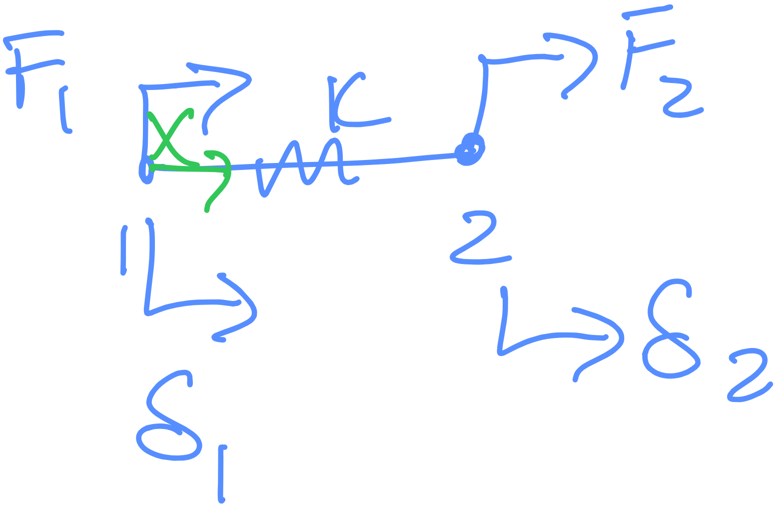
2D Trusses

1D Truss

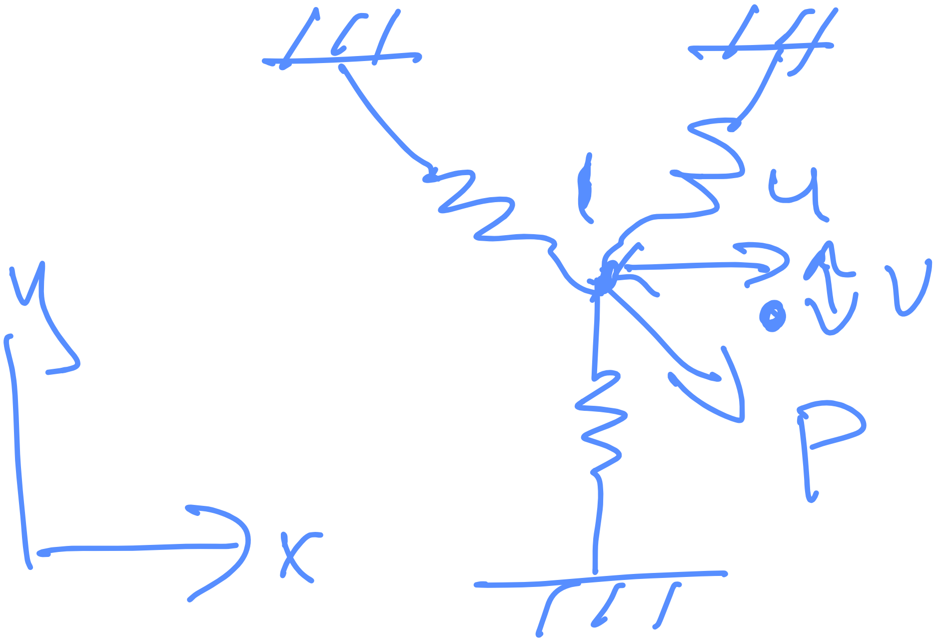


