SYDE 556/750
Simulating Neurobiological Systems
Lecture 2: Neurons

Andreas Stöckel

Based on lecture notes by
Chris Eliasmith and Terrence C. Stewart

January 9, 2020

Accompanying Readings: Chapter 2.1 and Chapter 4.1 of Neural Engineering

Contents

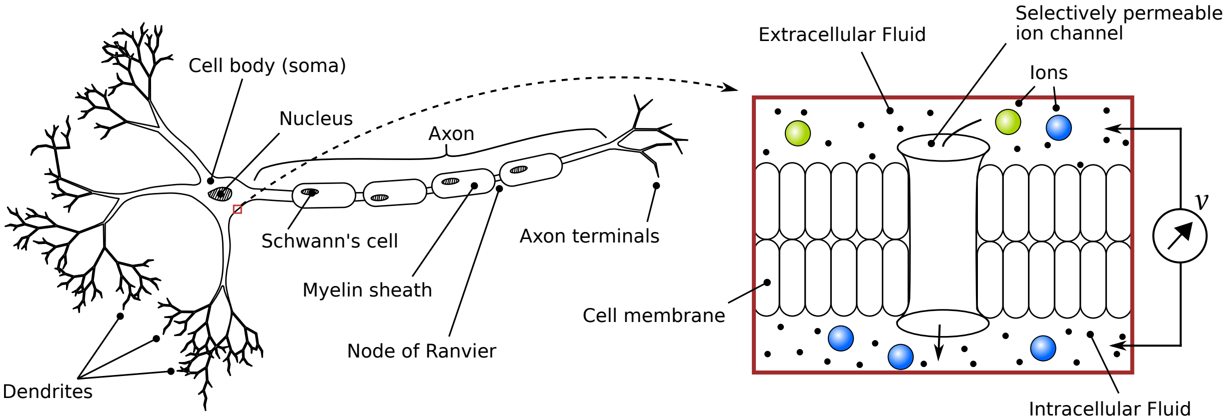


Figure 1: Illustration showing a text-book neuron, as well as a schematic cross-section through the cell membrane. Left part of the illustration from [2], adapted from [1]

1 Overview

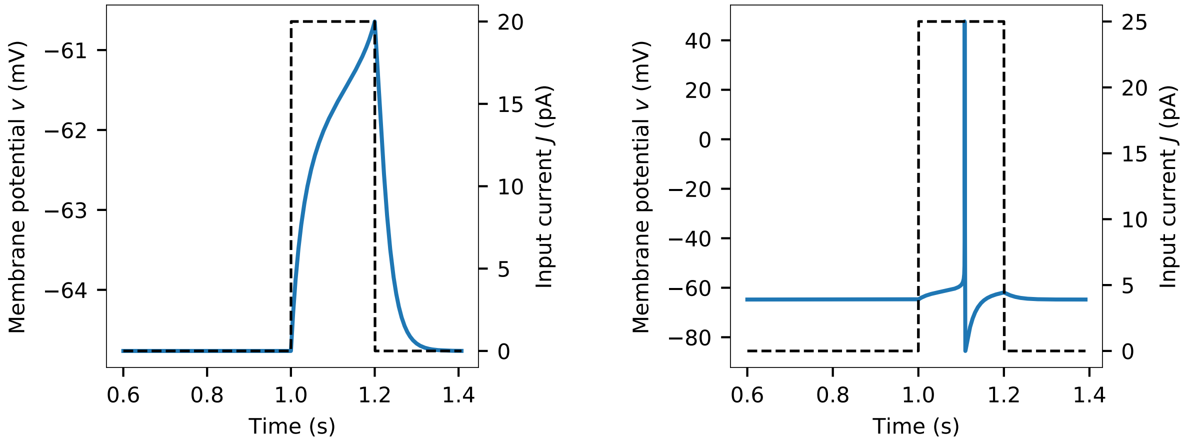
As we have discussed in the last lecture, we consider neurons to be the fundamental computational element in the nervous system. (Most) neurons communicate by generating action potentials (or spikes) that are sent to and received by post-synaptic neurons.

An important part of understanding nervous systems is thus to understand the "code" that is being exchanged between neurons. Unfortunately, there is no scientific consensus about what exactly "the neural code is". What we do have, are very detailed models of how individual neurons generate spikes. Thus, we will approach the problem of neural representation in two stages. First, in this lecture, we will have a look at single neurons and try to get an understanding of how neurons generate action potentials. Second, in the next two lectures, we will use information theory to think about what a potential neural representation could be.

