

**SYDE 556/750**

**Simulating Neurobiological Systems**  
**Lecture 3: Representations**

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January 14 & 16 & 21, 2020



UNIVERSITY OF  
**WATERLOO**

FACULTY OF  
ENGINEERING



# Visual Cortex

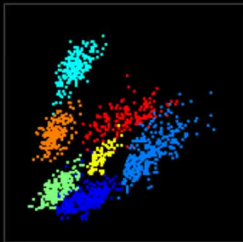


## Mapping receptive fields

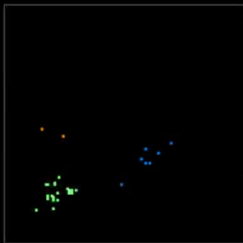
cell activity

behavior

overall



ongoing













































































# NEF Principle 1: Representation

## **NEF Principle 1 – Representation**

*Groups* (“populations”, or “ensembles”) of neurons *represent* represent values via nonlinear encoding and linear decoding.

# Lossless Codes

INTERNATIONAL ALPHABET FLAGS, PHONETIC ALPHABET, MORSE CODE AND SEMAPHORE ALPHABET																															
<b>A</b> ALFA  <small>Alphabet: A Z 10 10 10</small>	<b>B</b> BRAVO  <small>Alphabet: B Z 10 10 10</small>	<b>C</b> CHARLIE  <small>Alphabet: C Z 10 10 10</small>	<b>D</b> DELTA  <small>Alphabet: D Z 10 10 10</small>	<b>E</b> ECHO  <small>Alphabet: E Z 10 10 10</small>	<b>F</b> FOXTROT  <small>Alphabet: F Z 10 10 10</small>	<b>G</b> GOLF  <small>Alphabet: G Z 10 10 10</small>	<b>H</b> HOTEL  <small>Alphabet: H Z 10 10 10</small>	<b>I</b> INDIA  <small>Alphabet: I Z 10 10 10</small>	<b>J</b> JULIETT  <small>Alphabet: J Z 10 10 10</small>	<b>K</b> KILO  <small>Alphabet: K Z 10 10 10</small>	<b>L</b> LIMA  <small>Alphabet: L Z 10 10 10</small>	<b>M</b> MIKE  <small>Alphabet: M Z 10 10 10</small>	<b>N</b> NOVEMBER  <small>Alphabet: N Z 10 10 10</small>	<b>O</b> OSCAR  <small>Alphabet: O Z 10 10 10</small>	<b>P</b> PAPA  <small>Alphabet: P Z 10 10 10</small>	<b>Q</b> QUEBEC  <small>Alphabet: Q Z 10 10 10</small>	<b>R</b> ROMEO  <small>Alphabet: R Z 10 10 10</small>	<b>S</b> SIERRA  <small>Alphabet: S Z 10 10 10</small>	<b>T</b> TANGO  <small>Alphabet: T Z 10 10 10</small>	<b>U</b> UNIFORM  <small>Alphabet: U Z 10 10 10</small>	<b>V</b> VICTOR  <small>Alphabet: V Z 10 10 10</small>	<b>W</b> WHISKEY  <small>Alphabet: W Z 10 10 10</small>	<b>X</b> XRAY  <small>Alphabet: X Z 10 10 10</small>	<b>Y</b> YANKEE  <small>Alphabet: Y Z 10 10 10</small>	<b>Z</b> ZULU  <small>Alphabet: Z Z 10 10 10</small>	<b>Alphabet</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Hand</b>  <small>Hand: A Z 10 10 10</small>	<b>Error</b>  <small>Error: A Z 10 10 10</small>	<b>Reversal</b>  <small>Reversal: A Z 10 10 10</small>	<b>Answer</b>  <small>Answer: A Z 10 10 10</small>	<b>Doublet Sign</b>  <small>Doublet Sign: A Z 10 10 10</small>
NAVAL NUMERAL FLAGS, PHONETIC NUMERALS AND MORSE CODE																															
<b>1</b> ONE  <small>Alphabet: 1 Z 10 10 10</small>	<b>2</b> TWO  <small>Alphabet: 2 Z 10 10 10</small>	<b>3</b> THREE  <small>Alphabet: 3 Z 10 10 10</small>	<b>4</b> FOUR  <small>Alphabet: 4 Z 10 10 10</small>	<b>5</b> FIVE  <small>Alphabet: 5 Z 10 10 10</small>	<b>6</b> SIX  <small>Alphabet: 6 Z 10 10 10</small>	<b>7</b> SEVEN  <small>Alphabet: 7 Z 10 10 10</small>	<b>8</b> EIGHT  <small>Alphabet: 8 Z 10 10 10</small>	<b>9</b> NINE  <small>Alphabet: 9 Z 10 10 10</small>	<b>0</b> ZERO  <small>Alphabet: 0 Z 10 10 10</small>																						
INTERNATIONAL NUMERAL PENNANTS																															
<b>1</b> "Pennant One"  <small>Alphabet: 1 Z 10 10 10</small>	<b>2</b> "Pennant Two"  <small>Alphabet: 2 Z 10 10 10</small>	<b>3</b> "Pennant Three"  <small>Alphabet: 3 Z 10 10 10</small>	<b>4</b> "Pennant Four"  <small>Alphabet: 4 Z 10 10 10</small>	<b>5</b> "Pennant Five"  <small>Alphabet: 5 Z 10 10 10</small>	<b>6</b> "Pennant Six"  <small>Alphabet: 6 Z 10 10 10</small>	<b>7</b> "Pennant Seven"  <small>Alphabet: 7 Z 10 10 10</small>	<b>8</b> "Pennant Eight"  <small>Alphabet: 8 Z 10 10 10</small>	<b>9</b> "Pennant Nine"  <small>Alphabet: 9 Z 10 10 10</small>	<b>0</b> "Pennant Zero"  <small>Alphabet: 0 Z 10 10 10</small>																						
NAVAL SPECIAL FLAGS AND PENNANTS																															
<b>International Answer</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Code</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Black Pennant</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Corpus</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Designation</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Division</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Emergency</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Flotilla</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Formation</b>  <small>Alphabet: A Z 10 10 10</small>																							
<b>Interrogative</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Negative</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Preparative</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Port</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Speed</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Squadron</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Starboard</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Station</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Submarine</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Tow</b>  <small>Alphabet: A Z 10 10 10</small>																						
<b>First Substitute</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Second Substitute</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Third Substitute</b>  <small>Alphabet: A Z 10 10 10</small>	<b>Fourth Substitute</b>  <small>Alphabet: A Z 10 10 10</small>																												

