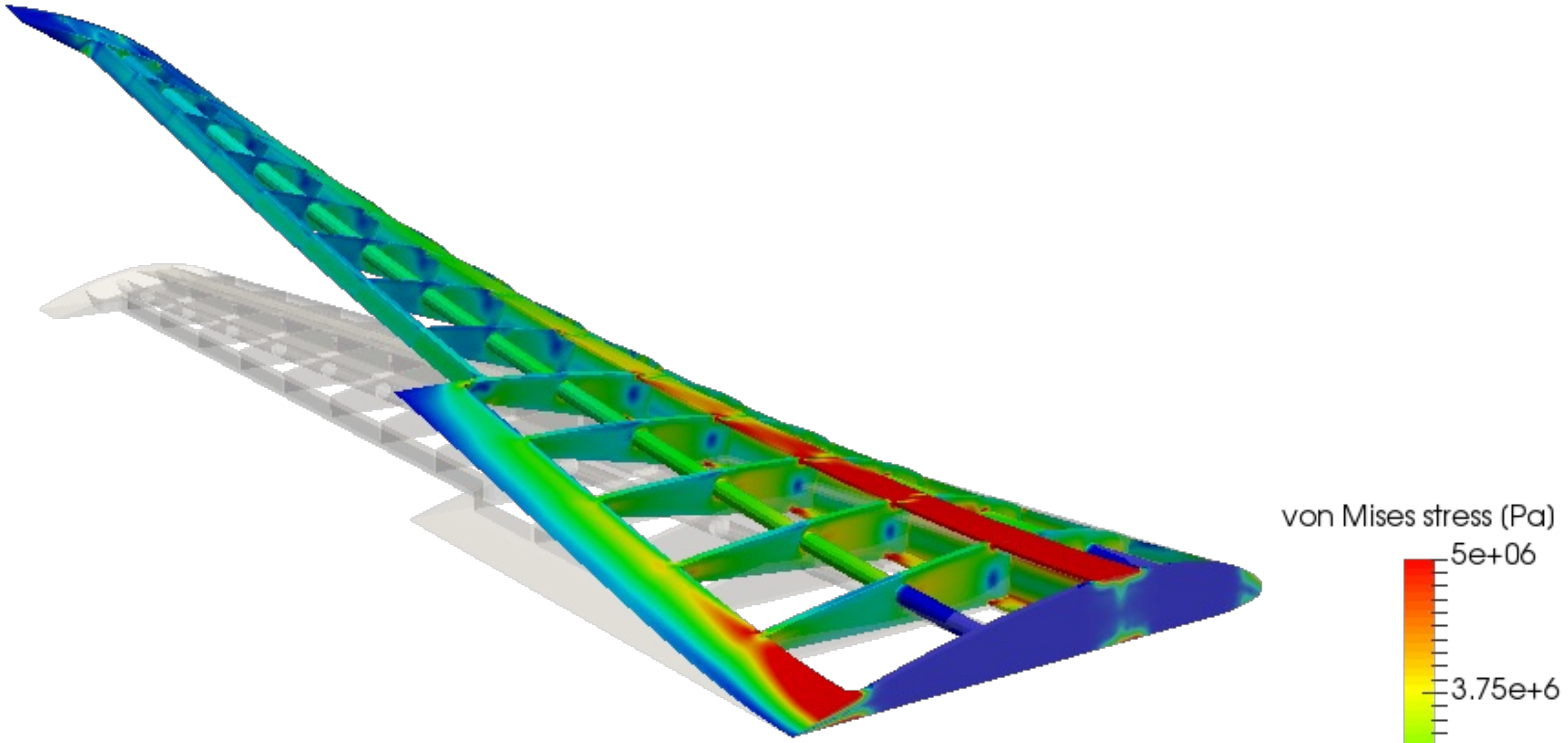
AEROSP 510 Winter 2023

Finite Element Methods in Engineering



Instructor
Prof. Nakhiah Goulbourne
FXB 3042
ngbourne@umich.edu

Lecture: Tuesdays & Thursdays 9-10:30am

Office Hours: Tuesdays 1:00pm – 2:30pm
Wednesdays 1:00 – 2:30pm

Objective: Introduction to theory and formulation of finite element method for solving problems in solid mechanics.

Textbook (Optional): Introduction of finite elements in engineering, Chandrupatla and Belegundu (Third edition)

Coding: Course will use Matlab programming for all assignments, exams, and final project.

COURSE PREREQUISITES

Intro to Solid Mechanics (Aero 315 or equivalent)

Senior undergraduate student or first year graduate student in engineering, or permission of instructor

Familiarity with calculus, differential equations, and linear algebra is essential

Grading: ~ 10 Homeworks, 30% of overall grade

Exam 1: February 14, 25%

Exam 2: April 4, 25%

Final Project: Report due Sunday, April 17 11:59PM, 20%

Computer Usage: Matlab

Other: Adhere to the College of Engineering Honor Code.

- Homeworks are usually due on Thursdays in class. Collaboration on homework is allowed/encouraged. But the final submittal should be your own hand-written work and/or computer code.

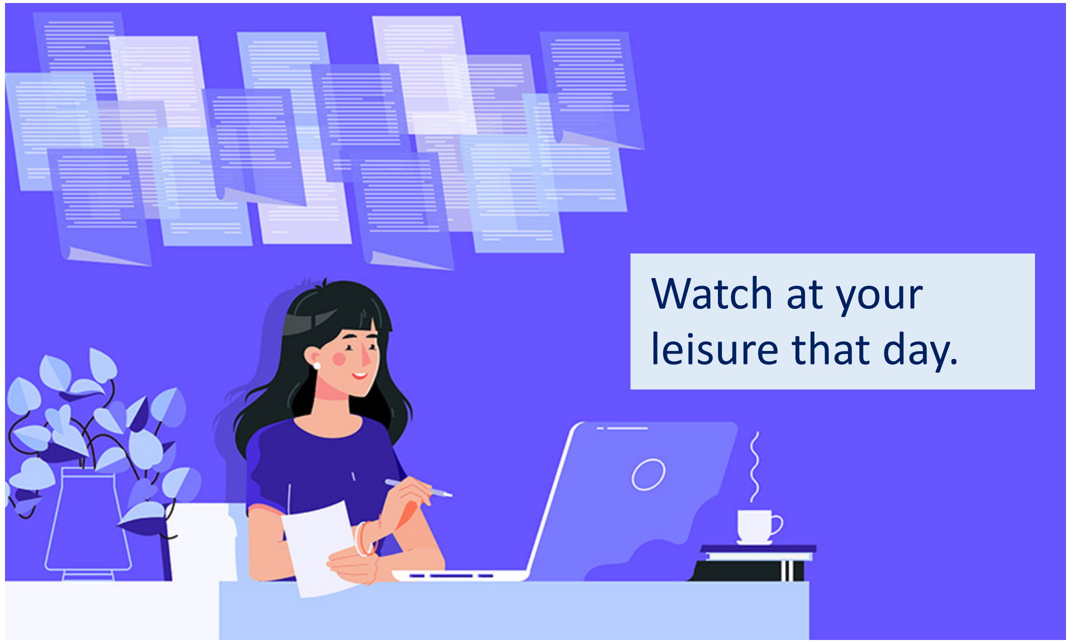
- First late homework allowed without penalty (within 4 days of deadline). Any subsequent late homework will be penalized by 20% for the first day and 50% thereafter. Homeworks are not accepted after 4 days from deadline.

- Exams are take home type and are open notes with Matlab component. No collaborations allowed.

COURSE SCHEDULE

No in person lecture on 3 dates:
January 19, March 24, and April 11

Pre-recorded video lectures will be posted by 9am on these 3 dates:
Uploaded to Canvas



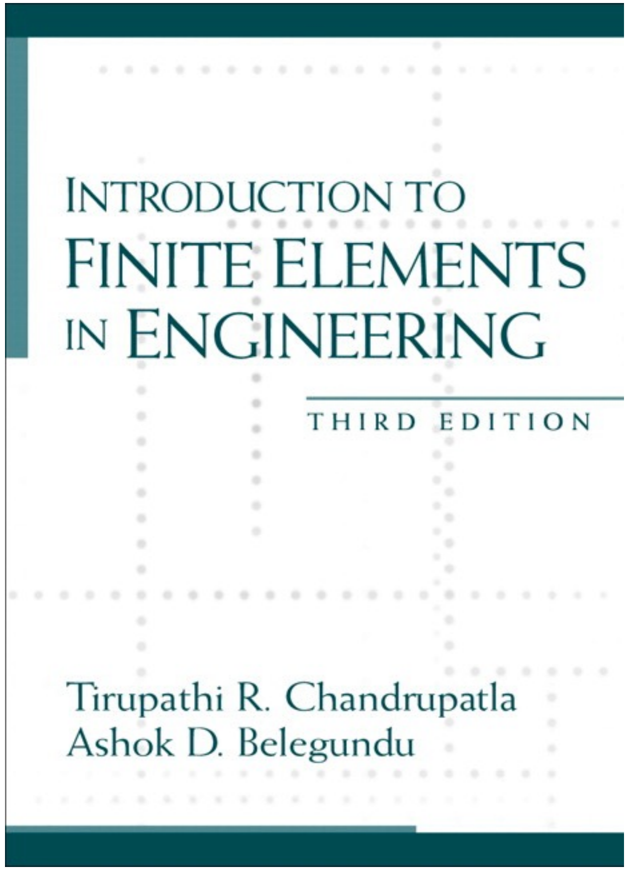
Communication with Instructor

For personal questions and issues (seeking resources/reporting absences): Use email

