

**SYDE 556/750**

**Simulating Neurobiological Systems**  
**Lecture 5: Feed-Forward Transformation**

Andreas Stöckel

January 30, 2020

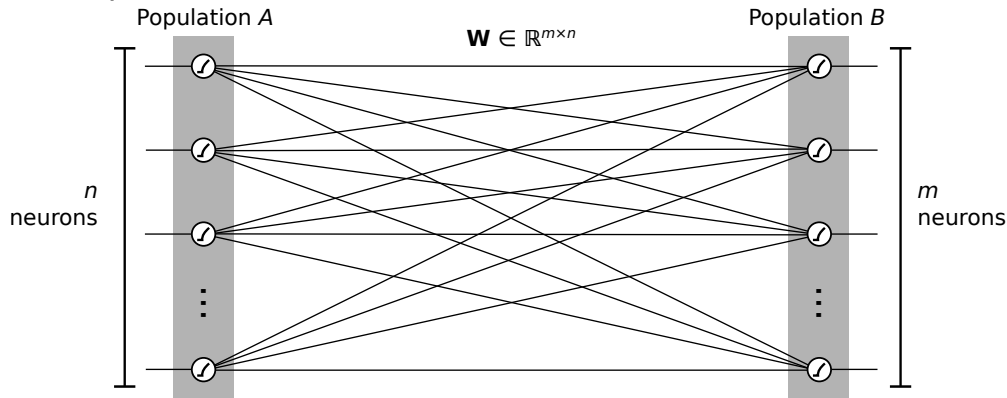


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ENGINEERING



## NEF Principle 2: Transformation

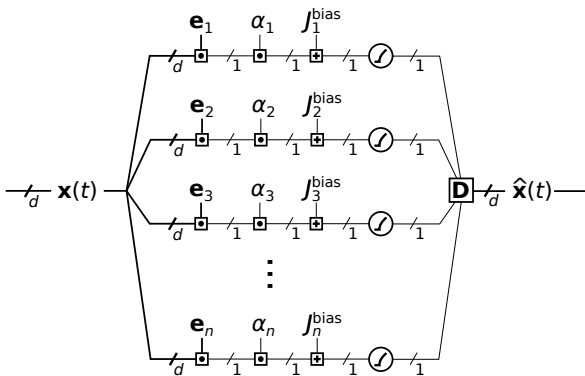


### NEF Principle 2 – Transformation

Connections between populations describe *transformations* of neural representations. Transformations are functions of the variables represented by neural populations.

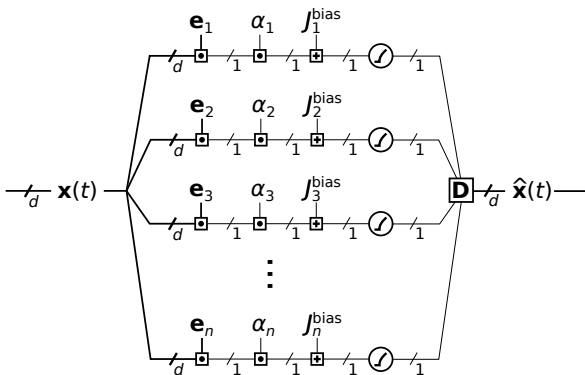
# A Tale of Two Populations (I)

Population A

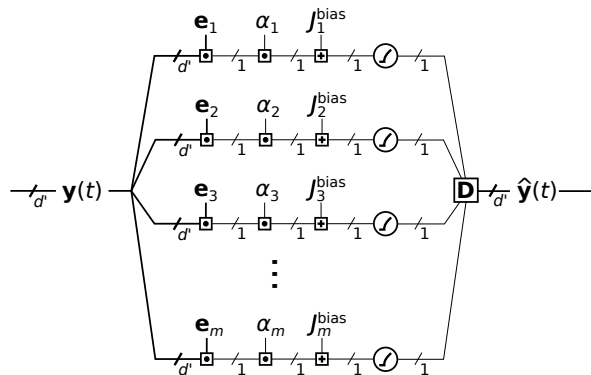


# A Tale of Two Populations (I)

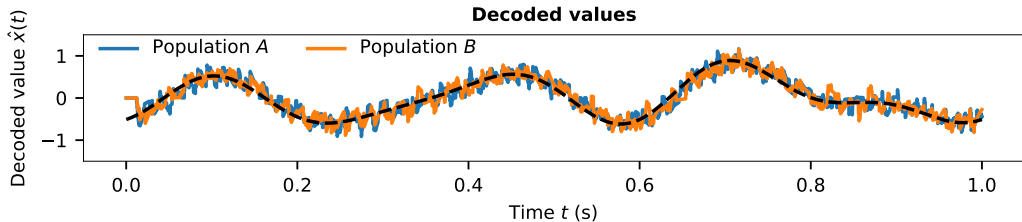
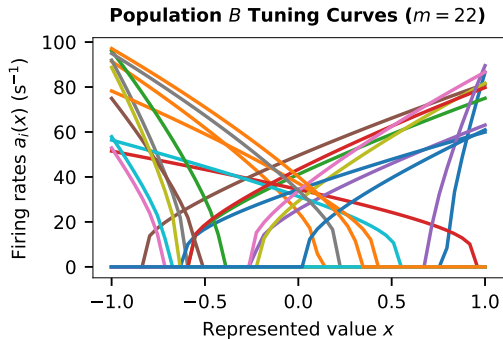
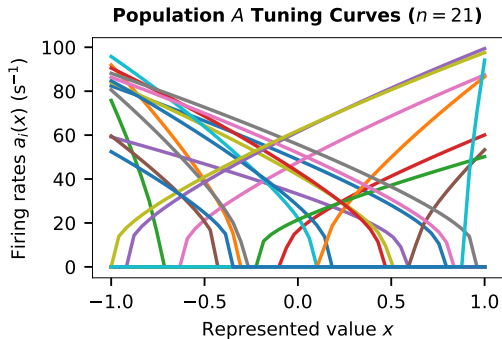
Population A



