15 students

Code in CANVAS : codelectures.m

The code follows this procedure except it used dof numbering as

B : 1 & 2

F : 3 & 4

C : 5 & 6

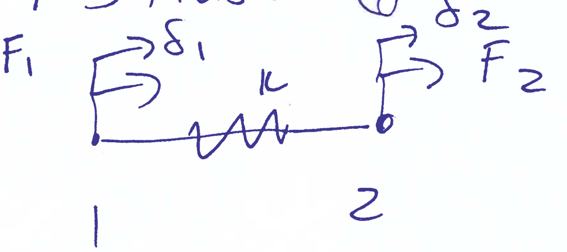
D : 7 & 8

STRESS FORMULA

$\sigma = \frac{E}{L}[-\cos\theta \; -\sin\theta \; \cos\theta \; \sin\theta]$

Thermal loads in 2D TRUSSES

1D Truss (from HW1)



Apply ΔT

$$F_1 + F_2 = 0$$

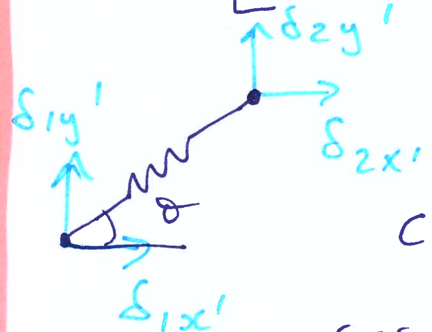
$$\sigma = E\left(\frac{\delta_2 - \delta_1}{L}\right) - E\alpha\Delta T$$

$$\sigma A = F_2$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} + \begin{bmatrix} EA\alpha\Delta T \\ -EA\alpha\Delta T \end{bmatrix}$$

2D TRUSS

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \underbrace{\begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}}_{T}\begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{2x'} \\ \delta_{2y'} \end{bmatrix}$$



$c = \cos\theta = \frac{x_2 - x_1}{L}$

$s = \sin\theta = \frac{y_2 - y_1}{L}$

$$\begin{bmatrix} F_{1x'} \\ F_{1y'} \\ F_{2x'} \\ F_{2y'} \end{bmatrix} = \underbrace{\begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix}}_{T^T}\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\begin{bmatrix} F_{1x'} \\ F_{1y'} \\ F_{2x'} \\ F_{2y'} \end{bmatrix} = \underset{T^T}{\begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix}} \left(\frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} + \begin{bmatrix} EA\alpha\Delta T \\ -EA\alpha\Delta T \end{bmatrix} \right)$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \rightarrow T \begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{2x'} \\ \delta_{2y'} \end{bmatrix}$$

$$\begin{bmatrix} F_{1x'} \\ F_{1y'} \\ F_{2x'} \\ F_{2y'} \end{bmatrix} = \underbrace{\frac{EA}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -sc & -s^2 & sc & s^2 \end{bmatrix}}_{K} \begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{2x'} \\ \delta_{2y'} \end{bmatrix} + EA\alpha\Delta T \begin{bmatrix} c \\ s \\ -c \\ -s \end{bmatrix}$$

Thermal Loads in 2D Truss

$$\boxed{K \begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{2x'} \\ \delta_{2y'} \end{bmatrix} = \begin{bmatrix} F_{1x'} \\ F_{1y'} \\ F_{2x'} \\ F_{2y'} \end{bmatrix} + EA\alpha\Delta T \begin{bmatrix} -c \\ -s \\ c \\ s \end{bmatrix}}$$

use external forces at the end

not assembled

when modifying the code, add $EA\alpha\Delta T \begin{bmatrix}-c\\-s\\c\\s\end{bmatrix}$ to the 'local force' in the for loop over Nelements.

