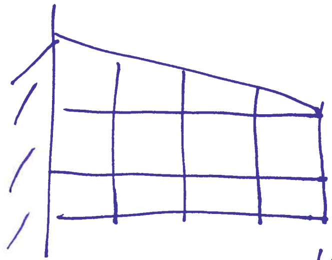
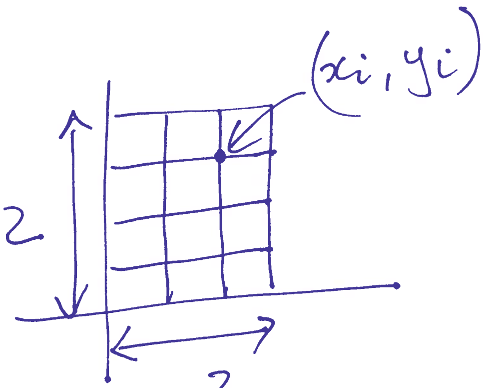
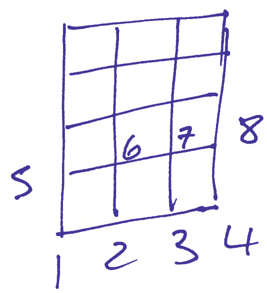
HW 8



In HW 8, you are asked to make a 2D mesh.

To mesh: Think of a logic to generate a square mesh (0,2)





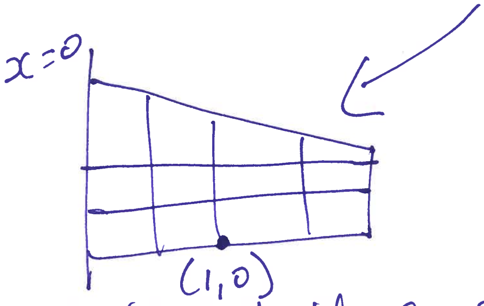
Number your mesh

You can use the function meshgrid in Matlab to create the mesh.

Then modify using:

$$y_i = y_i\left(\frac{1}{2}\left(2-\frac{x_i}{2}\right)\right)$$

'find' function in Matlab



Then, find the stress @ point (1,0)

Each node has dof = $(2i-1, 2i)$
where i = node number

<u>Objective</u> : Find stresses at (x,y) ← real element

<u>Approach</u> (approximate)

steps :

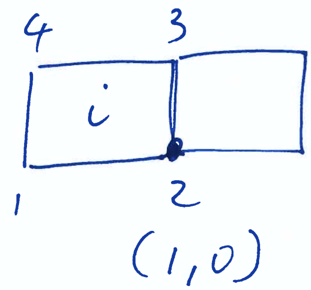
① Find ξ, η @ (x,y)

② Call your function from HW 6, p. 1 to find $dN\,dx \rightarrow$ B matrix

③ stress = $D * B * q$

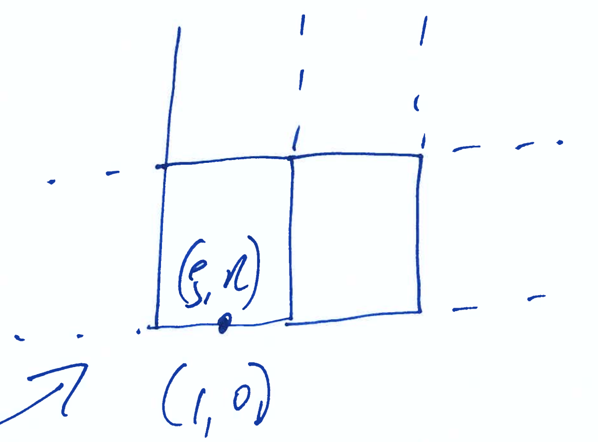
D ← elasticity matrix

$$q = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix}$$ for element 'i'



