \sqrt{(x-a)(b-x)}dx = \frac{2x-a-b}{4}\sqrt{(x-a)(b-x)} + \frac{(b-a)^2}{4}\arcsin\sqrt{\frac{x-a}{b-x}}+C, (a<b)$

三角函数的积分

1. $\int \tan x dx = -\ln|\cos x|+C$
2. $\int \cot x dx = \ln|\sin x|+C$
3. $\int \sec x dx = \ln\left|\tan(\frac{\pi}{4}+\frac{x}{2})\right|+C = \ln|\sec x+\tan x|+C$
4. $\int \csc x dx = \ln\left|\tan\frac{x}{2}\right|+C = \ln|\csc x-\cot x|+C$
5. $\int \sec^2 x dx = \tan x + C$
6. $\int \csc^2 x dx = -\cot x + C$
7. $\int \sec x \tan x dx = \sec x + C$
8. $\int \csc x \cot x dx = -\csc x + C$
9. $\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4}\sin 2x + C$
10. $\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4}\sin 2x + C$
11. $\int \sin^n x dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int\sin^{n-2}x dx$
12. $\int \cos^n x dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int\cos^{n-2}x dx$
13. $\int \frac{dx}{\sin^n x} = -\frac{1}{n-1}\frac{\cos x}{\sin^{n-1}x} + \frac{n-2}{n-1}\int\frac{dx}{\sin^{n-2}x}$
14. $\int \frac{dx}{\cos^n x} = \frac{1}{n-1}\frac{\sin x}{\cos^{n-1}x} + \frac{n-2}{n-1}\int\frac{dx}{\cos^{n-2}x}$
15. $$\int \cos^m x\sin^n x dx$$ $$= \frac{1}{m+n}\cos^{m-1}x\sin^{n+1}x + \frac{m-1}{m+n}\int\cos^{m-2}x\sin^n x dx$$ $$= -\frac{1}{m+n}\cos^{m+1}x\sin^{n-1}x + \frac{n-1}{m+1}\int\cos^m x\sin^{n-2}x dx$$
16. $\int \frac{dx}{a+b\sin x} = \begin{cases} \frac{2}{\sqrt{a^2-b^2}}\arctan\frac{a\tan\frac{x}{2}+b}{\sqrt{a^2-b^2}}+C & (a^2>b^2) \\ \frac{1}{\sqrt{b^2-a^2}}\ln\left|\frac{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan\frac{x}{2}+b+\sqrt{b^2-a^2}}\right|+C & (a^2<b^2) \end{cases}$
17. $\int \frac{dx}{a+b\cos x} = \begin{cases} \frac{2}{a+b}\sqrt{\frac{a+b}{a-b}}\arctan\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)+C & (a^2>b^2) \\ \frac{1}{a+b}\sqrt{\frac{a+b}{a-b}}\ln\left|\frac{\tan\frac{x}{2}+\sqrt{\frac{a+b}{b-a}}}{\tan\frac{x}{2}-\sqrt{\frac{a+b}{b-a}}}\right|+C & (a^2<b^2) \end{cases}$
18. $\int \frac{dx}{a^2\cos^2 x+b^2\sin^2 x} = \frac{1}{ab}\arctan\left(\frac{b}{a}\tan x\right)+C$
19. $\int \frac{dx}{a^2\cos^2 x-b^2\sin^2 x} = \frac{1}{2ab}\ln\left|\frac{b\tan x+a}{b\tan x-a}\right|+C$
20. $\int x\sin ax dx = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax + C$
21. $\int x^2\sin ax dx = -\frac{1}{a}x^2\cos ax + \frac{2}{a^2}x\sin ax + \frac{2}{a^3}\cos ax + C$
22. $\int x\cos ax dx = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax + C$
23. $\int x^2\cos ax dx = \frac{1}{a}x^2\sin ax + \frac{2}{a^2}x\cos ax - \frac{2}{a^3}\sin ax + C$

反三角函数的积分 (其中 $a>0$)