$F_2 = K(\delta_2 - \delta_1)$

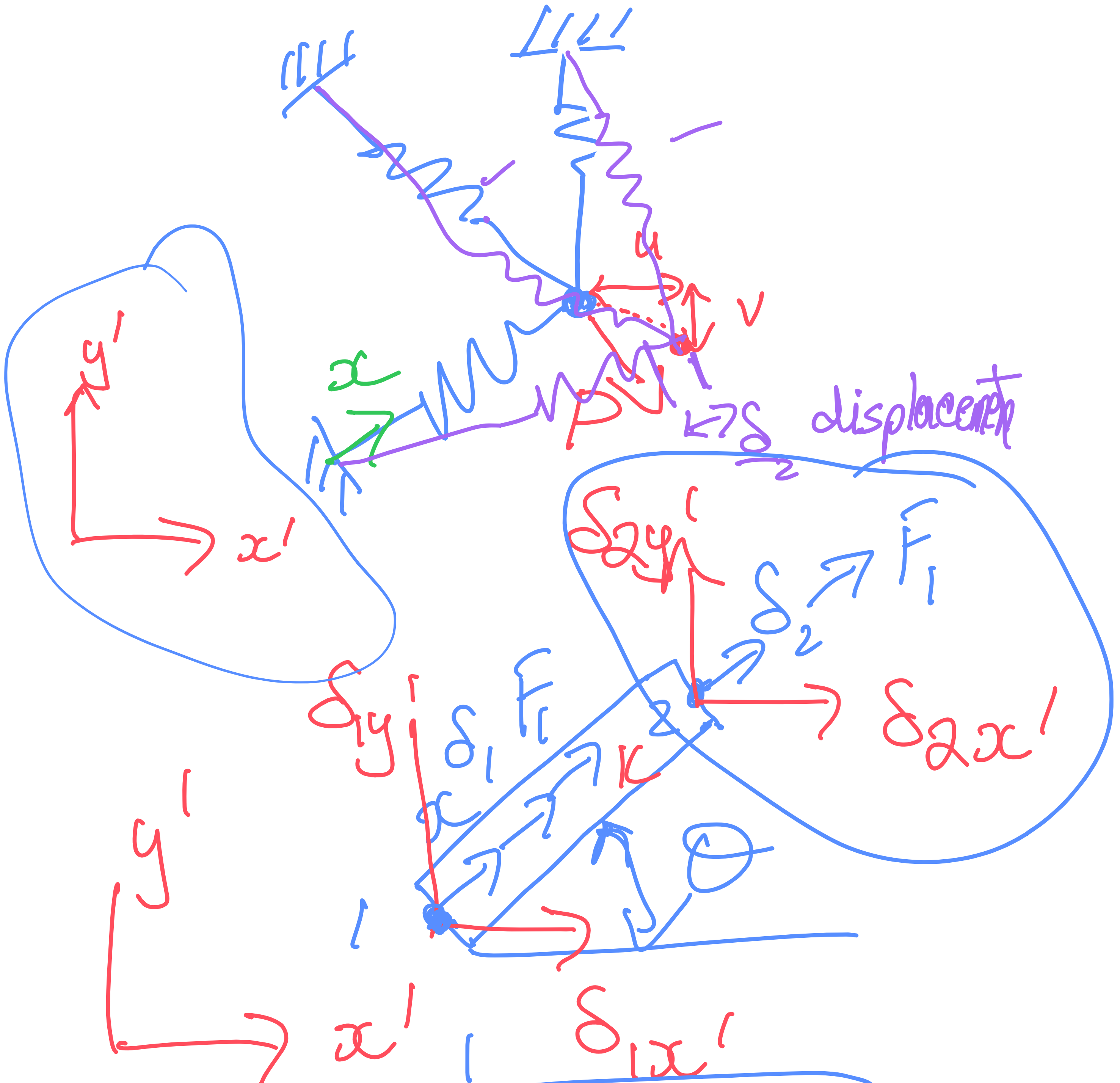
$F_1 = -F_2$ (equil)

