

SYDE 556/750

Simulating Neurobiological Systems
Lecture 6: Recurrent Dynamics

Andreas Stöckel

February 4 & 6 & 11, 2020

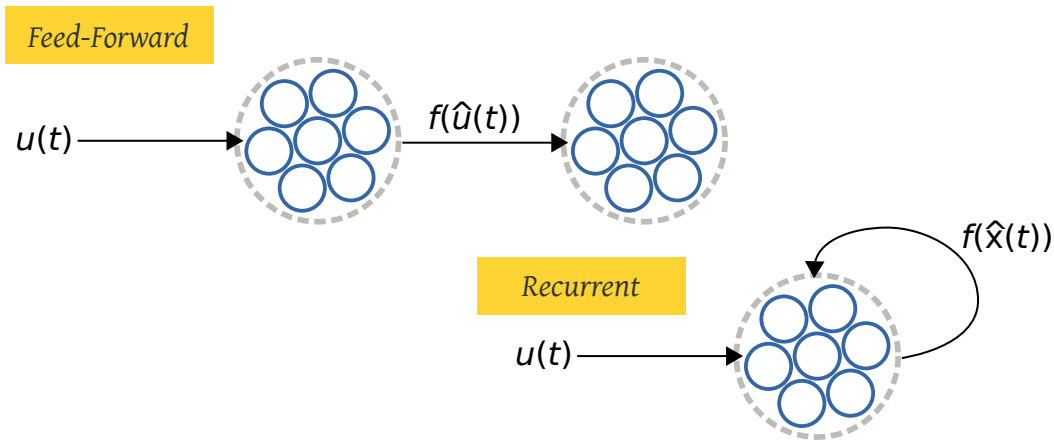


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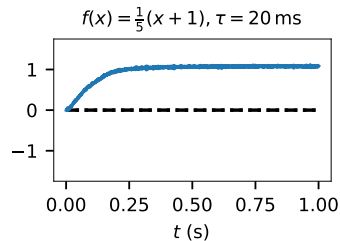
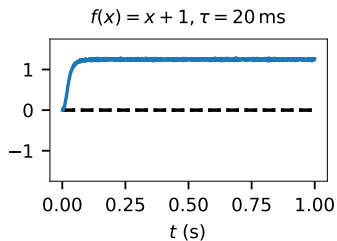
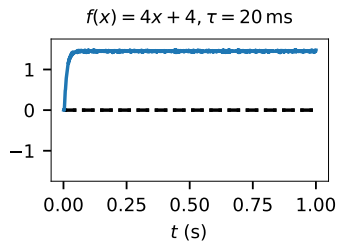
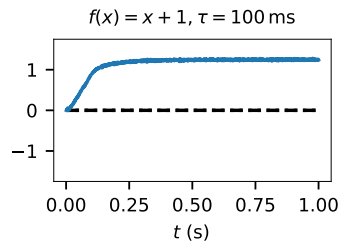
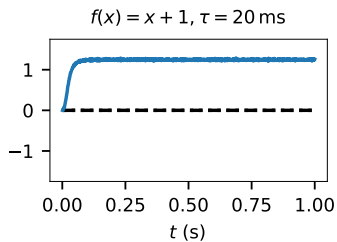
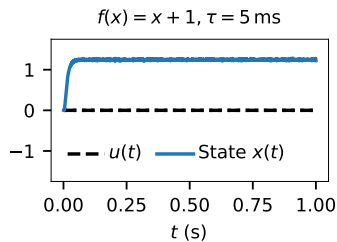
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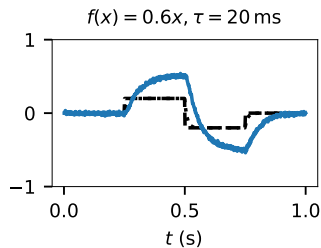
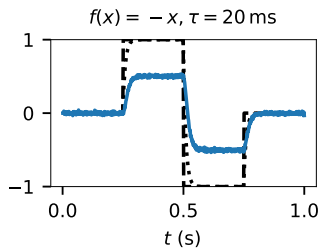
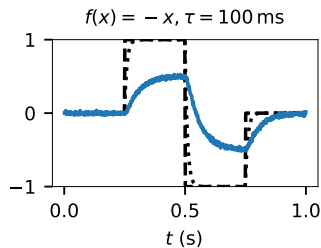
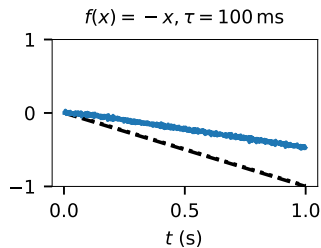
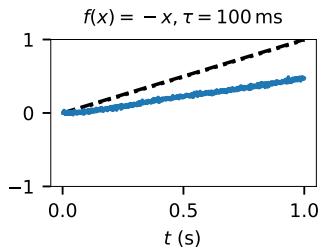
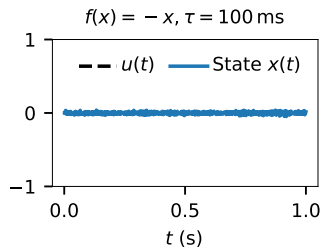
Feed Forward vs. Recurrent Connections



