SYDE 556/750

Simulating Neurobiological Systems
Lecture 6: Recurrent Dynamics

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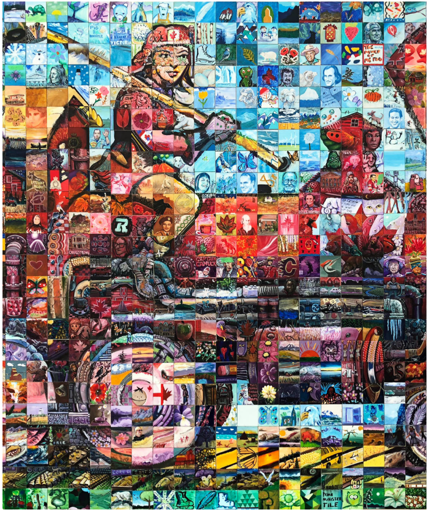

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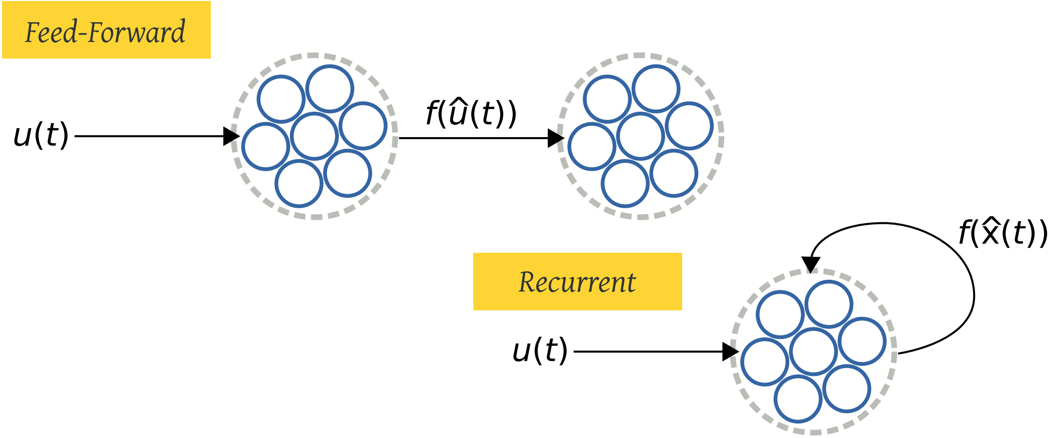
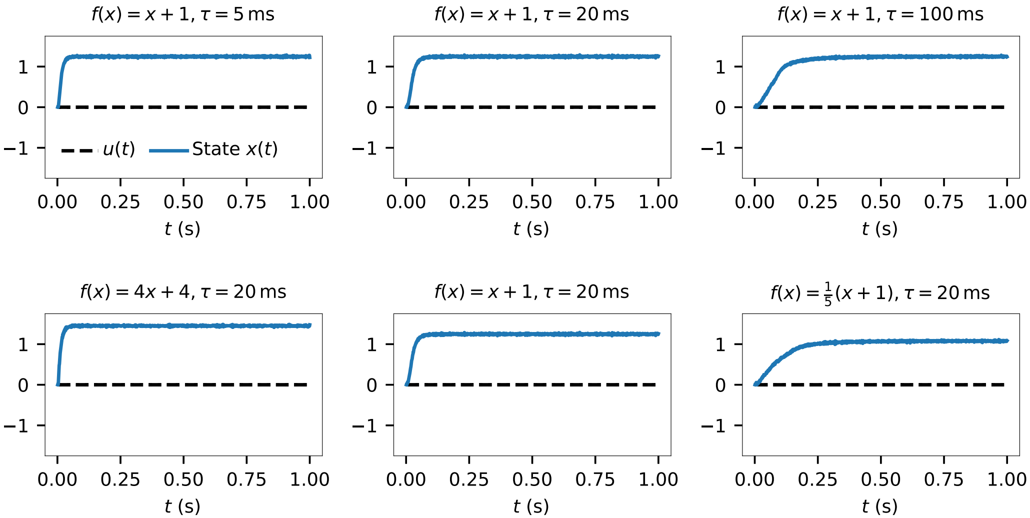
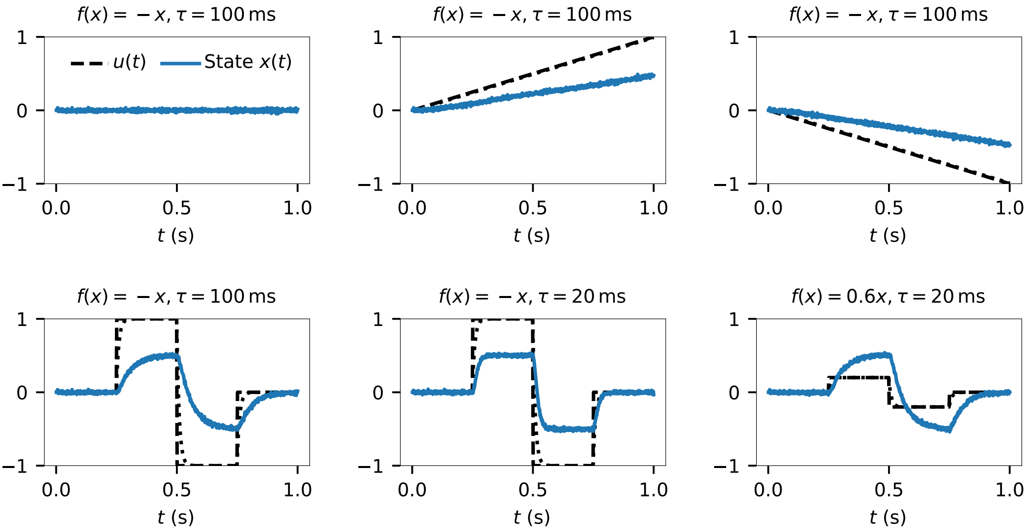
Feed Forward vs. Recurrent Connections