$F_1 = K(\delta_1 - \delta_2)$

2D Truss



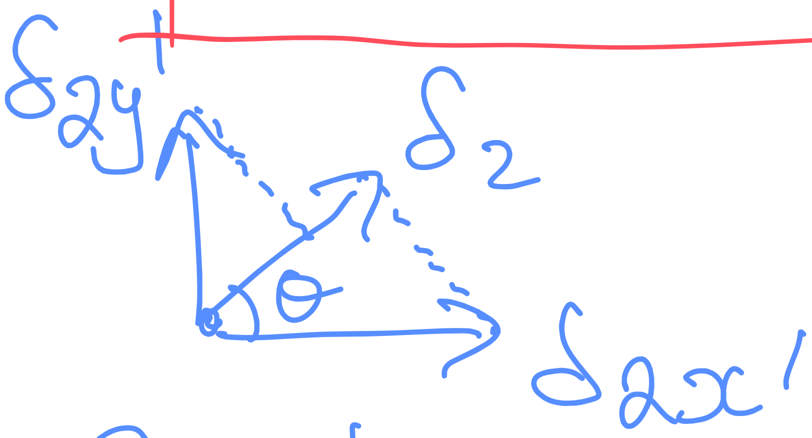
2D Truss



$$\boxed{\begin{aligned} F_2 &= K(\delta_2 - \delta_1) \\ F_1 &= K(\delta_1 - \delta_2) \end{aligned}}$$

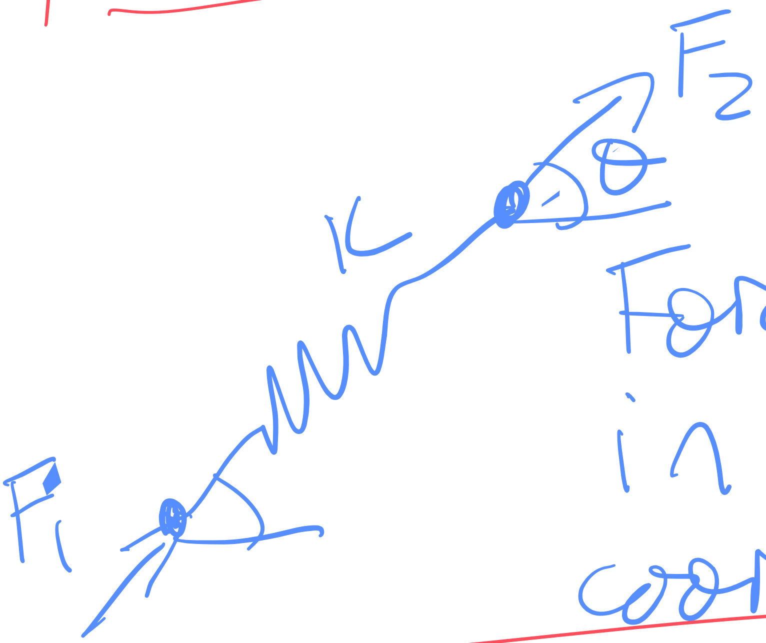
Force is only along the truss

$$\boxed{\delta_2 = \delta_{2x'}\cos\theta + \delta_{2y'}\sin\theta}$$



Similarly,

$$\boxed{\delta_1 = \delta_{1x'}\cos\theta + \delta_{1y'}\sin\theta}$$



Force on node 2 in $x'-y'$ coordinate system

$$\boxed{\underline{F}_2 = \begin{bmatrix} F_2\cos\theta \\ F_2\sin\theta \end{bmatrix} = \begin{bmatrix} F_{2x'} \\ F_{2y'} \end{bmatrix}}$$

Similarly →

$$\underline{F}_1 = \begin{bmatrix} F_1\cos\theta \\ F_1\sin\theta \end{bmatrix} = \begin{bmatrix} F_{1x'} \\ F_{1y'} \end{bmatrix}$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix}}_{T\text{ matrix}} \begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{2x'} \\ \delta_{2y'} \end{bmatrix}$$

Eq 1.

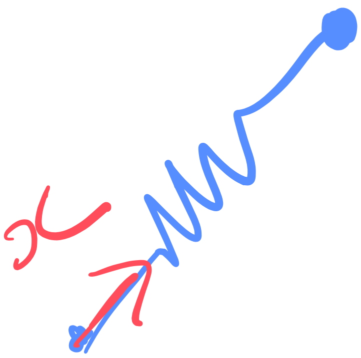
$$\begin{bmatrix} F_{1x'} \\ F_{1y'} \\ F_{2x'} \\ F_{2y'} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

Eq. 2.

