

**SYDE 556/750**

**Simulating Neurobiological Systems**  
**Lecture 4: Temporal Representations**

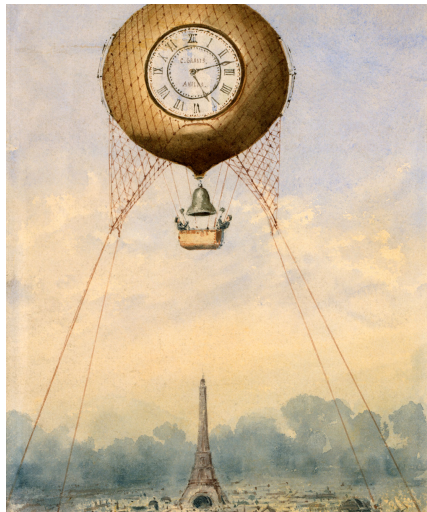
Andreas Stöckel

January 22 & 28, 2020

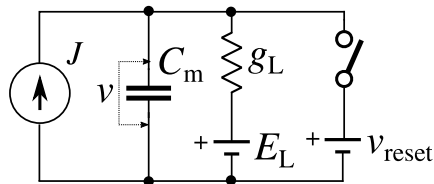
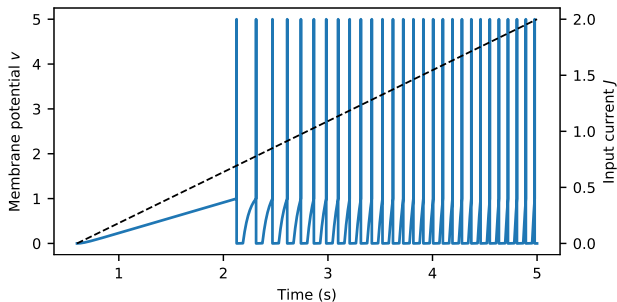


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## Reminder: The LIF Neuron



$$\frac{d}{dt}v(t) = -\frac{1}{\tau_{RC}}(v(t) - J),$$

$$v(t) \leftarrow \delta(t - t_{\text{th}}),$$

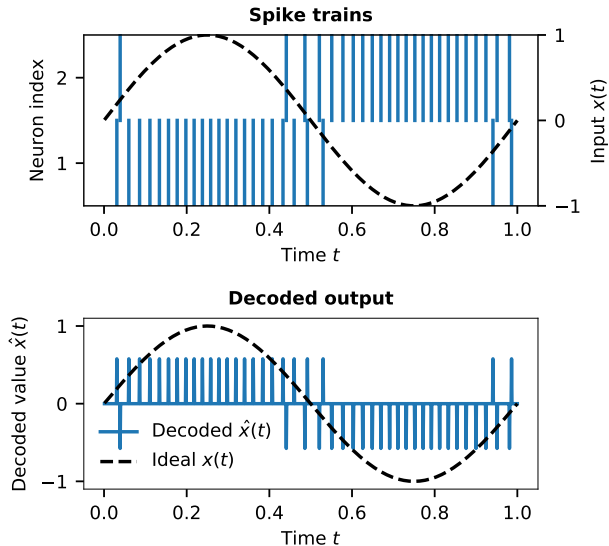
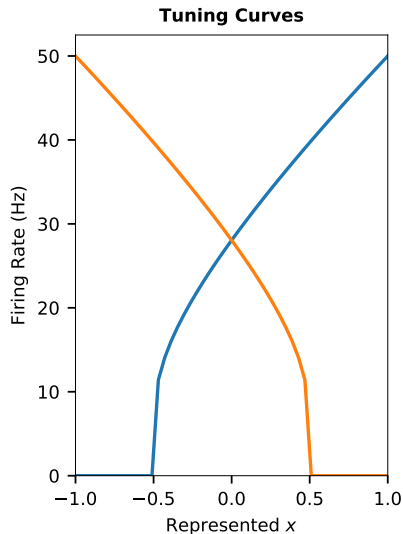
$$v(t) \leftarrow 0,$$