For questions on the homework or on understanding a concept: Attend office hours

Important: Instructor will not debug your code for you

COURSE TEXTBOOK



The textbook covers linear Finite Element Methods for static and dynamic problems in Structures

For more advanced topics use course notes

Computational Mechanics

Mature field focused on numerical methods for solving problems in mechanics

Structures

Fluids

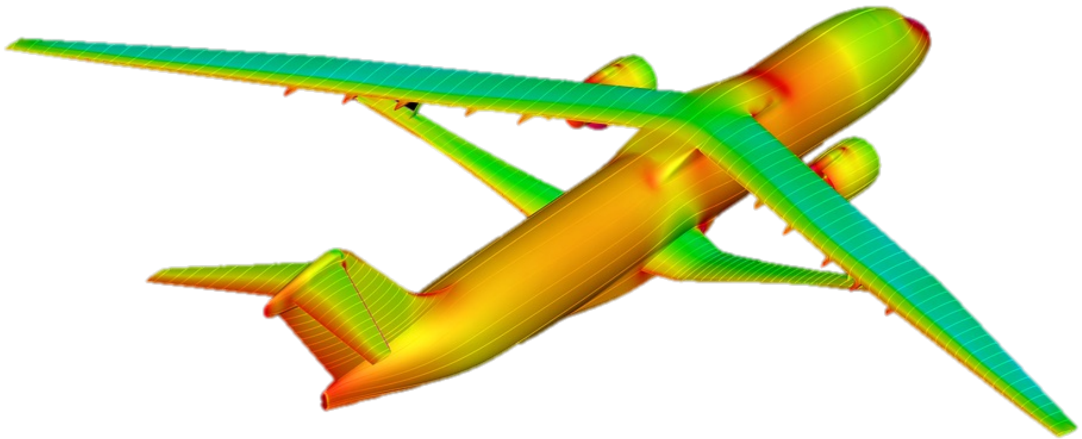


Illustration: FEA simulation

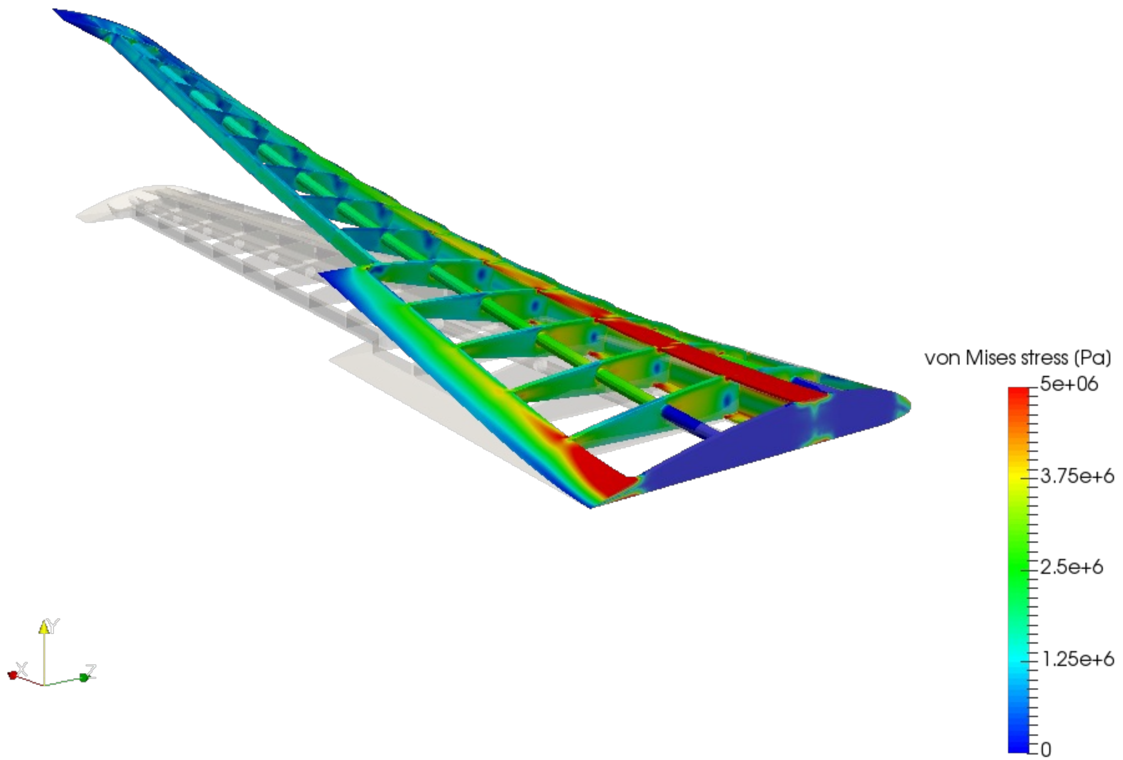
Image Credit: Dreamstime illustations

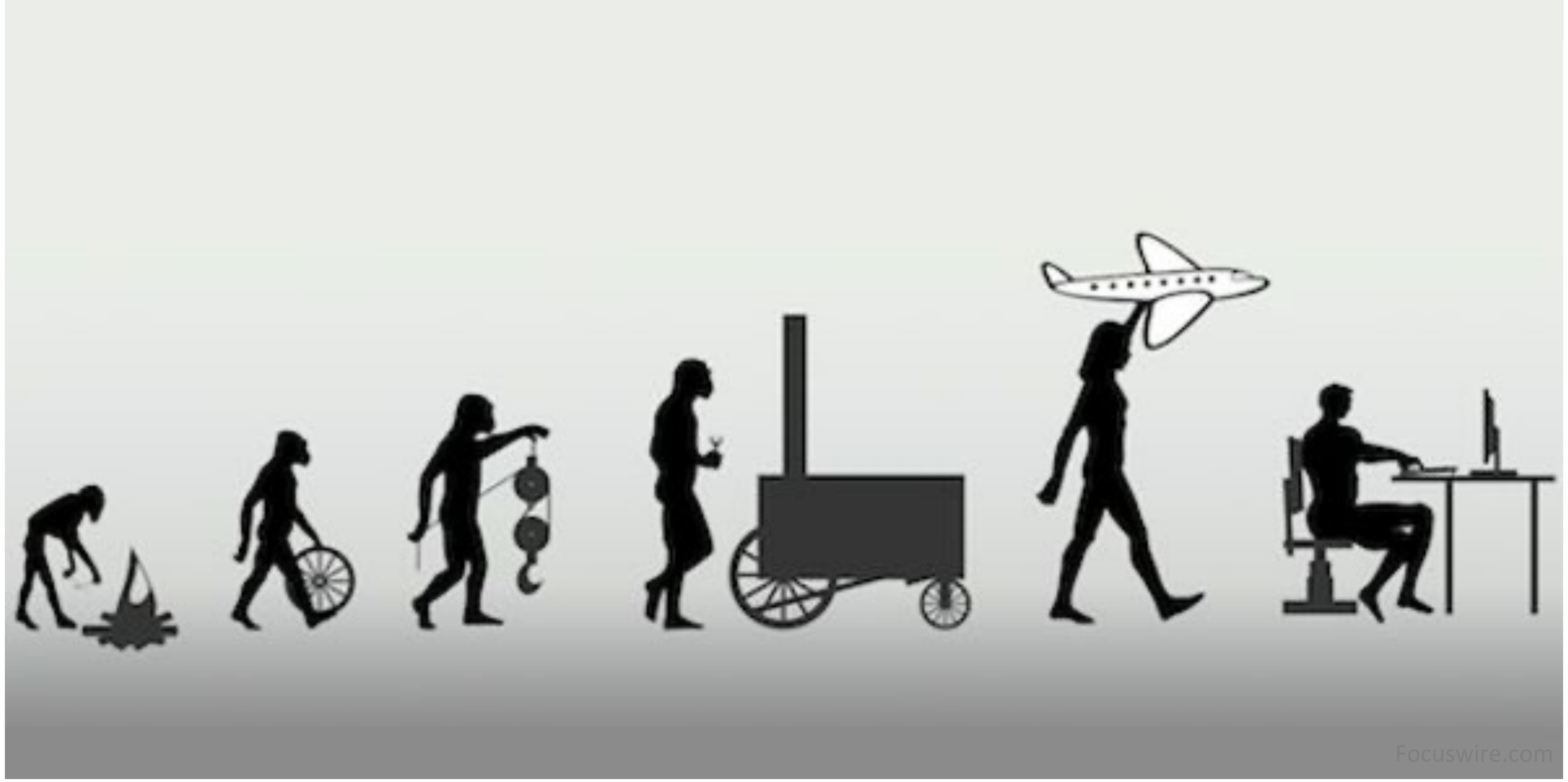
CFD simulation

Image Credit: NASA/Craig Hunter

Some Computational Mechanics Methods

FDM, Finite differences
Finite Volume methods
FEM, Finite Element Methods
BEM, Boundary Element Methods
Spectral methods
Mesh free methods
More…





Historical perspective on FEM

Who created FEM?