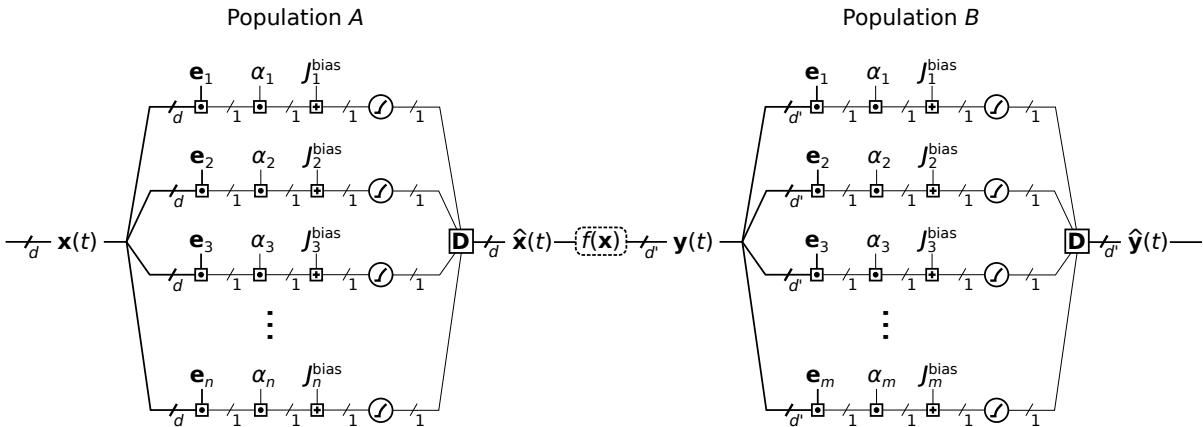
Population B



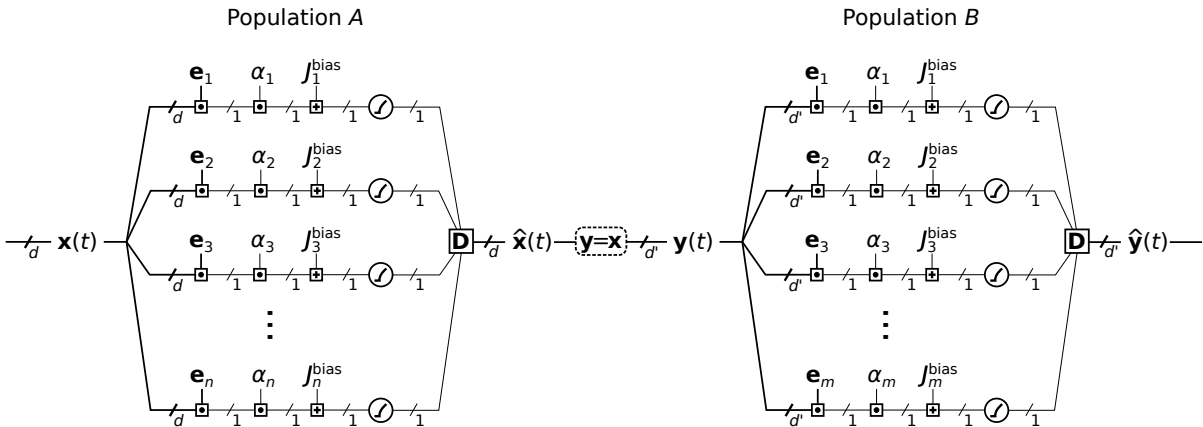
# Communication Channel Experiment: Same input signal



