Exam 2 Study

Sunday, April 2, 2023 2:14 PM

$$\frac{d^2u}{dx^2} - \frac{du}{dx} + u = 1$$

where $u(x=1) = 1$ & $\frac{du}{dx}(x=0) = 0$

We first obtain the weak form:

$$\int_{x_1}^{x_3} \left(\frac{d^2u}{dx^2} - \frac{du}{dx} + u - 1\right) w = 0$$

$$\int_{x_1}^{x_3} \frac{d^2u}{dx^2} w - \frac{du}{dx} w + uw - w = 0$$

$$N_1 = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} \qquad N_2 = \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} \qquad N_3 = \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

$$\int_{x_1}^{x_3} \frac{d^2u}{dx^2} w - \frac{du}{dx}\frac{dw}{dx} - \frac{du}{dx} w + uw - w = 0$$

$$\int_{x_1}^{x_3} \frac{d}{dx}\left[\frac{du}{dx} w\right] + \frac{du}{dx}\frac{dw}{dx} - \frac{du}{dx} w + uw - w = 0$$

$$\frac{du}{dx} w \Big|_{x_1}^{x_3} + \int_{x_1}^{x_3} \frac{du}{dx}\frac{dw}{dx} - \int_{x_1}^{x_3} \frac{du}{dx} w + \int_{x_1}^{x_3} uw - \int_{x_1}^{x_3} w = 0$$

$$\frac{du}{dx} w \Big|_{x_1}^{x_3} = \frac{du}{dx}\Big|_{x_3} w(x_3) - \frac{du}{dx}\Big|_{x_1} w(x_1)$$

$$= \left[w(x_1)\ 0\ w(x_3)\right] \begin{bmatrix} -\frac{du}{dx}\Big|_{x_1} \\ 0 \\ \frac{du}{dx}\Big|_{x_3} \end{bmatrix}$$

$$\int_{x_1}^{x_3} \frac{du}{dx}\frac{dw}{dx}\, dx$$

$$= \int_{x_1}^{x_3} \left(N_1' u(x_1) + N_2' u(x_2) + N_3' u(x_3)\right)\left(N_1' w(x_1) + N_2' w(x_2) + N_3' w(x_3)\right)$$

$$= \left[w(x_1)\ w(x_2)\ w(x_3)\right] \int_{x_1}^{x_3} \begin{bmatrix} N_1' \\ N_2' \\ N_3' \end{bmatrix} \left[N_1' + N_2' + N_3'\right] \begin{bmatrix} u(x_1) \\ u(x_2) \\ u(x_3) \end{bmatrix}$$

$$\int_{x_1}^{x_3} \frac{du}{dx} w = \int_{x_1}^{x_3} \left(N_1' u(x_1) + N_2' u(x_2) + N_3' u(x_3)\right)\left(N_1 w(x_1) + N_2 w(x_2) + N_3 w(x_3)\right)$$

$$= \left[w(x_1)\ \ w(x_2)\ \ w(x_3)\right] \int_{x_1}^{x_2} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \left[N_1'\ N_2'\ N_3'\right] \begin{bmatrix} u(x_1) \\ u(x_2) \\ u(x_3) \end{bmatrix}$$

$$\int_{x_1}^{x_3} uw\, dx$$

$$= \int_{x_1}^{x_3} \left(N_1 u(x_1) + N_2 u(x_2) + N_3 u(x_3)\right)\left(N_1 w(x_1) + N_2 w(x_2) + N_3 w(x_3)\right)$$

$$= \left[w(x_1)\ \ w(x_2)\ \ w(x_3)\right] \int^{x_3} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \left[N_1\ N_2\ N_3\right] \begin{bmatrix} u(x_1) \\ u(x_2) \\ u(x_3) \end{bmatrix} dx$$

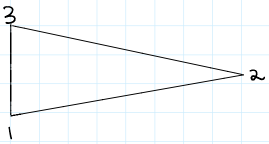
$$= \begin{bmatrix} w(x_1) & w(x_2) & w(x_3) \end{bmatrix} \int_{x_1} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} u(x_1) \\ u(x_2) \\ u(x_3) \end{bmatrix} dx$$

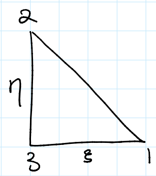
$$\int_{x_1}^{x_3} w \, dx$$

$$= \begin{bmatrix} w(x_1) & w(x_2) & w(x_3) \end{bmatrix} \int_{x_1}^{x_2} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} dx$$

recombining, we obtain:

$$\begin{bmatrix} \left.\frac{du}{dx}\right|_{x_1} \\ 0 \\ \left.\frac{du}{dx}\right|_{x_3} \end{bmatrix} - \int_{x_1}^{x_3} \left(\begin{bmatrix} N_1' \\ N_2' \\ N_3' \end{bmatrix} \begin{bmatrix} N_1' + N_2' + N_3' \end{bmatrix} \begin{bmatrix} u(x_1) \\ u(x_2) \\ u(x_3) \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \begin{bmatrix} N_1' & N_2' & N_3' \end{bmatrix} \begin{bmatrix} u(x_1) \\ u(x_2) \\ u(x_3) \end{bmatrix} - \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} u(x_1) \\ u(x_2) \\ u(x_3) \end{bmatrix} - \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \right) = 0$$





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