1. $\int \arcsin\frac{x}{a}dx = x\arcsin\frac{x}{a} + \sqrt{a^2-x^2}+C$
2. $\int x\arcsin\frac{x}{a}dx = (\frac{x^2}{2}-\frac{a^2}{4})\arcsin\frac{x}{a} + \frac{x}{4}\sqrt{x^2-x^2}+C$
3. $\int x^2\arcsin\frac{x}{a}dx = \frac{x^3}{3}\arcsin\frac{x}{a} + \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2}+C$
4. $\int \arccos\frac{x}{a}dx = x\ arccos\frac{x}{a} - \sqrt{a^2-x^2}+C$
5. $\int x\arccos\frac{x}{a}dx = (\frac{x^2}{2}-\frac{a^2}{4})\arccos\frac{x}{a} - \frac{x}{4}\sqrt{a^2-x^2}+C$
6. $\int x^2\arccos\frac{x}{a}dx = \frac{x^3}{3}\arccos\frac{x}{a} - \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2}+C$
7. $\int \arctan\frac{x}{a}dx = x\arctan\frac{x}{a} - \frac{a}{2}\ln(a^2+x^2)+C$
8. $\int x\arctan\frac{x}{a}dx = \frac{1}{2}(a^2+x^2)\arctan\frac{x}{a} - \frac{a}{2}x+C$
9. $\int x^2\arctan\frac{x}{a}dx = \frac{x^3}{3}\arctan\frac{x}{a} - \frac{a}{6}x^2 + \frac{a^3}{6}\ln(a^2+x^2)+C$

指数函数的积分

1. $\int a^x dx = \frac{1}{\ln a}a^x + C$
2. $\int e^{ax}dx = \frac{1}{a}a^{ax}+C$
3. $\int xe^{ax}dx = \frac{1}{a^2}(ax-1)a^{ax}+C$
4. $\int x^n e^{ax}dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}dx$
5. $\int xa^x dx = \frac{x}{\ln a}a^x - \frac{1}{(\ln a)^2}a^x + C$
6. $\int x^n a^x dx = \frac{1}{\ln a}x^n a^x - \frac{n}{\ln a}\int x^{n-1}a^x dx$
7. $\int e^{ax}\sin bx dx = \frac{1}{a^2+b^2}e^{ax}(a\sin bx - b\cos bx)+C$
8. $\int e^{ax}\cos bx dx = \frac{1}{a^2+b^2}e^{ax}(b\sin bx + a\cos bx)+C$
9. $\int e^{ax}\sin^n bx dx = \frac{1}{a^2+b^2n^2}e^{ax}\sin^{n-1}bx(a\sin bx - nb\cos bx) + \frac{n(n-1)b^2}{a^2+b^2n^2}\int e^{ax}\sin^{n-2}bx dx$
10. $\int e^{ax}\cos^n bx dx = \frac{1}{a^2+b^2n^2}e^{ax}\cos^{n-1}bx(a\cos bx + nb\sin bx) + \frac{n(n-1)b^2}{a^2+b^2n^2}\int e^{ax}\cos^{n-2}bx dx$

对数函数的积分

1. $\int \ln x dx = x\ln x - x + C$
2. $\int \frac{dx}{x\ln x} = \ln|\ln x|+C$
3. $\int x^n\ln x dx = \frac{1}{n+1}x^{n+1}(\ln x - \frac{1}{n+1})+C$
4. $\int (\ln x)^n dx = x(\ln x)^n - n\int(\ln x)^{n-1}dx$
5. $\int x^m(\ln x)^n dx = \frac{1}{m+1}x^{m+1}(\ln x)^n - \frac{n}{m+1}\int x^m(\ln x)^{n-1}dx$

STL 积分/求和 (need std::)

1. $\int_0^1 t^{x-1}(1-t)^{y-1}dt = beta(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$
2. $\int_0^{+\infty} t^{num-1}e^{-t}dt = tgamma(num) = e^{lgamma(num)} = \Gamma(num)$
3. $\int_0^{phi}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} = ellint\_1(k,phi)$
4. $\int_0^{phi}\sqrt{1-k^2\sin^2\theta}d\theta = ellint\_2(k,phi)$
5. $\int_{num}^{+\infty}\frac{e^{-t}}{t}dt = -expint(-num)$
6. $\sum_{n=1}^{+\infty} n^{-num} = riemann\_zeta(num)$
7. $\frac{2}{\sqrt{\pi}}\int_0^{arg} e^{-t^2}dt = erf(arg)$