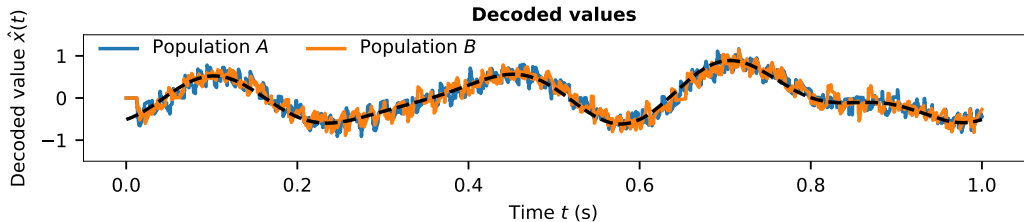
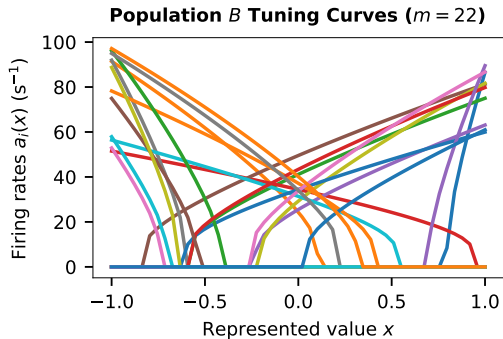
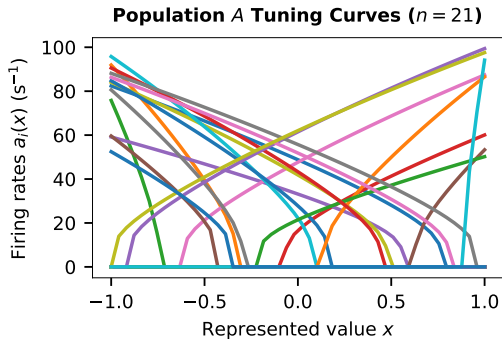
# A Tale of Two Populations (II)



## A Tale of Two Populations (II)

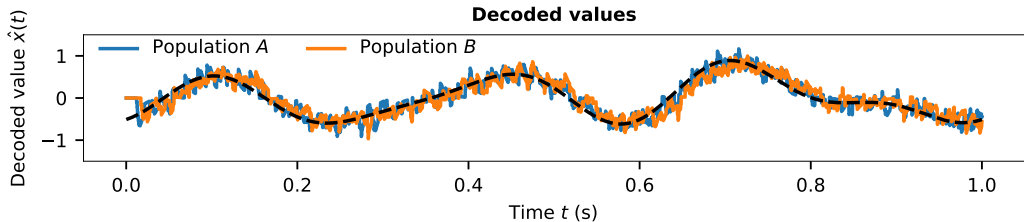
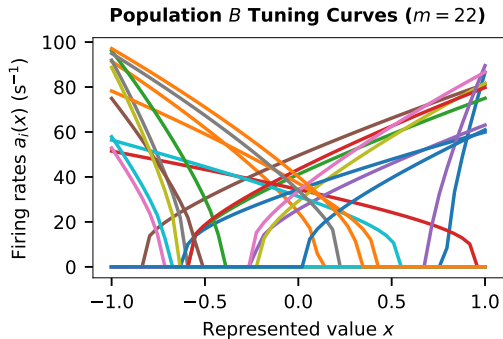
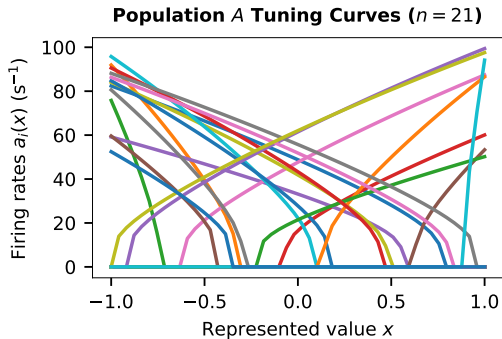