2 Spiking Neurons

Note: We're going to have a slightly closer look at biologically detailed spiking neuron models towards the end of the class. For now, we're skimming over the details a little. Feel free to have a look at [1] if you want to learn more about basic neurobiology.

Neurons are cells that specialise in the integration and transfer of electrical signals. Cells are generally separated from the environment by a thick, impermeable "barrier", the *cell membrane*, consisting of a bi-layer of lipid molecules. The cell membrane establishes an "intra-cellular" space that is isolated from the "extra-cellular" space. Both spaces are filled with a watery liquid, the *intra-cellular fluid* and *extra-cellular fluid*, respectively (fig. 1).



(a) Sub-threshold neuron

(b) Super-threshold neuron

Figure 2: Computer simulation of a Hodgkin Huxley model neuron [3, 4]. The blue line corresponds to the membrane potential v, the dashed line to the current J that is being injected into the neuron.

2.1 Qualitative neural behaviour

When we stick a sharp electrode into a neuron (akin to the “single electrode recording”), we can measure a difference in electrical potential, i.e. a voltage v between the inside and the outside of the cell (fig. 1). We call this initial voltage the *resting potential* E_L.[1]

Instead of just measuring this potential we may also inject an external current into the neuron by hooking it up to a current source (i.e. a power supply that regulates current instead of voltage). When doing this, we find four things:

1. The cell acts like a *capacitor*, i.e., the voltage increases while we’re injecting a current (fig. 2a).
2. The capacitor is *leaky*. As soon as we stop injecting a current, the voltage collapses back to the resting potential v_rest injecting a current (fig. 2a).
3. As soon as the voltage surpasses a certain value, the *threshold potential* v_th, the cell will generate a spike.
4. Shortly after the spike has been produced, the voltage drops below the resting potential. During this period, the *refractory period* of length τ_ref we cannot get the neuron to spike again, even if we apply large input current J.

Note: Imporatantly, neurons in biology are *dynamical systems*, i.e., they possess a behaviour that evolves over time. *Artificial neurons* (see below) are time-independent. They are mathematical functions that take an input and “immediately” map it onto an output.

[1] The weird symbol “E_L” stems the more precise name of this potential, the “**L**eak channel **E**quilibrium potential”.

2.2 The Leaky Integrate and Fire Neuron

We can qualitatively summarize this behaviour in a very simple model, the so called "Leaky Integrate and Fire" neuron model. This model was first proposed by the French scientist Louis Lapicque in 1907 [5, 6].

Sub-threshold behaviour First, we have the *sub-threshold* behaviour, a so called leaky integrator.

$$\frac{\mathrm{d}}{\mathrm{d}t} v(t) = \frac{1}{C_\mathrm{m}} \big(g_\mathrm{L}(E_\mathrm{L} - v(t)) + J \big) , \quad \text{if } v(t) < v_\mathrm{th} \,. \tag{1}$$

This differential equation corresponds to a capacitor with capacity C_m that is charged with a current J and that slowly discharges to a potential v_reset over a resistor with conductance (the inverse of the resistance) $g_\mathrm{L} = \frac{1}{R}$ (the *leak conductance*).

Super-threshold behaviour Second, we have the *super-threshold behaviour*, the spike production and refractory period. Assume $v(t) = v_\mathrm{th}$ at $t = t_\mathrm{th}$. Then

$$\begin{aligned} v(t) &= \delta(t - t_\mathrm{th}) \,, && \text{if } t = t_\mathrm{th} \,, \\ v(t) &= v_\mathrm{reset} \,, && \text{if } t > t_\mathrm{th} \text{ and } t \geq t_\mathrm{th} + \tau_\mathrm{ref} \,, \end{aligned} \tag{2}$$

where $\delta(t)$ is the Dirac delta function, i.e., the function defined as

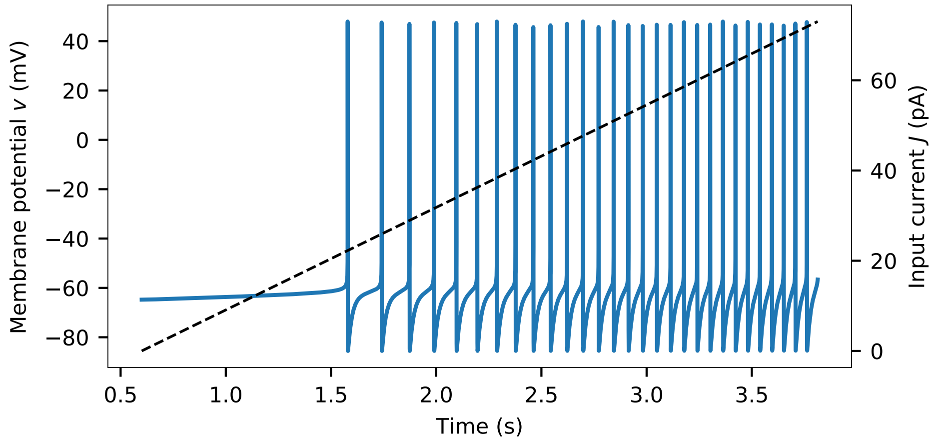
$$\delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \,, \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t) \, \mathrm{d}t = 1 \,.$$

