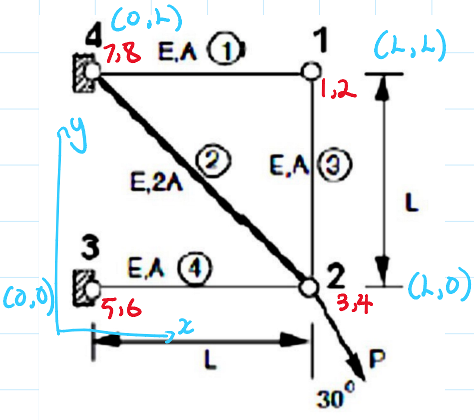
Homework 2

Monday, January 30, 2023 8:57 PM



We first define the node-connectivity matrix

	N_1	N_2
①	4	→ 1
②	4	→ 2
③	1	→ 2
④	3	→ 2

We refer to the 2D Truss Formula

$$k = \frac{EA}{L} = \begin{bmatrix} \begin{pmatrix} c^2 & cs \\ sc & s^2 \end{pmatrix} & \begin{pmatrix} -c^2 & -cs \\ -sc & -s^2 \end{pmatrix} \\ \begin{pmatrix} -c^2 & -cs \\ -sc & -s^2 \end{pmatrix} & \begin{pmatrix} c^2 & cs \\ sc & s^2 \end{pmatrix} \end{bmatrix}$$

where: $c = \cos\theta$

$s = \sin(\theta)$

For element ①: $\theta = 0$ $4 \rightarrow 1$

①:

$$k_{①} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccc} 7 & 8 & 1 & 2 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 7 \\ 8 \\ 1 \\ 2 \end{array}$$

For element ②: $4 \rightarrow 2$

$90° + 45° = 135°$ $\theta = 135°$

$$k = \frac{2EA}{\ldots} \begin{array}{c} \begin{array}{cccc} 7 & 8 & 3 & 4 \end{array} \\ \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{\ldots} & \frac{1}{\ldots} & \frac{1}{\ldots} & -\frac{1}{\ldots} \end{bmatrix} \end{array} \begin{array}{c} 7 \\ 8 \end{array} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccc} 7 & 8 & 3 & 4 \end{array} \\ \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} & \frac{1}{\sqrt{2}} \\ -1/\ldots & \frac{1}{\ldots} & \frac{1}{\ldots} & -\frac{1}{\ldots} \end{bmatrix} \end{array} \begin{array}{c} 7 \\ 8 \end{array}$$

$$K_{②} = \frac{2EA}{1.414L}\begin{array}{c} \\ \left[\begin{array}{cccc} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{array}\right] \end{array}\begin{array}{c} 7 \\ 8 \\ 3 \\ 4 \end{array} = \frac{EA}{L}\left[\begin{array}{cccc} 1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} & \frac{1}{\sqrt{2}} \\ -1/\sqrt{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array}\right]\begin{array}{c} 7 \\ 8 \\ 3 \\ 4 \end{array}$$

For element ③ : $1 \to 2$

$\theta = 90$

$$K_{③} = \frac{EA}{L}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

For element ④ $3 \to 2$

$$K_{④} = \frac{EA}{L}\begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 5 \\ 6 \\ 3 \\ 4 \end{array}$$

$$\left[\begin{array}{c} 0 \\ 0 \\ P\cos(60) \\ -P\sin(60) \\ R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{array}\right] = \frac{EA}{L}\begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.707 & -0.707 & -1 & 0 & -.7071 & 0.7071 \\ 0 & -1 & -0.7071 & 1.707 & 0 & 0 & 0.7071 & -0.707 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -0.7071 & 0.7071 & 0 & 0 & 1.7071 & -0.7071 \\ 0 & 0 & 0.7071 & -0.7071 & 0 & 0 & -0.7071 & 0.7071 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}\left[\begin{array}{c} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{array}\right]$$

b)

$$\left[\begin{array}{c} 0 \\ 0 \\ \vdots \end{array}\right] \quad \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ & & & & & & & \end{array}\right. \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \quad \left[\begin{array}{c} u_{1x} \\ u_{1y} \\ \vdots \end{array}\right]$$

$$\begin{bmatrix} 0 \\ P\cos(60) \\ -P\sin(60) \\ R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.707 & -0.707 & -1 & 0 & -.7071 & 0.7071 \\ 0 & -1 & -0.7071 & 1.707 & 0 & 0 & 0.7071 & -0.7071 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -0.7071 & 0.7071 & 0 & 0 & 1.7071 & -0.7071 \\ 0 & 0 & 0.7071 & -0.7071 & 0 & 0 & -0.7071 & 0.7071 \end{bmatrix} \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{bmatrix} u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ P\cos(60) \\ -P\sin(60) \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1.707 & -0.707 \\ 0 & -1 & -0.707 & 1.707 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{bmatrix}$$

$$U = \begin{bmatrix} 0 \\ -0.2273 \\ -0.0523 \\ -0.2273 \end{bmatrix} \cdot 10^{-4}$$