# Communication Channel Experiment: Populations in series

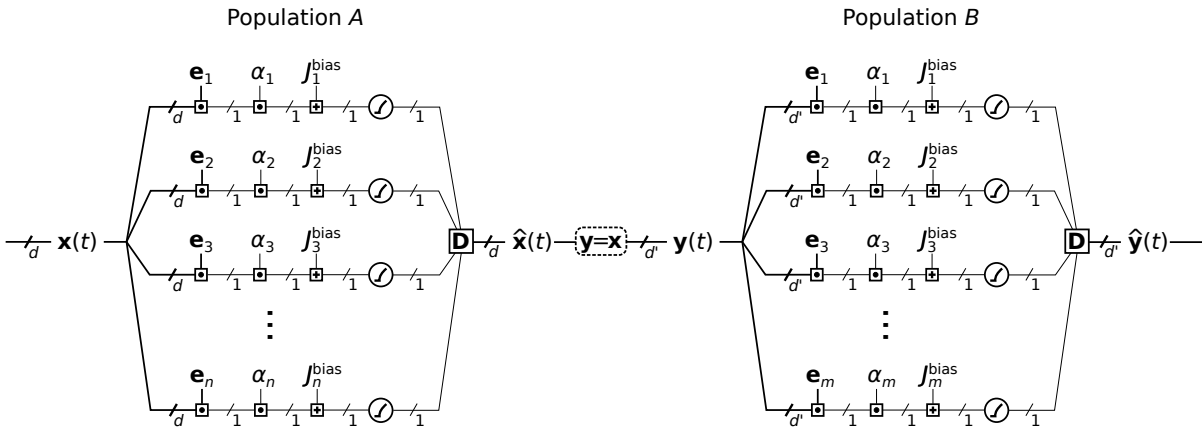




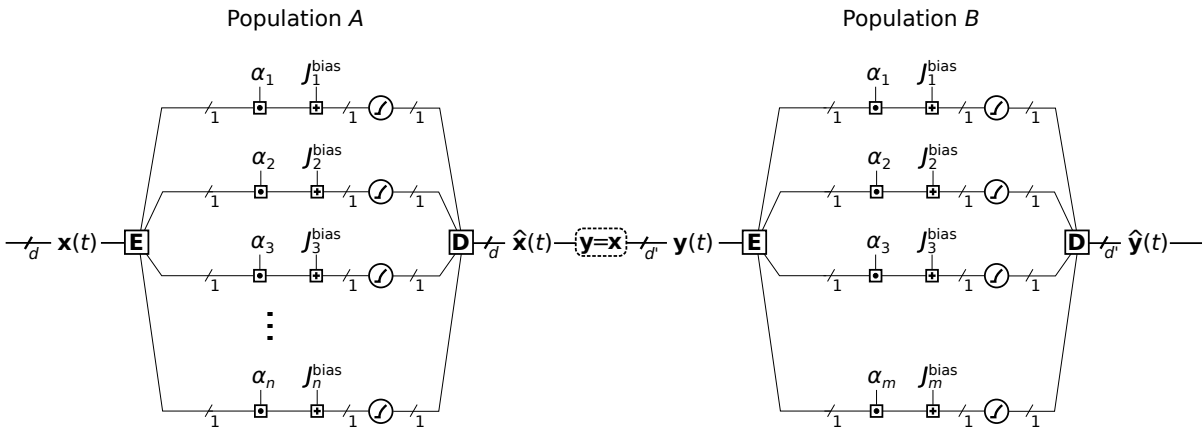
# Communication Channel Experiment: Populations in series



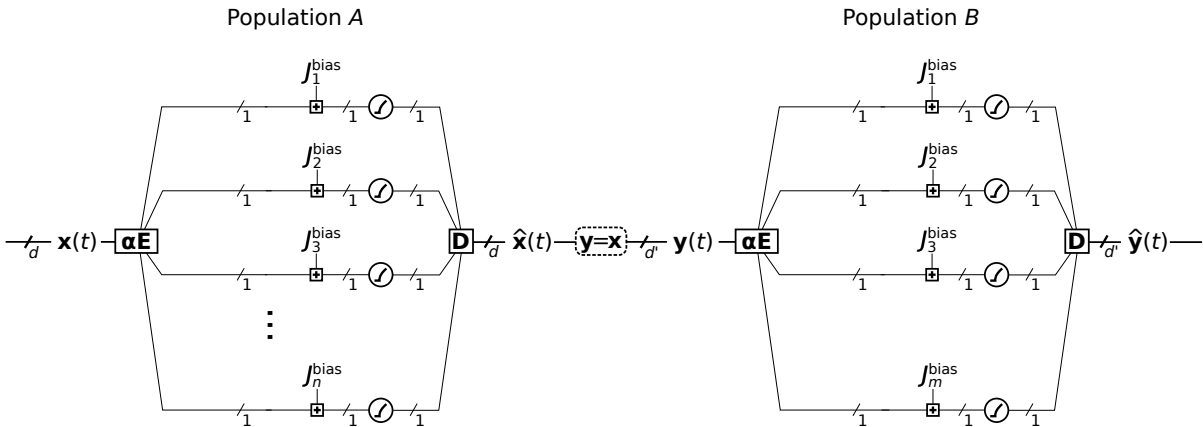
# Computing Synaptic Weights: Step 1 – Encoding Matrix



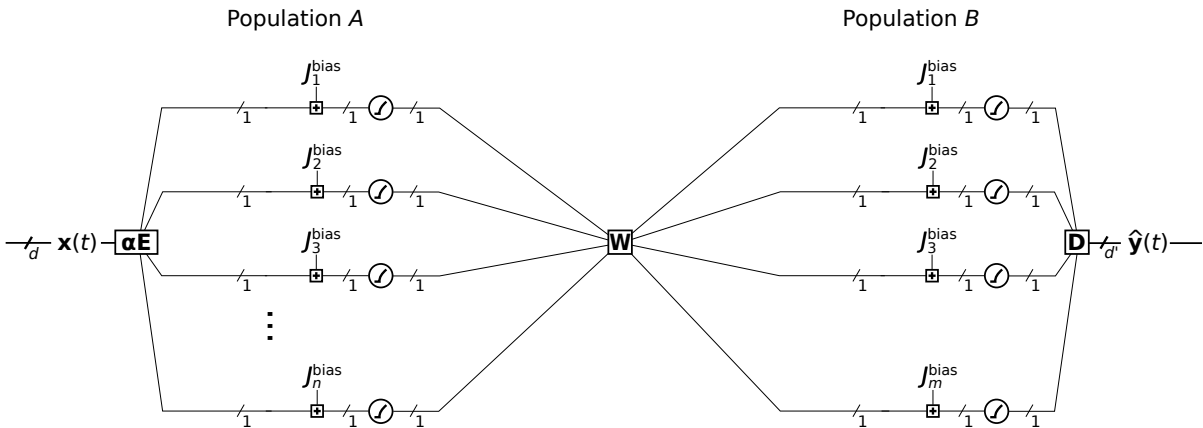
# Computing Synaptic Weights: Step 1 – Encoding Matrix



