SYDE 556/750

**Simulating Neurobiological Systems
Lecture 3: Representations**

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UNIVERSITY OF
WATERLOO | **FACULTY OF
ENGINEERING**

Visual Cortex
Mapping receptive fields

cell activity

overall

ongoing

behavior

NEF Principle 1: Representation

> **NEF Principle 1 – Representation**
> *Groups* ("populations", or "ensembles") of neurons *represent* represent values via nonlinear encoding and linear decoding.

Lossless Codes

Encoding: $\mathbf{a} = f(\mathbf{x})$

Decoding: $\mathbf{x} = f^{-1}(\mathbf{a})$

Binary numbers: Nonlinear encoding, linear decoding

- ▶ Represent a natural number between 0 and $2^n - 1$ as n binary digits.

- ▶ **Nonlinear encoding**

$$a_i = \left(f(x)\right)_i = \begin{cases} 1 & \text{if } x - 2^i \left\lfloor \frac{x}{2^i} \right\rfloor > 2^{i-1}\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

- ▶ **Linear decoding**

$$x = f^{-1}(\mathbf{a}) = \sum_{i=0}^{n-1} 2^i a_i = \mathbf{F}\mathbf{a} = \begin{pmatrix} 1 & 2 & \dots & 2^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix}\,.$$

- ▶ This is a **distributed code**. But, **not robust** against additive noise!

Lossy codes

▶ **Lossy code**
Inverse f^{-1} does not exist, instead *approximate* the represented value

$$\text{Encoding:} \quad \mathbf{a} = f(\mathbf{x}) \qquad\qquad \text{Decoding:} \quad \mathbf{x} \approx g(\mathbf{a})$$

▶ **Examples**
  ▶ Audio, image, and video coding schemes
    (MP3, JPEG, H.264)
  ▶ Basis transformation onto first *n* principal components (PCA)
  ▶ **Neural Representations**

