

**SYDE 556/750**

**Simulating Neurobiological Systems**  
**Lecture 7: Analysing Representations**

Terry Stewart

October 25, 2021

- ▶ Slide design: Andreas Stöckel
- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith



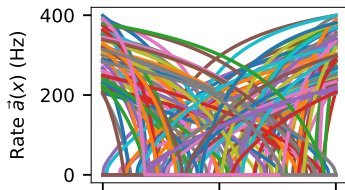
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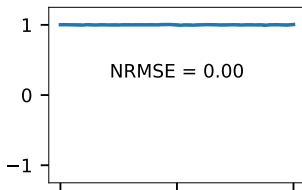


# Decoding Polynomials

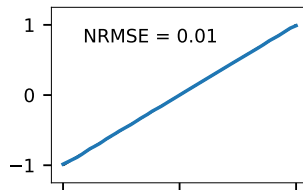
**Tuning curves ( $n = 128$ )**



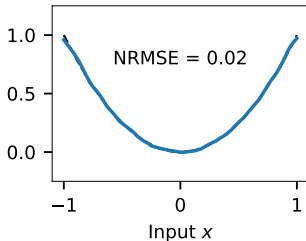
$f(x) = 1$



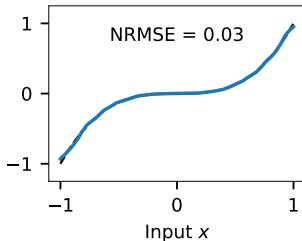
$f(x) = x$



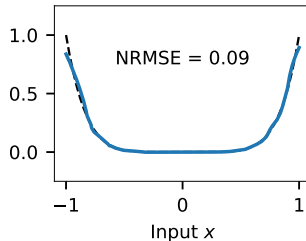
$f(x) = x^2$



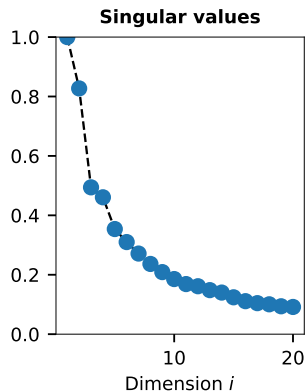
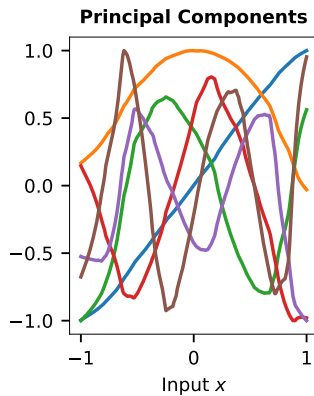
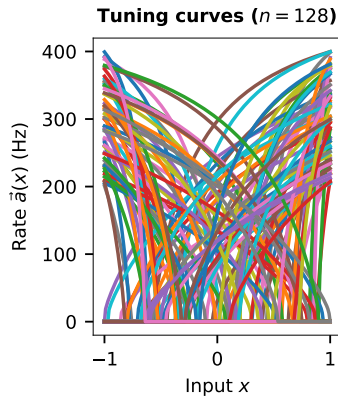
$f(x) = x^3$



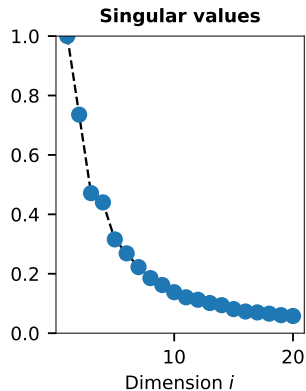
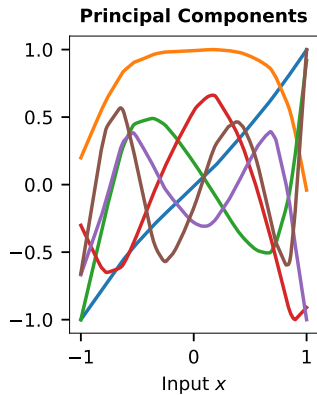
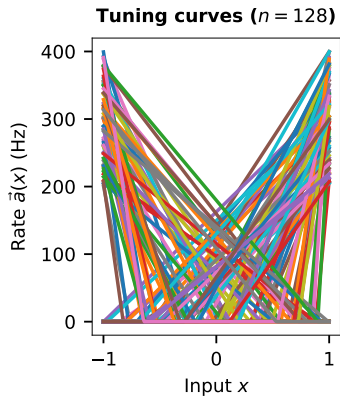
$f(x) = x^6$