i.e. local force $= EA\alpha\Delta T \begin{bmatrix}-c\\-s\\c\\s\end{bmatrix}$

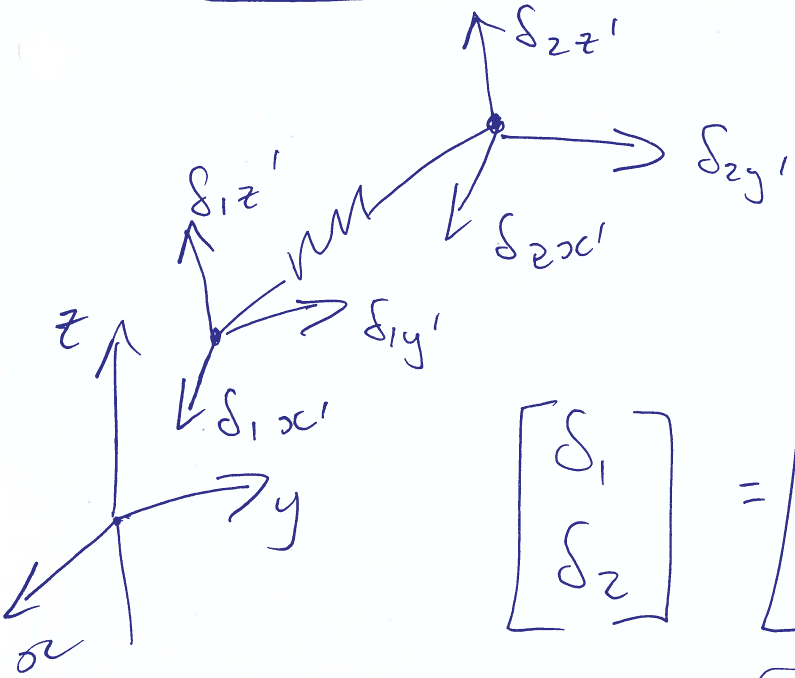
Post process: stress

$$\sigma = \frac{E}{L}(\delta_2 - \delta_1) - E\alpha\Delta T$$

$$\sigma = \frac{E}{L}\begin{bmatrix}-c & -s & c & s\end{bmatrix}\begin{bmatrix}\delta_{1x'}\\ \delta_{1y'}\\ \delta_{2x'}\\ \delta_{2y'}\end{bmatrix} - \underbrace{E\alpha\Delta T}_{\text{add this to the code}}$$

$$\delta_2 = c\,\delta_{2x'} + s\,\delta_{2y'}$$

3D TRUSSES



$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \underbrace{\begin{bmatrix} l & m & n & 0 & 0 & 0 \\ 0 & 0 & 0 & l & m & n \end{bmatrix}}_{T} \begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{1z'} \\ \delta_{2x'} \\ \delta_{2y'} \\ \delta_{2z'} \end{bmatrix}$$

$$l = \frac{x_2 - x_1}{Length}$$

l, m, n - direction cosines

$$m = \frac{y_2 - y_1}{Length}$$

$$n = \frac{z_2 - z_1}{Length}$$

$$\begin{bmatrix} F_{1x'} \\ F_{1y'} \\ F_{1z'} \\ F_{2x'} \\ F_{2y'} \\ F_{2z'} \end{bmatrix} = T^T \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} l & 0 \\ m & 0 \\ n & 0 \\ 0 & l \\ 0 & m \\ 0 & n \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$$

$$\begin{bmatrix} F_{1x'} \\ F_{1y'} \\ F_{1z'} \\ F_{2x'} \\ F_{2y'} \\ F_{2z'} \end{bmatrix} = \underbrace{T^T \left[\frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right] T}_{\text{stiffness matrix } K} \begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{1z'} \\ \delta_{2x'} \\ \delta_{2y'} \\ \delta_{2z'} \end{bmatrix}$$