## Computing Synaptic Weights: Step 2 – Scaled Encoding Matrix

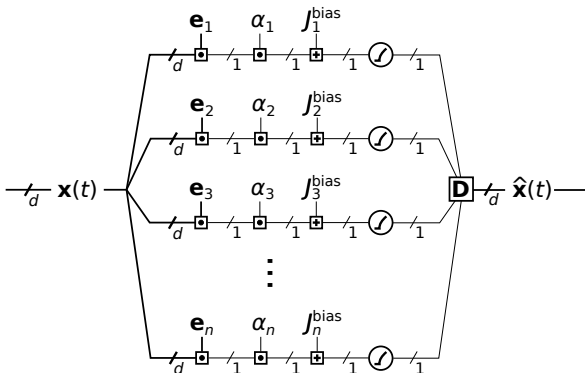


# Computing Synaptic Weights: Step 3 – $\mathbf{W} = \mathbf{E}\mathbf{D}$

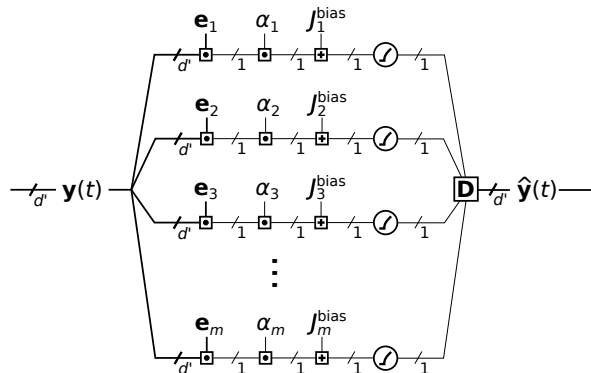


# Computing Functions

Population A

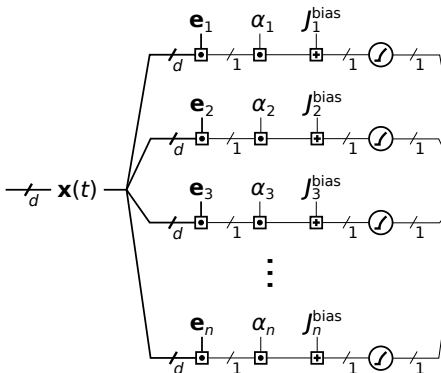


Population B

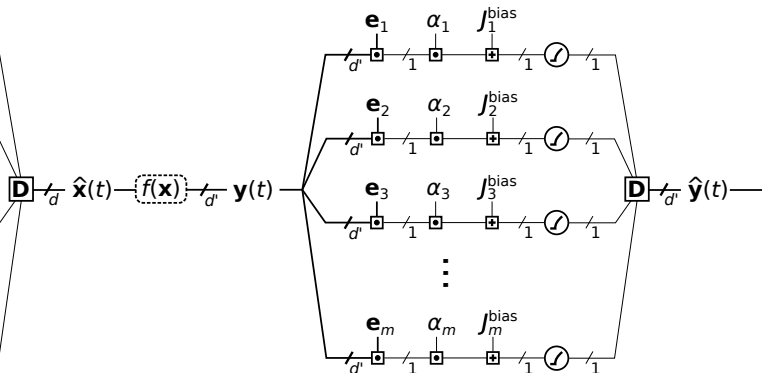


# Computing Functions

Population A

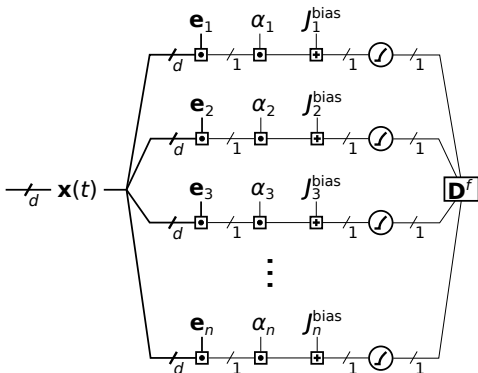


Population B

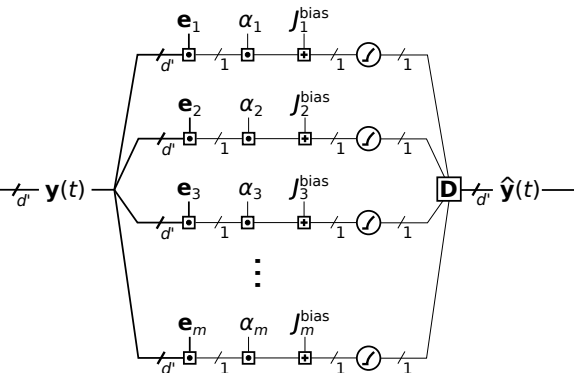


# Computing Functions

Population A



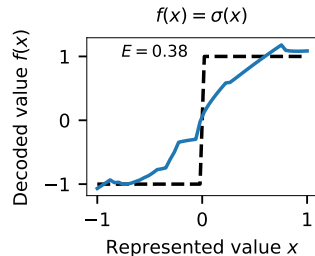
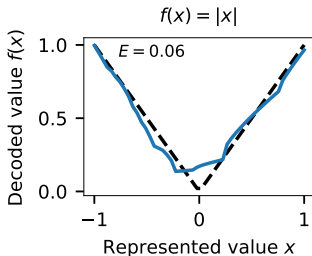
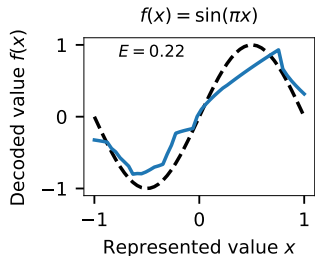
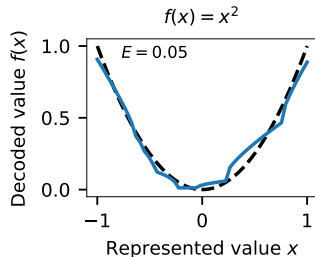
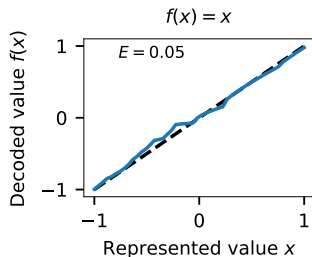