# LIF Tuning Curve Principal Components

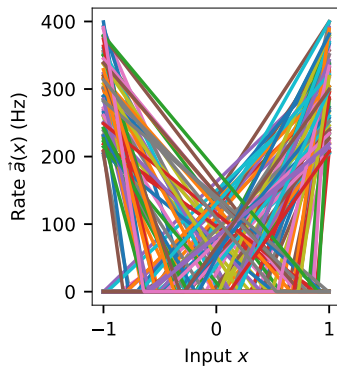


# ReLU Tuning Curve Principal Components

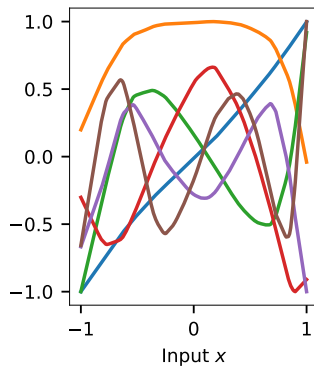


# ReLU Tuning Curve Principal Components

**Tuning curves ( $n = 128$ )**

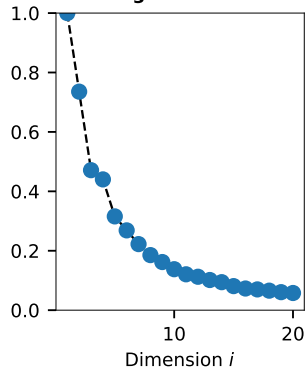


**Principal Components**

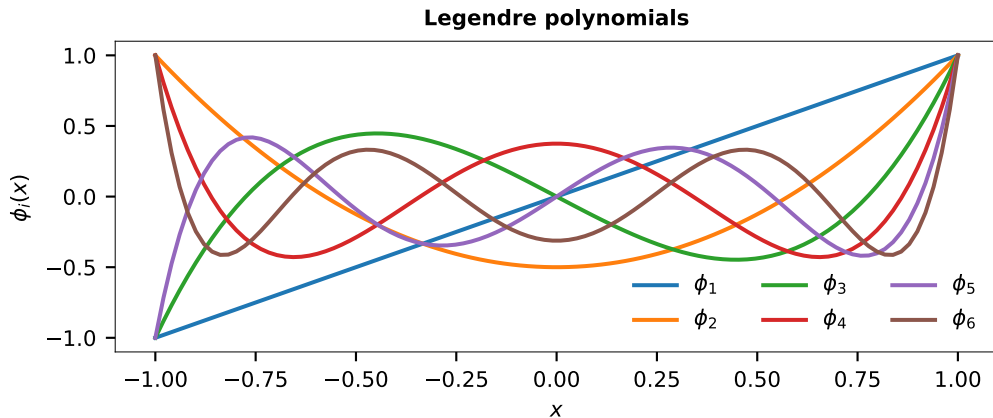


$\approx$  Legendre Basis

**Singular values**

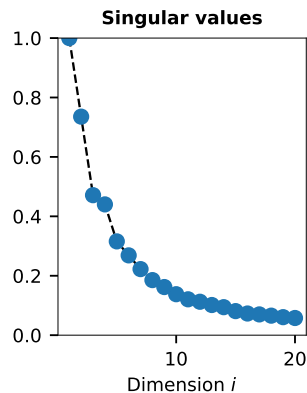
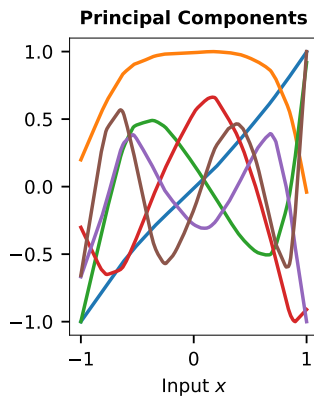
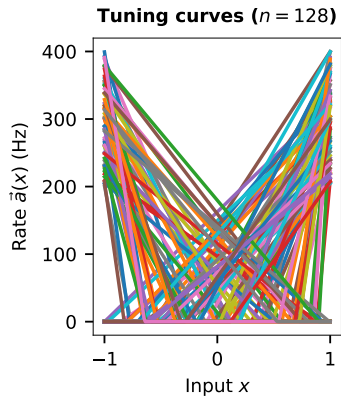