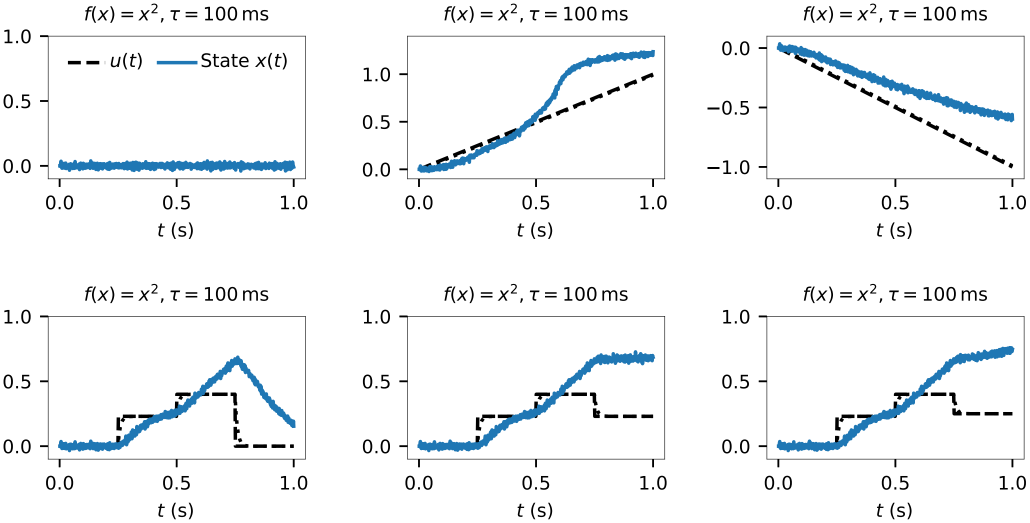
Recurrence Experiments (I)



Recurrence Experiments (II)



Recurrence Experiments (III)



NEF Principle 3: Dynamics

Time-Invariant Dynamical System

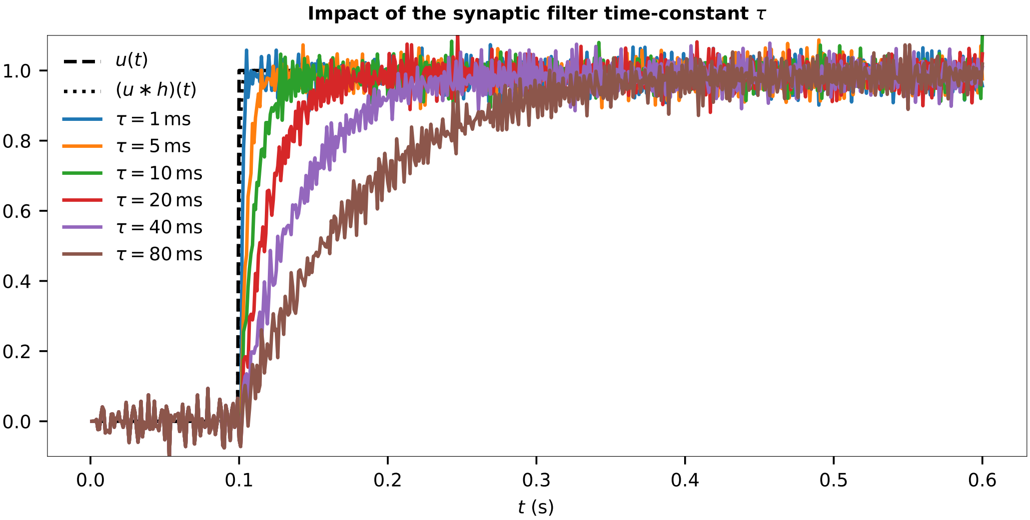
$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = f(\mathbf{x}(t), \mathbf{u}(t))$$