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Encoding:  $a = f(x)$

Decoding:  $x = f^{-1}(a)$

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- This is a **distributed code**. But, **not robust** against additive noise!

# Lossy codes

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Inverse  $f^{-1}$  does not exist, instead *approximate* the represented value

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- ▶ Audio, image, and video coding schemes (MP3, JPEG, H.264)

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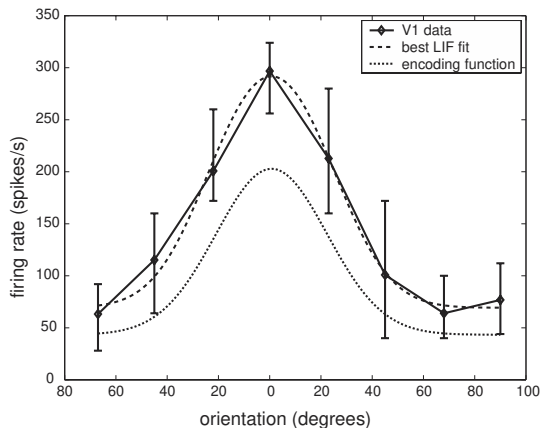
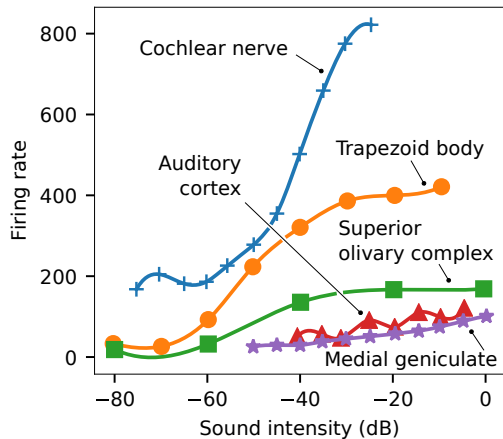
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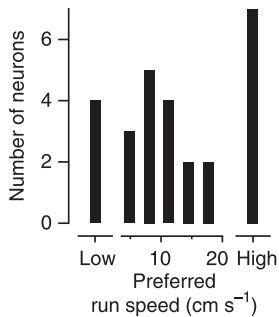
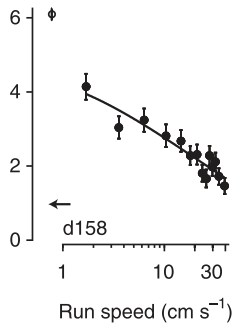
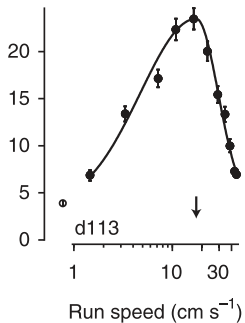
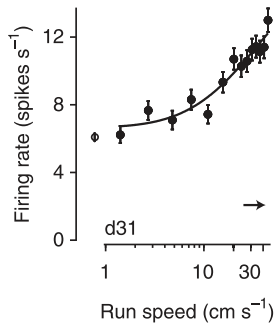
- ▶ **Examples**

