AEROSP 590: Entropy Condition

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1 Entropy Condition - Origin and Formulation

1.1 Origin

The earliest work for entropy condition could be traced to the Rankine-Hugoniot Jump Conditions. Assuming that u is a piecewise continuous weak solution, then it follows that across a line of discontinuity:

$$f(u_R) - f(u_L) = S(u_R - u_L) \tag{1}$$

where S is the speed of propagation of the discontinuity, and u_L and u_R are the states on the left and on the right of the discontinuity.

When we are solving hyperbolic partial differential equations, we are typically inclined into solving the weak form of the equation. Strong form of the equation must be valid at every material point. The weak form, on the other hand, only has to be met on an average sense, which requies additional boundary conditions and laws to narrow down the solution. The class of all weak solutions is too wide in the sens that there is no uniqueness for the initial value problem, and an additional principle is needed for determining a physically relevant solution. We cast the physically relevant solution as a limit of solutions with some dissipation:

$$u_t + f(u)_x = \epsilon \left[(u)u_x\right]_x \tag{2}$$

Oleinik[*reference pending*] has shown that discontinuities of such admissble solutions can be characterized by the following condition:

$$\frac{f(u) - f(u_L)}{u - u_L} \geq S \geq \frac{f(u) - f(u_R)}{u - u_R} \tag{3}$$

This is known as the *entropy condition.* Oleinik in his paper, has shown that weak solutions satisfying the above condition are *uniquely* determined by their initial data.