$$\text{if } v(t) < 1,$$

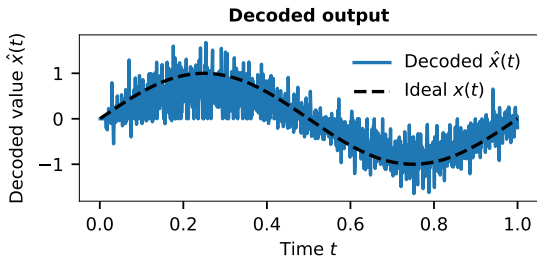
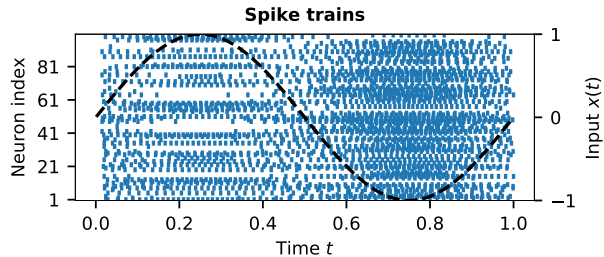
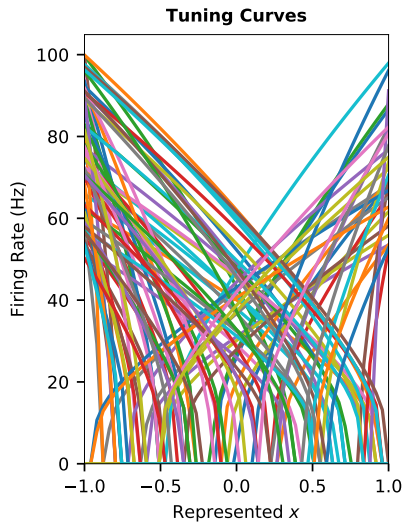
$$\text{if } t = t_{\text{th}},$$

$$\text{if } t > t_{\text{th}} \text{ and } t \geq t_{\text{th}} + \tau_{\text{ref}},$$

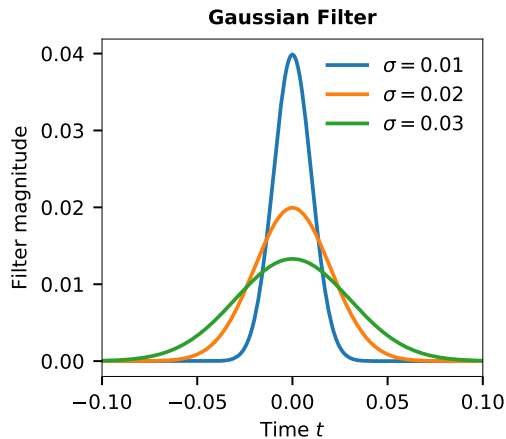
# Temporal Decoding of Two Neurons



# Temporal Decoding of One Hundred Neurons



# Filtering by Convolution



## Gaussian Filter

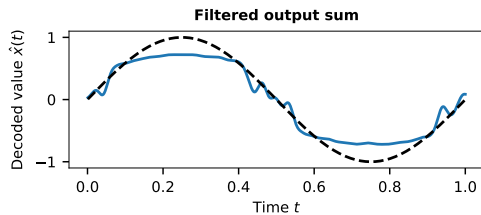
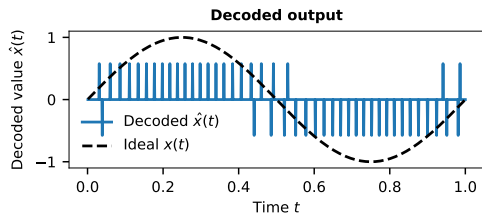
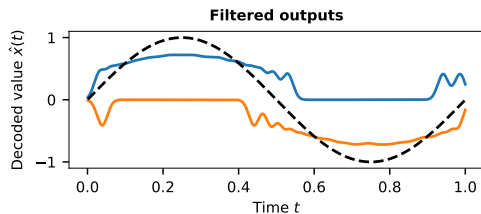
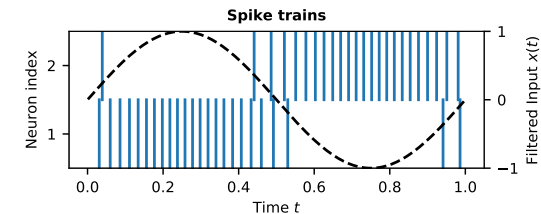
$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

where  $c$  chosen s.t.  $\int_{-\infty}^{\infty} h(t) dt = 1$

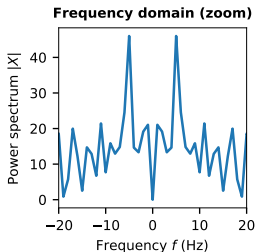
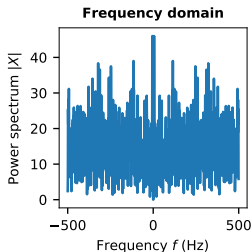
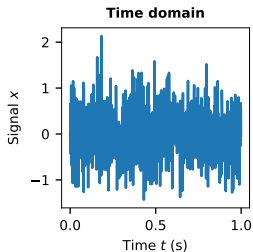
## Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - \tau)g(\tau) dt'$$