T^T

T : Transformation matrix

T^T : transpose of T



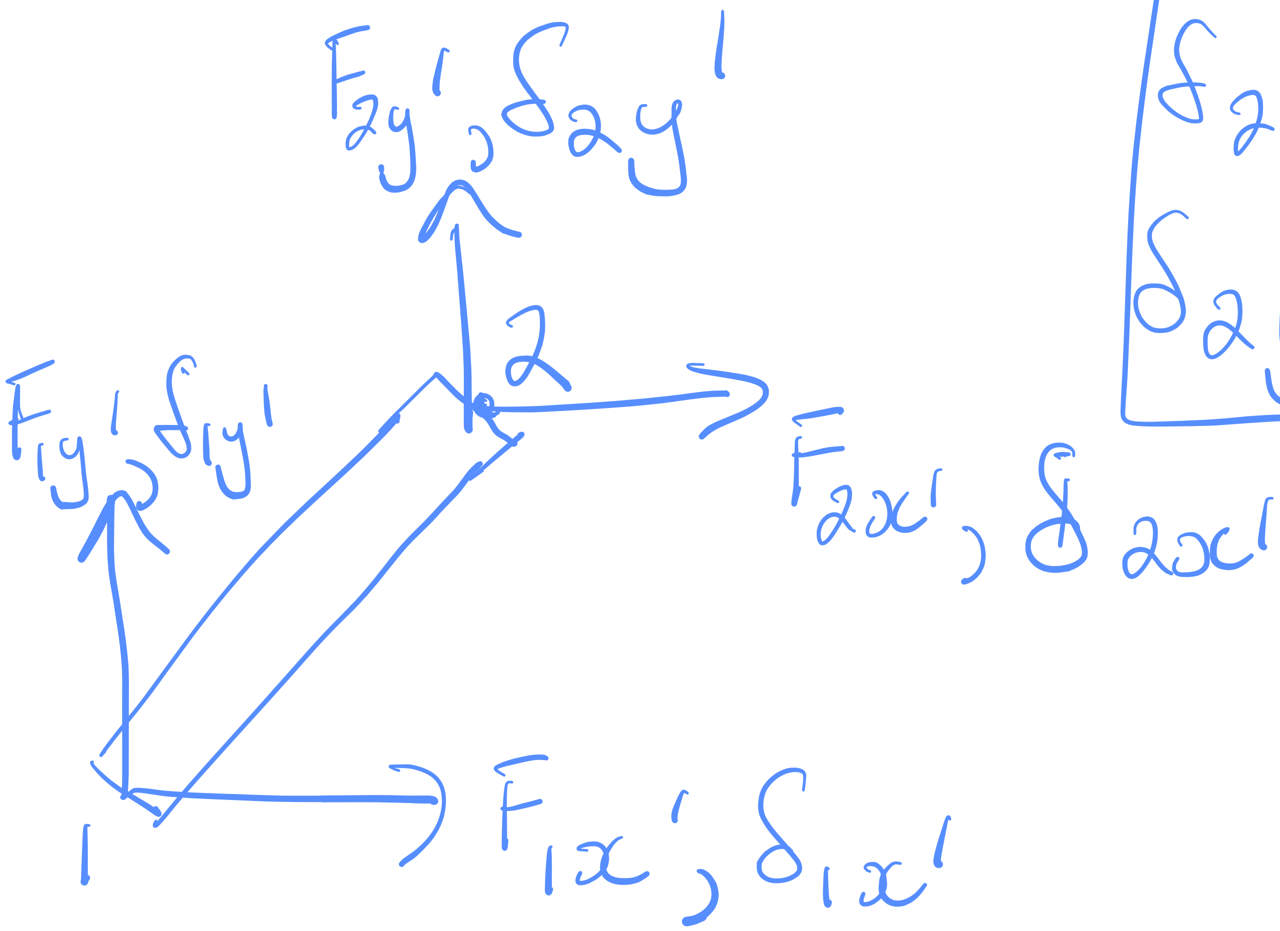
$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{k} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = K\,T \begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{2x'} \\ \delta_{2y'} \end{bmatrix}$$

Using Eq 2.