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- ▶ **Neural Representations**

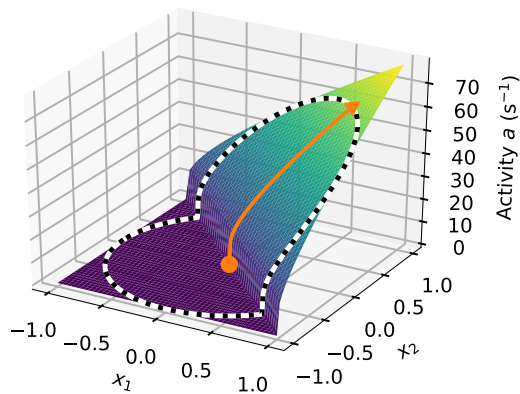
# Tuning curves (I)



## Tuning curves (II)

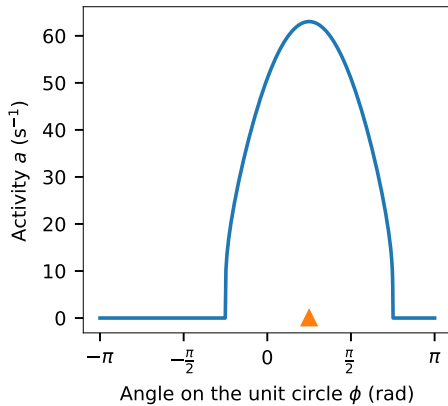
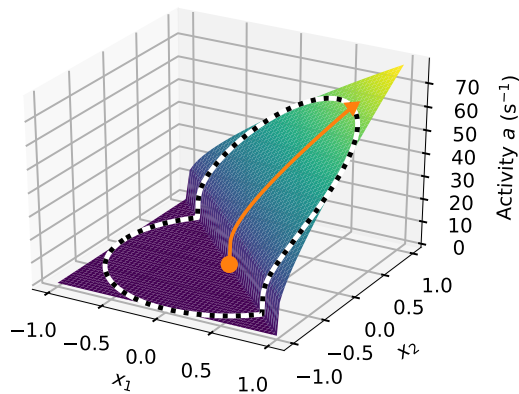


## Preferred Directions in Higher Dimensions: Representing 2D Values

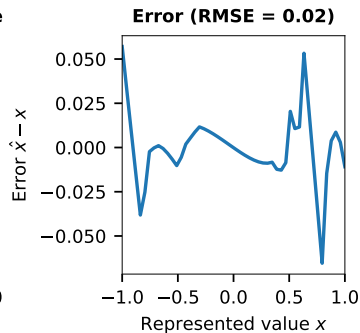
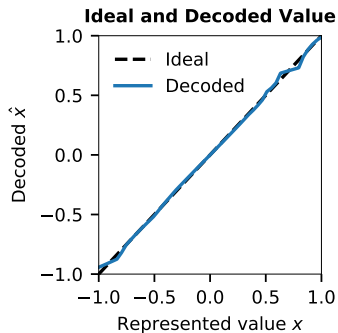
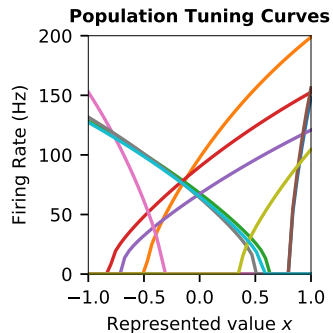