Structural engineering is the art of molding materials we do not really understand into shapes we cannot really analyze, so as to withstand forces we cannot really assess, in such a way that the public does not really suspect.—E. H. Brown, Structural Analysis

From Boeing to Berkeley: structural engineers, the cold war, and the origins of finite element analysis

Arms Race

Cold War History

Berlin Blockade

Cuban Missile Crisis

Perestroika

Détente

Suez Crisis

Berlin Wall

THE HISTORY CHANNEL
WHERE THE PAST COMES ALIVE

U-2 Spy Incident

Nato and Warsaw Pact

The Space Race

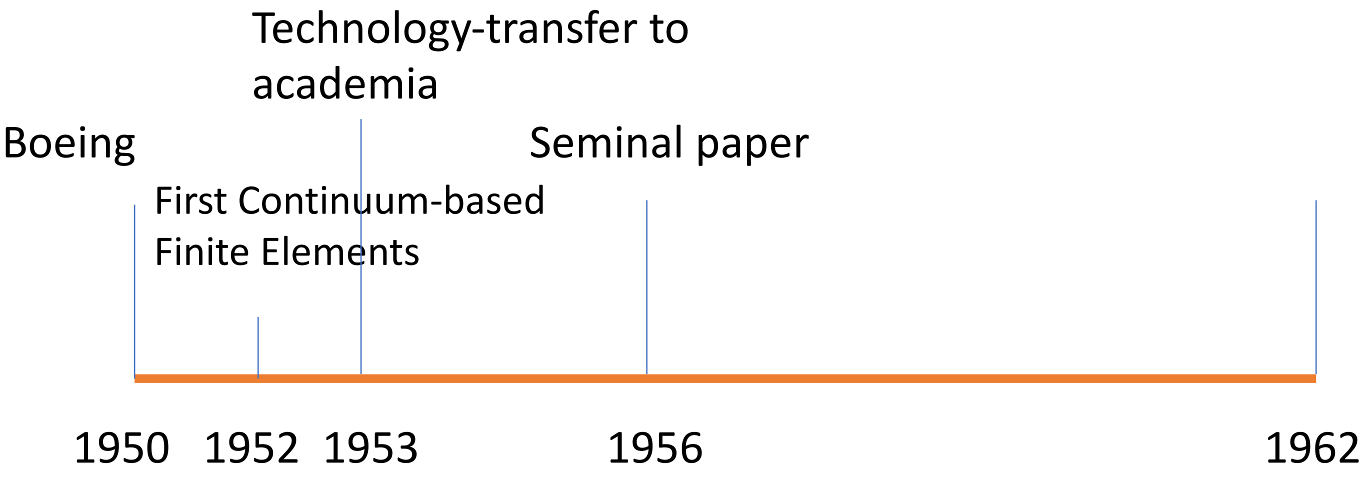
Berlin Airlift

Who created FEM?

Better to ask who created FEM that we use?

A Brief Historical Sketch…

M.J. (Jon) Turner at Boeing – perfected and generalized the Direct Stiffness Method and convinced Boeing to throw resources into it whilst other companies used the Force Method



Martin : Aeronautics and Astronautics Professor
Clough: Structural Engineering Professor

JOURNAL OF THE AERONAUTICAL SCIENCES

VOLUME 23 SEPTEMBER, 1956 NUMBER 9

Stiffness and Deflection Analysis of Complex Structures

M. J. TURNER,* R. W. CLOUGH,† H. C. MARTIN,‡ AND L. J. TOPP**

ABSTRACT

tion on static air loads, and theoretical analysis of aeroelastic effects on stability and control. This is a problem of exceptional difficulty when thin wings and tail surfaces of low aspect ratio, either swept or unswept, are involved.

It is recognized that camber bending (or rib bending) is a significant feature of the vibration modes of the newer configurations, even of the low-order modes; in order to encompass these characteristics it seems likely that the load-deflection relations of a practical structure must be expressed in the form of either deflection or stiffness influence coefficients. One approach is to employ structural models and to determine the influence coefficients experimentally; it is anticipated that the experimental method will be employed extensively in the future, either in lieu of or as a final check on the result of analysis. However, elaborate

The Academics: R.W. Clough (UC Berkeley), H.C. Martin (U. Washington), J.H. Argyris (Imperial College), O.C. Zienkiewitz (University of Wales, Swansea)

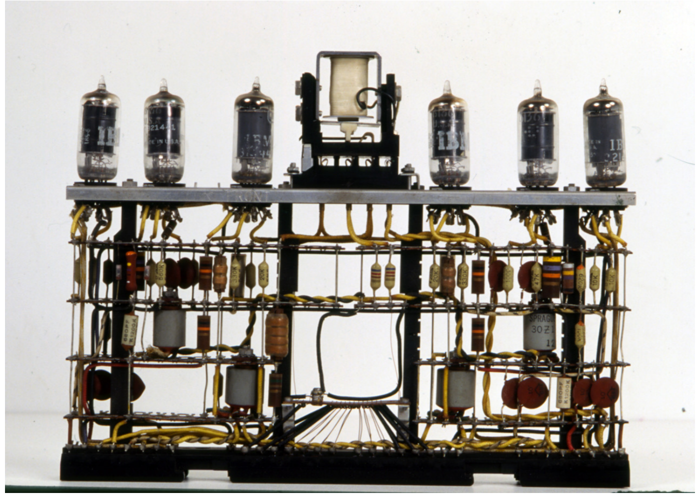
FEM PRECURSORS

Euler – piecewise linear function with discontinuous derivatives at nodes.

R. Courant – 1941 presentation to American Math Society and article on variational treatment of PDEs

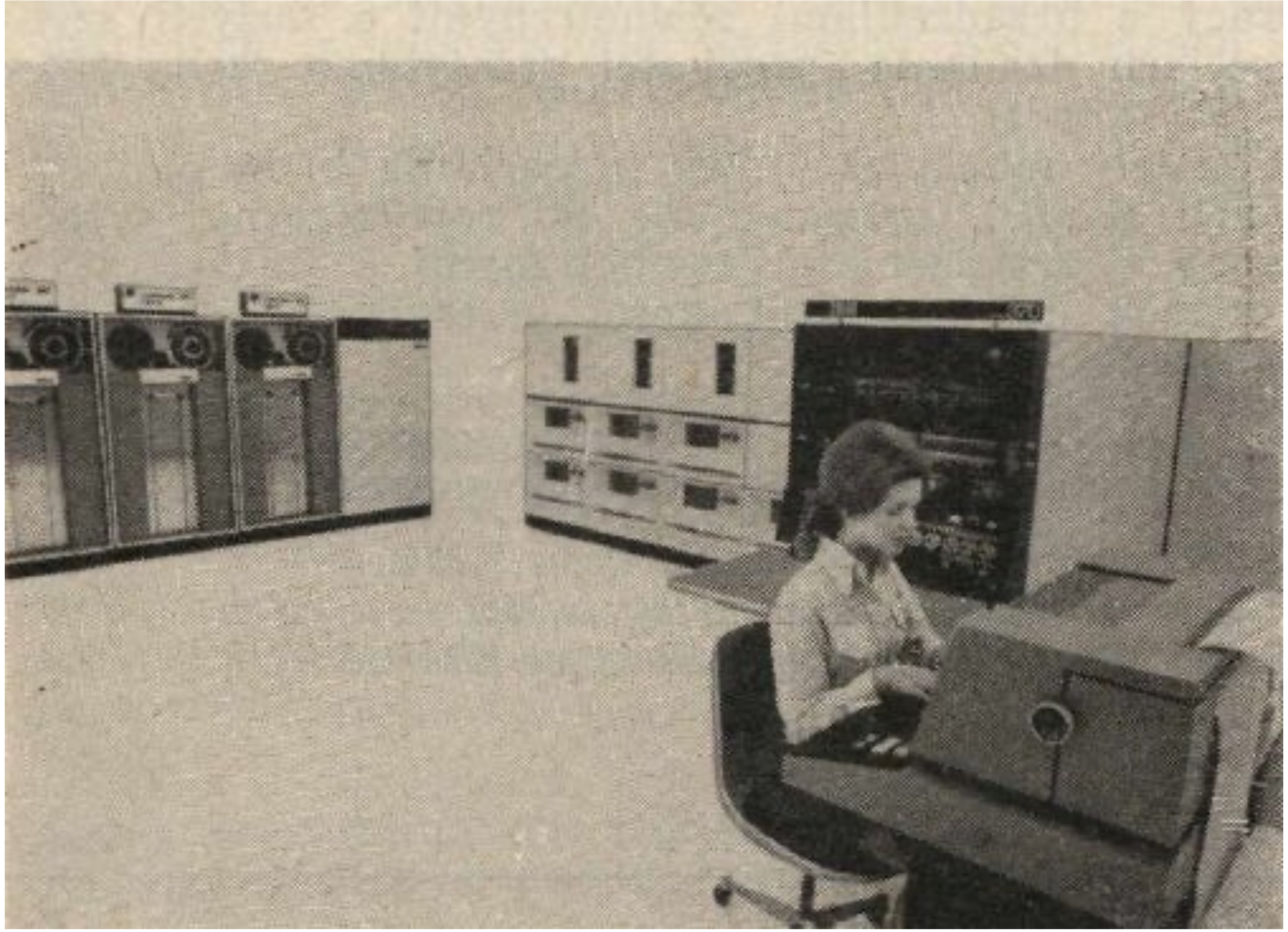
Turner et al. seminal paper cites Levy and Schuerch as precursors who presented a method for calculating stiffness matrix of entire structures that might be amenable to "high speed digital computing machines".