Normalized equations For our modelling purposes, we don't really care about the exact values of v_th, v_reset and E_L. We can just normalise these voltages, i.e., assume that $v_\mathrm{th} = 1$, and $v_\mathrm{reset} = E_\mathrm{L} = 0$.

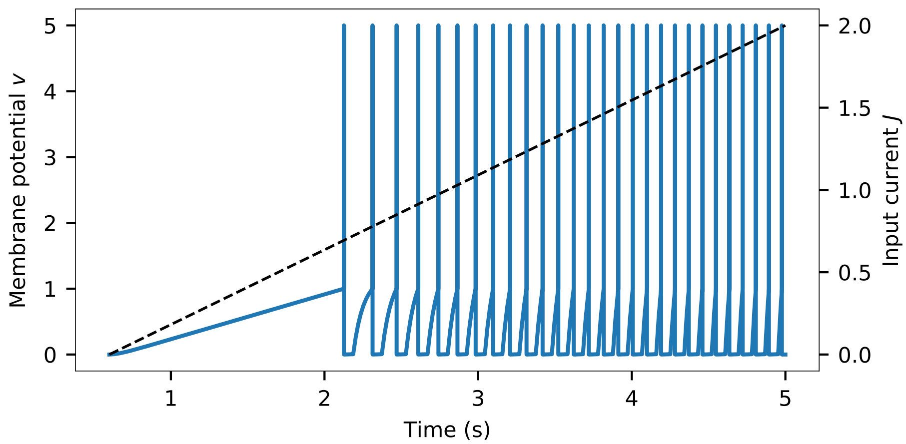
We can rewrite eqs. (1) and (2) as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} v(t) &= -\frac{1}{\tau_\mathrm{RC}} \big(v(t) - RJ \big) , && \text{if } v(t) < v_\mathrm{th} \,. \\ v(t) &= \delta(t - t_\mathrm{th}) \,, && \text{if } t = t_\mathrm{th} \,, \\ v(t) &= 0 \,, && \text{if } t > t_\mathrm{th} \text{ and } t \geq t_\mathrm{th} + \tau_\mathrm{ref} \,, \end{aligned} \tag{3}$$

where, $\tau_\mathrm{RC} = C_\mathrm{m} R$ and $R = \frac{1}{g_\mathrm{L}}$.



(a) Hodgkin-Huxley model neuron



(b) Normalized Leaky Integrate-and-Fire neuron

Figure 3: Effects of a current ramp on an Hodgkin-Huxley type model neuron and a (normalized) LIF neuron. As above, the blue line is the membrane potential, the dashed line is the input current.

2.3 Characterizing the LIF Neuron

3 Artificial Neurons

References

[1] E. Kandel et al. *Principles of Neural Science*. 5th ed. McGraw-Hill Education, 2012.

[2] Andreas Stöckel. “Design Space Exploration of Associative Memories Using Spiking Neurons with Respect to Neuromorphic Hardware Implementations”. MA thesis. Germany: Bielefeld University, 2015.

[3] Alan L. Hodgkin and Andrew F. Huxley. “A Quantitative Description of Membrane Current and Its Application to Conduction and Excitation in Nerve”. In: *The Journal of Physiology* 117.4 (1952), pp. 500–544.

[4] Roger D. Traub and Richard Miles. *Neuronal Networks of the Hippocampus*. Vol. 777. Cambridge University Press, 1991.

[5] Louis Lapicque. “Recherches quantitatives sur lexcitation électrique des nerfs traitée comme une polarization”. In: *Journal de Physiologie et de Pathologie Generale* 9 (1907), pp. 620–635.

[6] L.F Abbott. “Lapicques Introduction of the Integrate-and-Fire Model Neuron (1907)”. In: *Brain Research Bulletin* 50.5 (1999), pp. 303–304. ISSN: 0361-9230. DOI: `https://doi.org/10.1016/S0361-9230(99)00161-6`. URL: `http://www.sciencedirect.com/science/article/pii/S0361923099001616`.