3D TRUSS

$$K = \begin{bmatrix} \frac{EA}{L} \begin{bmatrix} l^2 & ml & nl \\ ml & m^2 & mn \\ nl & mn & n^2 \end{bmatrix} & -k \\ -k & k \end{bmatrix}$$

local stiffness

k

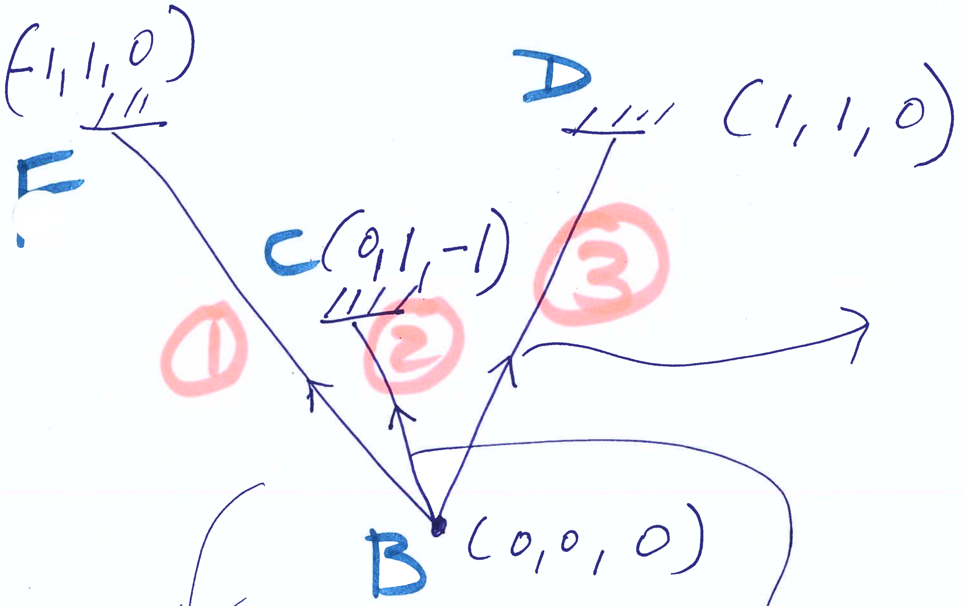
6x6 matrix

code

$$K = \begin{array}{c|c} & \begin{array}{cc} \text{node 1} & \text{node 2} \end{array} \\ \begin{array}{c} \text{node 1} \\ \text{node 2} \end{array} & \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \end{array}$$

$\longrightarrow$ assemble

$$k = \frac{EA}{L} \begin{bmatrix} l^2 & ml & nl \\ ml & m^2 & mn \\ nl & mn & n^2 \end{bmatrix}$$



Truss 3

$$\begin{array}{c|cc} & B & D \\ \hline B & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{bmatrix} k^{3} & -k^{3} \\ -k^{3} & k^{3} \end{bmatrix}$}} \\ D & & \end{array}$$

Truss 1

$$\begin{array}{c|cc} & B & F \\ \hline B & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{bmatrix} k^{1} & -k^{1} \\ -k^{1} & k^{1} \end{bmatrix}$}} \\ F & & \end{array}$$

Truss 2

$$\begin{array}{c|cc} & B & C \\ \hline B & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{bmatrix} k^{2} & -k^{2} \\ -k^{2} & k^{2} \end{bmatrix}$}} \\ C & & \end{array}$$

Each of these is a 6x6 matrix

Assemble

Block assembly!

3x3 matrices will be summed

k^{1} is a 3x3 matrix

$$\begin{array}{c|cccc} & B & C & D & F \\ \hline B & \multicolumn{4}{c}{\multirow{4}{*}{$\left[\begin{array}{c|c|c|c} k^{1}+k^{2}+k^{3} & -k^{2} & -k^{3} & -k^{1} \\ \hline -k^{2} & k^{2} & 0 & 0 \\ \hline -k^{3} & 0 & k^{3} & 0 \\ \hline -k^{1} & 0 & 0 & k^{1} \end{array}\right]$}} \\ C & & & & \\ D & & & & \\ F & & & & \end{array}$$

3x3 zero matri

CODE I posted (changes for 3D)
dof = 3, stress formula, coordinate matrix

Thermal loads (3D)

add $E A \alpha \Delta T \begin{bmatrix} -l \\ -m \\ -n \\ l \\ m \\ n \end{bmatrix}$ to local force