IBM 704, large digital mainframe computer introduced in 1957



IBM 704, vacuum-tube circuit module

$10 Million Masterpiece (adjusted for inflation)



IBM's System/370 Model 145 in 1970
500 KB of RAM, 233 megabytes of hard disk space, and ran at 2.5 MHz.

Your iphone is ~ 1000 times faster and 80 times more storage

Matrix Structural Analysis

Digital Computer

Variational Approximation Theory

Driving application was aerospace structures. Only rich aerospace companies could afford the splurge amidst the cold war.

HISTORICAL HIGHLIGHTS…

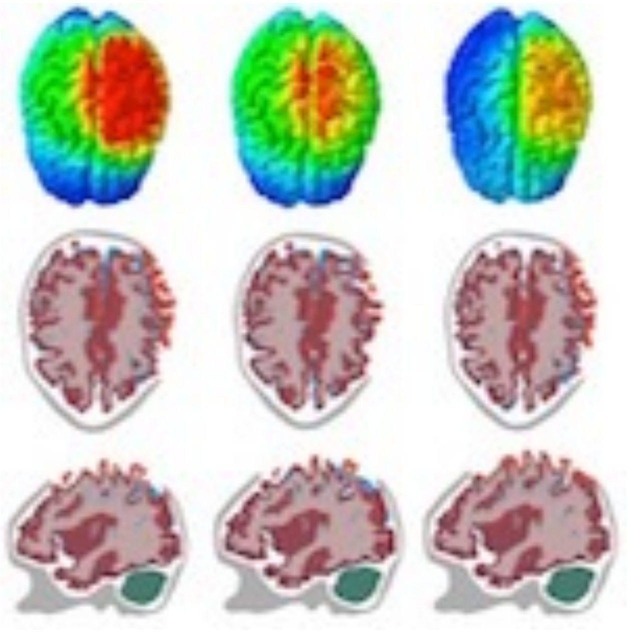
Golden Age 1962 -1972 – variational generation

- first textbook, mathematical exposition, numerical integration techniques, shape functions,…, applications to nonstructural problems, displacement-based methods dominate

Post 1972 – new formulations, technology updates, Hughes and Bathe textbooks, commercial codes

The 1980s – many more approaches emerge and the community diversifies

What is FEM?



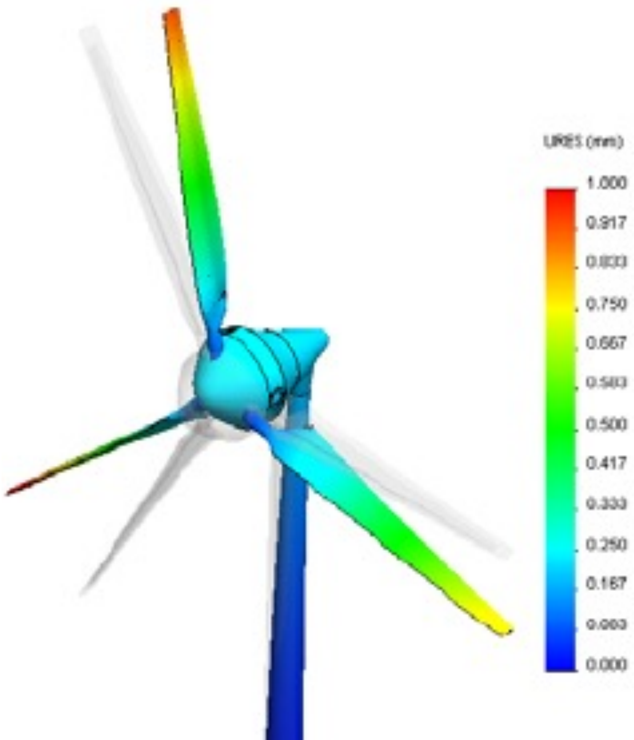
A computational approach for solving problems that are governed by differential equations.

The mathematical equations behind FEM allow us to create a simulation.

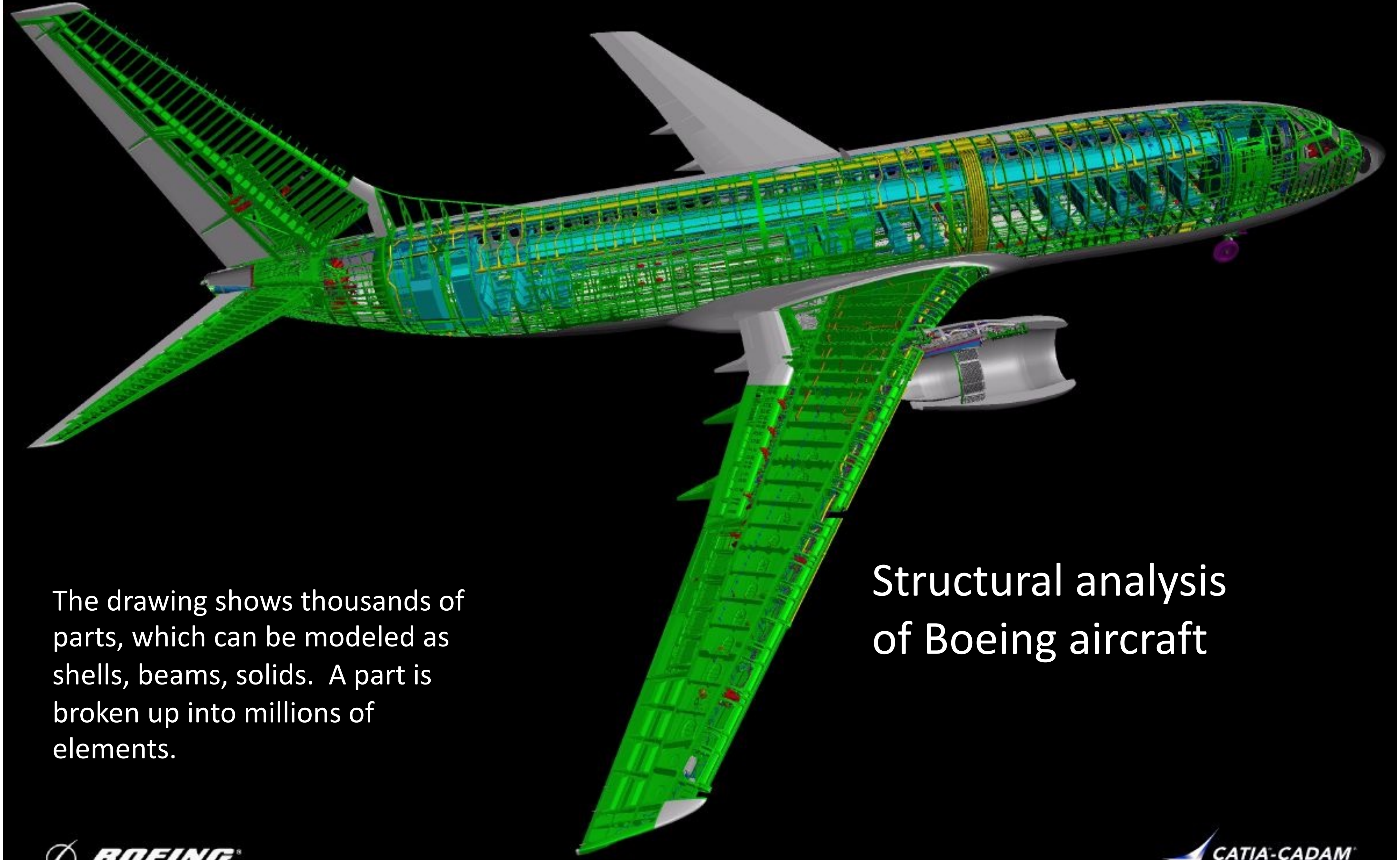
FEM allows us to solve structural behavior without having to manufacture and test every design and every part. This cuts down on cost and time.

FEM has wide application in structural and fluid behavior, thermal transport, wave propagation, biomechanics.

Important tool for engineering design and analysis.