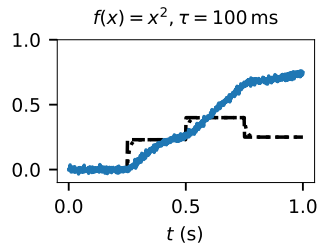
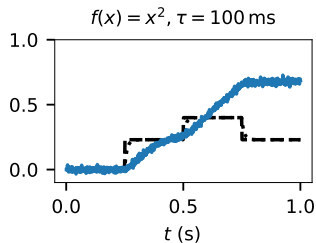
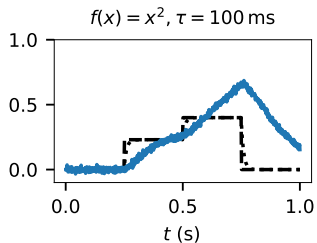
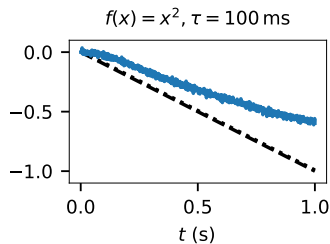
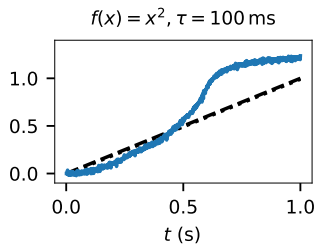
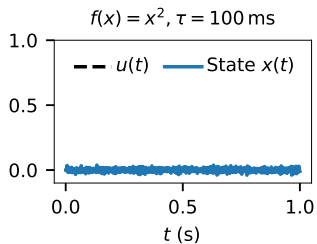
Recurrence Experiments (I)



Recurrence Experiments (II)



Recurrence Experiments (III)



NEF Principle 3: Dynamics

Time-Invariant Dynamical System

$$\frac{d\mathbf{x}(t)}{dt} = f(\mathbf{x}(t), \mathbf{u}(t))$$

Linear Time-Invariant (LTI)