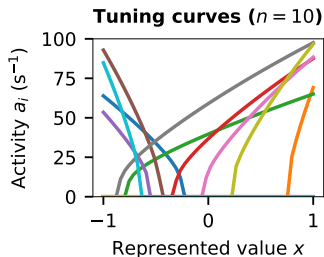
Population B



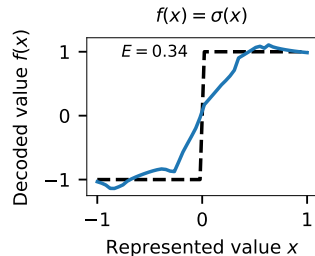
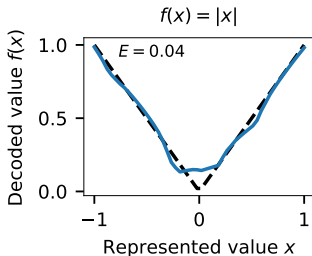
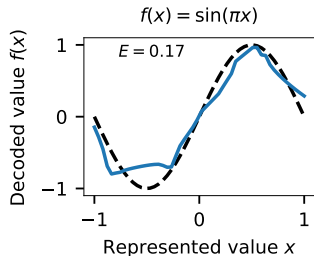
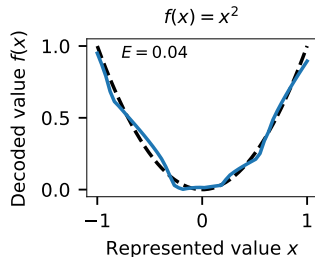
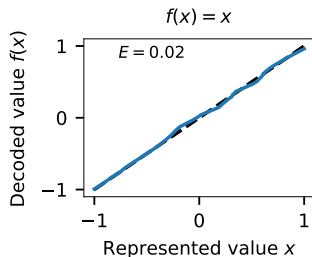
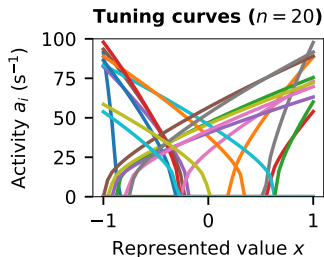
**Function Decoder**  $\mathbf{D}^f = ((\mathbf{A}\mathbf{A}^\top + N\sigma^2\mathbf{I})^{-1}\mathbf{A}\mathbf{Y}^\top)^\top$ , where  $(\mathbf{Y})_{ik} = (f(\mathbf{x}_k))_i$



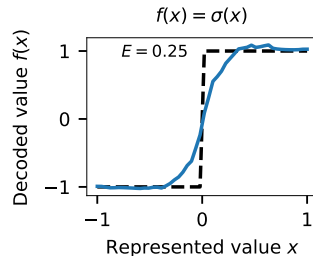
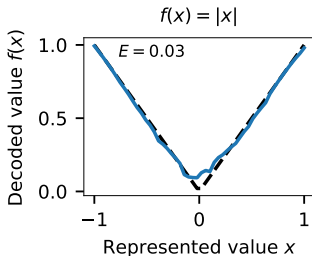
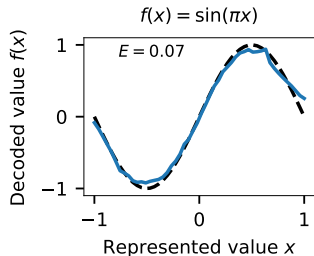
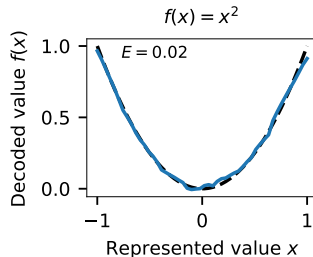
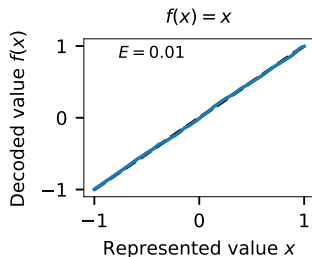
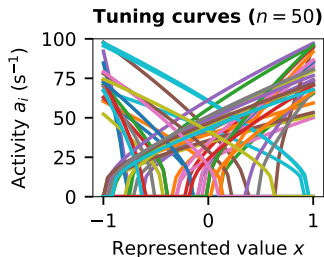
# Decoding Functions – Using a Few Neurons



