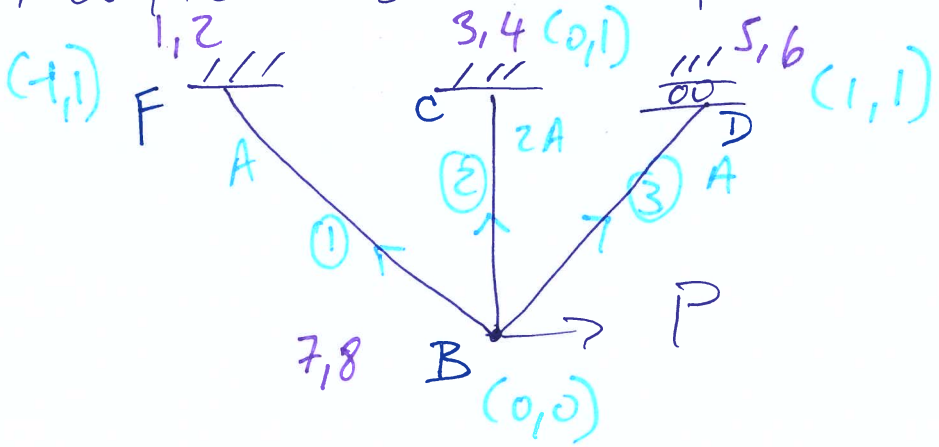
Now, let's solve the problem



$P = 10^3 N$

$A = 10^{-2} m^2$

$E = 10^7 Pa$

$L = 1$

Find displacement: u_{Bx}, u_{By}, u_{Dx}

①	B → F
②	B → C
③	B → D

Formula

$$K = \frac{EA}{L}$$

c : $\cos\theta$

s : $\sin\theta$

2D TRUSS

$$\begin{bmatrix} \begin{pmatrix} c^2 & cs \\ sc & s^2 \end{pmatrix} & \begin{pmatrix} -c^2 & -sc \\ -sc & -s^2 \end{pmatrix} \\ \begin{pmatrix} -c^2 & -sc \\ -sc & -s^2 \end{pmatrix} & \begin{pmatrix} c^2 & cs \\ sc & s^2 \end{pmatrix} \end{bmatrix}$$

1,2

TRUSS ①

	x_2	y_2
F	-1	1

$\sqrt{2}$

135°

7,8

B

x_1	y_1
0	0

CODE

$$\cos\theta = \frac{x_2 - x_1}{\text{Length}} = -\frac{1}{\sqrt{2}}$$

$$\sin\theta = \frac{y_2 - y_1}{\text{Length}} = \frac{1}{\sqrt{2}}$$

BY HAND → OR

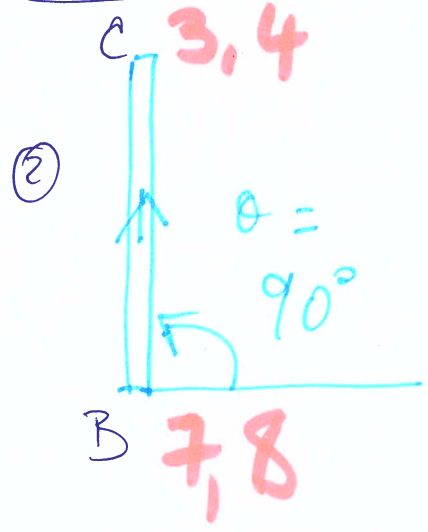
$\cos(135) = -\frac{1}{\sqrt{2}}$

$\sin 135 = 1/\sqrt{2}$

$$K = \frac{EA}{\sqrt{2}}$$

	7	8	1	2	
7	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	7
8	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	8
1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1
2	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2

TRUSS ②



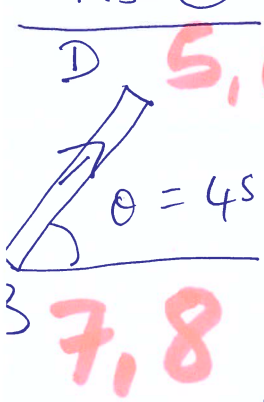
$$K = \frac{EA}{\sqrt{2}}$$

	7	8	3	4	
7	0	0	0	0	7
8	0	$2\sqrt{2}$	0	$-2\sqrt{2}$	8
3	0	0	0	0	3
4	0	$-2\sqrt{2}$	0	$2\sqrt{2}$	4

$\cos 90 = 0$

$\sin 90 = 1$

TRUSS ③



$$K = \frac{EA}{\sqrt{2}}$$

	7	8	5	6	
7	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	7
8	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	8
5	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	5
6	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	6