$$\begin{bmatrix} F_{1x'} \\ F_{1y'} \\ F_{2x'} \\ F_{2y'} \end{bmatrix} = T^T \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$= T^T K T \begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{2x'} \\ \delta_{2y'} \end{bmatrix}$$



$$\begin{bmatrix} F_{1x}' \\ F_{1y}' \\ F_{2x}' \\ F_{2y}' \end{bmatrix} = \left[T^T K T \right] \begin{bmatrix} \delta_{1x}' \\ \delta_{1y}' \\ \delta_{2x}' \\ \delta_{2y}' \end{bmatrix}$$

new transformed local stiffness matrix

$T^T K T$: $[4 \times 2][2 \times 2][2 \times 4]$

local : 4×4

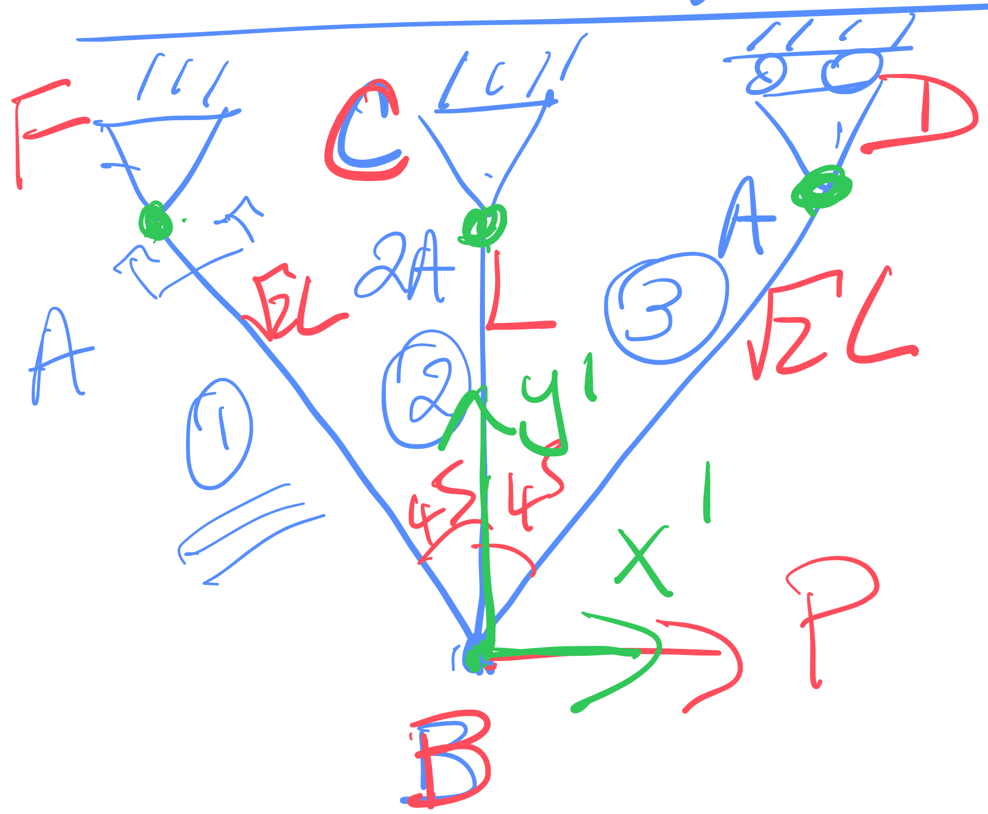
stiffness matrix for 2D trusses

$$T^T K T = k \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$$k = \frac{EA}{L}$$