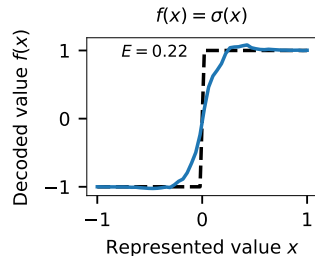
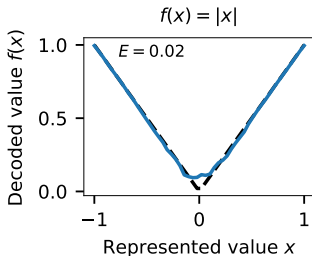
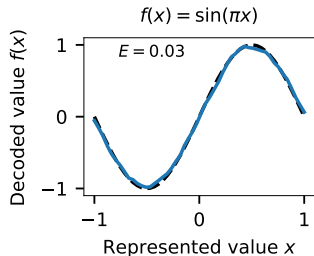
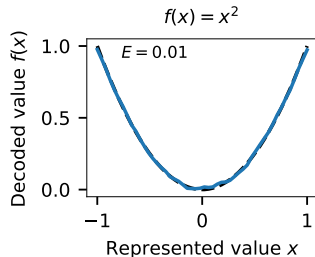
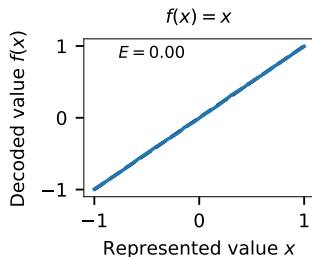
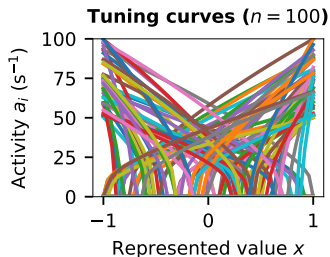
# Decoding Functions – Using More Neurons



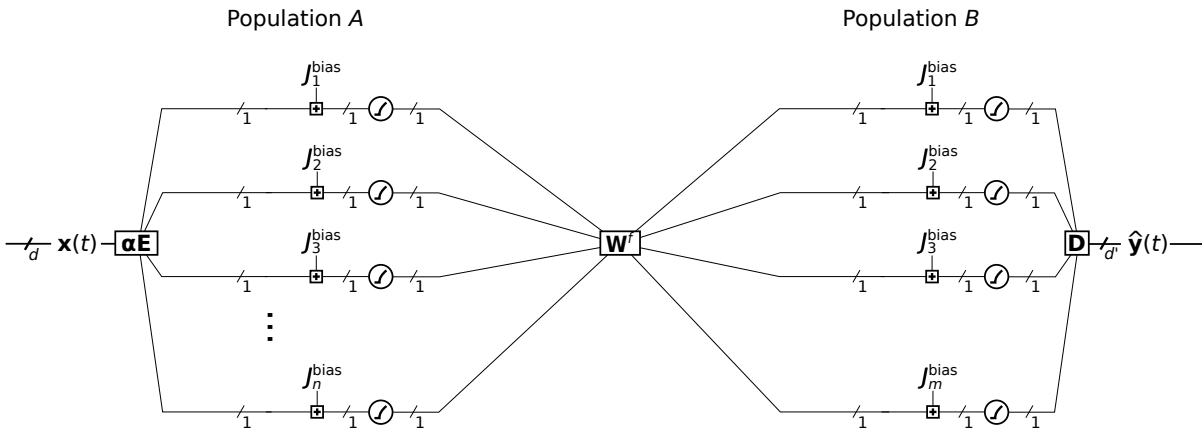
# Decoding Functions – Using More Neurons



# Decoding Functions – Using More Neurons



# Computing Functions – Weight Matrix



$$\mathbf{W}^f = \mathbf{E} \mathbf{D}^f$$

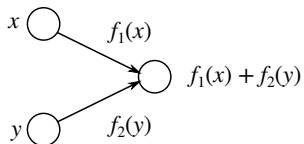
# Computing Multivariate Functions

○ Homogenous population    ⊗ Heterogenous population

→ Linear connection    —| Inh. connection    —● Exc. connection

## Linear Superposition

$$W^{f_1} \mathbf{a}_1(\mathbf{x}) + W^{f_2} \mathbf{a}_2(\mathbf{y})$$



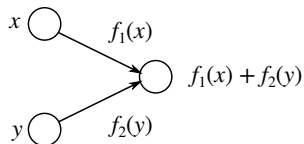
# Computing Multivariate Functions

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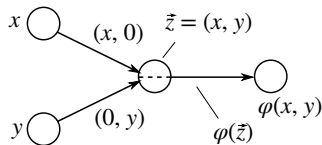
## Linear Superposition

