SYDE 556/750

Simulating Neurobiological Systems
Lecture 3: Representations

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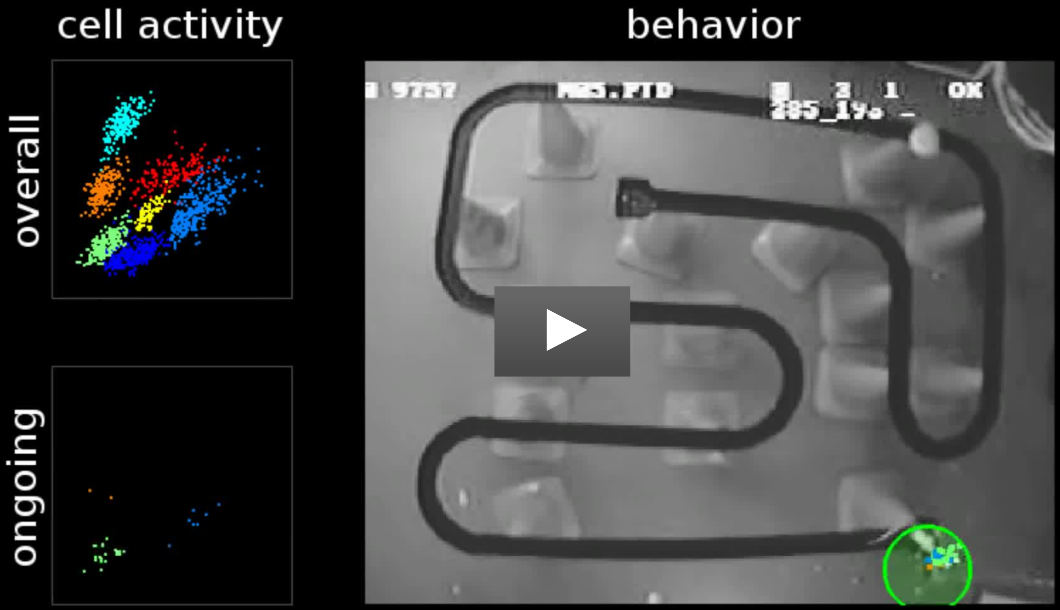
January 14 & 16 & 21, 2020

UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

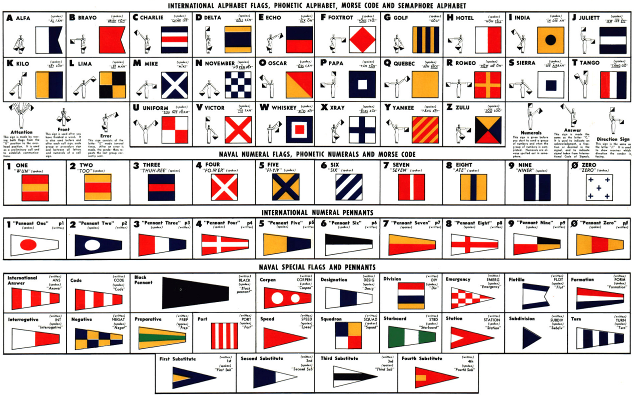

Visual Cortex
Mapping receptive fields


cell activity
behavior
overall
ongoing

NEF Principle 1: Representation

NEF Principle 1 – Representation
Groups ("populations", or "ensembles") of neurons *represent* represent values via nonlinear encoding and linear decoding.

Lossless Codes



A	• —	U	• • —
B	— • • •	V	• • • —
C	— • — •	W	• — —
D	— • •	X	— • • —
E	•	Y	— • — —
F	• • — •	Z	— — • •
G	— — •		
H	• • • •		
I	• •		
J	• — — —		
K	— • —	1	• — — — —
L	• — • •	2	• • — — —
M	— —	3	• • • — —
N	— •	4	• • • • —
O	— — —	5	• • • • •
P	• — — •	6	— • • • •
Q	— — • —	7	— — • • •
R	• — •	8	— — — • •
S	• • •	9	— — — — •
T	—	0	— — — — —

Encoding: $\mathbf{a} = f(\mathbf{x})$ Decoding: $\mathbf{x} = f^{-1}(\mathbf{a})$

Binary numbers: Nonlinear encoding, linear decoding

- Represent a natural number between 0 and $2^n - 1$ as n binary digits.