## Reminder: Legendre Polynomials



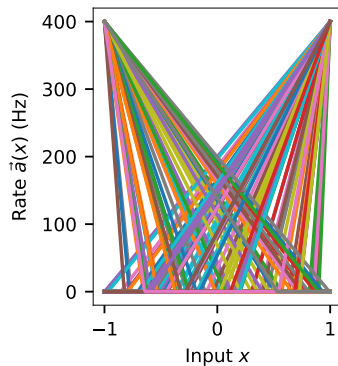
$$\varphi_i(x) = \frac{1}{2^i} \sum_{k=0}^i \binom{i}{k}^2 (x-1)^{i-k} (x+1)^k$$

# Modifying the Basis – Same Maximum Rate (I)

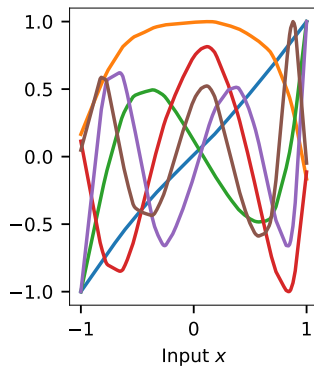


## Modifying the Basis – Same Maximum Rate (I)

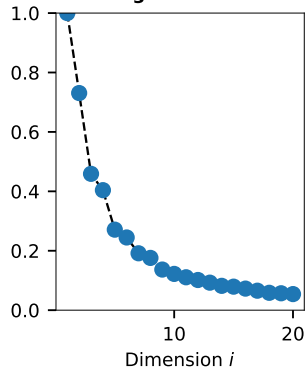
**Tuning curves ( $n = 128$ )**



**Principal Components**

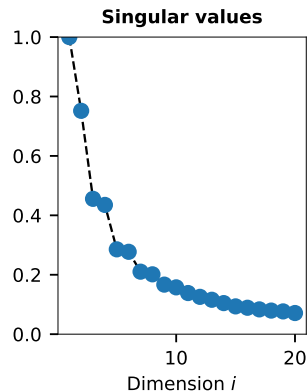
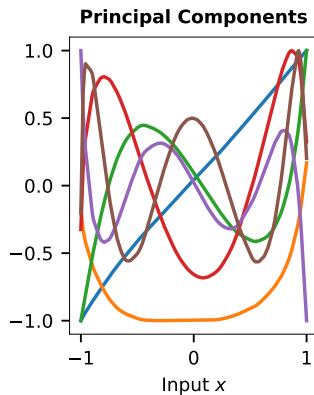
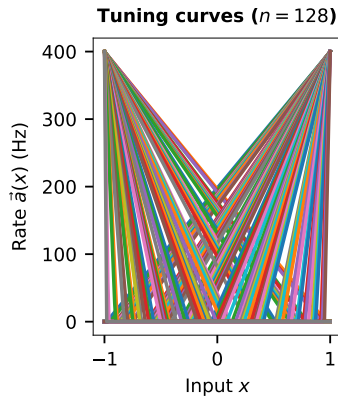