For element 'i', $\xi = 1$, $\eta = -1$

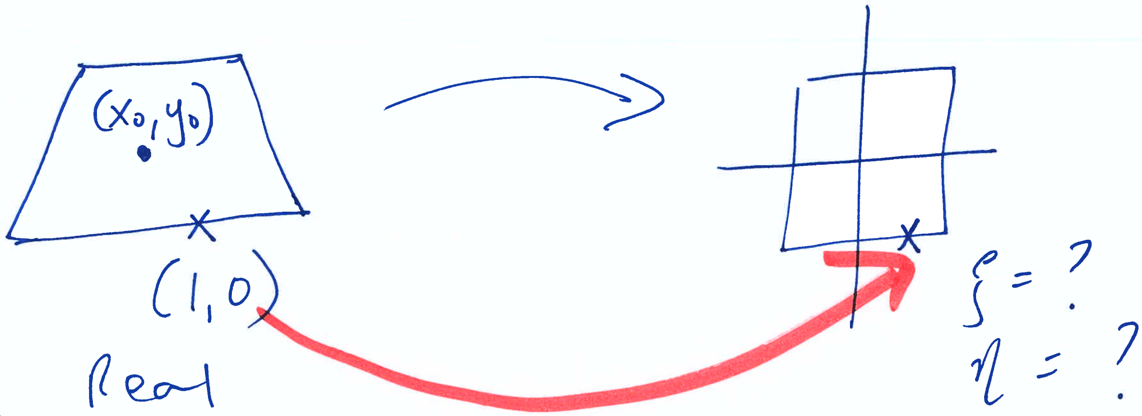
Let's say the point of interest is not at a node. For example,



$$J = \begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{pmatrix}_{(0,0)}$$

@ $\xi = 0$

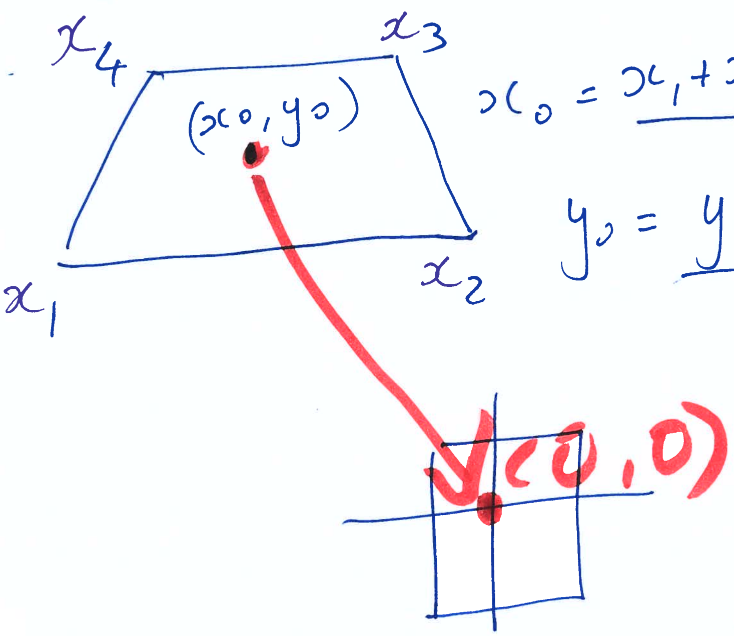
$\eta = 0$



$$\overbrace{\begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{pmatrix}}^{J^T}\Bigg|_{@\,\xi=0,\ \eta=0} \begin{bmatrix} \Delta \xi \\ \Delta \eta \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \begin{array}{l} 1 \ \ x - x_0 \\ 0 \ \ y - y_0 \end{array}$$

$$\Delta x = x - x_0$$
$$\Delta y = y - y_0$$

$$\frac{\partial x}{\partial \xi} \Delta \xi + \frac{\partial x}{\partial \eta} \Delta \eta = \Delta x$$



$$x_0 = \frac{x_1 + x_2 + x_3 + x_4}{4}$$
$$y_0 = \frac{y_1 + y_2 + y_3 + y_4}{4}$$

(x_0, y_0) is the center of the real element

To find stress at an arbitrary point (x,y)

→ Find element containing the arbitrary point (with coordinates x_i, y_i)