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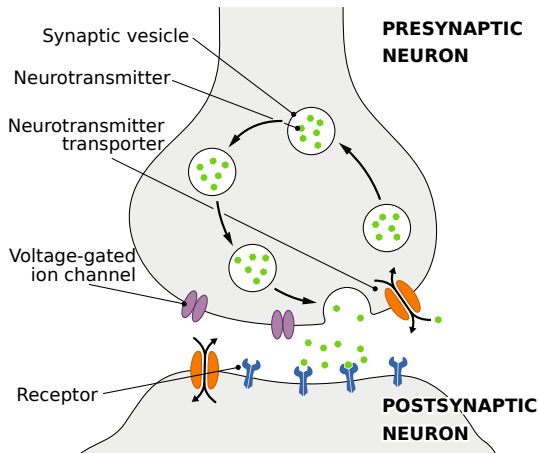


# Decoding Without Taking Noise Into Account



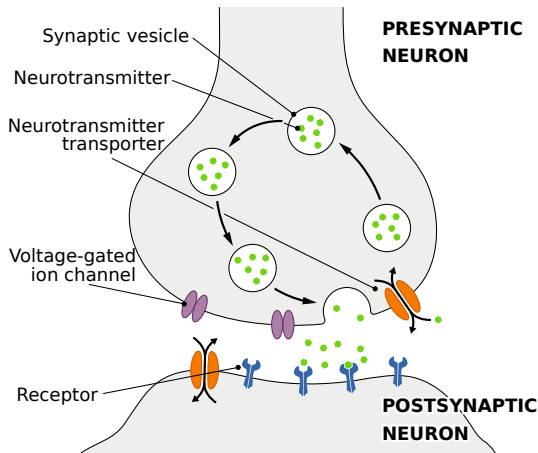
# Sources of Noise in Biological Neural Networks

- ▶ **Axonal jitter**  
Active axonal spike propagation
- ▶ **Vesicle release failure**  
10-30% of pre-synaptic events cause post-synaptic current
- ▶ **Neurotransmitter per vesicle**  
Varying amounts of neurotransmitter
- ▶ **Ion channel noise**  
Ion-channels are “binary”, stochastic
- ▶ **Thermal noise**
- ▶ **Network effects**  
Simple, noise-free inhibitory/excitatory networks produce irregular spike trains



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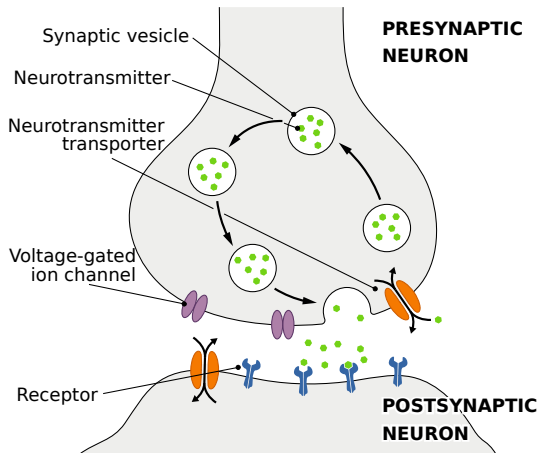
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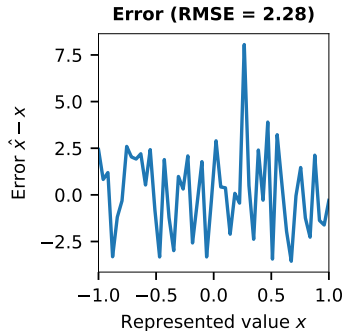
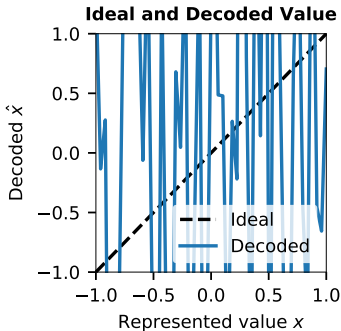
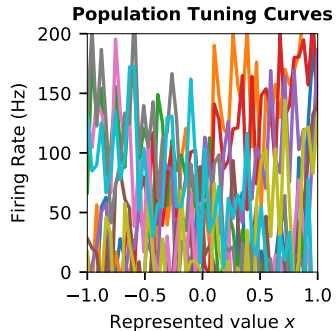
- ▶ **How to model?** Gaussian noise