s : $\sin\theta$
c : $\cos\theta$

Example of a 2D truss



Trusses @ 45°
3 Trusses
1, 2, 3

① : $E, A, \sqrt{2}L \Rightarrow k_1 = \frac{EA}{\sqrt{2}L}$

② : $E, 2A, L \Rightarrow k_2 = \frac{2EA}{L}$

③ : $E, A, \sqrt{2}L \Rightarrow k_3 = \frac{EA}{\sqrt{2}L}$

Step 1: Mesh

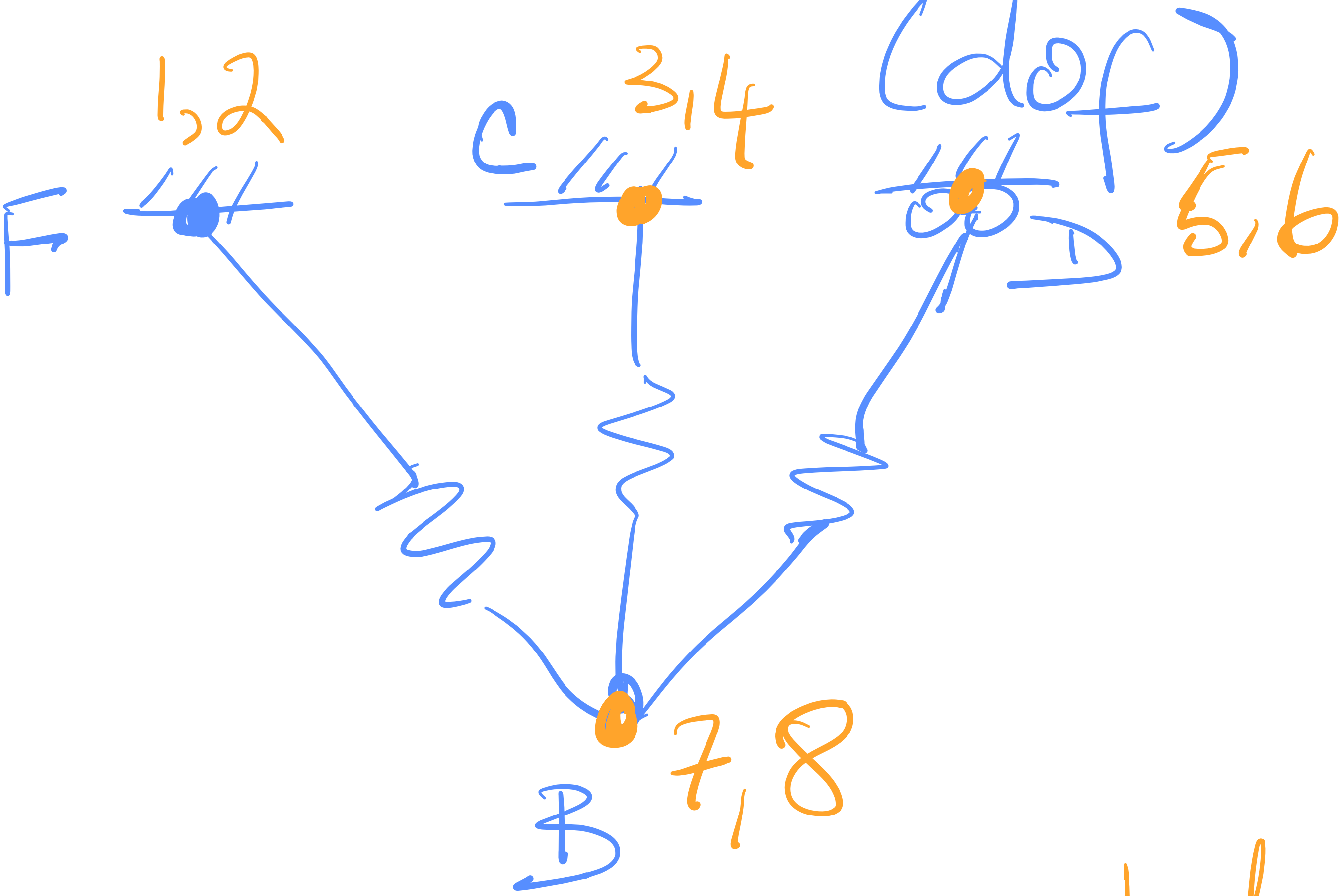
Coordinate Matrix

$$\begin{array}{c|cc} & x & y \\ B & 0 & 0 \\ F & - & L \\ C & 0 & L \\ D & L & L \end{array}$$

Connectivity Matrix

$$\begin{array}{c|cc} & \text{node1} & \text{node2} \\ ① & B & F \\ ② & B & C \\ ③ & B & D \end{array}$$