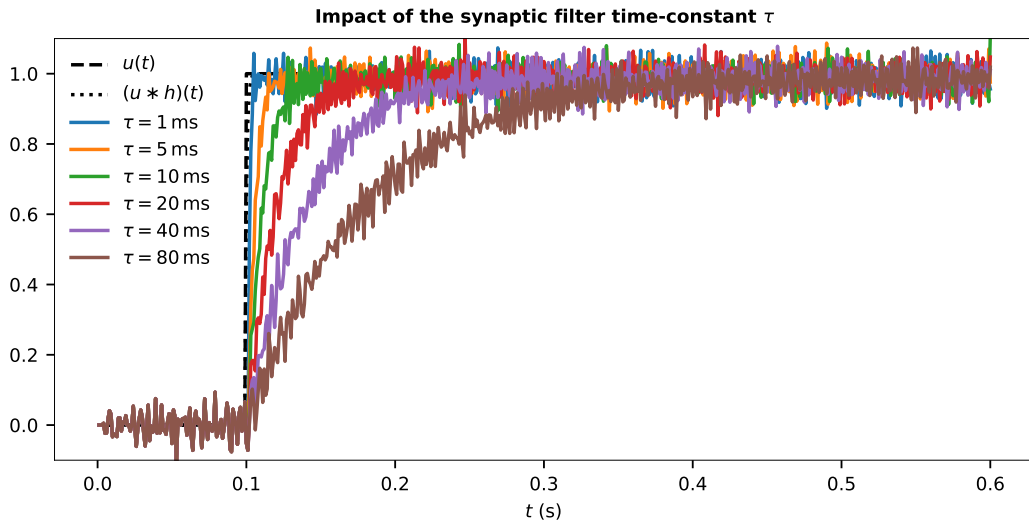
Dynamical System

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

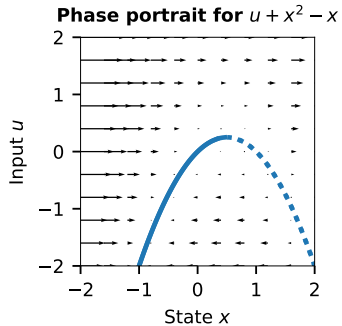
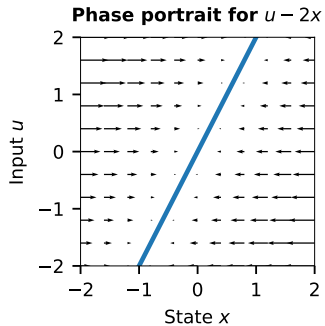
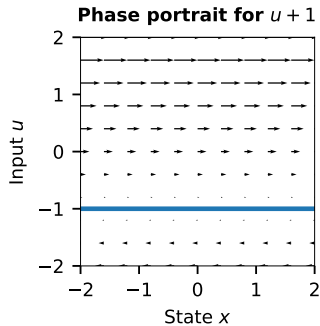
NEF Principle 3 – Dynamics

Neural dynamics are characterized by considering neural representations as control theoretic state variables. We can use control theory (and dynamical systems theory) to analyse and construct these systems.

Making Sense of Dynamics

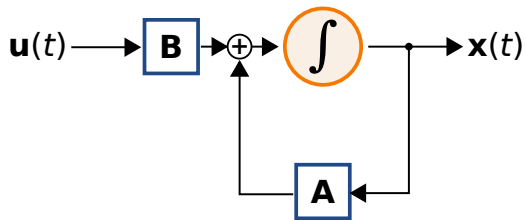


Phase Portraits

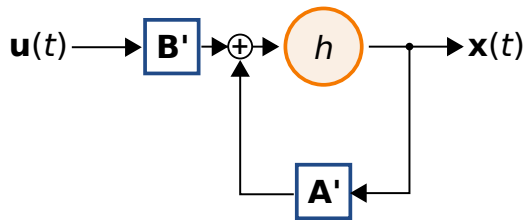


Implementing Dynamics using a Neural Ensemble

Evaluating an LTI
using an Integrator



Evaluating an LTI
using a Synaptic Filter



Implementing Dynamical Systems as a Neural Ensemble

LTI System

$$\phi(\mathbf{u}, \mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\phi'(\mathbf{u}, \mathbf{x}) = \mathbf{A}'\mathbf{x} + \mathbf{B}'\mathbf{u}$$

$$\mathbf{A}' = \tau\mathbf{A} + \mathbf{I}$$

$$\mathbf{B}' = \tau\mathbf{B}.$$

Additive Time-Invariant System

$$\phi(\mathbf{u}, \mathbf{x}) = f(\mathbf{x}) + g(\mathbf{u})$$

$$\phi'(\mathbf{u}, \mathbf{x}) = f'(\mathbf{x}) + g'(\mathbf{u})$$

$$f'(\mathbf{x}) = \tau f(\mathbf{x}) + \mathbf{x}$$

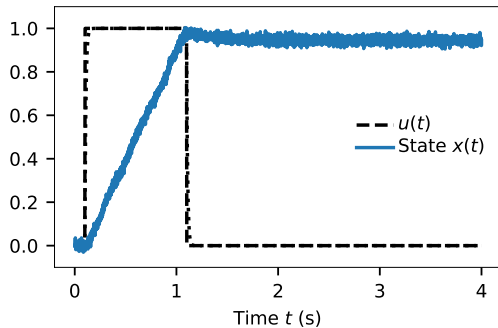
$$g'(\mathbf{u}) = \tau g(\mathbf{u})$$

“General” Recipe

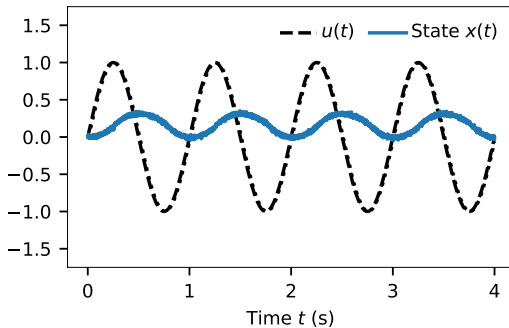
Scale the original dynamics by τ , add feedback \mathbf{x}