$\cos\theta = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$

Assembly

Global stiffness matrix

$\frac{EA}{\sqrt{2}}$

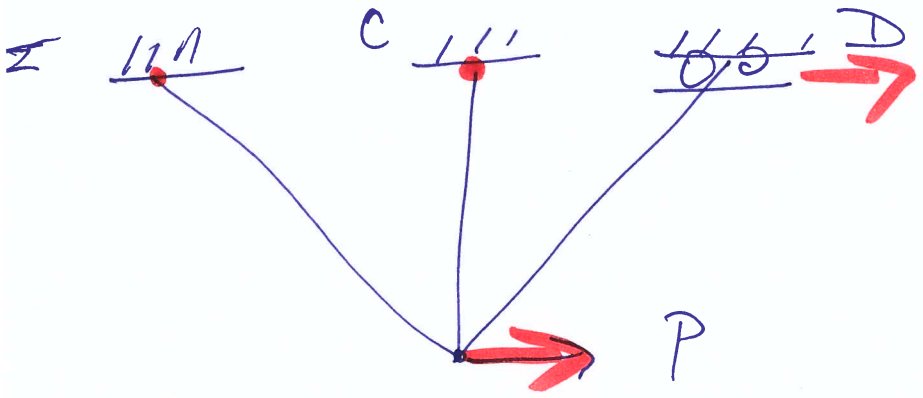
	1	2	3	4	5	6	7	8
1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
2	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
3	0	0	0	0	0	0	0	0
4	0	0	0	$2\sqrt{2}$	0	0	0	$-2\sqrt{2}$
5	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
6	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
7	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}+0+\frac{1}{2}$	$-\frac{1}{2}+0+\frac{1}{2}$
8	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-2\sqrt{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}+0+\frac{1}{2}$	$\frac{1}{2}+2\sqrt{2}+\frac{1}{2}$

$$
\begin{bmatrix} R_{Fx} \\ R_{Fy} \\ R_{Cx} \\ R_{Cy} \\ 0 \\ R_{Dy} \\ P \\ 0 \end{bmatrix}
\quad \text{Roller} \rightarrow 0
$$

$\frac{EA}{\sqrt{2}} =$

$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
0	0	0	0	0	0	0	0
0	0	0	$2\sqrt{2}$	0	0	0	$-2\sqrt{2}$
0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}+0+\frac{1}{2}$	$-\frac{1}{2}+0+\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{2}$	0	$-2\sqrt{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}+0+\frac{1}{2}$	$\frac{1}{2}+2\sqrt{2}+\frac{1}{2}$

$$
\begin{bmatrix} u_{Fx} \\ u_{Fy} \\ u_{Cx} \\ u_{Cy} \\ u_{Dx} \\ u_{Dy} \\ u_{Bx} \\ u_{By} \end{bmatrix}
$$



Apply B.C.'s : delete rows & columns [1, 2, 3, 4, 6]

$$\begin{bmatrix} 0 \\ P \\ 0 \end{bmatrix} = \frac{EA}{\sqrt{2}} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}+0+\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 1+2\sqrt{2} \end{bmatrix} \begin{bmatrix} u_{Dx} \\ u_{Bx} \\ u_{By} \end{bmatrix}$$

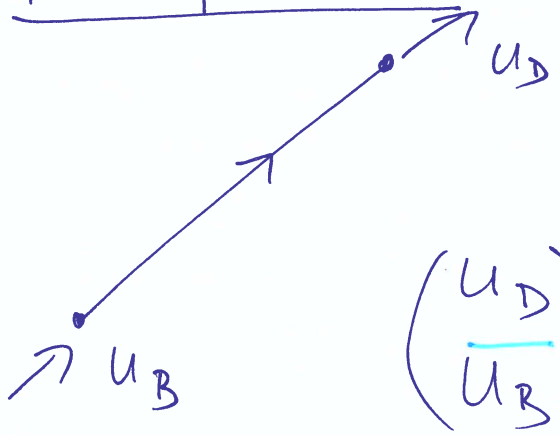
Use: $P = 10^3$ & $A = 10^{-2}$ & solve

$E = 10^7$

$u_{Dx} = 0.038\ m$

$u_{Bx} = 0.033\ m$ $\qquad u_{By} = 0.005\ m$

Post process



$$T = E\left(\frac{u_D - u_B}{L}\right)$$

$$\begin{pmatrix} u_D \\ u_B \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{pmatrix} \begin{bmatrix} u_{Dx} \\ u_{Dy} \\ u_{Bx} \\ u_{By} \end{bmatrix}$$

transformation matrix

$$\sigma = \frac{E}{L}\left(c\,u_{Bx} + s\,u_{By} - c\,u_{Bx} - s\,u_{By} \right)$$

STRESS FORMULA

$$\sigma = \frac{E}{L}\begin{bmatrix} -\cos\theta & -\sin\theta & \cos\theta & \sin\theta \end{bmatrix}\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{bmatrix}$$

where B = node 1

D = node 2

$u_{Bx} = u_{1x}$

$u_{By} = u_{1y}$

STRESS ①

F = node 2

135°

B = node 1

$$\sigma = \frac{E}{\sqrt{2}}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} u_{Bx} \\ u_{By} \\ u_{Fx} \\ u_{Fy} \end{bmatrix}$$

$u_{Fx} = 0$, $u_{Fy} = 0$

$$\sigma^{①} = \frac{E}{\sqrt{2}}\left(\frac{u_{Bx}}{\sqrt{2}} - \frac{u_{By}}{\sqrt{2}} \right) = 141.4 \text{ kPa}$$

$E = 10^7$, $u_{Bx} = 0.033$, $u_{By} = 0.005$

$\sigma^{②} = -50$ kPa

$\sigma^{③} = 0 \Leftarrow$ roller