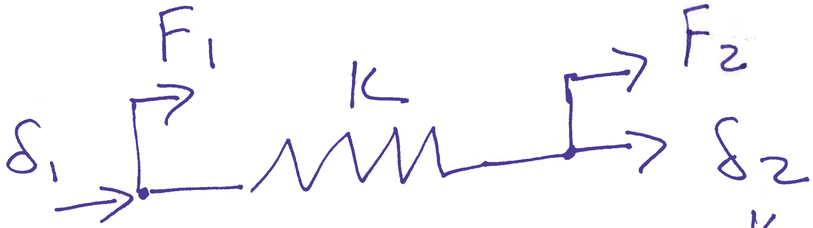
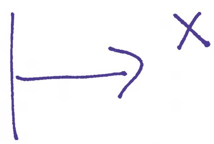
Direct Method with Truss Elements.

A 1D truss element gives a good introduction to several FEM ideas. A 1D truss element is equivalent to a spring

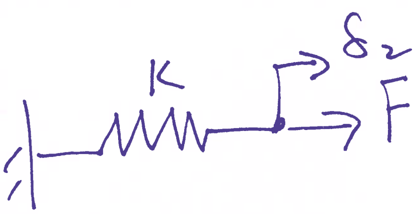


K spring constant
δ_1, δ_2 displacement



$$F = K(\delta_2 - \delta_1)$$

F force



For static equilibrium

$$\Sigma F = 0$$

$$F = K\delta_2$$

We will be deriving FEM for a truss system i.e. system of springs

K : spring constant

TRUSS $\equiv$ SPRING

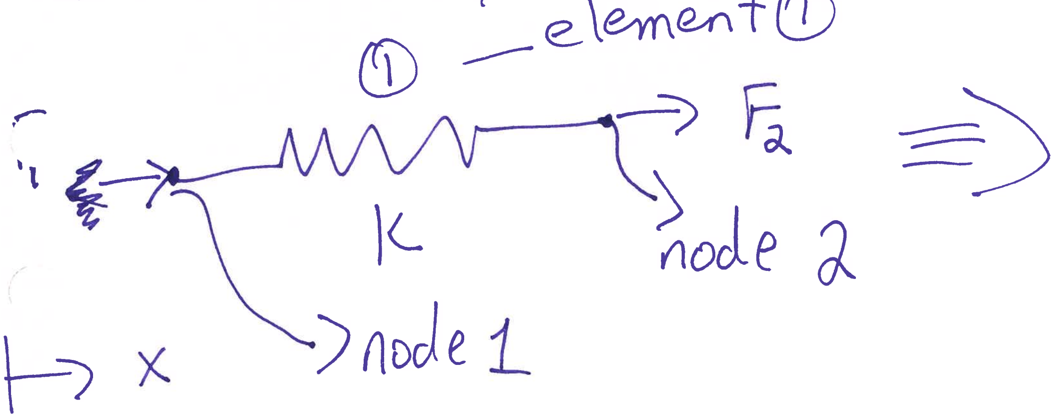
K : stiffness of a truss element $= \frac{EA}{L}$

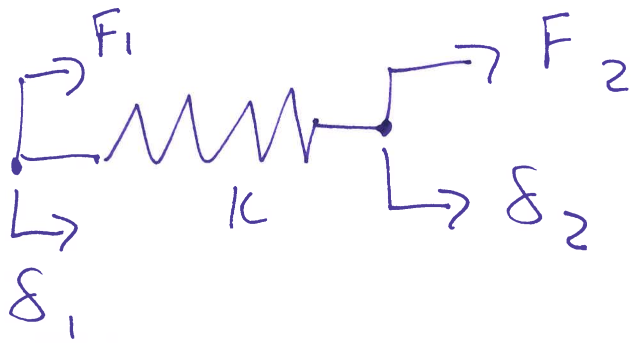
Consider a truss element - a bar:

cylinder

L : length
A : cross-sectional area
E : elastic modulus

In static equilibrium, $\Sigma F = 0$





$$F_1 + F_2 = 0$$

$$F_1 = -F_2$$

Constitutive equation for the spring using

1) $F_2 = K(\delta_2 - \delta_1)$

2) $F_1 = K(\delta_1 - \delta_2)$

In matrix form

$$\underset{\text{'local' force vector}}{\begin{bmatrix} F_1^{①} \\ F_2^{①} \end{bmatrix}} = \underbrace{\begin{bmatrix} K^{①} & -K^{①} \\ -K^{①} & K^{①} \end{bmatrix}}_{\text{local stiffness matrix}} \underset{\text{local disp. vector}}{\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}}$$

These are the equations for the truss element ①