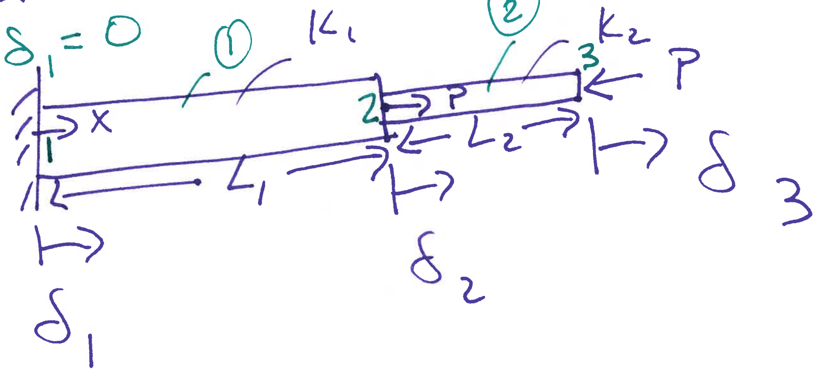
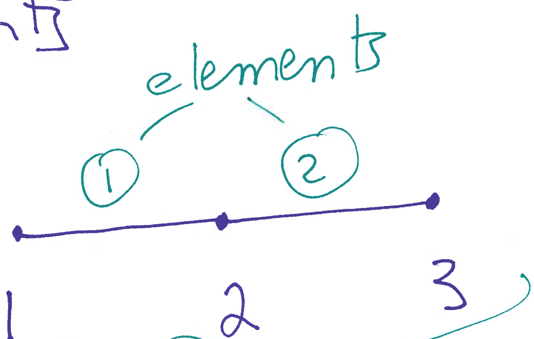
Consider a Truss system



1. Find δ_2 & δ_3 the displacements
2. Find stress distribution

<u>Step 1</u>: Create the 'mesh'

- This is a computer generated partition of an object into components called elements



$x_1 = 0$ $\quad x_2 = L_1$ $\quad x_3 = L_1 + L_2$

global nodes

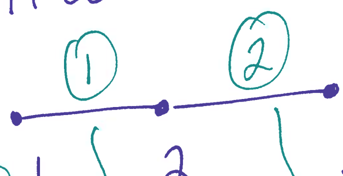
Form the coordinate matrix

$$x = \begin{bmatrix} 0 \\ L_1 \\ L_1 + L_2 \end{bmatrix} \begin{array}{l} \leftarrow \text{node 1} \\ \leftarrow \text{node 2} \\ \leftarrow \text{node 3} \end{array}$$

Step 2.
Connectivity matrix

$$
\begin{array}{cc}
① & \\
② &
\end{array}
\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}
$$

Step 3.
Elemental Matrix Equations



For element ①

$$
\begin{bmatrix} F_1^{①} \\ F_2^{①} \end{bmatrix} = \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}
$$

For element ②

$$
\begin{array}{c} 2 \\ 3 \end{array}
\begin{bmatrix} F_2^{②} \\ F_3^{②} \end{bmatrix} = \begin{bmatrix} K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix}
$$

$F_2^{①} + F_2^{②}$ = net external force on node 2

Step 4.
Assemble global equations

$$
\begin{array}{c} 1 \\ 2 \\ 3 \end{array}
\underbrace{\begin{bmatrix} \cancel{F_1\,?} \\ F_2\ {+P} \\ F_3\ {-P} \end{bmatrix}}_{\text{external forces}}
=
\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\underbrace{\begin{bmatrix} \cancel{K_1} & \cancel{-K_1} & \cancel{0} \\ \cancel{-K_1} & K_1+K_2 & -K_2 \\ \cancel{0} & -K_2 & K_2 \end{bmatrix}}_{\text{3x3 global stiffness matrix}} \end{array}
\begin{bmatrix} \cancel{\delta_1} \to 0 \\ \delta_2 \\ \delta_3 \end{bmatrix}
$$

Size : # DOF X # DOF

Step 5.

Apply Boundary Conditions & Solve

B.C.

→1. $\delta_1 = 0$ ⟹ apply by deleting col. & row 1 of the global system of equations

→2. Apply external forces

$F_2 = P$

$F_3 = -P$

$$\begin{bmatrix} P \\ -P \end{bmatrix} = \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix}$$

Solve by inverting

$$\begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix}^{-1} \begin{bmatrix} P \\ -P \end{bmatrix} = \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix}$$

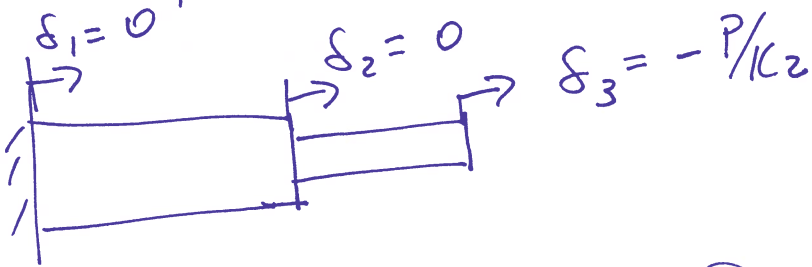
Output

$$\boxed{\begin{aligned} \delta_2 &= 0 \\ \delta_3 &= -\frac{P}{K_2} \end{aligned}}$$