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- **Linear decoding**

$$x = f^{-1}(\mathbf{a}) = \sum_{i=0}^{n-1} 2^i a_i = \mathbf{F}\mathbf{a} = \begin{pmatrix} 1 & 2 & \dots & 2^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix}\,.$$

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- This is a **distributed code**. But, **not robust** against additive noise!

Lossy codes

- **Lossy code**
  Inverse f^{-1} does not exist, instead *approximate* the represented value

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- **Examples**
  - Audio, image, and video coding schemes
    (MP3, JPEG, H.264)
  - Basis transformation onto first n principal components (PCA)

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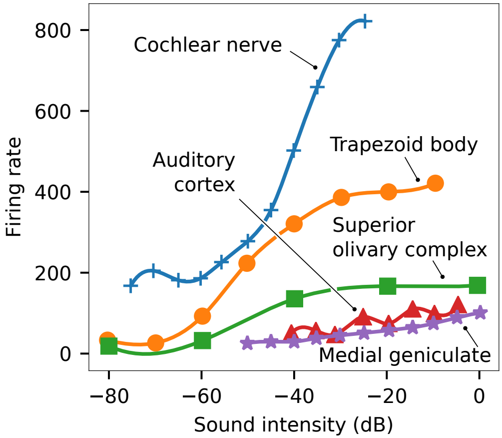
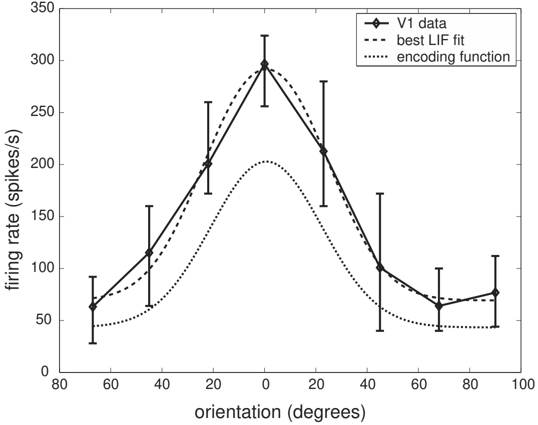
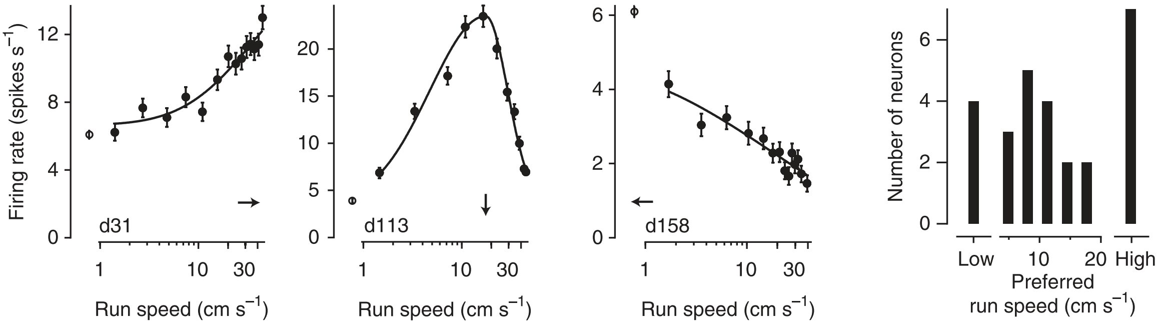
- **Examples**
  - Audio, image, and video coding schemes
    (MP3, JPEG, H.264)
  - Basis transformation onto first n principal components (PCA)
  - **Neural Representations**