Numbering of degrees of freedom

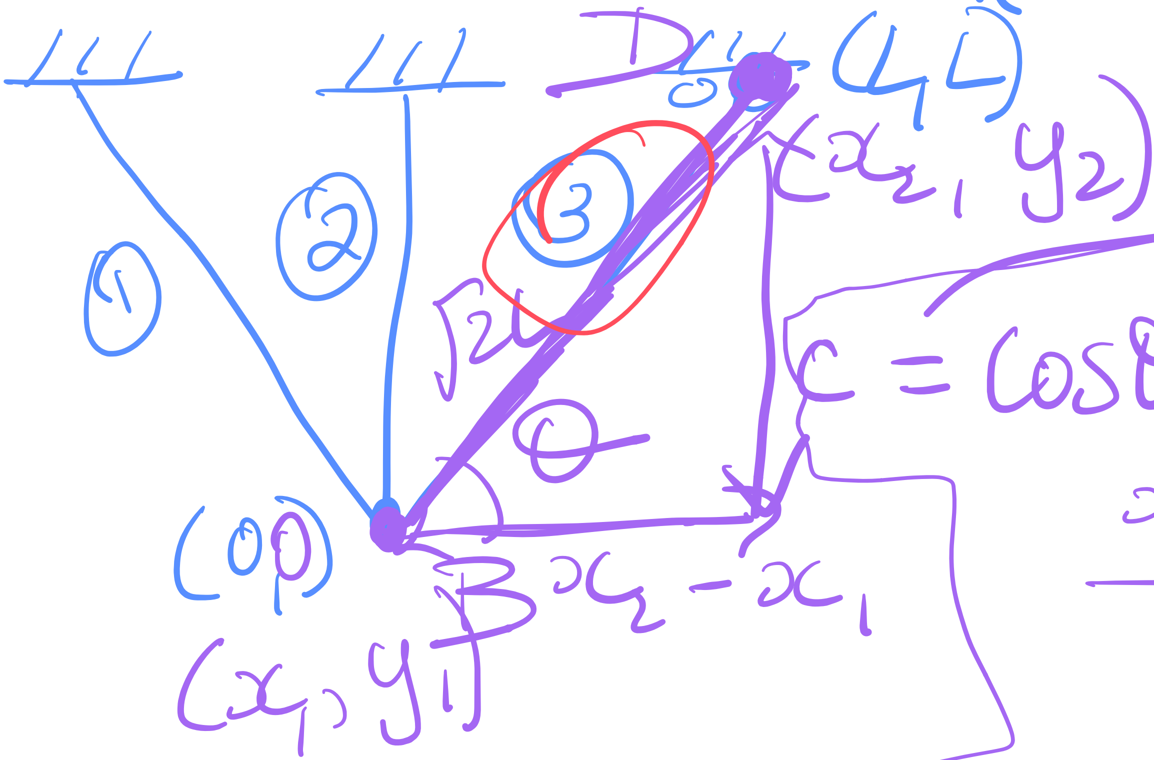


1 : x-displacement of F

2 : y-displacement of F

	x-dof	y-dof	
B	7	8	
C	3	4	
D	5	6	u_{Dx}, u_{Dy}
F	1	2	u_{Fx}, u_{Fy}

Step 2 Assemble global matrix



$$c = \cos\theta = \frac{x_2 - x_1}{\sqrt{2}L} = \frac{L-0}{\sqrt{2}L} = \frac{1}{\sqrt{2}}$$

$$s = \sin\theta = \frac{y_2 - y_1}{\sqrt{2}L} = \frac{L-0}{\sqrt{2}L} = \frac{1}{\sqrt{2}}$$