Linear Time-Invariant (LTI) Dynamical System

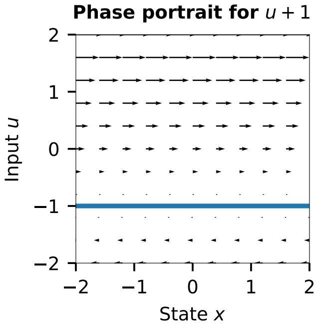
$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

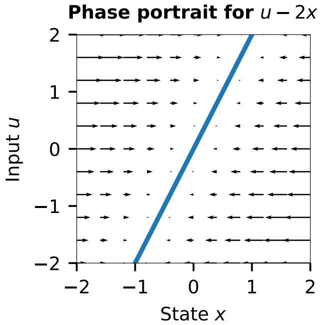
NEF Principle 3 – Dynamics
Neural dynamics are characterized by considering neural representations as control theoretic state variables. We can use control theory (and dynamical systems theory) to analyse and construct these systems.

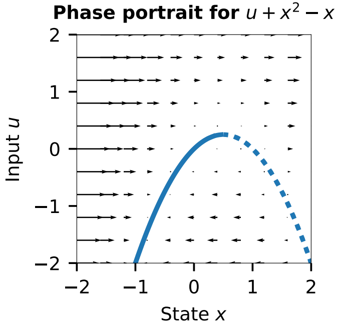
Making Sense of Dynamics



Phase Portraits

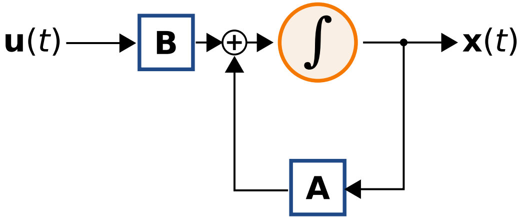




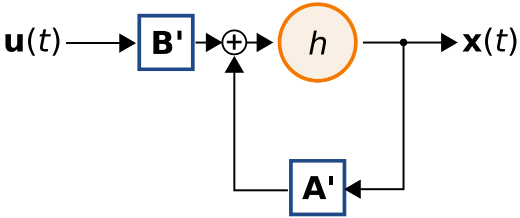


Implementing Dynamics using a Neural Ensemble

Evaluating an LTI
using an Integrator



Evaluating an LTI
using a Synaptic Filter



Implementing Dynamical Systems as a Neural Ensemble

LTI System

$$
\begin{aligned}
\phi(\mathbf{u}, \mathbf{x}) &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\
\phi'(\mathbf{u}, \mathbf{x}) &= \mathbf{A}'\mathbf{x} + \mathbf{B}'\mathbf{u} \\
\mathbf{A}' &= \tau\mathbf{A} + \mathbf{I} \\
\mathbf{B}' &= \tau\mathbf{B}\,.
\end{aligned}
$$

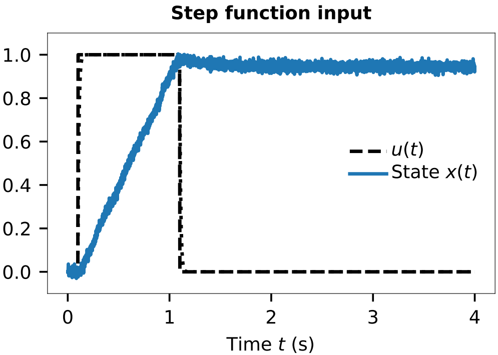
Additive Time-Invariant System

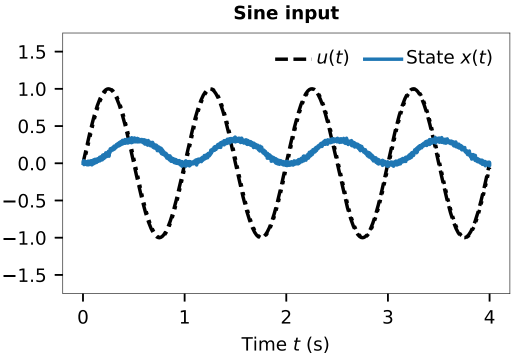
$$
\begin{aligned}
\phi(\mathbf{u}, \mathbf{x}) &= f(\mathbf{x}) + g(\mathbf{u}) \\
\phi'(\mathbf{u}, \mathbf{x}) &= f'(\mathbf{x}) + g'(\mathbf{u}) \\
f'(\mathbf{x}) &= \tau f(\mathbf{x}) + \mathbf{x} \\
g'(\mathbf{u}) &= \tau g(\mathbf{u})
\end{aligned}
$$