Tuning curves (I)

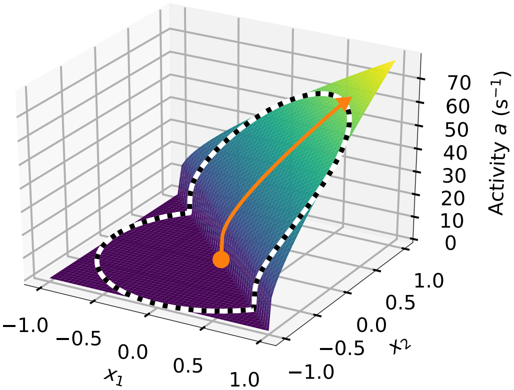




Tuning curves (II)



Preferred Directions in Higher Dimensions: Representing 2D Values



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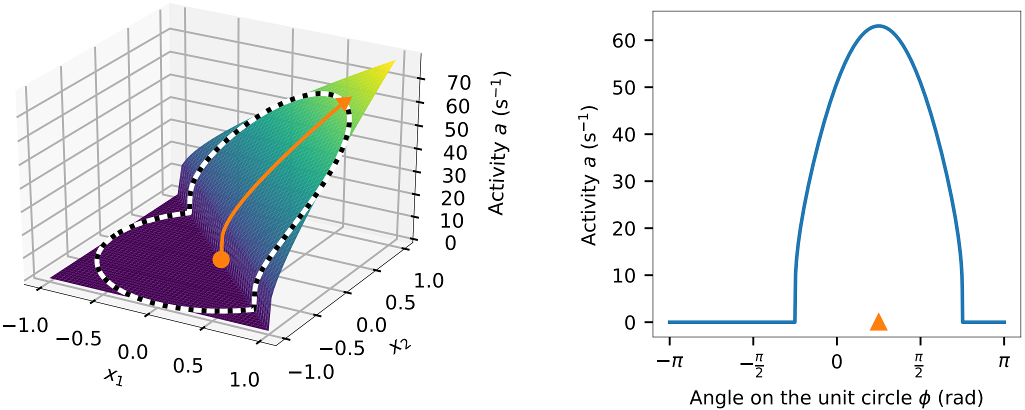
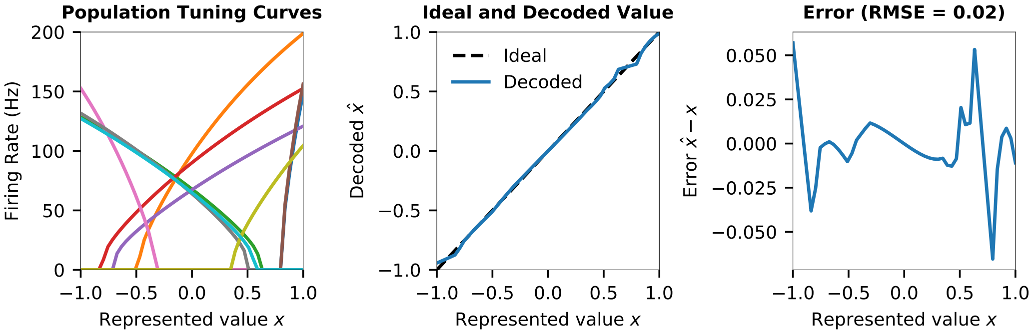
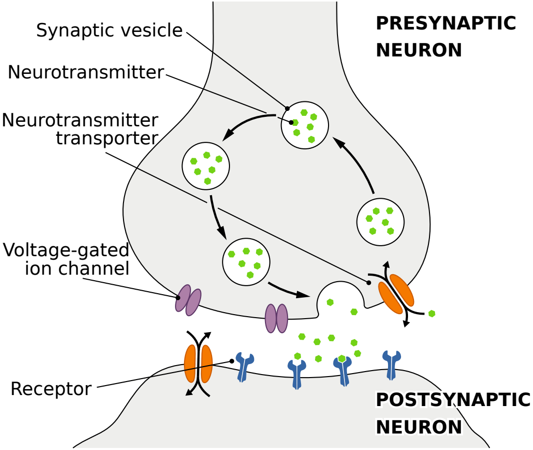


