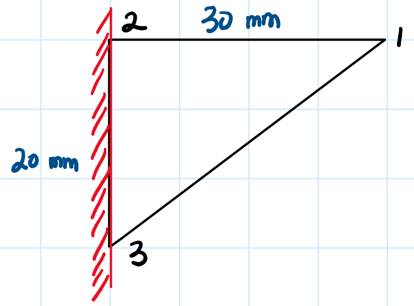
Homework 6

Wednesday, March 15, 2023 3:52 AM

1.



$t = 30\,mm$

$E = 70{,}000\,MPA$

$\nu = 0.3$

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \cancel{q_3} \\ \cancel{q_4} \\ \cancel{q_5} \\ \cancel{q_6} \end{bmatrix} \qquad B = \frac{1}{x_{13}y_{23} - x_{23}y_{13}} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 0.7 & 0 \\ 0 & 0 & 0.35 \end{bmatrix} \cdot \frac{E}{1-\nu^2}$$

	x	y
n_1	30	20
n_2	0	20
n_3	0	0

in mm

$$B = \frac{1}{(30)(20)} \begin{bmatrix} 20 & 0 \\ 0 & 0 \\ 0 & 20 \end{bmatrix} \cdot \frac{1}{1000}$$

$$B = \begin{bmatrix} 0.0333 & 0 \\ 0 & 0 \\ 0 & 0.0333 \end{bmatrix}$$ with mm or $$\begin{bmatrix} 33.33 & 0 \\ 0 & 0 \\ 0 & 33.33 \end{bmatrix}$$ with m.

$A^e = (0.03)(0.02)\frac{1}{2}$

$= 0.0003\,m^2$

$\boxed{k = B^T D B A^e h}$

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$A = (0.03)(0.02)\frac{1}{2}$

$= 0.0003\ m^2$

$h = 10\ mm = 0.01\ m$

Local force vector then becomes:

$$F_{Local} = \begin{bmatrix} 50 \\ -100 \\ \cancel{R_{2x}} \\ \cancel{R_{2y}} \\ \cancel{R_{3x}} \\ \cancel{R_{3y}} \end{bmatrix}$$

$$k\, q = F_{Local}$$

$$\begin{bmatrix} 0.256\,E{-3} & 0 \\ 0 & 0.08974\,E{-3} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0.0195 \cdot 10^{-5} \\ -0.1114 \cdot 10^{-5} \end{bmatrix} m$$

b) $\tau = D B q$

$$= \begin{bmatrix} 0.5 \\ 0.15 \\ -1 \end{bmatrix} \cdot 10^6 \ Pa$$

c) The body force addition in this case is:

$$f_y = 10^6 y^2 \ N/m^3$$

We know the following formula from lecture:

$$f^{local}_{body\ force} = \frac{t^e A^e}{3} \begin{bmatrix} b_x \\ b_y \\ b_x \\ b_y \\ b_x \\ b_y \end{bmatrix} \begin{array}{l} \left.\vphantom{\begin{matrix} b_x \\ b_y \end{matrix}}\right] \text{node 1} \\ \left.\vphantom{\begin{matrix} b_x \\ b_y \end{matrix}}\right] \text{node 2} \\ \left.\vphantom{\begin{matrix} b_x \\ b_y \end{matrix}}\right] \text{node 3} \end{array}$$

in our case:

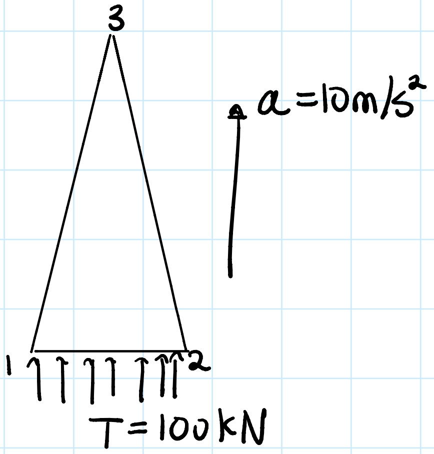
$$b_y = 10^6 (N_1 y_1 + N_2 y_2 + N_3 y_3)^2$$

$$= 10^6 (\xi y_1 + \eta y_2 + (1 - \xi - \eta) y_3)^2$$

$$b_x = 0$$

$$F_{global} = \begin{bmatrix} 50 \\ \\ -100 \end{bmatrix} + \begin{bmatrix} 0 \\ \\ 10^6 (\xi y_1 + \eta y_2 + (1 - \xi - \eta) y_3)^2 \end{bmatrix}$$

2.



For this problem, we could assume that vertex 3 is fixed.

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ \cancel{q_5} \\ \cancel{q_6} \end{bmatrix}$$

	x	y
n_1	-0.5	0
n_2	0.5	0
n_3	0	10

$$B = \frac{1}{x_{13}y_{23} - x_{23}y_{13}} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$= \frac{1}{(-0.5)(-10)-(0.5)(-10)} \begin{bmatrix} -10 & 0 & 10 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & -0.5 & 0 & 1 \\ -0.5 & -10 & -0.5 & 10 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & -0.05 & 0 & 0.1 \\ -0.05 & -1 & -0.05 & 1 & 0.1 & 0 \end{bmatrix}$$

$$D = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \cdot 10^{11}$$

We have a distributed force on the bottom due to thrust and an internal body force due to acceleration.

For thrust, we know total force, and it is a constant. Therefore, we divide it by half and supply to node 1, 2.

For acceleration, we calculate that

$$\begin{aligned} F_y &= m a \\ &= A \cdot t \cdot \rho \cdot a \\ &= (1 \cdot 10/2) \cdot 1 \cdot (2000) \cdot (10) \\ &= 1E5 \text{ Newtons.} \end{aligned}$$

We follow the body force formula:

$$f = \frac{t \cdot A}{3}\begin{bmatrix} b_x \\ b_y \\ b_x \\ b_y \\ b_x \\ b_y \end{bmatrix}$$

$$\begin{bmatrix} f_{by} \\ f_{bx} \\ f_{by} \end{bmatrix}$$

The global force vector then becomes:

$$f_{global} = \frac{(1)(5)}{3} \underbrace{\begin{bmatrix} 0 \\ 1E5 \\ 0 \\ 1E5 \\ 0 \\ 1E5 \end{bmatrix}}_{\text{Body Force}} + \underbrace{\begin{bmatrix} 0 \\ 50E3 \\ 0 \\ 50E3 \\ 0 \\ 0 \end{bmatrix}}_{\text{Thrust}}$$

Then:

$$B^T D B A^e t \quad q = f_{global}$$

$$1 \cdot 10^{11} \cdot \begin{bmatrix} 5.0062 & 0.12 & -5.0 & -0.12 \\ 0.12 & 2.51 & 0.125 & -2.48 \\ -4.99 & 0.125 & 5.006 & -0.125 \\ -0.125 & -2.4875 & -0.125 & 2.512 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2.1666 \\ 0 \\ 2.1666 \end{bmatrix} \cdot 1E5$$

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} -0.0216 \\ 0.86774 \\ -0.0216 \\ 0.86774 \end{bmatrix} \cdot 10^{-4} \text{ m}$$

We only look for deformation in the y-direction.

Therefore, the total change in height is:

Therefore, the total change in height is:

$$\boxed{0.86774 \cdot 10^{-4}\, m}$$