## NEF Principle 0: Noise

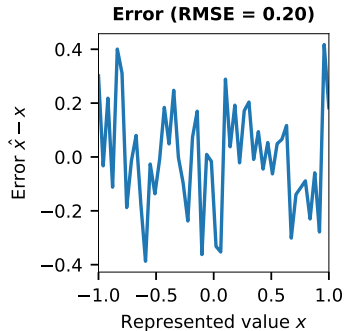
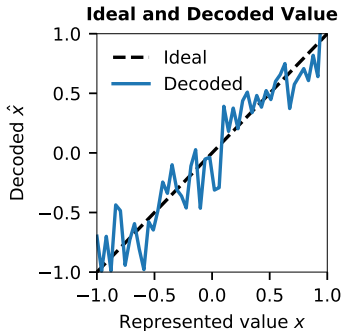
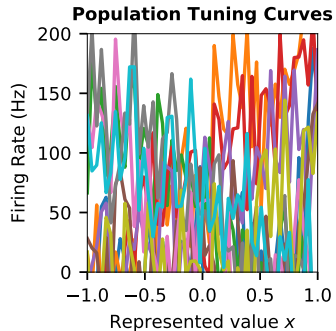
### **NEF Principle 0 – Noise**

Biological neural systems are subject to significant amounts of noise from various sources. Any analysis of such systems must take the effects of noise into account.

# Decoding Noisy $\mathbf{A}$ Without Taking Noise Into Account



# Decoding Noisy $\mathbf{A}$ Accounting for Noise



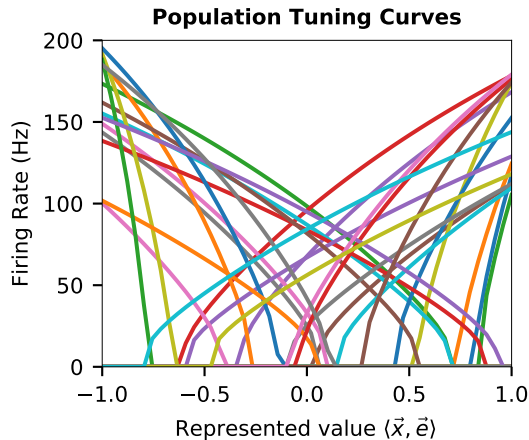


# Summary: Building a model of neural representation (Encoding)

## Encoding

- ▶ Select  $d$ , possible range  $\mathbf{x} \in \mathbb{X}$ , usually  $\mathbb{X} = \{\mathbf{x} \mid \|\mathbf{x}\| \leq r, \mathbf{x} \in \mathbb{R}^d\}$  ( $r = 1$ )
- ▶ Select number of neurons  $n$
- ▶ Select tuning curves, maximum rates  
 $\Rightarrow \mathbf{e}_i, \alpha_i, J_i^{\text{bias}}$ 
  - ▶ Sample  $\mathbf{e}_i$  from unit-sphere
  - ▶ Uniformly distribute  $x$ -intercept, maximum rate
- ▶ Encoding equation:

$$a_i(\mathbf{x}) = G[\alpha_i \langle \mathbf{e}_i, \mathbf{x} \rangle + J_i^{\text{bias}}]$$



## Summary: Building a model of neural representation (Decoding)

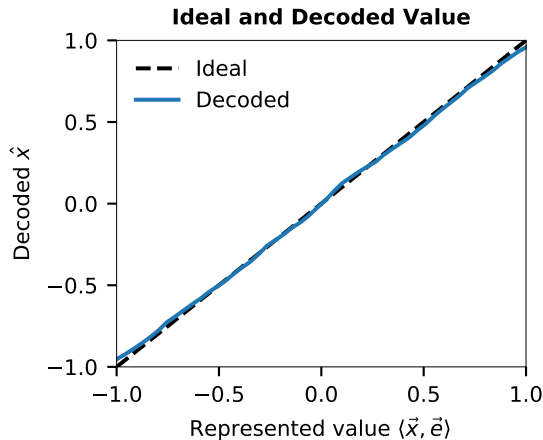
### Decoding

- ▶ Uniformly sample  $N$  samples from  $\mathbb{X}$ ,  $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_N)$
- ▶ Compute  $\mathbf{A}$ , where  $(\mathbf{A})_{ik} = a_i(\mathbf{x}_k)$
- ▶ Decoder computation:

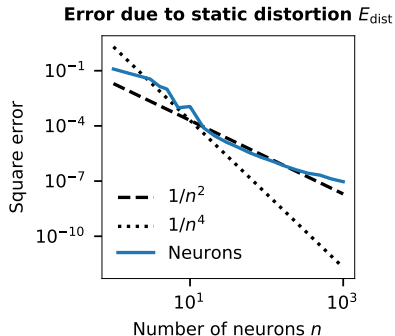
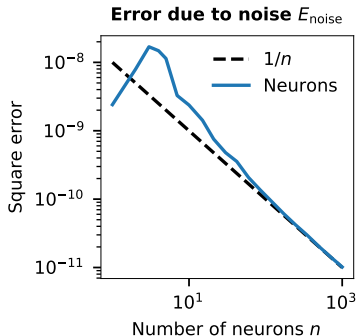
$$\mathbf{D}^\top = (\mathbf{A}\mathbf{A}^\top + N\sigma^2\mathbf{I})^{-1} \mathbf{A}\mathbf{X}^\top$$

- ▶ Decoding equation:

$$\hat{\mathbf{X}} = \mathbf{D}\mathbf{A}$$

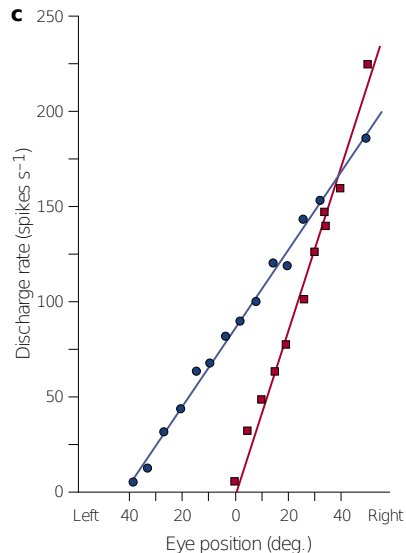
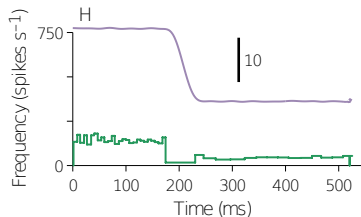
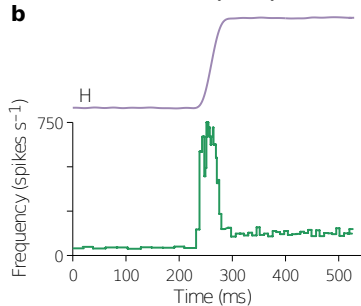
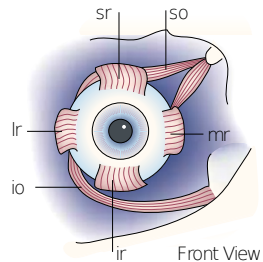
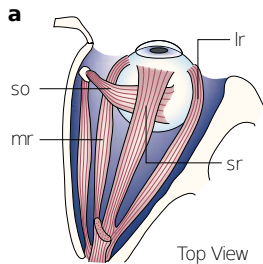


# Analysing Sources of Errors



$$E = \underbrace{\frac{1}{2} \int_{-1}^1 \left( x - \sum_{i=1}^n d_i a_i(x) \right)^2 dx}_{E_{\text{dist}}} + \underbrace{\frac{1}{2} \sigma^2 \sum_{i=1}^n d_i^2}_{E_{\text{noise}}}$$

## Example: Horizontal Eye Position (1D)



## Example: Horizontal Eye Position (1D) (cont.)

### ► Step 1: System Description

- What is being represented?
  - $x$  is the horizontal eye position
- What is the tuning curve shape?
  - Linear, low  $\tau_{\text{ref}}$ , high  $\tau_{\text{RC}}$
  - $e_i \in \{1, -1\}$
  - Firing rates up to  $300 \text{ s}^{-1}$