Decoding Without Taking Noise Into Account



Sources of Noise in Biological Neural Networks

- **Axonal jitter**
  Active axonal spike propagation
- **Vesicle release failure**
  10-30% of pre-synaptic events cause post-synaptic current
- **Neurotransmitter per vesicle**
  Varying amounts of neurotransmitter
- **Ion channel noise**
  Ion-channels are “binary”, stochastic
- **Thermal noise**
- **Network effects**
  Simple, noise-free inhibitory/excitatory networks produce irregular spike trains



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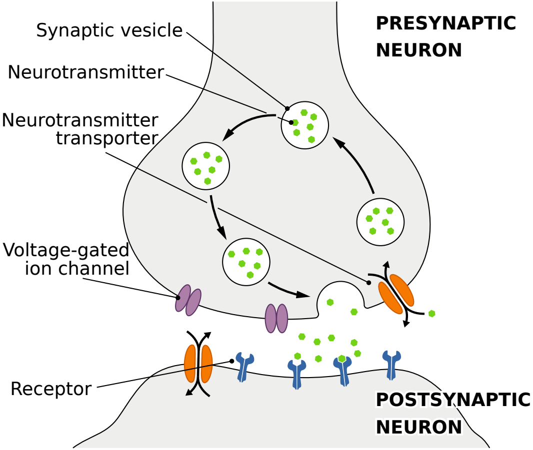

- **How to model?**

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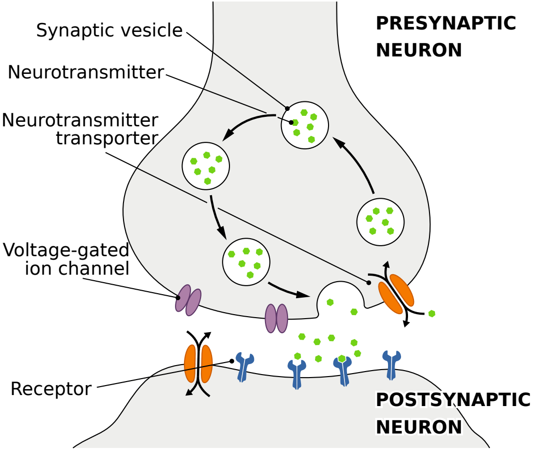
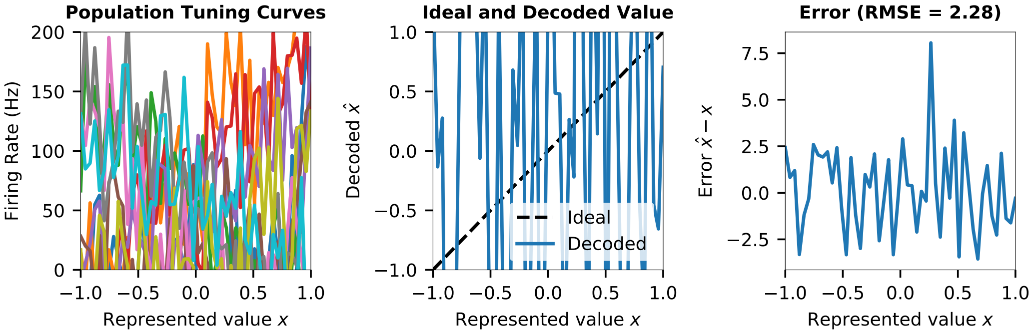