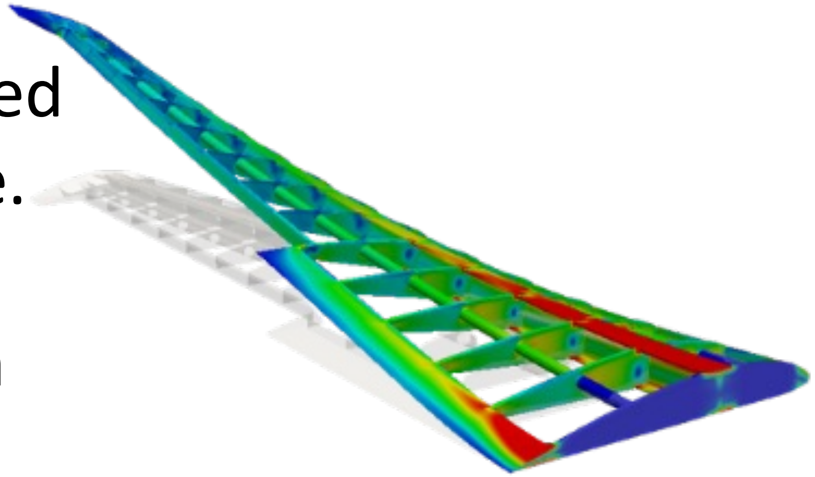
Structural analysis of Boeing aircraft



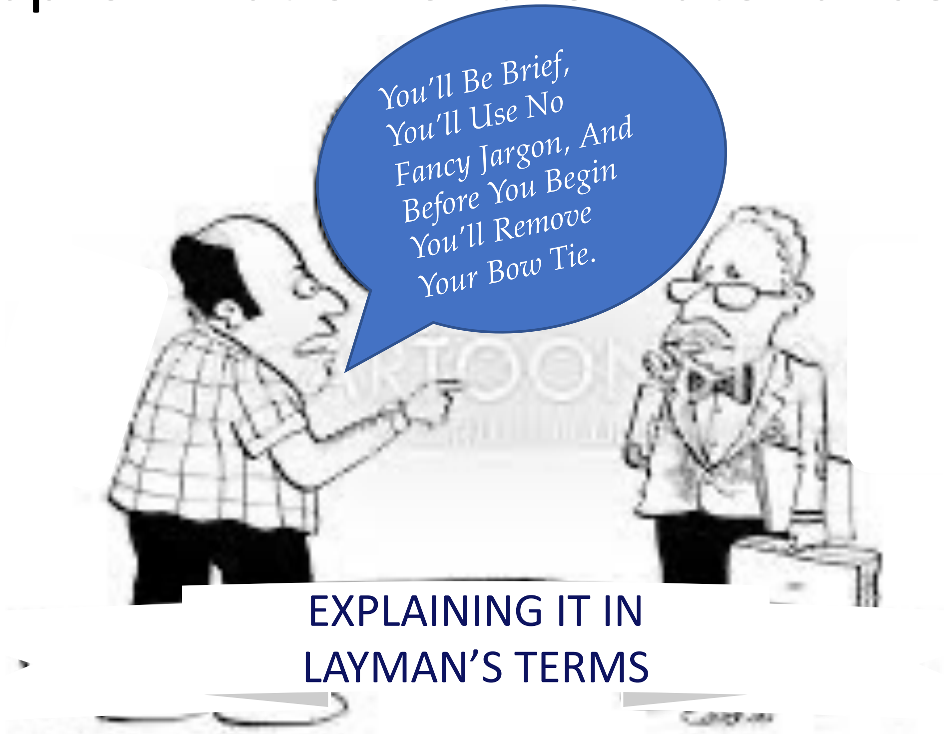
The drawing shows thousands of parts, which can be modeled as shells, beams, solids. A part is broken up into millions of elements.

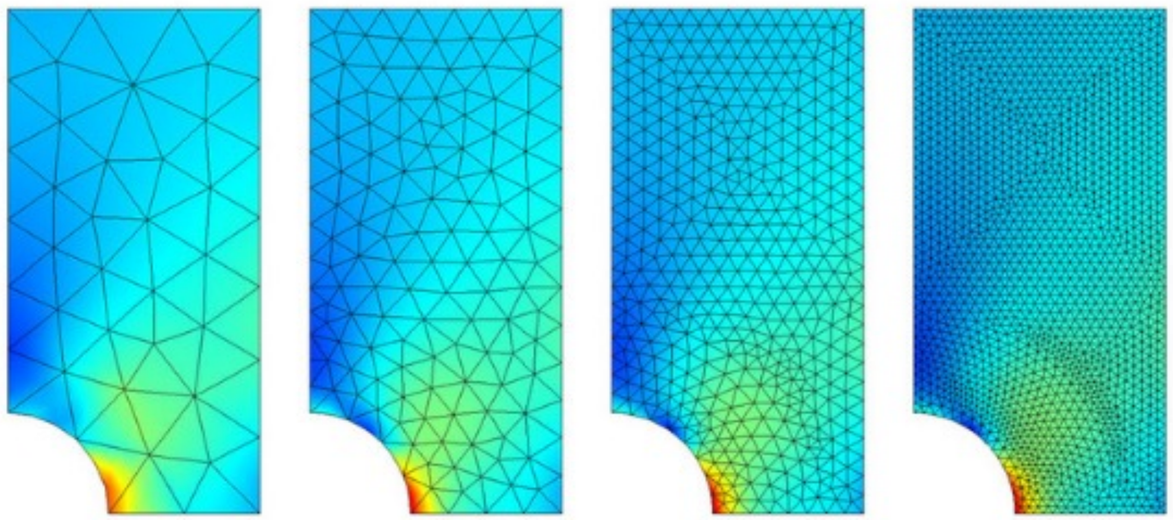
What is FEM?

Breaks up a structure into a large number of finite elements, which are much simpler, more easily solved for loads and stresses than the geometry as a whole.



Each element is then ‘summed up’ to provide a high accuracy approximation of the material behavior.



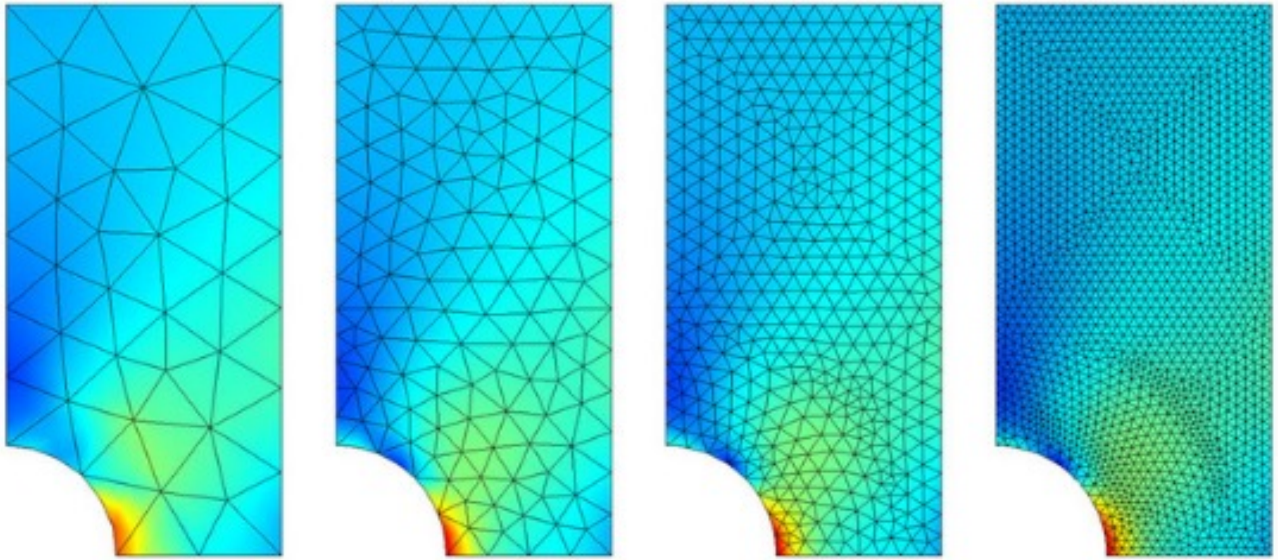


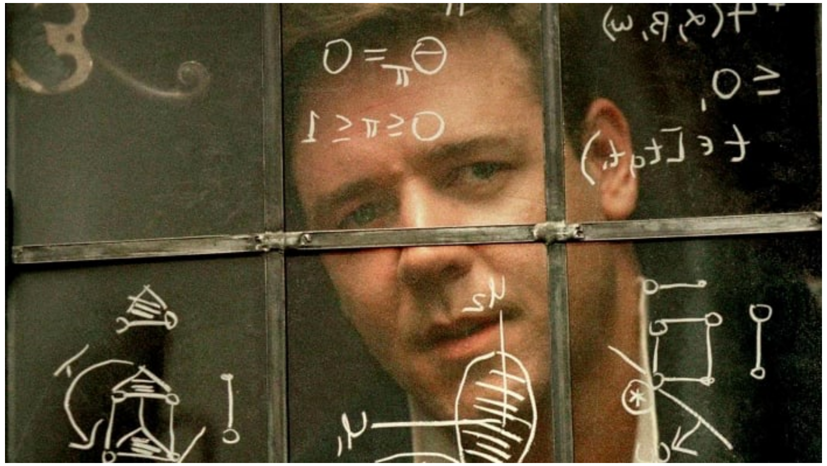
What is FEM?