"General" Recipe

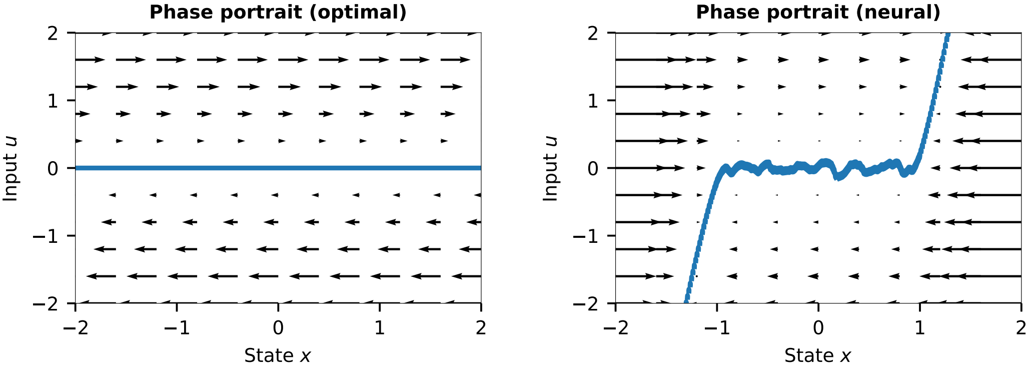
Scale the original dynamics by τ, add feedback $\mathbf{x}$

Integrator Example (I)

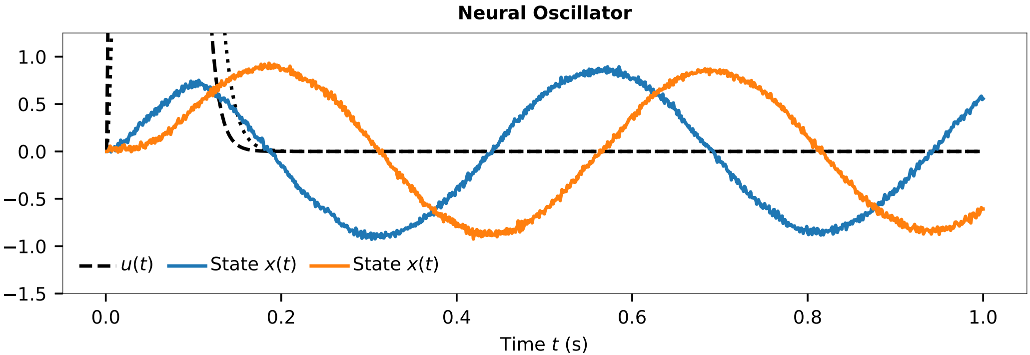




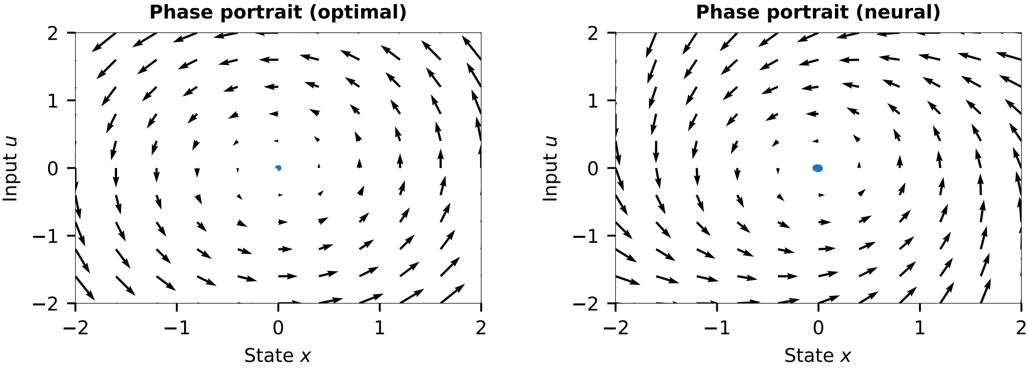
Integrator Example (II)