- **How to model?** Gaussian noise

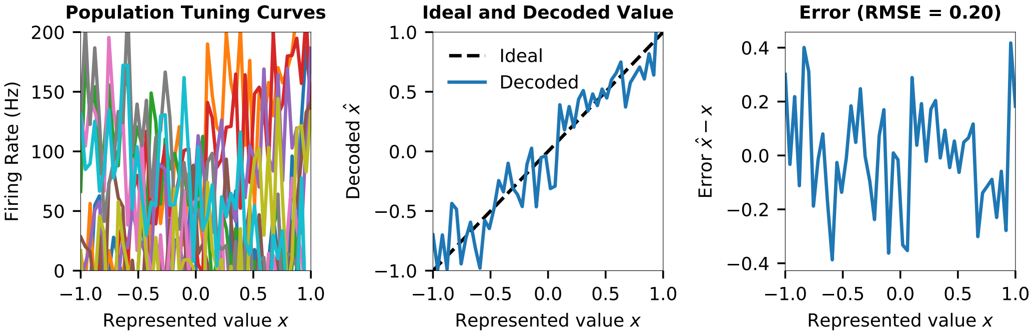
NEF Principle 0: Noise

NEF Principle 0 – Noise
Biological neural systems are subject to significant amounts of noise from various sources. Any analysis of such systems must take the effects of noise into account.

Decoding Noisy $\mathbf{A}$ Without Taking Noise Into Account



Decoding Noisy $\mathbf{A}$ Accounting for Noise

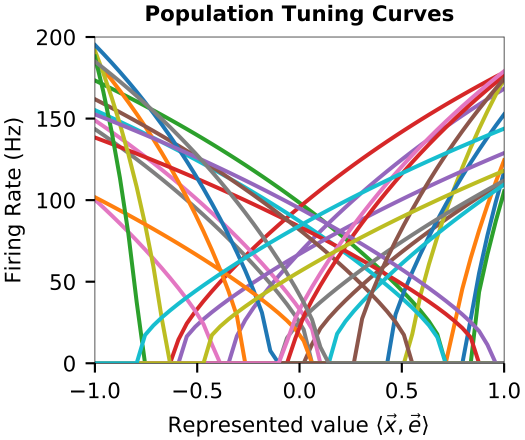


Summary: Building a model of neural representation (Encoding)

Encoding

- Select d, possible range $\mathbf{x} \in \mathbb{X}$, usually $\mathbb{X} = \left\{\mathbf{x} \mid \|\mathbf{x}\| \leq r, \mathbf{x} \in \mathbb{R}^d\right\}$ $(r = 1)$
- Select number of neurons n
- Select tuning curves, maximum rates $\Rightarrow \mathbf{e}_i,\ \alpha_i,\ J_i^{\mathrm{bias}}$
  - Sample $\mathbf{e}_i$ from unit-sphere
  - Uniformly distribute $\alpha_i,\ J_i^{\mathrm{bias}}$
- Encoding equation:

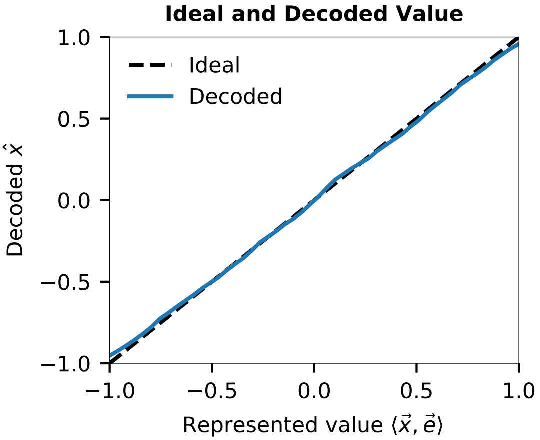
$$a_i(\mathbf{x}) = G\left[\alpha_i \langle \mathbf{e}_i, \mathbf{x} \rangle + J_i^{\mathrm{bias}}\right]$$