The general formula for stress in bars are of the following:

$$\sigma = \frac{E}{L} \begin{bmatrix} -\cos\theta & -\sin\theta & \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{bmatrix}$$

where [node 1 & 2] are defined via the node connectivity matrix:

	N_1 N_2
①	4 → 1
②	4 → 2
③	1 → 2
④	3 → 2

We then summarize the length and area for each element in the following chart:

	N_1 N_2	Area	Length
①	$4 \to 1$	1E-4	1
②	$4 \to 2$	2E-4	$\sqrt{2}$
③	$1 \to 2$	1E-4	1
④	$3 \to 2$	1E-4	1

We also know that, $F_{axial} = \tau_{element} A$. Therefore, the axial force for each element are:

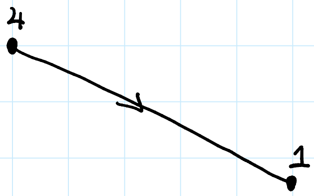
Element ①

$\theta = 0°$

$$\boxed{\begin{aligned} \tau &= 0 \text{ Pa} \\ F_{Axial} &= 0 \text{ N} \end{aligned}}$$

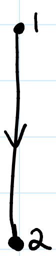
Element ②

$\theta = 135°$



$$\boxed{\begin{aligned} \tau &= 6.1237E5 \text{ Pa} \\ F_{Axial} &= \tau \cdot A \\ &= \tau \cdot 2 \cdot (1E-4) \\ &= 122.47 \text{N} \end{aligned}}$$

Element ③



$\theta = 90°$

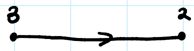
$$\boxed{\begin{aligned} \tau &= 0 \text{ Pa} \\ F_{axial} &= 0 \text{N} \end{aligned}}$$

Element ④

$\theta = 0°$

Element ④

$\theta = 0°$



$$\boxed{\begin{aligned} \sigma &= -3.6603\,E5\ Pa \\ F_{axial} &= -36.6025\ N \end{aligned}}$$

We now calculate the reaction force:

$$\begin{bmatrix} 0 \\ 0 \\ P\cos(30) \\ P\sin(30) \\ R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{bmatrix} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.707 & -0.707 & -1 & 0 & -.7071 & 0.7071 \\ 0 & -1 & -0.7071 & 1.707 & 0 & 0 & 0.7071 & -0.7071 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -0.7071 & 0.7071 & 0 & 0 & 1.7071 & -0.7071 \\ 0 & 0 & 0.7071 & -0.7071 & 0 & 0 & -0.7071 & 0.7071 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{bmatrix} 0 \\ -0.4873 \\ 0.5229 \\ -0.4873 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} * 10^{-5}$$

We subbed in the displacements solved previously and we can calculate the reaction forces by:

$$\begin{bmatrix} R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -0.7071 & 0.7071 & 0 & 0 & 1.7071 & -0.7071 \\ 0 & 0 & 0.7071 & -0.7071 & 0 & 0 & -0.7071 & 0.7071 \end{bmatrix} \begin{array}{c} 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{bmatrix} 0 \\ -0.4873 \\ 0.5229 \\ -0.4873 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} * 10^{-5}$$

$$\boxed{\begin{bmatrix} R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{bmatrix} = \begin{bmatrix} 36.6025 \\ 0 \\ -86.6025 \\ 86.6025 \end{bmatrix}}$$

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c) To modify the code, we add the following expressions

$$\begin{bmatrix} F_{ix}' \end{bmatrix} \qquad \begin{bmatrix} -c \end{bmatrix}$$

To modify the code, we used the following expressions

$$F_{local} = \begin{bmatrix} F_1 x' \\ F_1 y' \\ F_2 x' \\ F_2 y' \end{bmatrix} + EA\alpha\Delta T \begin{bmatrix} -c \\ -s \\ c \\ s \end{bmatrix}$$

$$\sigma = \frac{E}{L}\begin{bmatrix} -c & -s & c & s \end{bmatrix}\begin{bmatrix} \delta_{1x'} \\ \delta_{1y'} \\ \delta_{2x'} \\ \delta_{2y'} \end{bmatrix} - EA\alpha\Delta T$$

The answer after modifying the code is:

$$\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.2702 \\ 0.4948 \\ -0.2298 \end{bmatrix} \cdot 10^{-3}\ m$$

$$\begin{bmatrix} \sigma_{①} \\ \sigma_{②} \\ \sigma_{③} \\ \sigma_{④} \end{bmatrix} = \begin{bmatrix} 3.4996 \\ 2.5354 \\ 3.4996 \\ 3.4630 \end{bmatrix} \cdot 10^{7}\ Pa$$