Tuning curves (I)
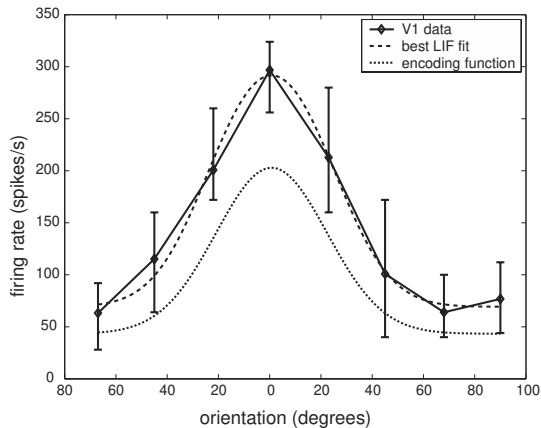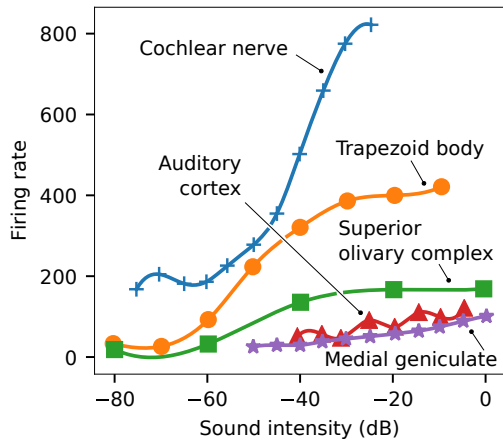
Tuning curves (II)
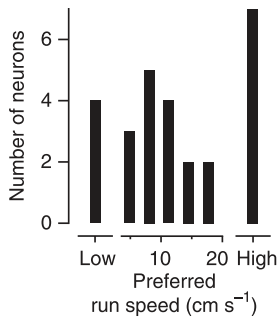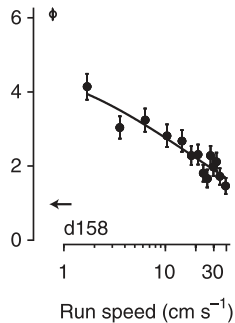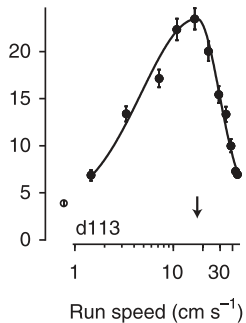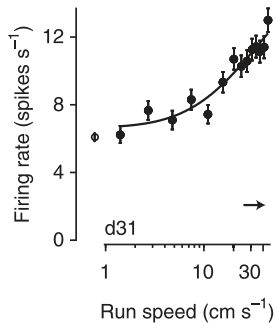
Preferred Directions in Higher Dimensions: Representing 2D Values
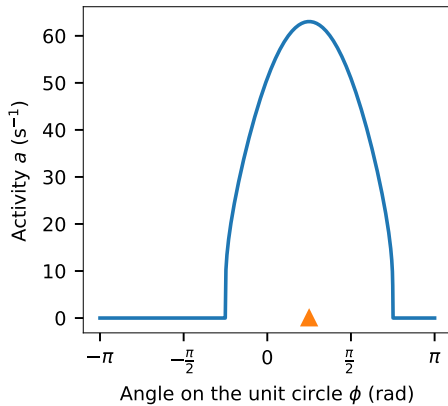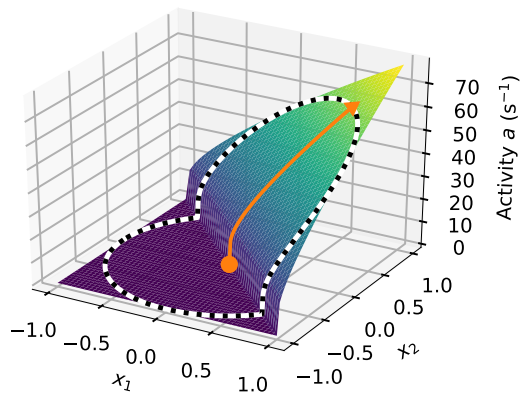
Decoding Without Taking Noise Into Account
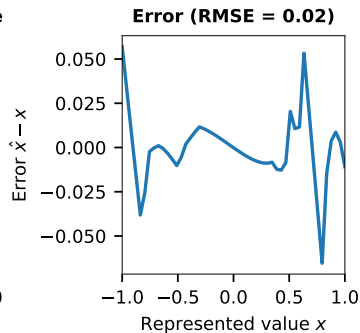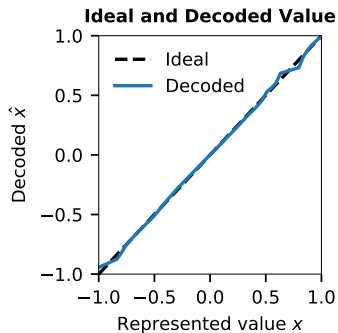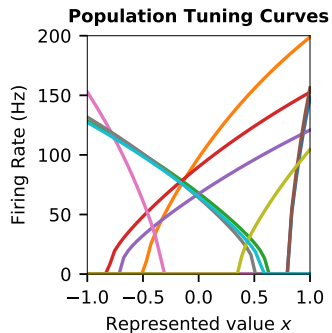
Sources of Noise in Biological Neural Networks

- **Axonal jitter**
  Active axonal spike propagation

- **Vesicle release failure**
  10-30% of pre-synaptic events cause post-synaptic current

- **Neurotransmitter per vesicle**
  Varying amounts of neurotransmitter

- **Ion channel noise**
  Ion-channels are "binary", stochastic

- **Thermal noise**

- **Network effects**
  Simple, noise-free inhibitory/excitatory networks produce irregular spike trains
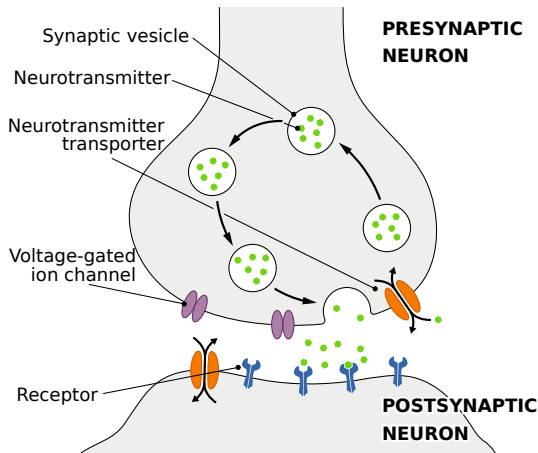
- **How to model?** Gaussian noise

NEF Principle 0: Noise

NEF Principle 0 – Noise
Biological neural systems are subject to significant amounts of noise from various sources. Any analysis of such systems must take the effects of noise into account.