Integrator Example (I)

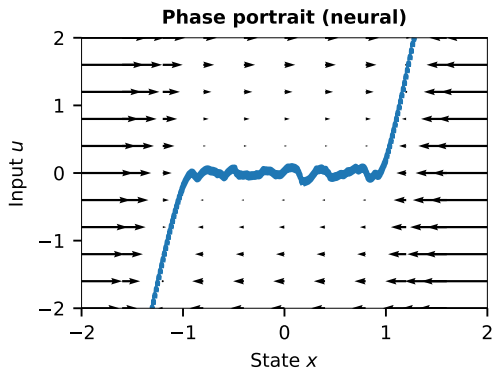
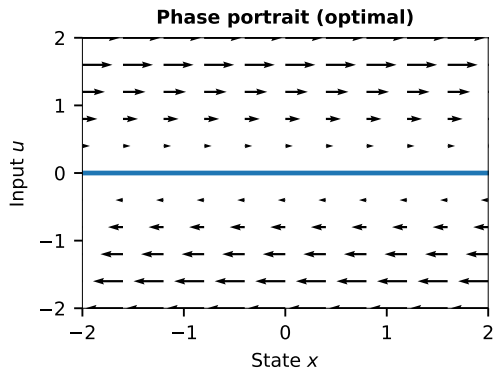
Step function input



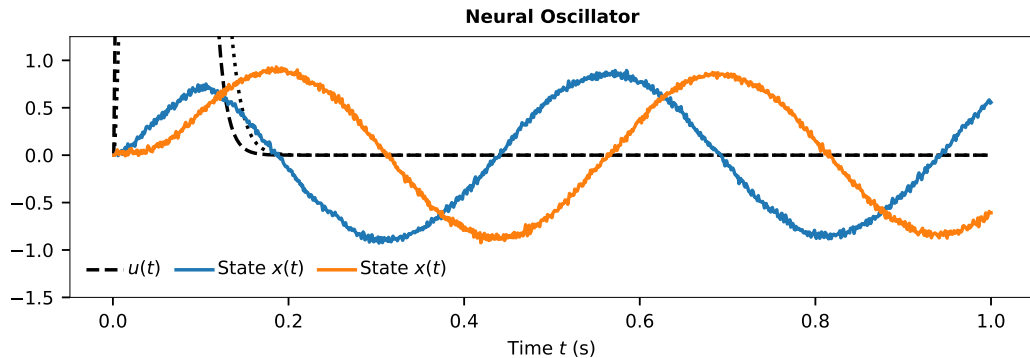
Sine input



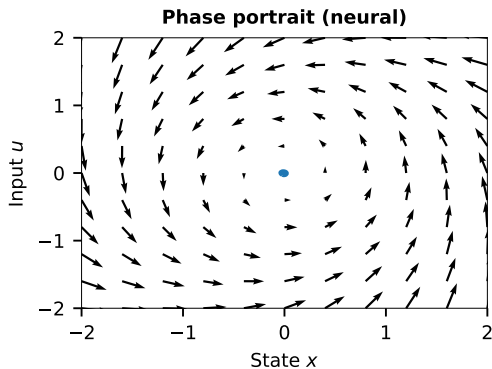
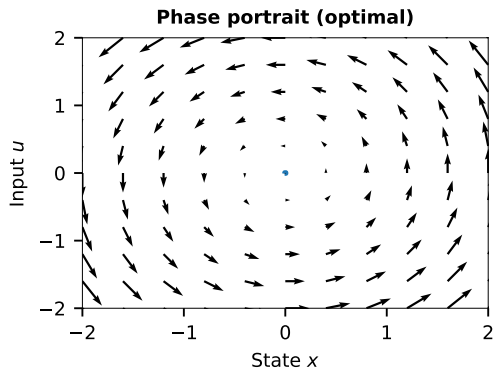
Integrator Example (II)



Oscillator Example (I)

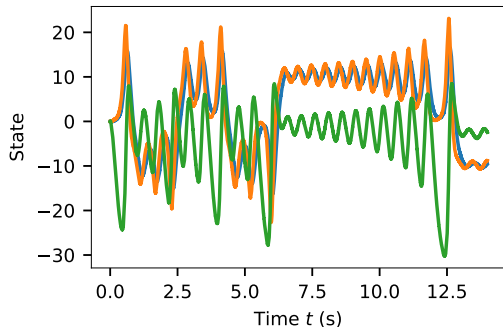


Oscillator Example (II)