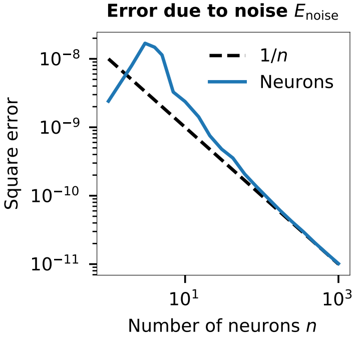
Summary: Building a model of neural representation (Decoding)

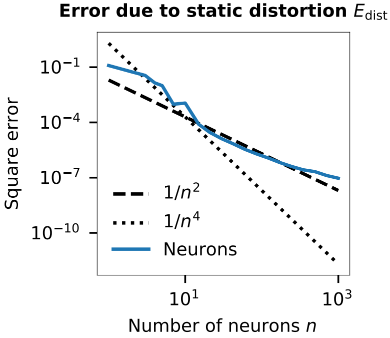
Decoding

- Uniformly sample N samples from $\mathbb{X}$, $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$
- Compute $\mathbf{A}$, where $(\mathbf{A}) = a_i()$
- Decoder computation:
$$\mathbf{D} = \left(\mathbf{A}\mathbf{A}^{\mathrm{T}} + N\sigma^2\mathbf{I}\right)\mathbf{A}\mathbf{X}^T$$
- Decoding equation:
$$\hat{\mathbf{X}} = \mathbf{D}\mathbf{A}$$



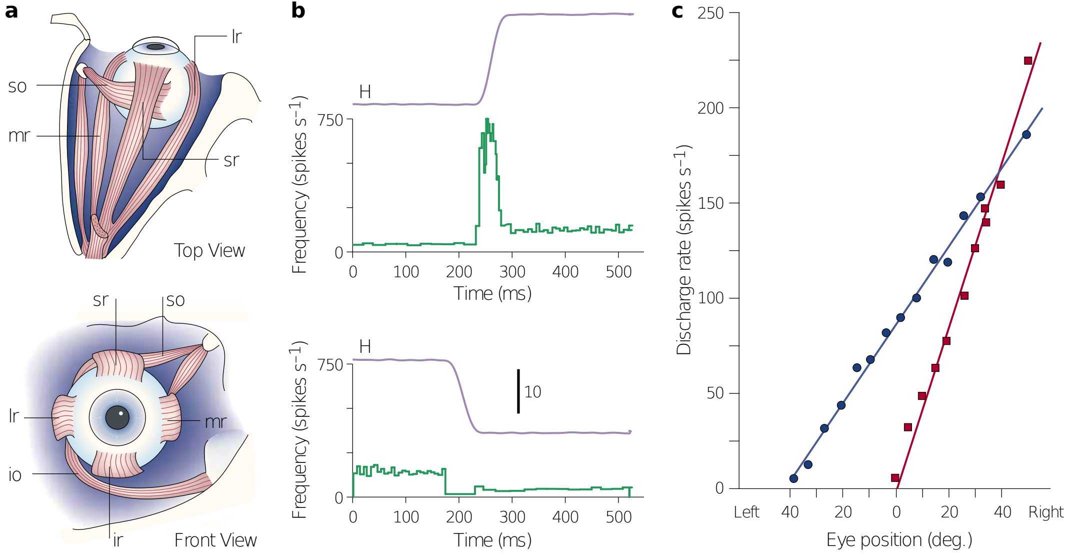
Analysing Sources of Errors





$$E = \underbrace{\frac{1}{2}\int_{-1}^{1}\left(x - \sum_{i=1}^{n} d_i a_i(x)\right)^2 \mathrm{d}x}_{E_{\text{dist}}} + \underbrace{\frac{1}{2}\sigma^2 \sum_{i=1}^{n} d_i^2}_{E_{\text{noise}}}$$