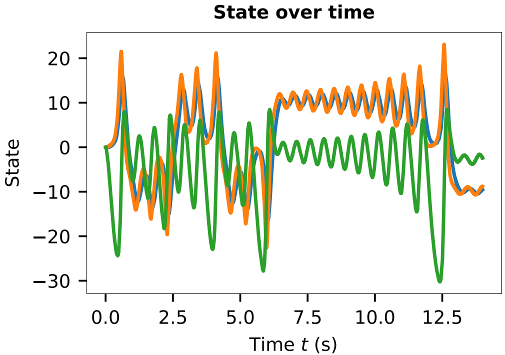
Oscillator Example (I)

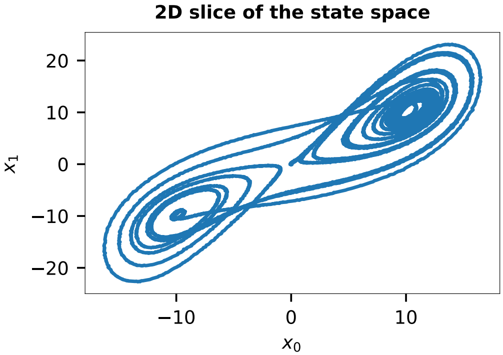


Oscillator Example (II)



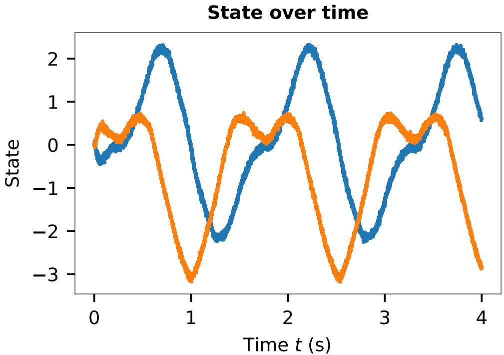
Lorentz Attractor

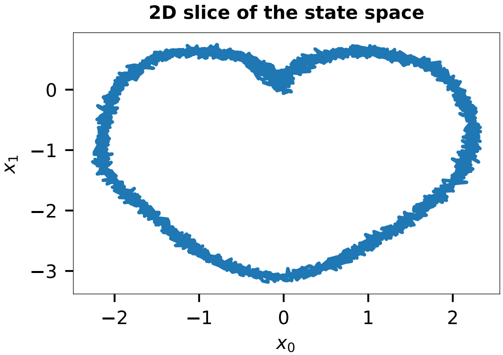




$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \begin{pmatrix} 10x_2(t) - 10x_1(t) \\ -x_1(t)x_3(t) - x_2(t) \\ x_1(t)x_2(t) - \frac{8}{3}(x_3(t) + 28) - 28 \end{pmatrix}$$

Heart Shape


State over time
State
2
1
0
−1
−2
−3
0
1
2
3
4
Time t (s)


2D slice of the state space
x_1
0
−1
−2
−3
−2
−1
0
1
2
x_0

Horizontal Eye Control

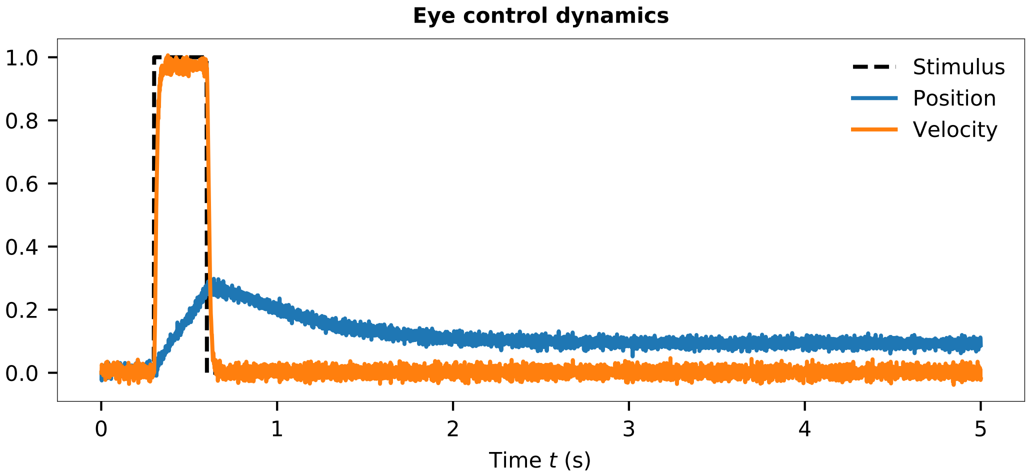


Image sources

Title slide
"The Canada 150 Mosaic Mural"
Author: Mosaic Canada Murals.
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