Decoding Noisy **A** Without Taking Noise Into Account
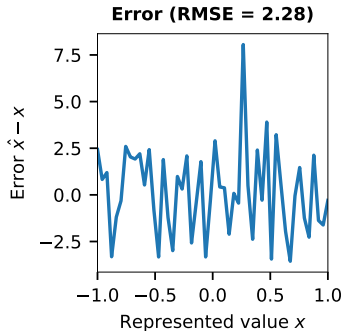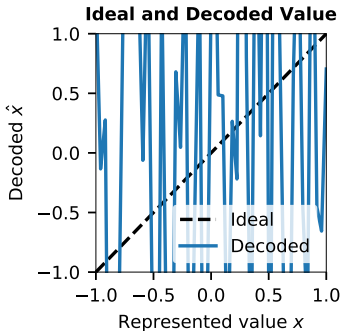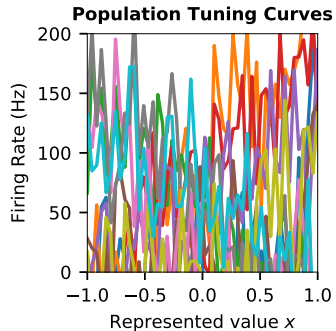
Decoding Noisy **A** Accounting for Noise
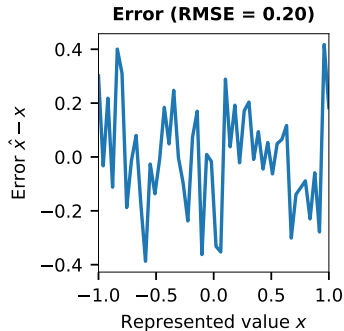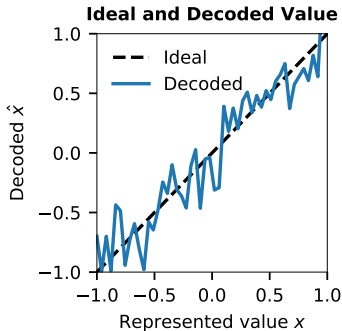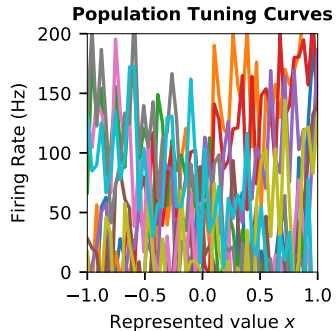
Summary: Building a model of neural representation (Encoding)

Encoding

▶ Select d, possible range $\mathbf{x} \in \mathbb{X}$, usually $\mathbb{X} = \left\{ \mathbf{x} \mid \|\mathbf{x}\| \leq r, \mathbf{x} \in \mathbb{R}^d \right\}$ ($r = 1$)

▶ Select number of neurons n

▶ Select tuning curves, maximum rates $\Rightarrow \mathbf{e}_i,\ \alpha_i,\ J_i^{\text{bias}}$

    ▶ Sample $\mathbf{e}_i$ from unit-sphere

    ▶ Uniformly distribute $\alpha_i,\ J_i^{\text{bias}}$

▶ Encoding equation:

$$a_i(\mathbf{x}) = G\left[\alpha_i \langle \mathbf{e}_i, \mathbf{x} \rangle + J_i^{\text{bias}} \right]$$

Population Tuning Curves

Firing Rate (Hz) — Represented value $\langle \vec{x}, \vec{e} \rangle$
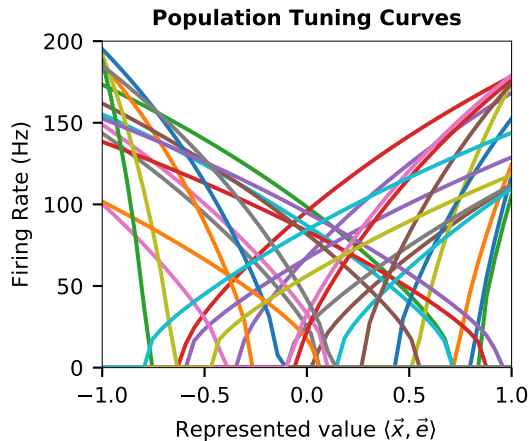
Summary: Building a model of neural representation (Decoding)

Decoding

- Uniformly sample N samples from $\mathbb{X}$, $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$

- Compute $\mathbf{A}$, where $(\mathbf{A}) = a_i()$

- Decoder computation:
$$\mathbf{D} = (\mathbf{A}\mathbf{A}^{\mathrm{T}} + N\sigma^2\mathbf{I})\mathbf{A}\mathbf{X}^{T}$$