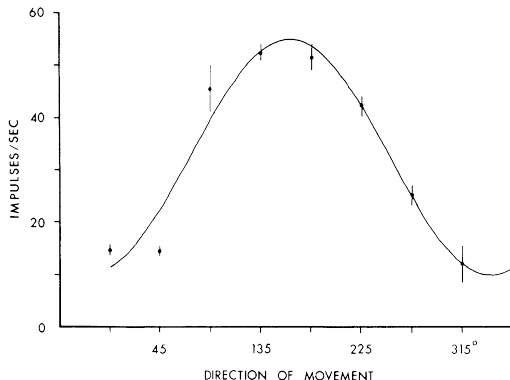
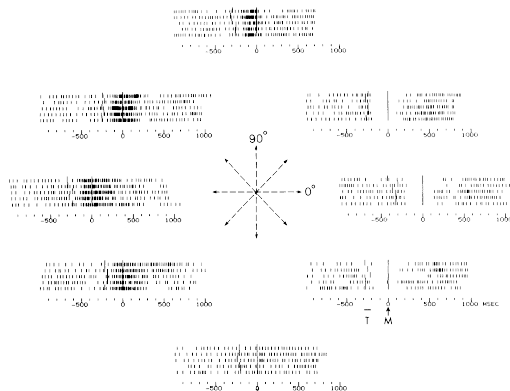
### ► Step 2: Design Specification

- Range of values
  - $\mathbb{X} = [-60, 60]$
- Amount of noise
  - About 20% of  $\max(\mathbf{A})$

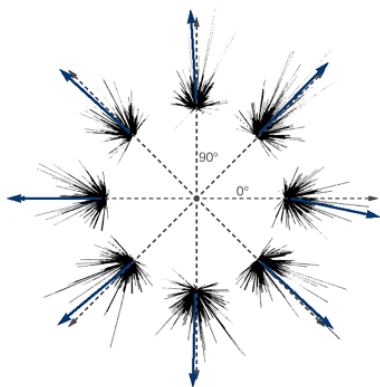
### ► Step 3: Implementation

- Choose tuning curve parameters
- Compute decoders

# Example: Arm Movements (2D)



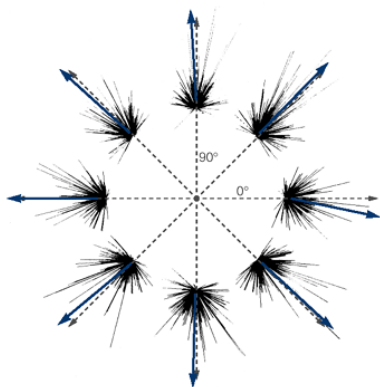
## Example: Arm Movements (2D) (cont.)



- ▶ Experiment by Georgopoulos et al., 1982
- ▶ Preferred arm movement directions  $\mathbf{e}_i$
- ▶ **Idea:** *Population Vectors*, decode using

$$\hat{\mathbf{x}} = \sum_{i=1}^n a_i(\mathbf{x}) \mathbf{e}_i = \mathbf{E} \mathbf{A}$$

## Example: Arm Movements (2D) (cont.)



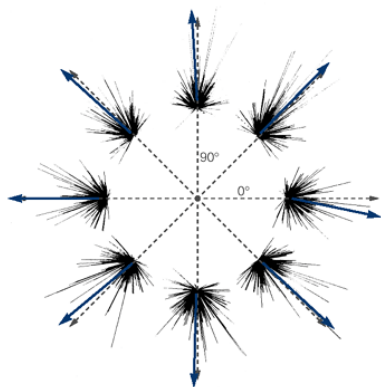
- ▶ Experiment by Georgopoulos et al., 1982
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- ▶ **Idea:** *Population Vectors*, decode using

$$\hat{\mathbf{x}} = \sum_{i=1}^n a_i(\mathbf{x}) \mathbf{e}_i = \mathbf{E} \mathbf{A}$$

- ⊕ Good direction estimate



## Example: Arm Movements (2D) (cont.)

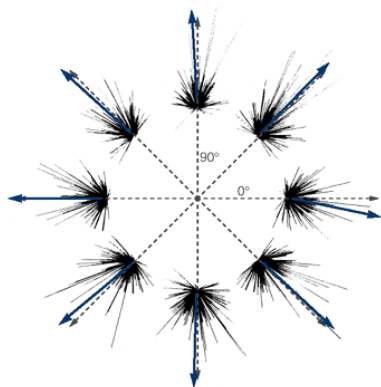


- ▶ Experiment by Georgopoulos et al., 1982
- ▶ Preferred arm movement directions  $\mathbf{e}_i$
- ▶ **Idea:** *Population Vectors*, decode using

$$\hat{\mathbf{x}} = \sum_{i=1}^n a_i(\mathbf{x}) \mathbf{e}_i = \mathbf{E} \mathbf{A}$$