$$W^{f_1} \mathbf{a}_1(\mathbf{x}) + W^{f_2} \mathbf{a}_2(\mathbf{y})$$



## Nonlinear Functions

Multi-dimensional  $\mathbf{z}$



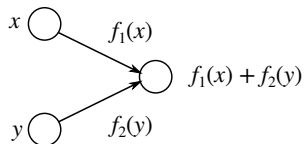
# Computing Multivariate Functions

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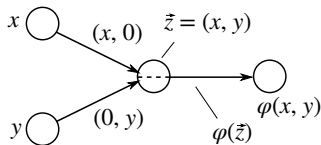
## Linear Superposition

$$W^{f_1} \mathbf{a}_1(\mathbf{x}) + W^{f_2} \mathbf{a}_2(\mathbf{y})$$



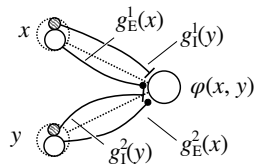
## Nonlinear Functions

Multi-dimensional  $\mathbf{z}$



## (Dendritic Computation)

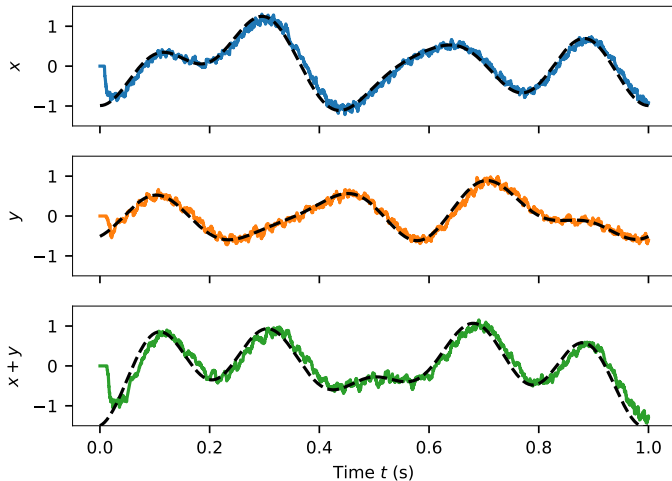
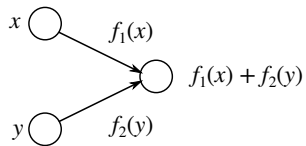
Exploit dendritic nonlinearity





# Computing Multivariate Functions – Linear Superposition

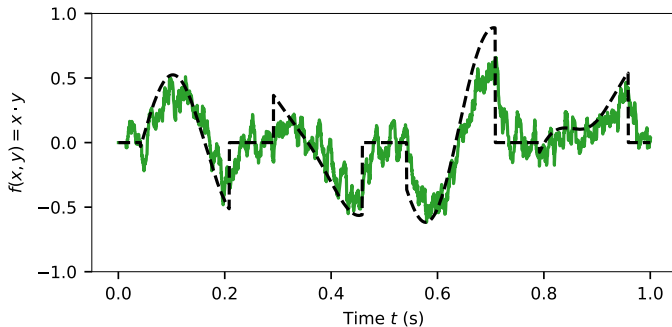
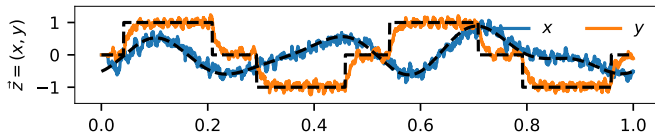
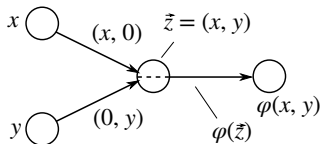
## Linear Superposition



# Computing Multivariate Functions – Multiplication

## Nonlinear Functions

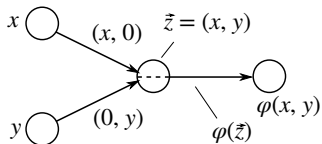
Multi-dimensional  $\mathbf{z}$



# Computing Multivariate Functions – Multiplication

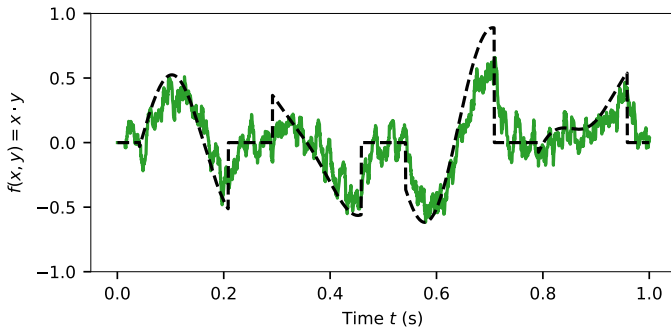
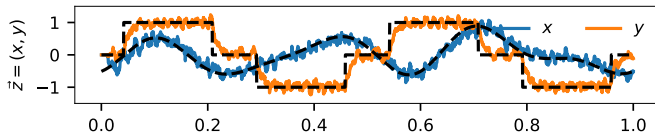
## Nonlinear Functions

Multi-dimensional  $\mathbf{z}$



Multiplication is useful...

- Gating of signals
- Attention effects
- Binding
- Statistical inference



# Image sources

## **Title slide**

“Yellow Butterfly”

Author: Albert Bierstadt, circa 1890.

From Wikimedia.