**Singular values**



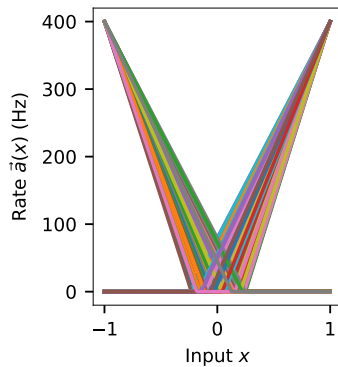


## Modifying the Basis – Equidistant $x$ -Intercepts (II)

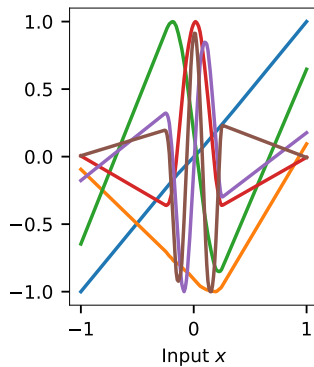


## Modifying the Basis – Limited x-Intercepts (III)

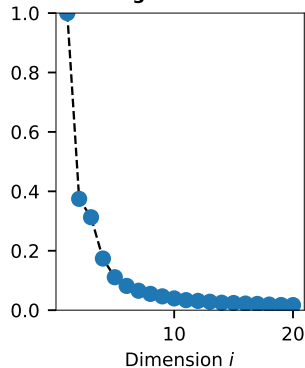
**Tuning curves ( $n = 128$ )**



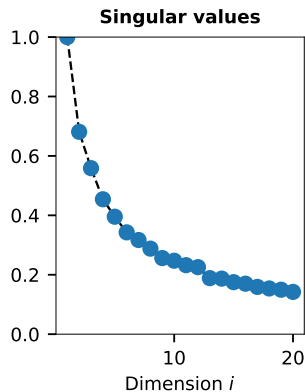
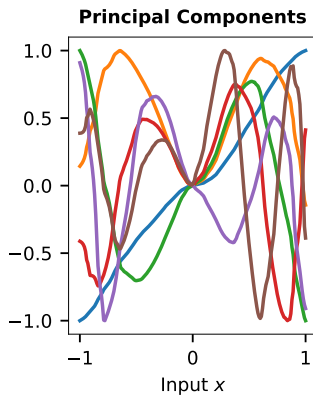
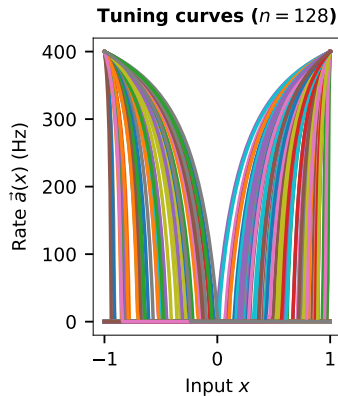
**Principal Components**



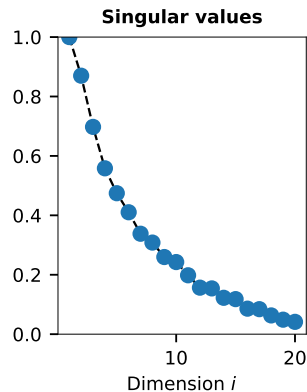
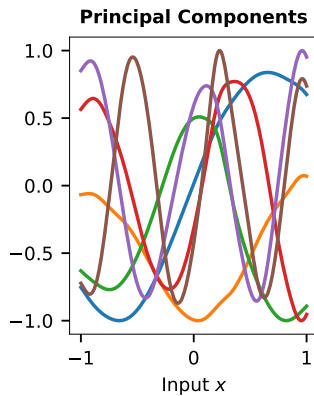
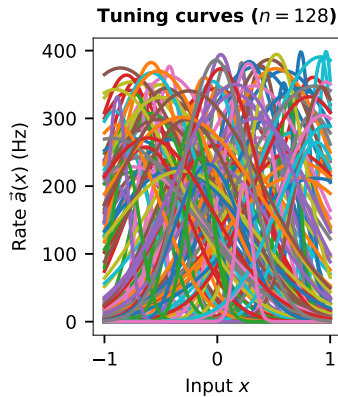
**Singular values**