- ⊕ Good direction estimate
- ⊖ Cannot reconstruct magnitude

## Example: Arm Movements (2D) (cont.)



- ▶ Experiment by Georgopoulos et al., 1982
- ▶ Preferred arm movement directions  $\mathbf{e}_i$
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$$\hat{\mathbf{x}} = \sum_{i=1}^n a_i(\mathbf{x}) \mathbf{e}_i = \mathbf{E} \mathbf{A}$$

- ⊕ Good direction estimate
- ⊖ Cannot reconstruct magnitude

**The NEF does not use population vectors!**

## Example: Arm Movements (2D) (cont.)

### ► Step 1: System Description

- What is being represented?
  - $\mathbf{x}$  the movement direction (or hand position)
- What is the tuning curve shape?
  - Bell-shaped
  - Encoders are randomly distributed along the unit circle
  - Firing rates up to  $60\text{ s}^{-1}$

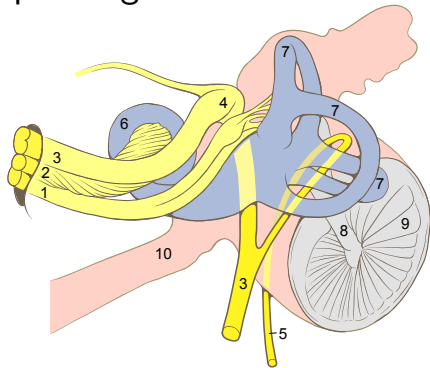
### ► Step 2: Design Specification

- Range of values
  - $\mathbb{X} = \{\mathbf{x} \mid \|\mathbf{x}\| \leq r, \mathbf{x} \in \mathbb{R}^2\}$
- Amount of noise
  - About 20% of  $\max(\mathbf{A})$

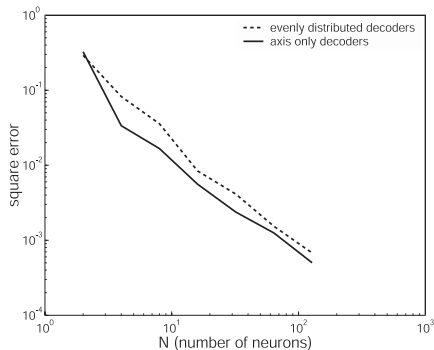
### ► Step 3: Implementation

- Choose tuning curve parameters
- Compute decoders

## Example: Higher Dimensional Representation

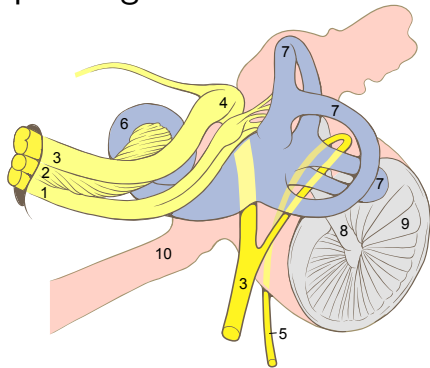


- ▶ Vestibular system senses head acceleration in 3D
- ▶ Axis aligned, must choose  $\mathbf{e}_i \in \{[1, 0, 0], [-1, 0, 0], \dots, [0, 0, -1]\}$

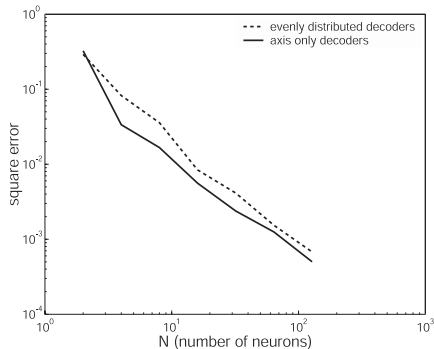


- ▶ Same as three 1D populations
- ▶ Slightly lower precision

## Example: Higher Dimensional Representation



- ▶ Vestibular system senses head acceleration in 3D
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- ▶ Same as three 1D populations
- ▶ Slightly lower precision
- ▶ **Encoders affect accuracy**

# Administration

- ▶ **Assignment 1 has been released.**

The due date has been adjusted to January, 30.

- ▶ Some new potential times for office hours

Mon 15:30–16:30, Mon 16:30–17:30, Tue 15:00–16:00,

Thu 11:30–12:30 (current slot), Thu 12:30–13:30

# Image sources

## **Title slide**

“The Ultimate painting.”

Author: Clark Richert.

From Wikimedia.