- FEM introduces a piecewise approximation to the governing equations

- It discretizes the whole domain in elements and writes approximate equations for each element

- It then assembles the local equations

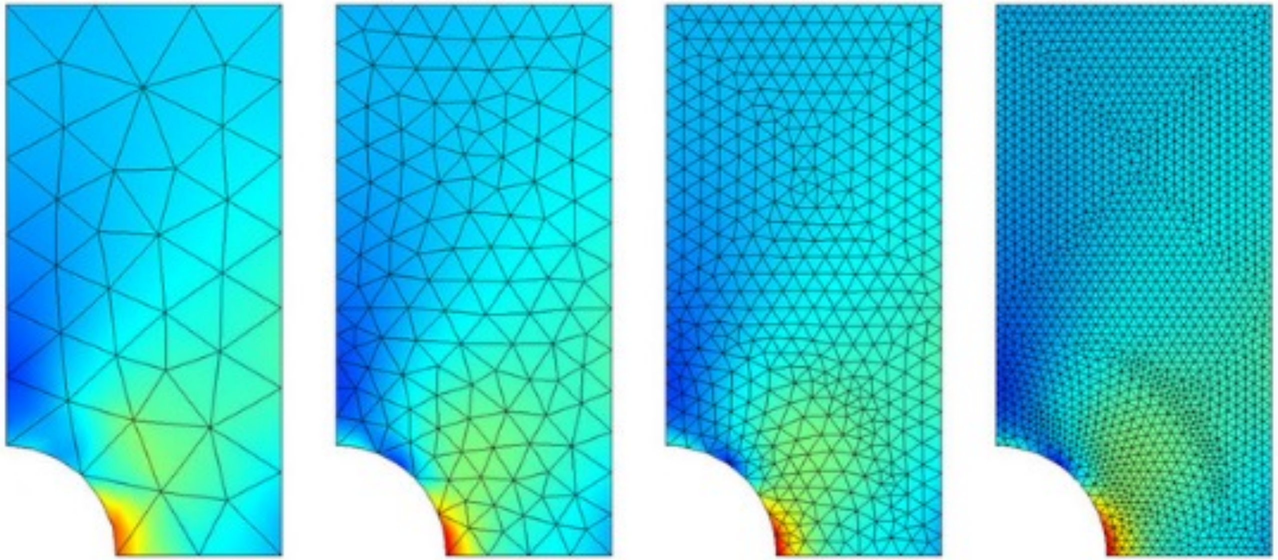




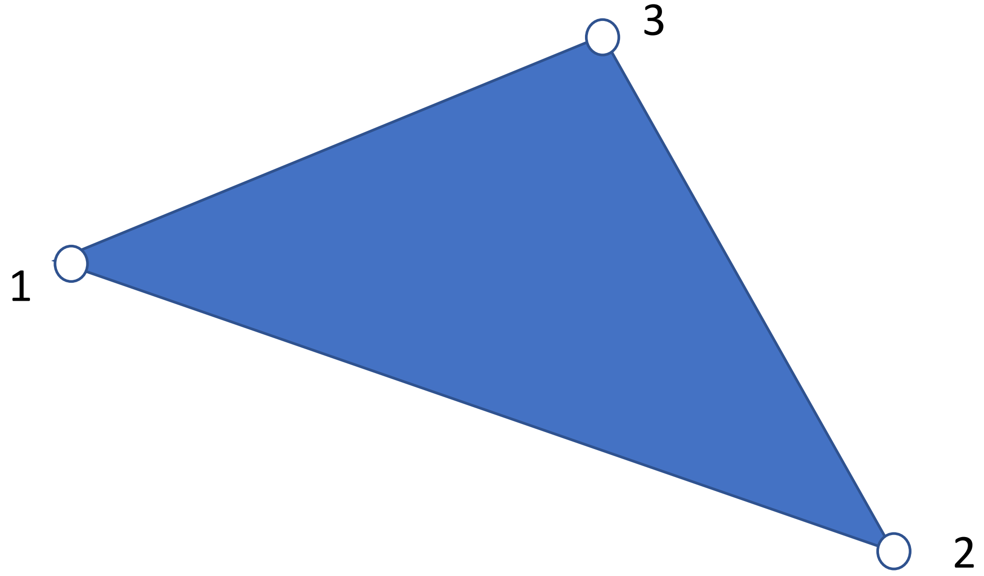
Continuum Mechanics Problems

We need to approximate the main unknowns (infinite number) with a finite number of unknowns.

$$v(x,t) \in B$$



In each discrete element of the domain, we interpolate the unknown fields using their values at the nodes of the element.



The nodes can be at the boundaries or inside the element.

Approximating the Governing Equations

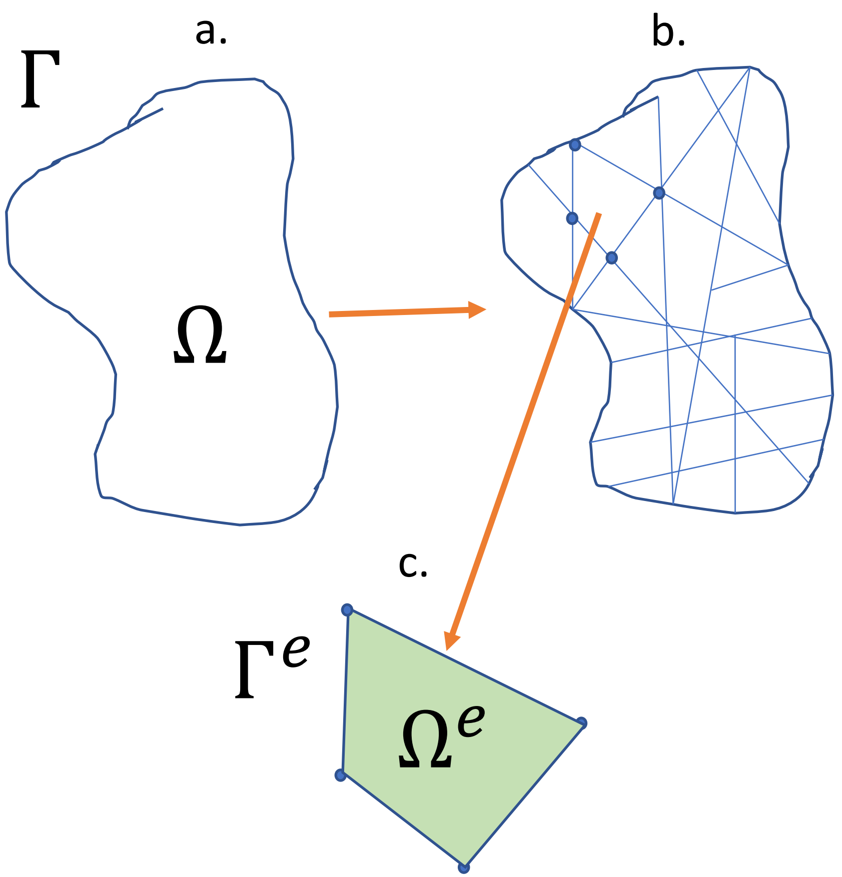
There is a common mathematical theory underlying how we locally approximate the governing equations in each element

- Direct approach (trusses, beams, etc.)
- Variational approach
- Galerkin approach (weighted residual)

These methods transform the original differential equation governing the physics of the problem into an algebraic problem of the form [A]{x}={b}

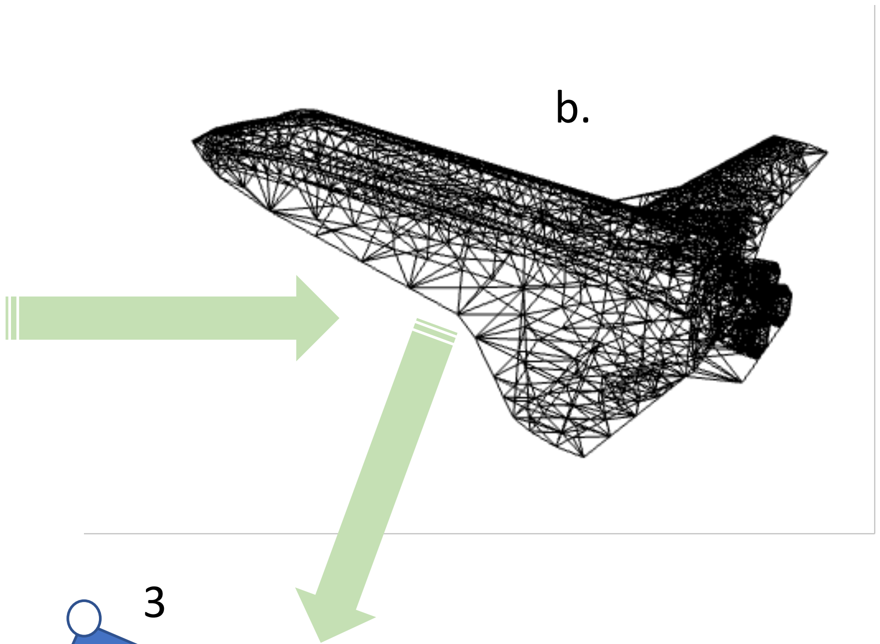
Overview of the Steps in FEM

Step 1. Mesh generation – Discretize the domain in ‘elements’

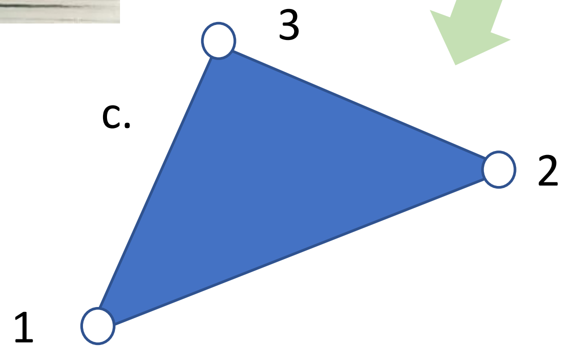


a.

b.