## Modifying the Basis – Symmetric Tuning Curves (IV)

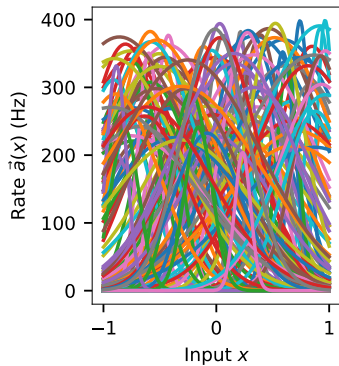


# Gaussian Tuning Curve Principal Components

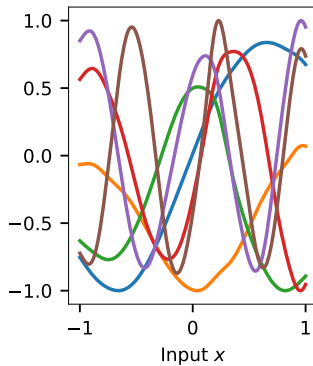


# Gaussian Tuning Curve Principal Components

**Tuning curves ( $n = 128$ )**

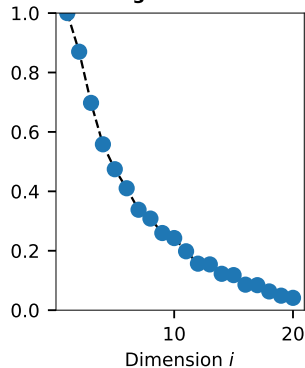


**Principal Components**



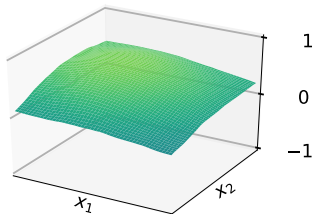
$\approx$  Fourier Basis

**Singular values**

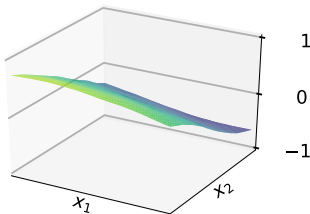


# PCA of 2D Tuning Curves

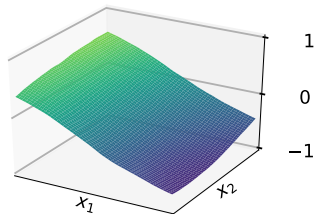
Principal component 1



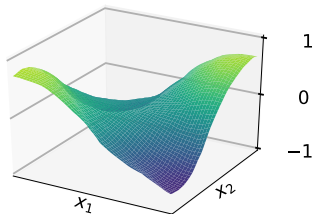
Principal component 2



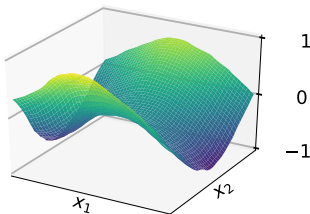
Principal component 3



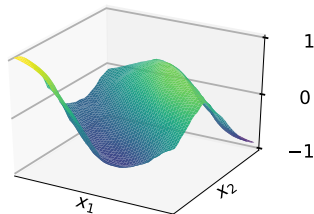
Principal component 4



Principal component 5

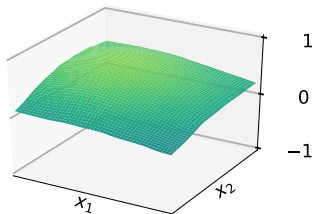


Principal component 6

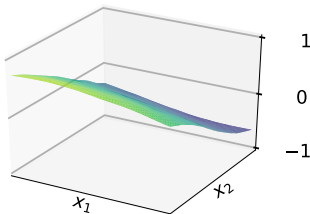


# PCA of 2D Tuning Curves

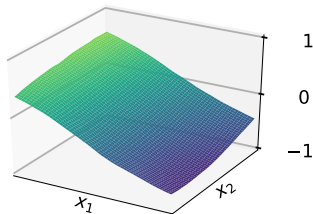
Principal component 1



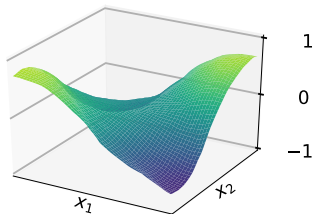
Principal component 2



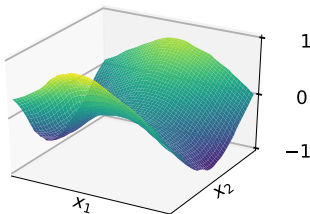
Principal component 3



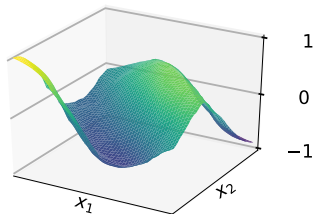
Principal component 4



Principal component 5



Principal component 6



Combination of 2D Polynomials

# Conclusions

- ▶ Can use **PCA** to find the basis functions underlying neural representations
- ▶ **Singular values** inversely proportional to noise
- ▶ **Basis function shape** depends on
  - ▶ x-intercept distributions
  - ▶ Neuron response curve  $G[J]$
- ▶ Finding optimal tuning curves for computing particular functions  
⇒ Full network optimization (must use gradient descent)



# Image sources

## **Title slide**

Maurice Denis: Homage to Cézanne, 1900  
From Wikimedia.