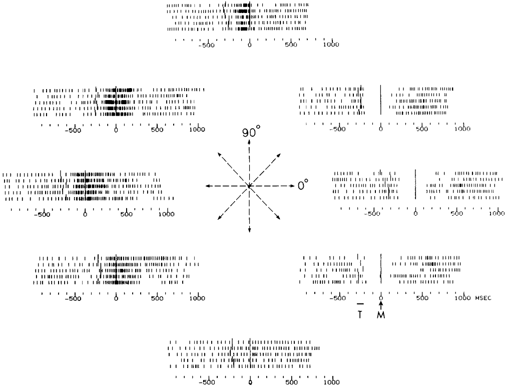
Example: Horizontal Eye Position (1D)

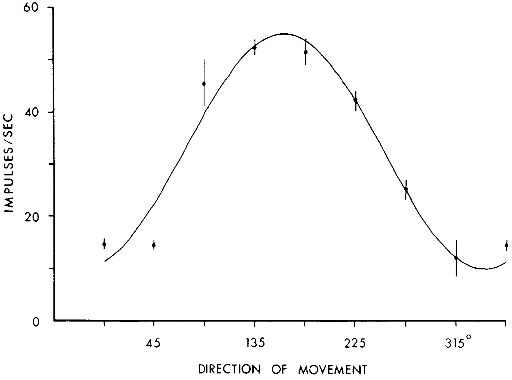


Example: Horizontal Eye Position (1D) (cont.)

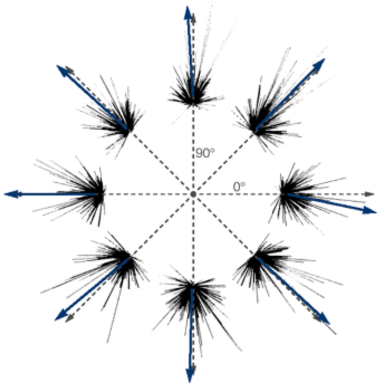
- **Step 1: System Description**
  - What is being represented?
    - x is the horizontal eye position
  - What is the tuning curve shape?
    - Linear, low τ_{ref}, high τ_{RC}
    - $e_i \in \{1, -1\}$
    - Firing rates up to $300\,\mathrm{s}^{-1}$
- **Step 2: Design Specification**
  - Range of values
    - $\mathbb{X} = [-60, 60]$
  - Amount of noise
    - About 20% of $\max(\mathbf{A})$
- **Step 3: Implementation**
  - Choose tuning curve parameters
  - Compute decoders

Example: Arm Movements (2D)





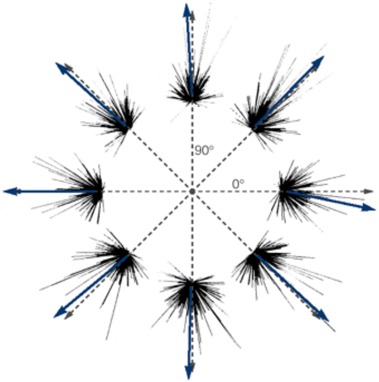
Example: Arm Movements (2D) (cont.)



- Experiment by Georgopoulos et al., 1982
- Preferred arm movement directions $\mathbf{e}_i$
- **Idea:** *Population Vectors*, decode using

$$\hat{\mathbf{x}} = \sum_{i=1}^{n} a_i(\mathbf{x})\mathbf{e}_i = \mathbf{E}\mathbf{A}$$

Example: Arm Movements (2D) (cont.)



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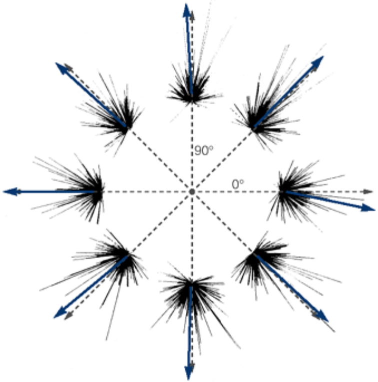

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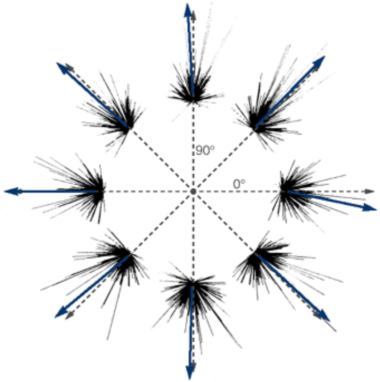

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The NEF does not use population vectors!