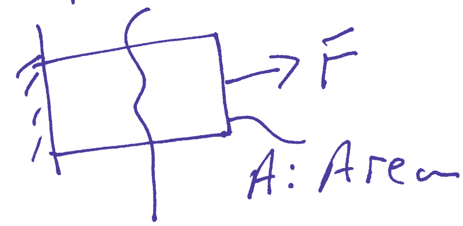
Given $\delta_1 = 0$

Step 6.

Post process to calculate the stresses



Recall



$$\sigma = \frac{F}{A}$$

$F^{(1)}$: internal force in bar (1)

$F^{(2)}$: internal force in bar (2)

$$F^{(1)} = K_1(\cancel{\delta_2}^{\,0} - \cancel{\delta_1}^{\,0})$$

$$\sigma^{(1)} = \frac{F^{(1)}}{A_1}$$

$$\boxed{\sigma^{(1)} = 0}$$

$$F^{(2)} = K_2(\delta_3 - \cancel{\delta_2}^{\,0})$$

$$\sigma^{(2)} = \frac{F^{(2)}}{A_2}$$

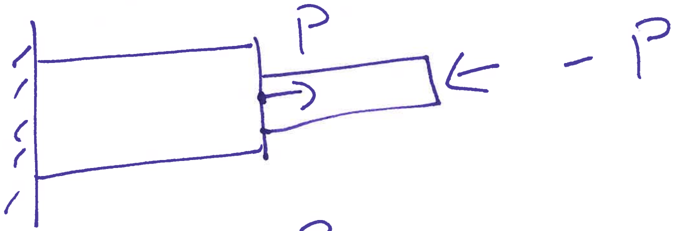
$$\sigma^{(2)} = \frac{\cancel{K_2}\left(-\frac{P}{\cancel{K_2}}\right)}{A_2}$$

$$\sigma^{(2)} = -\frac{P}{A_2}$$

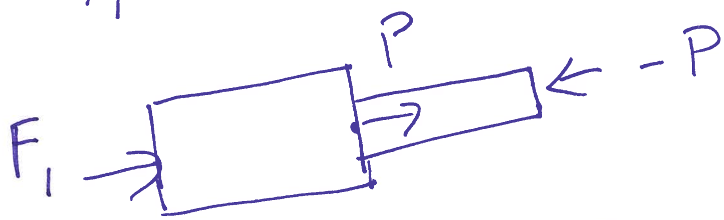
F_1 is the reaction force on the wall

$$\underline{F_1 = 0}$$

Check



Global FBD



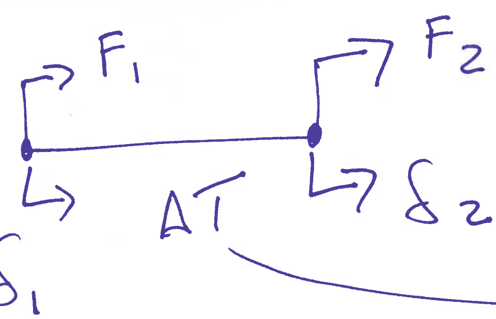
$$F_1 + P - P = 0$$

Global Equilibrium

$$\boxed{F_1 = 0}$$

Thermal Loads

Recall



$$K = \frac{EA}{L}$$

What if we apply a change in temperature?

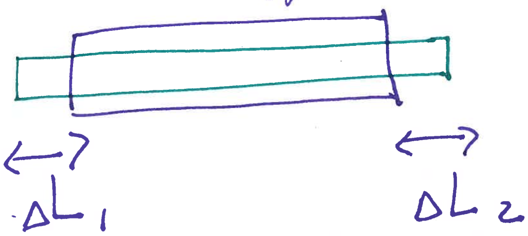
$$F_2 = K(\delta_2 - \delta_1)$$

Temperature change: ΔT

α: thermal expansion coefficient (K^{-1})

1 Unconstrained thermal expansion

$$\varepsilon^{thermal} = \alpha \Delta T$$



→ unconstrained thermal strain does not cause stress

Constitutive Relationship

$$\sigma = E\,(\text{total strain} - \text{thermal strain})$$

$$= E\left(\frac{\delta_2 - \delta_1}{L} - \alpha \Delta T\right)$$

$$F_2 = \sigma A = \frac{EA}{L}(\delta_2 - \delta_1) - EA\alpha\Delta T$$

From equilibrium $\Rightarrow F_1 = -F_2$

$$F_1 = \frac{EA}{L}(\delta_1 - \delta_2) + EA\alpha\Delta T$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} + \begin{bmatrix} EA\alpha\Delta T \\ -EA\alpha\Delta T \end{bmatrix}$$

$$k = \frac{EA}{L}$$