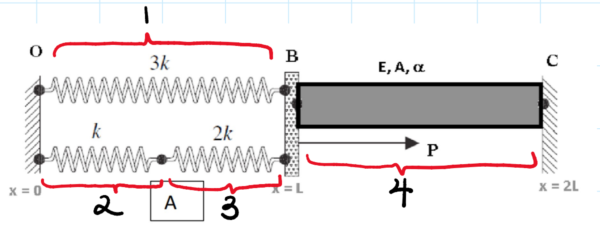
Homework 1

Thursday, January 19, 2023   4:12 PM



We have thermal expansion in the truss structure and an applied force P on point B.

We first define the node-connectivity matrix.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{cc} \text{Left} & \text{Right} \\ \left[\begin{array}{c} O \\ O \\ A \\ B \end{array}\right. & \left.\begin{array}{c} B \\ A \\ B \\ C \end{array}\right] \end{array}$$

For each local element, we write:

$$1: \begin{array}{c|cc} & O & B \\ \hline O & 3k & -3k \\ B & -3k & 3k \end{array} \quad \begin{bmatrix} \delta_O \\ \delta_B \end{bmatrix} = \begin{bmatrix} F_O^1 \\ F_B^1 \end{bmatrix}$$

$$2: \begin{array}{c|cc} & O & A \\ \hline O & k & -k \\ A & -k & k \end{array} \quad \begin{bmatrix} \delta_O \\ \delta_A \end{bmatrix} = \begin{bmatrix} F_O^2 \\ F_A^2 \end{bmatrix}$$

$$3: \begin{array}{c|cc} & A & B \\ \hline A & 2k & -2k \\ B & -2k & 2k \end{array} \quad \begin{bmatrix} \delta_A \\ \delta_B \end{bmatrix} = \begin{bmatrix} F_A^3 \\ F_B^3 \end{bmatrix}$$

$$4: \begin{array}{c|cc} & B & C \\ \hline B & \frac{EA}{L} & -\frac{EA}{L} \\ & EA & EA \end{array} \quad \begin{bmatrix} \delta_B \\ \end{bmatrix} = \begin{bmatrix} F_B^4 \\ \end{bmatrix}$$

$$
\begin{array}{c} \\ C \end{array}
\begin{bmatrix} \cdots & \cdots \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}
\begin{bmatrix} \vdots \\ \delta_C \end{bmatrix}
=
\begin{bmatrix} \vdots \\ F_C^4 \end{bmatrix}
$$

We now assemble forces:

node O : $F_O^1 + F_O^2 = R_O$ → reaction force at O

node A : $F_A^2 + F_A^3 = 0$ → no external force on A

node B : $F_B^1 + F_B^3 + F_B^4 = -EA\alpha\Delta T + P$ → Heat expansion + applied load

node C : $F_C^4 = R_C + EA\alpha\Delta T$ → Heat expansion and reaction at C

$$
\begin{array}{c|cccc}
 & O & A & B & C \\
\hline
O & 3k+k & -k & -3k & 0 \\
A & -k & k+2k & -2k & 0 \\
B & -3k & -2k & 3k+2k+\frac{EA}{L} & -\frac{EA}{L} \\
C & 0 & 0 & -\frac{EA}{L} & \frac{EA}{L}
\end{array}
\quad
\begin{bmatrix} \delta_O \\ \delta_A \\ \delta_B \\ \delta_C \end{bmatrix}
=
\begin{bmatrix} R_O \\ 0 \\ -EA\alpha\Delta T + P \\ R_C + EA\alpha\Delta T \end{bmatrix}
$$

a) Finding displacement

$$
\begin{array}{c|cccc}
 & O & A & B & C \\
\hline
O & 3k+k & -k & -3k & 0 \\
A & -k & k+2k & -2k & 0 \\
B & -3k & -2k & 3k+2k+\frac{EA}{L} & -\frac{EA}{L} \\
C & 0 & 0 & -\frac{EA}{L} & \frac{EA}{L}
\end{array}
\quad
\begin{bmatrix} \delta_O \to 0 \\ \delta_A \\ \delta_B \\ \delta_C \to 0 \end{bmatrix}
=
\begin{bmatrix} R_O \\ 0 \\ -EA\alpha\Delta T + P \\ R_C + EA\alpha\Delta T \end{bmatrix}
$$