→ Call your function from HW6, p.1 to find J at $(0,0)$ for this element

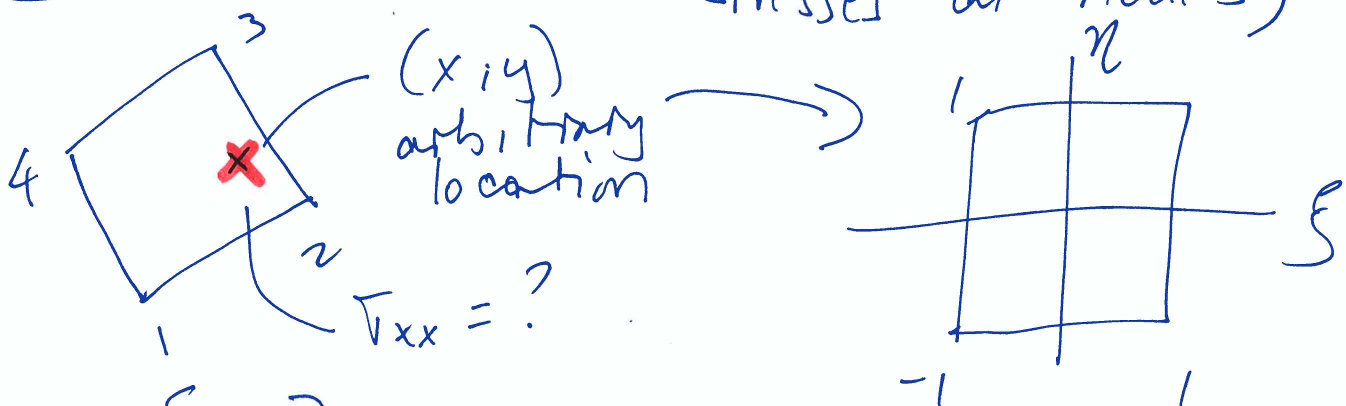
→ use

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \left(J^T\right)^{-1} \begin{bmatrix} x - \frac{\sum x_i}{4} \\ y - \frac{\sum y_i}{4} \end{bmatrix}$$

→ Call function again at ξ, η

→ $\sigma = D B q$

<u>Local</u> smoothing (process to compute stresses at nodes)



$$\sigma = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}$$

$$\underset{3\times 3}{D}\ \underset{3\times 8}{B}\ \underset{8\times 1}{q}$$

$\sigma = D B q$?

↑ is a function of ξ, η

We need to find ξ, η at this location x, y

$$x = x_1 N_1 + x_2 N_2 + x_3 N_3 + x_4 N_4$$

$$y = y_1 N_1 + y_2 N_2 + y_3 N_3 + y_4 N_4$$

We have 2 equations for ξ & η

$$N_1 = \frac{(1+\xi)(1+\eta)}{4}$$

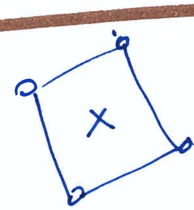
— But we don't solve these for ξ & η because the equations are nonlinear

I Approximate appro~~acha~~ach to find ξ & η

Given x, y & coord. Find ξ, η

$$\boxed{J^{-T} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} = \begin{bmatrix} \xi \\ \eta \end{bmatrix}}$$

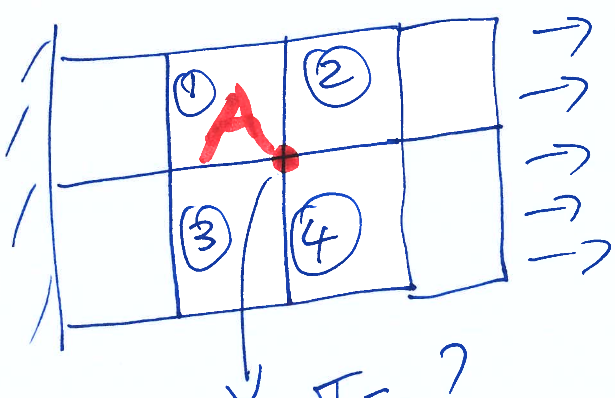
J @ $\xi = 0$, $\eta = 0$



use (x_0, y_0) mid point of real element

$$\frac{\sum x_i}{4}$$

II Local Smoothing



$\sigma = ?$

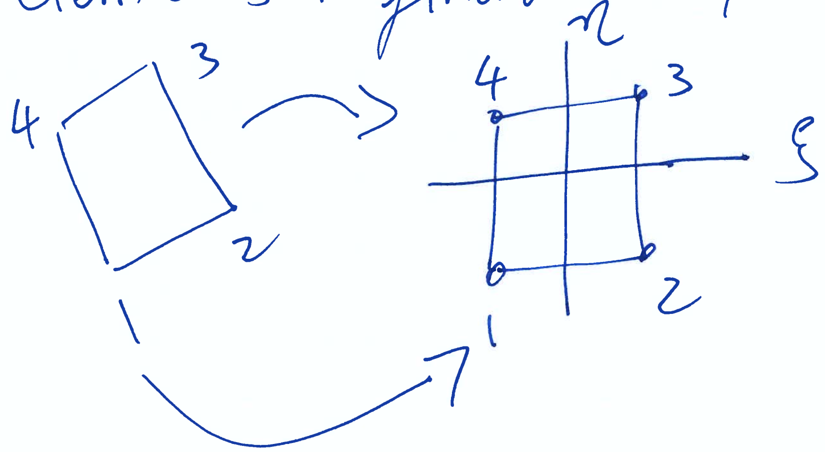
Let's say we want to find the stress σ at node A.

4 elements share this node.

1, 2, 3, 4

Each element will give a different value of stress at A.

In this case, we average the stresses from all 4 elements to find a unique stress at A.