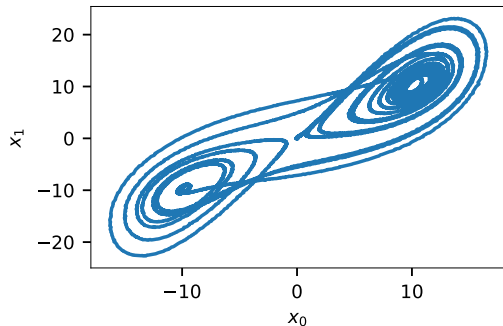


Lorentz Attractor

State over time



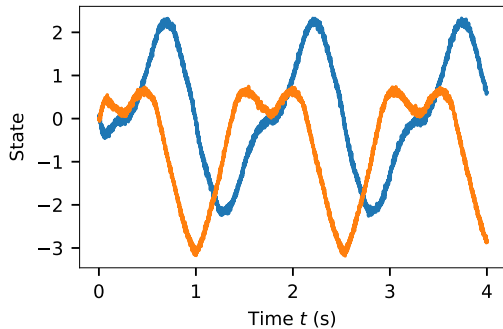
2D slice of the state space



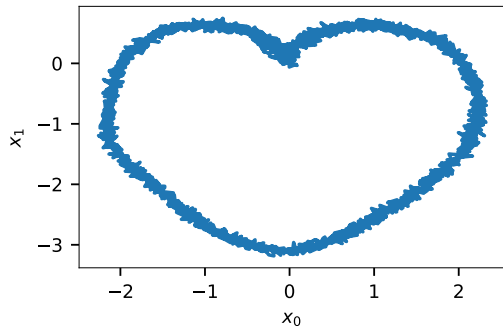
$$\frac{d\mathbf{x}(t)}{dt} = \begin{pmatrix} 10x_2(t) - 10x_1(t) \\ -x_1(t)x_3(t) - x_2(t) \\ x_1(t)x_2(t) - \frac{8}{3}(x_3(t) + 28) - 28 \end{pmatrix}$$

Heart Shape

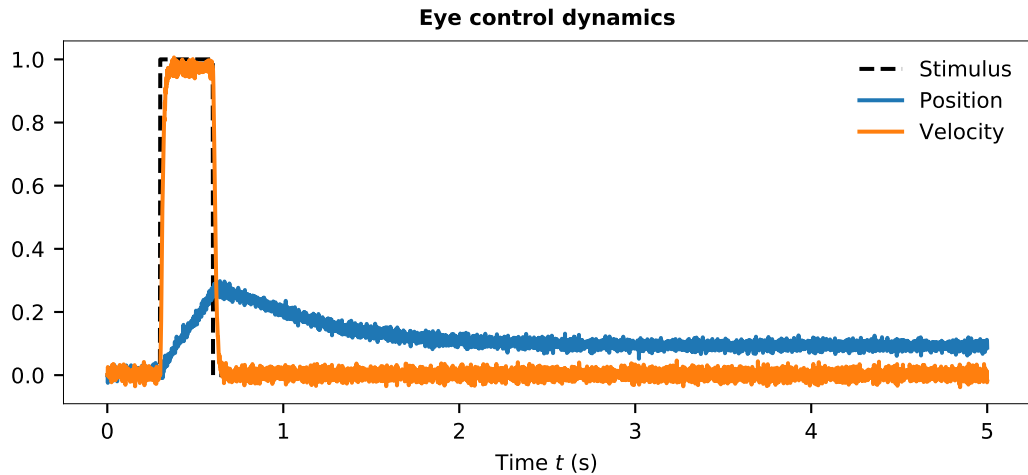
State over time



2D slice of the state space



Horizontal Eye Control



Administrative Remarks

- ▶ **Project proposals** due Friday, February 14
<http://compneuro.uwaterloo.ca/courses/syde-750/syde-556-possible-projects.html>
- ▶ **Assignment 2** due Thursday, February 13
- ▶ **Assignment 3** will be released later today, February 11 (due in three weeks)
- ▶ Some adjustments to the schedule

Image sources

Title slide

“The Canada 150 Mosaic Mural”

Author: Mosaic Canada Murals.

From Wikimedia.