Example: Arm Movements (2D) (cont.)

- **Step 1: System Description**
  - What is being represented?
    - $\mathbf{x}$ the movement direction (or hand position)
  - What is the tuning curve shape?
    - Bell-shaped
    - Encoders are randomly distributed along the unit circle
    - Firing rates up to $60\,\mathrm{s}^{-1}$
- **Step 2: Design Specification**
  - Range of values
    - $\mathbb{X} = \{\mathbf{x} \mid \|\mathbf{x}\| \leq r, \mathbf{x} \in \mathbb{R}^2\}$
  - Amount of noise
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- **Step 3: Implementation**
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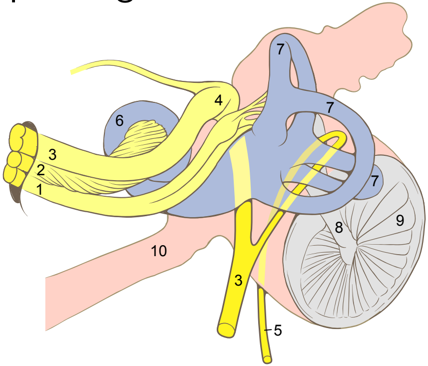
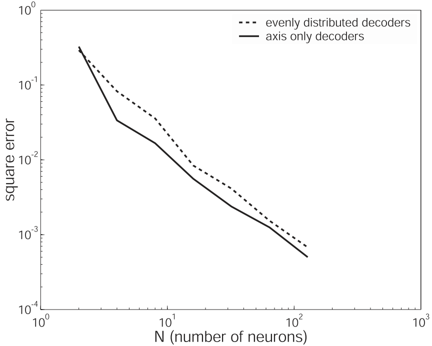
Example: Higher Dimensional Representation





- Vestibular system senses head acceleration in 3D
- Axis aligned, must choose $\mathbf{e}_i \in \{[1,0,0],[-1,0,0],\ldots,[0,0,-1]\}$

- Same as three 1D populations
- Slightly lower precision

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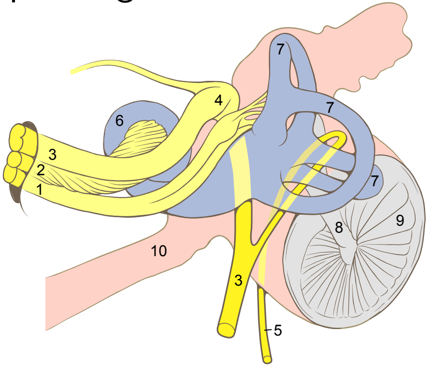

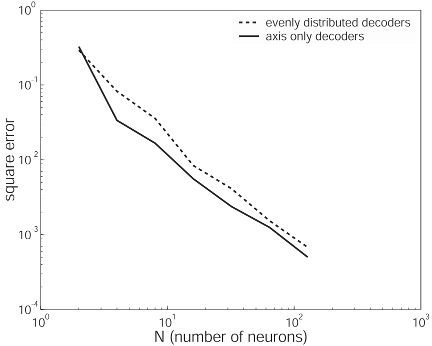

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- Slightly lower precision
- **Encoders affect accuracy**

Administration

- **Assignment 1 has been released.**

  The due date has been adjusted to January, 30.

- Some new potential times for office hours

  Mon 15:30-16:30, Mon 16:30-17:30, Tue 15:00-16:00,

  Thu 11:30-12:30 (current slot), Thu 12:30-13:30

Image sources

Title slide
"The Ultimate painting."
Author: Clark Richert.
From Wikimedia.