$$T^T K T$$

$$T = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}$$

$$K = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix}$$

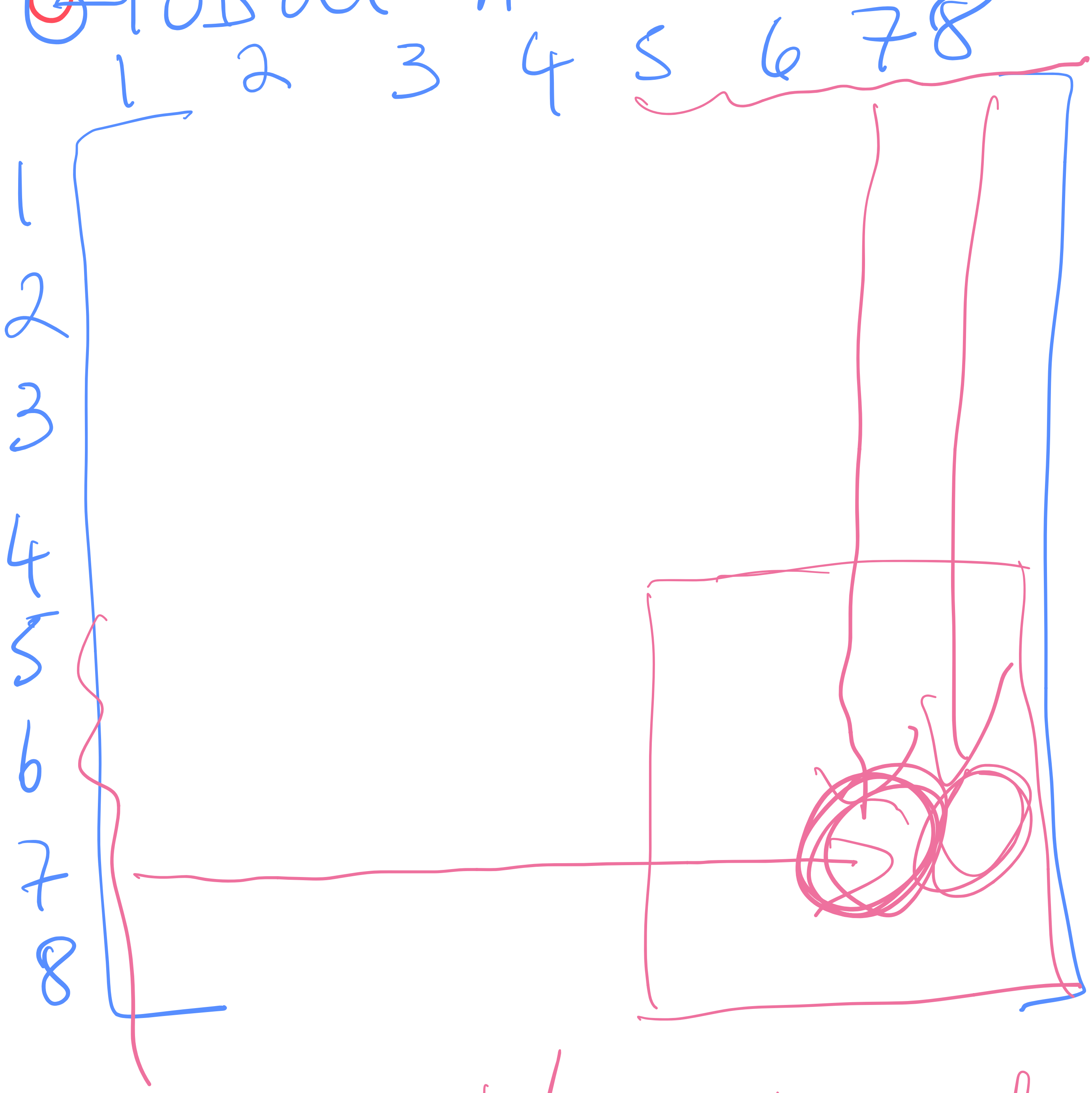
$$k_3 = \frac{EA}{\sqrt{2} L}$$

$$T^T K T =$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{EA}{\sqrt{2}L} & -\frac{EA}{\sqrt{2}L} \\ -\frac{EA}{\sqrt{2}L} & \frac{EA}{\sqrt{2}L} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{array}{cc} & \\ B & \begin{matrix} 7 \\ 8 \end{matrix} \\ D & \begin{matrix} 5 \\ 6 \end{matrix} \end{array} = \frac{EA}{\sqrt{2}L} \begin{array}{c} \begin{matrix} 7 & \quad 8 & \quad 5 & \quad 6 \end{matrix} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array}$$

Global matrix



Now let's solve the problem