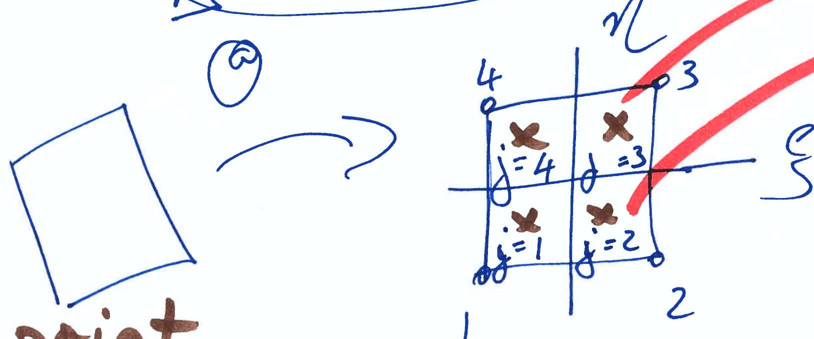


we compute stress at integration points

$(\xi = \pm\frac{1}{\sqrt{3}},\ \eta = \pm\frac{1}{\sqrt{3}})$

$\sigma = DBq$

@



$j = 1, 2, 3, 4$ are the 4 integration points

int. point

$$\sigma_{xx}^{j} = N_1^{j}\,\sigma_{xx1} + N_2^{j}\,\sigma_{xx2} + N_3^{j}\,\sigma_{xx3} + N_4^{j}\,\sigma_{xx4}$$

stress @ int. point j

stress @ node 1

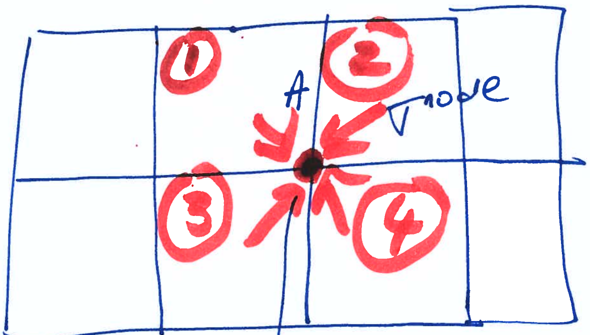
stress @ node 2

shape func @ integration point j

Write similar equations for all 4 integration points.

$$\underbrace{\begin{bmatrix}\sigma_{xx}^{1}\\ \sigma_{xx}^{2}\\ \sigma_{xx}^{3}\\ \sigma_{xx}^{4}\end{bmatrix}}_{\text{Known}\ \left[\sigma^{int.pts}\right]} = \underbrace{\begin{bmatrix} N_1^1 & N_2^1 & N_3^1 & N_4^1\\ N_1^2 & N_2^2 & N_3^2 & N_4^2\\ N_1^3 & N_2^3 & N_3^3 & N_4^3\\ N_1^4 & N_2^4 & N_3^4 & N_4^4\end{bmatrix}}_{\text{Known}\ \ M\ \text{matrix}} \underbrace{\begin{bmatrix}\sigma_{xx1}\\ \sigma_{xx2}\\ \sigma_{xx3}\\ \sigma_{xx4}\end{bmatrix}}_{\text{Unknown}\ \left[\sigma^{nodes}\right]}$$

$$\left[\sigma^{nodes}\right] = M^{-1}\left[\sigma^{int.pts}\right]$$



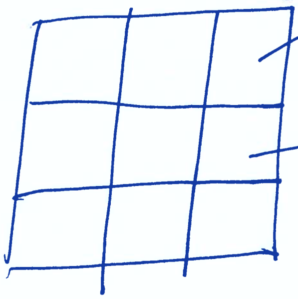
Smoothing will average σ^{node} that came from all elements that contain that node.

$$\begin{bmatrix}25\\ 20.5\\ 22\\ 23\end{bmatrix}$$

← stress @ node A from element 1

← stress @ node A from element 2

Global smoothing



local force ~~stiffness~~ local stiffness

$$\sigma^{intpt} = M^{(1)} \sigma^{node}$$

$$\sigma^{intpt} = M^{(2)} \sigma^{node}$$

Assemble these equations to get a global M matrix

$$(M^{global})(\sigma^{nodes}) = \sigma^{intpts}.$$

→ Solve to find σ^{nodes} at all nodes

→ Repeat this for every component of the stress matrix.

Galerkin smoothing

$$\sigma^{int.pt} = M \sigma^{node}$$

$$M^T \sigma^{intpt.} = M^T M \sigma^{node}$$

$$\sigma^{node} = \underbrace{(M^T M)}_{\text{symmetrix matrix}}^{-1} M^T \sigma^{int.pt.}$$

taking a 'pseudo inverse'

symmetrix matrix → easier to invert compared to 'M'