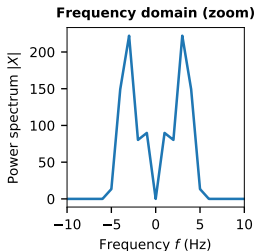
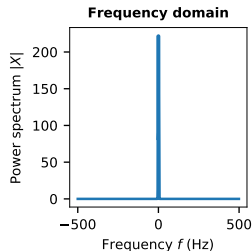
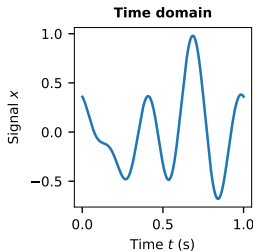
# Filtering a Spike Train



# Random Signals

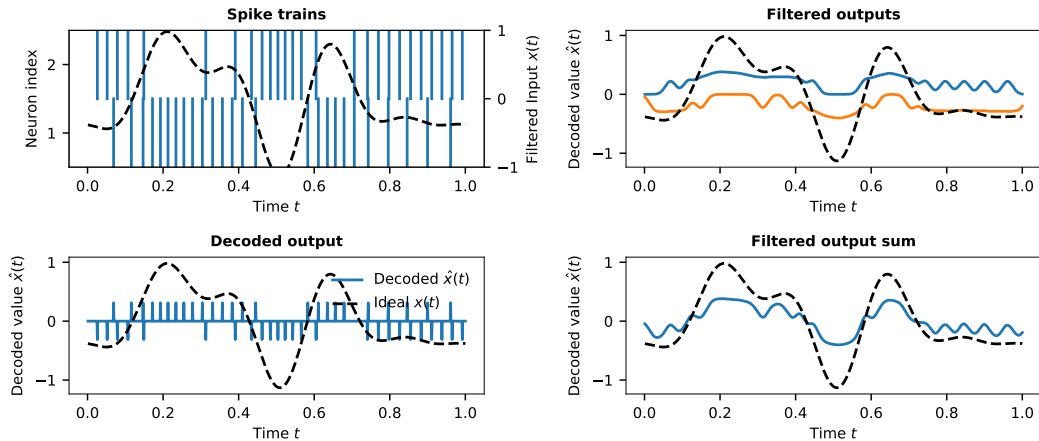


**White Noise**  
(zero mean)



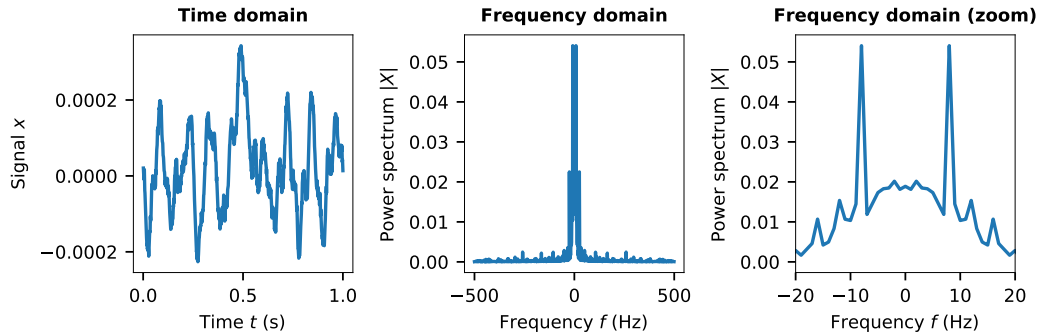
**Bandlimited**  
White Noise  
(zero mean,  
5 Hz bandwidth)