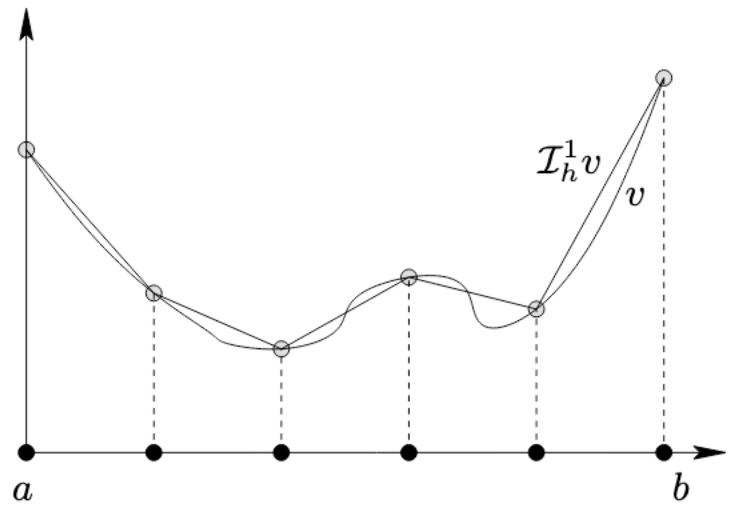
c.



Overview of the Steps in FEM

Step 2. In each element, use interpolation functions to approximate your unknown fields in terms of 'nodal values'

One-dimensional interpolation



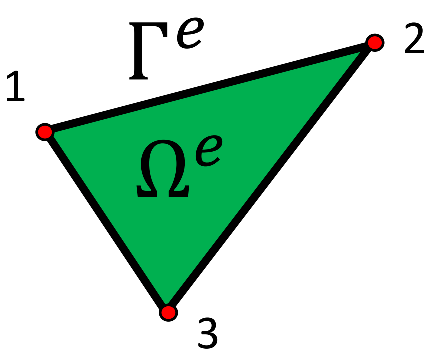
Interpolation by continuous, piecewise linear functions.

Overview of the Steps in FEM

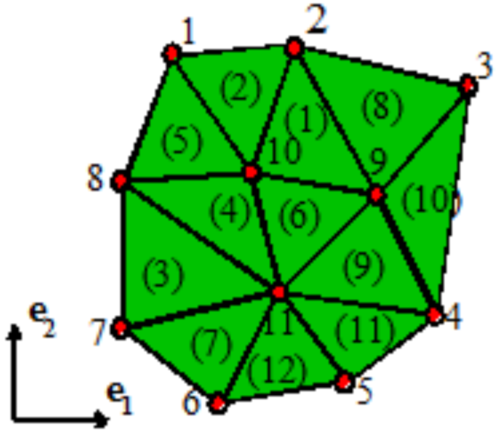
Step 3. Calculate local form of the governing equations, for example, the Galerkin approximation

- Each triangular element has 3 nodes: 1,2,3

Local View



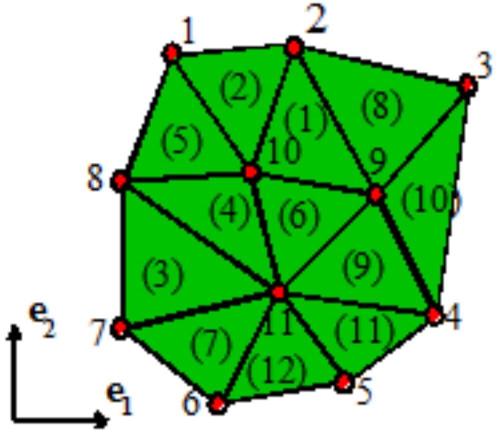
Global View



Overview of the Steps in FEM

Step 4. Assemble the local equations in a global set of algebraic equations of the form [K]{u}={F}

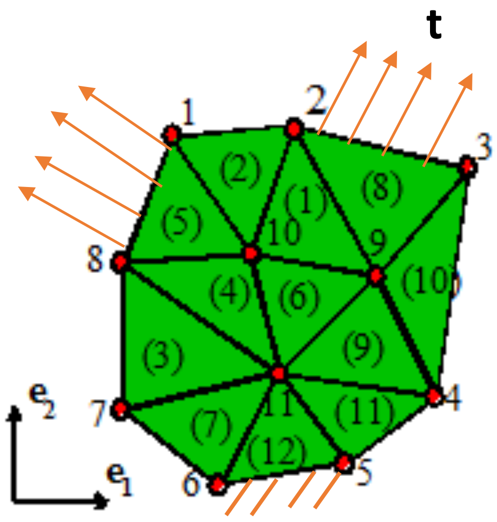
- Each triangular element has 3 nodes: 1,2,3
- To assemble, we will need to relate the local node numbers to global node numbers
- You will create a matrix that represents this information, known as the element connectivity matrix [T]



Overview of the Steps in FEM

Step 5. Apply the Boundary Conditions

- This can be a complicated step especially if the B.Cs are not expressed in terms of the unknown field (for example, traction/loads when unknowns are displacements
- Essential BCs, Natural, Mixed etc.



Step 6. Solve the Resulting System of Algebraic Equations

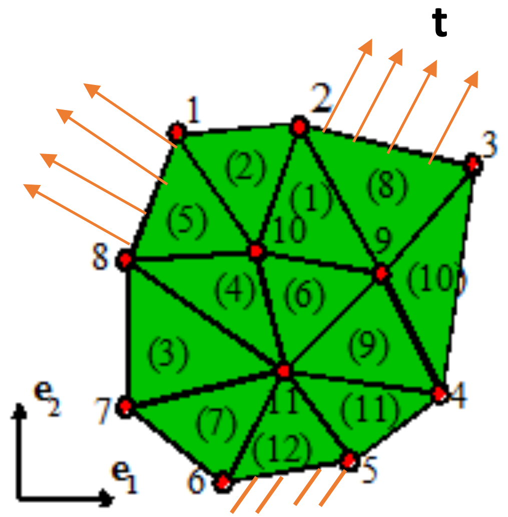
- Not a real focus in this course, but know that this is an important numerical step in terms of CPU cost

Overview of the Steps in FEM

Step 5. Apply the Boundary Conditions (BCs)

Step 6. Solve the resulting System of Algebraic Equations

Step 7. Post-process the results, for example, compute the stresses once the displacements are known



Linear and Nonlinear FEM

FEM is valuable for problems with no analytical solutions (most all engineering problems)

- Deformation problems with complex geometries and complex boundary conditions, and nonlinear material behavior
- Solving Navier-Stokes equations

This course focuses on Linear FEM (applicable for structural mechanics, linear static problems, heat transfer, dynamics and vibrations, basic fluid mechanics)

AERO 510 – IN THIS COURSE…

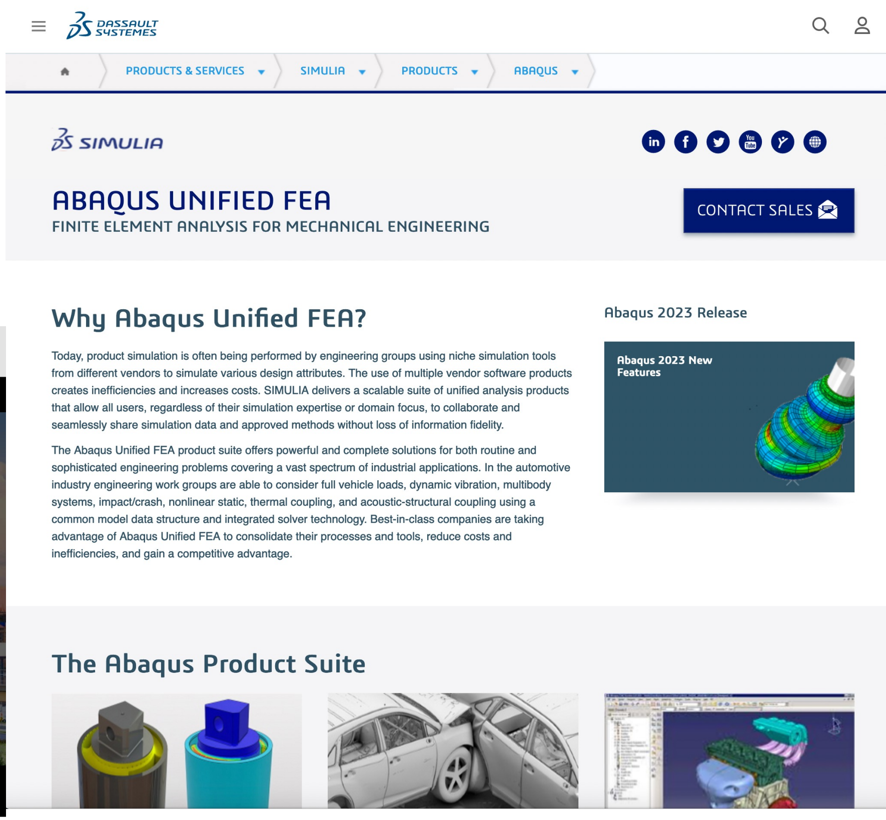
1. We will start by breaking a mechanical structure into components (elements), assemble the system equations, and proceed to the solution process and post-processing steps

