

Entropy Stability

(Your name here)

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Dig the literature to provide answers to the following questions. You cannot refer to any paper published after 1985. You can read more recent papers - but they should be pointing to earlier works.

1. Who came up with the entropy condition? Oldest readable reference you can find. Where does that condition come from? What are the motivations?
2. Find two first-order numerical fluxes that do satisfy this condition, under certain assumptions. Success stories (did they bring tangible improvements)? There are more than two.
3. Philip Roe is a legend in CFD, in part for his 'Roe flux'.
 - (a) Find the paper where he first introduced it.
 - (b) How did Phil Roe derive the 'Roe average'? What was he after that led to it?
4. One approach to entropy-stability was introduced by Tadmor in 1987 (read [1]). What distinguishes it from others?
5. It took quite some time for Tadmor's ideas to gain popularity. Moon mentioned work by Fisher and Carpenter in 2013, where robustness potential for higher-order schemes was demonstrated for the first time. But Phil Roe made key contributions in 2006 (see [2]). Discuss them
 - Why is there a logarithmic average in his flux?

Consider the 1D compressible Euler equations for a perfect gas,

1. Derive the conservation equation for entropy (ρs).
2. How would you formalize the derivation you just carried out? Say you have a system of conservation laws with \mathbf{u} and \mathbf{f} , and you are asked to derive the conservation equation (assuming there is one) for a scalar valued function $U = U(\mathbf{u})$.
3. Does the number of spatial dimensions matter?
4. Is (ρs) a convex function of mass, momentum and total energy? Define convexity first.
5. What are numerical and theoretical benefits of convexity?
6. Prove that the original Roe flux is not entropy-stable, using Tadmor's framework [1] and a code implementation of the Roe flux.
 - People were already aware that the Roe flux was not entropy-stable. A correction called 'entropy fix' was introduced. Find the paper that first introduced it.
 - Is the Roe flux with the entropy-fix entropy-stable ?

References

- [1] Tadmor, E. : The numerical viscosity of entropy stable schemes for systems of conservation laws. I., *Mathematics of Computation*, volume 49, 1987 pages 91-103.
- [2] Roe, P. L. : Affordable, entropy-consistent, Euler flux functions, Eleventh International Conference on Hyperbolic Problems Theory, Numerics, Applications, Lyon 2006. Link: https://www.math.umd.edu/~tadmor/references/files/Roe_Affordable_entropy_Hyp2006.pdf