# Filtering a Spike Train for a Random Signal



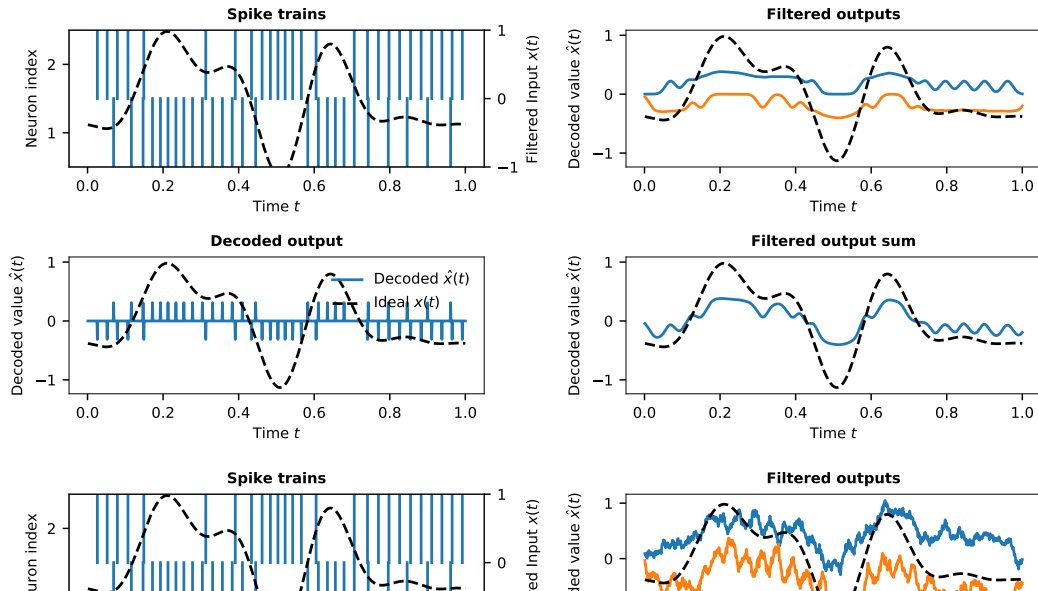


# Optimal Filter

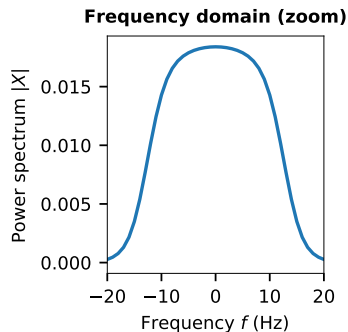
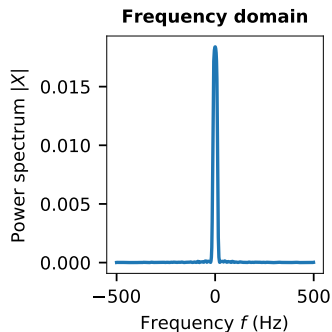
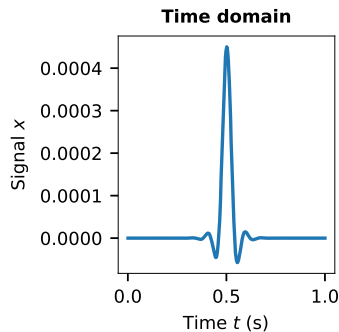


$$H(\omega) = \frac{X(\omega)\overline{R(\omega)}}{|R(\omega)|^2}$$

# Filtering a Spike Train for a Random Signal (Optimal Filter)

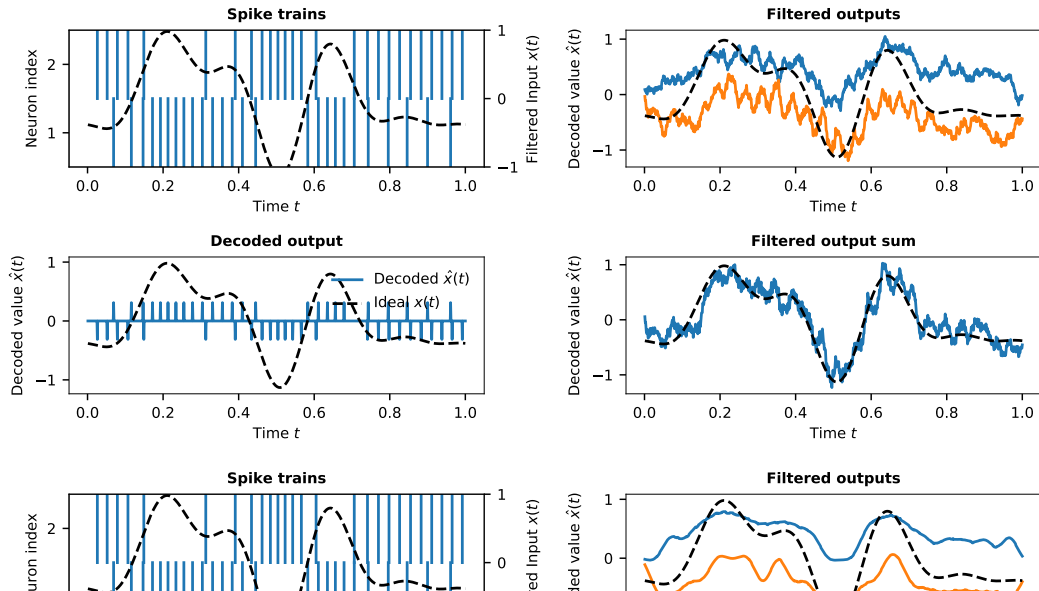


# Optimal Filter (Improved)



$$H(\omega) = \frac{X(\omega)\overline{R(\omega)} * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$

# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



# Image sources

## **Title slide**

“Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France.”

Author: Camille Grávis, between 1889 and 1900.

From Wikimedia.