- Decoding equation:
$$\hat{\mathbf{X}} = \mathbf{D}\mathbf{A}$$

Ideal and Decoded Value
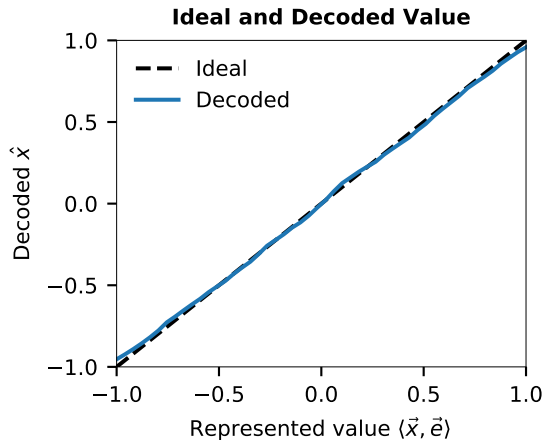
Example: Horizontal Eye Position (1D)
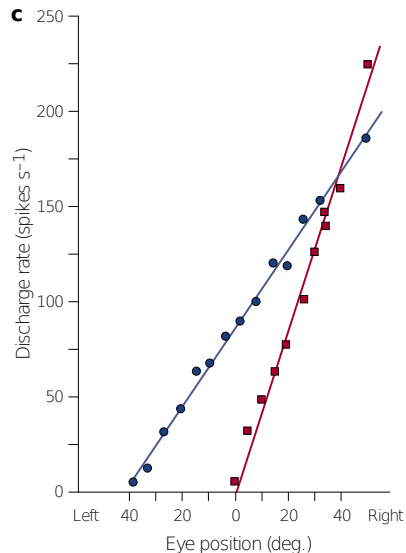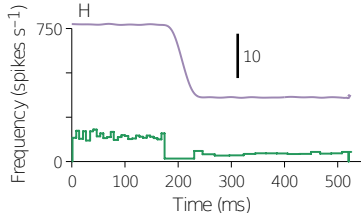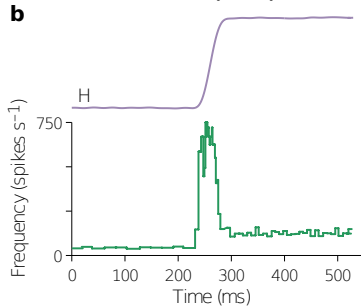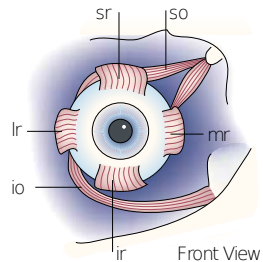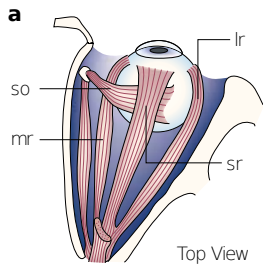
Example: Horizontal Eye Position (1D) (cont.)

▶ **Step 1: System Description**

    ▶ What is being represented?

        ▶ x is the horizontal eye position

    ▶ What is the tuning curve shape?