We are left with the following system:

$$\begin{bmatrix} k+2k & -2k \\ -2k & 3k+2k+\frac{EA}{L} \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_B \end{bmatrix} = \begin{bmatrix} 0 \\ -EA\alpha\Delta T + P \end{bmatrix}$$

This is a system of equation

$$\left[\begin{aligned} \delta_A &= \frac{2L(P-AE\alpha\Delta T)}{11Lk+3AE} \\ \delta_B &= \frac{3L(P-AE\alpha\Delta T)}{11Lk+3AE} \end{aligned}\right]$$

We can solve this with MATLAB

b)

$$\tau^{Truss} = E\left(\frac{\delta_C - \delta_B}{L}\right) - E\alpha\Delta T$$

$$= E\left(\frac{-3\cancel{L}(P-AE\alpha\Delta T)}{(11Lk+3AE)\cdot\cancel{L}}\right) - E\alpha\Delta T$$

$$\boxed{\tau^{Truss} = E\left(\frac{-3(P-AE\alpha\Delta T)}{(11Lk+3AE)}\right) - E\alpha\Delta T}$$

c)

$$\begin{array}{c|c} & \begin{matrix} O & A & B & C \end{matrix} \\ \begin{matrix} O \\ A \\ B \\ C \end{matrix} & \begin{bmatrix} 3k+k & -k & -3k & 0 \\ -k & k+2k & -2k & 0 \\ -3k & -2k & 3k+2k+\frac{EA}{L} & -\frac{EA}{L} \\ 0 & 0 & -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \end{array} \begin{bmatrix} 0 \\ \delta_A \\ \delta_B \\ 0 \end{bmatrix} = \begin{bmatrix} R_o \\ 0 \\ -EA\alpha\Delta T + P \\ R_c + EA\alpha\Delta T \end{bmatrix}$$

We subsitute our results for δ_A & δ_B obtained above & solve for R_o & R_c

$$
\begin{array}{c c}
 & \begin{array}{cccc} \quad O \quad & \quad A \quad & \quad B \quad & \quad C \quad \end{array} \\
\begin{array}{c} O \\ A \\ B \\ C \end{array} &
\begin{bmatrix}
3k+k & -k & -3k & 0 \\
-k & k+2k & -2k & 0 \\
-3k & -2k & 3k+2k+\frac{EA}{L} & -\frac{EA}{L} \\
0 & 0 & -\frac{EA}{L} & \frac{EA}{L}
\end{bmatrix}
\end{array}
\begin{bmatrix}
0 \\
\frac{2L(P-AE\alpha\Delta T)}{11Lk+3AE} \\
\frac{3L(P-AE\alpha\Delta T)}{11Lk+3AE} \\
0
\end{bmatrix}
=
\begin{bmatrix}
R_o \\
0 \\
-EA\alpha\Delta T + P \\
R_c + EA\alpha\Delta T
\end{bmatrix}
$$

$$-k\left(\frac{2L(P-AE\alpha\Delta T)}{11Lk+3AE}\right) - 3k\left(\frac{3L(P-AE\alpha\Delta T)}{11Lk+3AE}\right) = R_o$$

$$\boxed{R_o = -k\left(\frac{2L(P-AE\alpha\Delta T)}{11Lk+3AE}\right) - 3k\left(\frac{3L(P-AE\alpha\Delta T)}{11Lk+3AE}\right)}$$

$$-\frac{EA}{L}\left(\frac{3L(P-AE\alpha\Delta T)}{11Lk+3AE}\right) = R_c + EA\alpha\Delta T$$

$$\boxed{R_c = -\frac{EA}{L}\left(\frac{3L(P-AE\alpha\Delta T)}{11Lk+3AE}\right) - EA\alpha\Delta T}$$