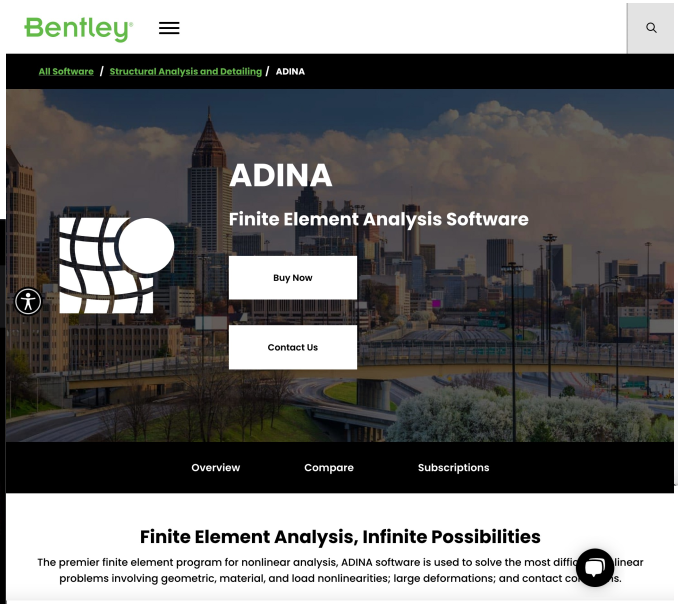
2. We will delve into the mathematical backbone of FEM using Galerkin approximations and discretizing the differential equations (FEM is deeply mathematical - an integral part of the course)

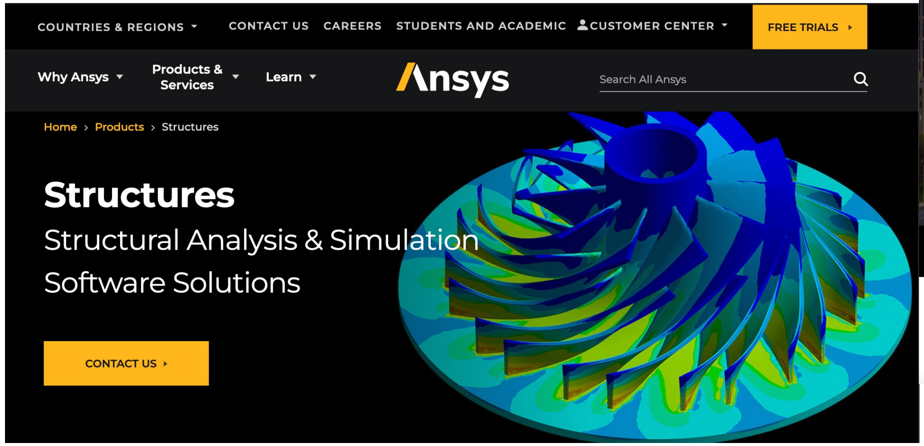
3. We will connect the methods where appropriate (minimum potential energy, principle of virtual work etc.)

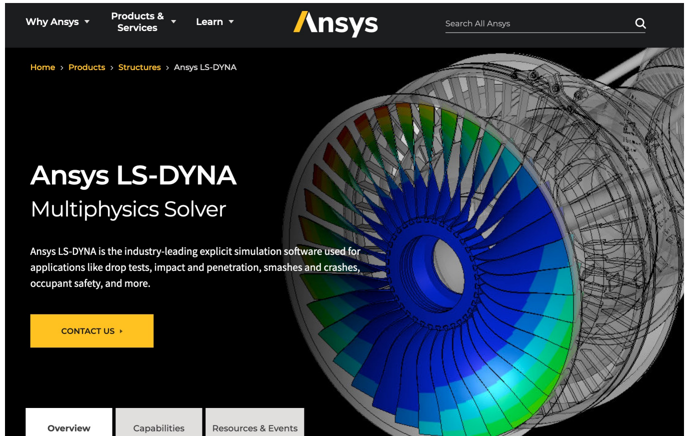
Commercial FEM tools

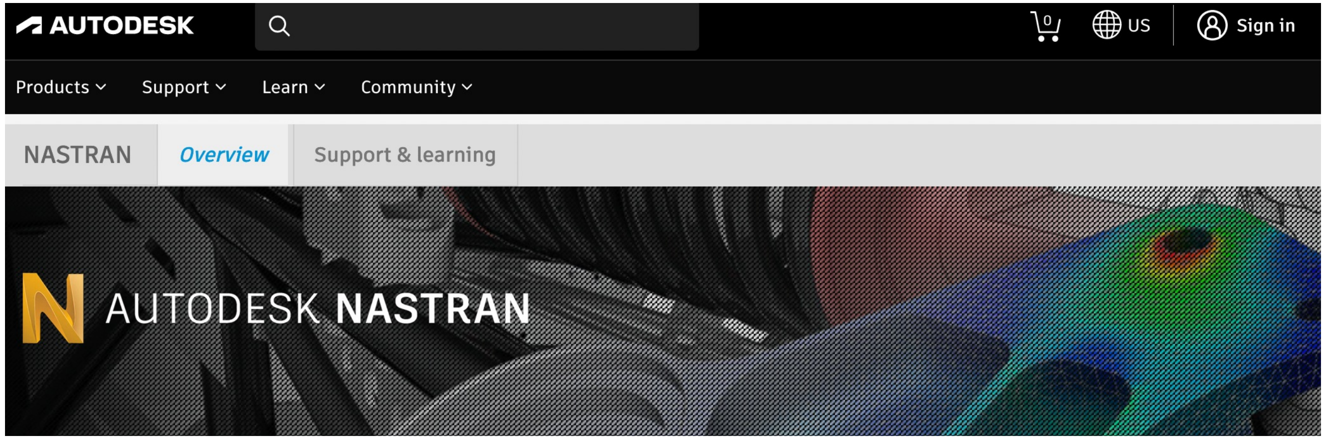
- Interface with CAD databases
- Userfriendly GUIs
- Black box for user-specified problems
- Lots of finite element choices and options
- Multiphysics coupling capability







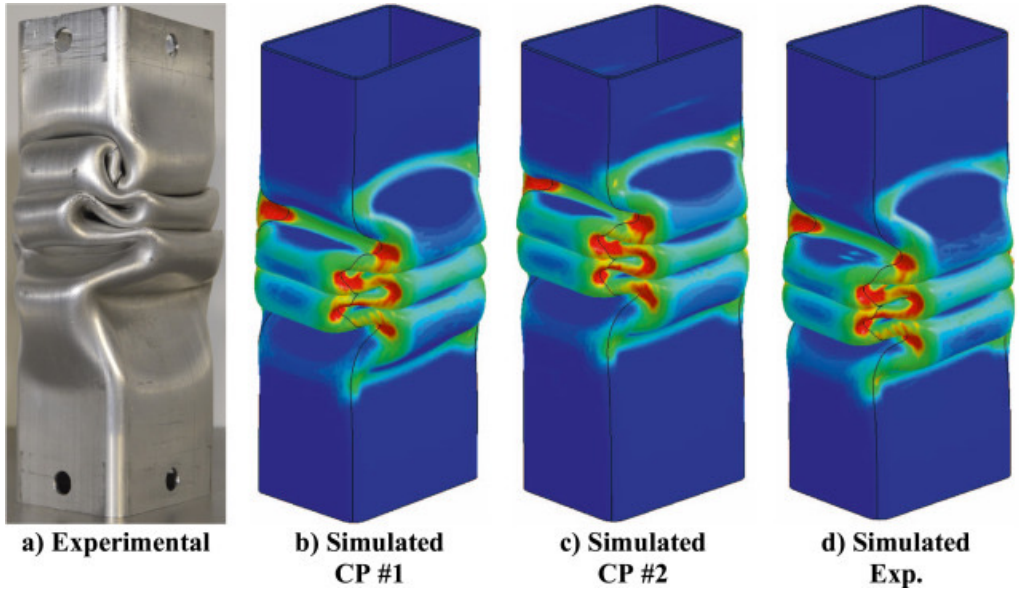




Matlab use in the course

This course is a computational mechanics course. The homeworks will help you *learn* the numerical method, so you will use a lot of Matlab.

The Matlab programs are not perfect. In fact, the commercial codes are not written in Matlab. So what are the benefits?



- You learn the fundamentals
- You learn numerical methods by doing numerical methods
- In depth experience of how FEM works to solve engineering problems
- Integrates pieces from other courses.
- You will be expected to understand the problems and the codes in fine detail, being able to use, modify, and extend them

Some of you will move on to advanced topics in your PhD research or when you go to work for Space-X, ABAQUS, Ford, or Sandia!

Your course questions

1. How reliant is it on previous AE 315 knowledge?
2. What kinds of problems can we solve at the end of this course?
3. Why Matlab?/ What's the minimum version of Matlab for the course?
4. **Ratio of handwritten work to coding?/What's the balance of written/code component of the HW/Exams?/ Is it coding intensive?/ Sample code provided?/ Are there any formatting requirements for the class i.e. hw/code? HW pdf format? Roughly written? Report?/ How long to spend per homework? Length/difficulty of homeworks?/How long is the exam?/Length of exams?/How hard are homeworks and exams? Are they graded on a curve?**