        ▶ Linear, low τ_{ref}, high τ_{RC}

        ▶ $e_i \in \{1, -1\}$

        ▶ Firing rates up to $300\,\mathrm{s}^{-1}$

▶ **Step 2: Design Specification**

    ▶ Range of values

        ▶ $\mathbb{X} = [-60, 60]$

    ▶ Amount of noise

        ▶ About 20% of $\max(\mathbf{A})$

▶ **Step 3: Implementation**

    ▶ Choose tuning curve parameters

    ▶ Compute decoders

Example: Arm Movements (2D)
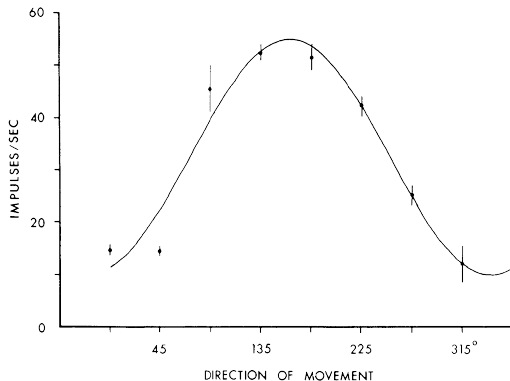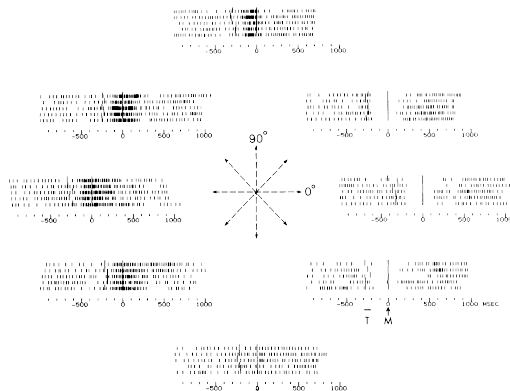
Example: Arm Movements (2D) (cont.)
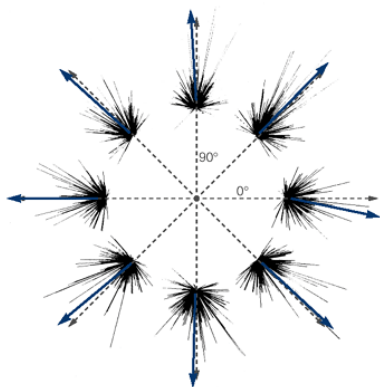
- ▶ Experiment by Georgopoulos et al., 1982
- ▶ Preferred arm movement directions $\mathbf{e}_i$
- ▶ **Idea:** *Population Vectors*, decode using

$$\hat{\mathbf{x}} = \sum_{i=1}^{n} a_i(\mathbf{x})\mathbf{e}_i = \mathbf{E}\mathbf{A}$$

- ➕ Good direction estimate
- ➖ Cannot reconstruct magnitude

The NEF does not use population vectors!

Example: Arm Movements (2D) (cont.)

▶ **Step 1: System Description**
  ▶ What is being represented?
    ▶ $\mathbf{x}$ the movement direction (or hand position)

  ▶ What is the tuning curve shape?
    ▶ Bell-shaped

    ▶ Encoders are randomly distributed along the unit circle

    ▶ Firing rates up to $60\,\mathrm{s}^{-1}$

▶ **Step 2: Design Specification**
  ▶ Range of values
    ▶ $\mathbb{X} = \{\mathbf{x} \mid \|\mathbf{x}\| \leq r, \mathbf{x} \in \mathbb{R}^2\}$

  ▶ Amount of noise
    ▶ About 20% of $\max(\mathbf{A})$

▶ **Step 3: Implementation**
  ▶ Choose tuning curve parameters
  ▶ Compute decoders

Example: Higher Dimensional Representation
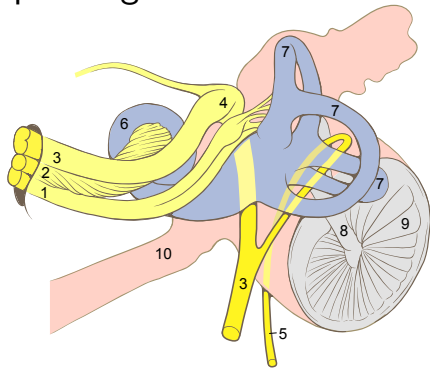
- ▶ Vestibular system senses head acceleration in 3D
- ▶ Axis aligned, must choose $\mathbf{e}_i \in \{[1,0,0],[-1,0,0],\ldots,[0,0,-1]\}$

- ▶ Same as three 1D populations
- ▶ Slightly lower precision
- ▶ **Encoders affect accuracy**

Administration

▶ **Assignment 1 has been released.**

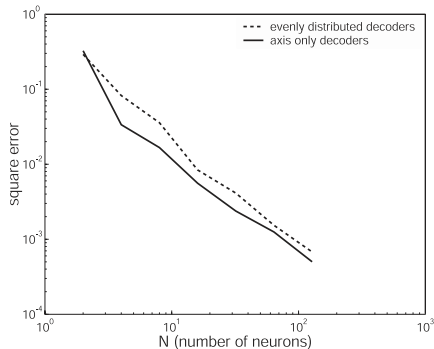The due date has been adjusted to January, 30.

▶ Some new potential times for office hours

`Mon 15:30-16:30, Mon 16:30-17:30, Tue 15:00-16:00,`

`Thu 11:30-12:30` (current slot)`, Thu 12:30-13:30`

Image sources

Title slide
"The Ultimate painting."
Author: Clark Richert.
From Wikimedia.