SYDE 556/750

Simulating Neurobiological Systems
Lecture 4: Temporal Representations
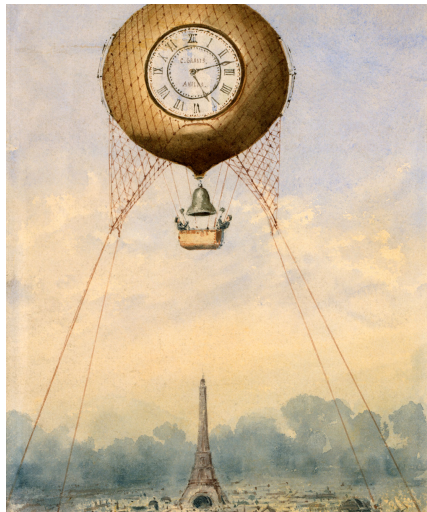
Andreas Stöckel

January 22 & 28, 2020

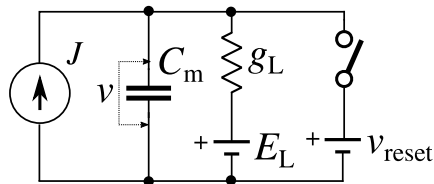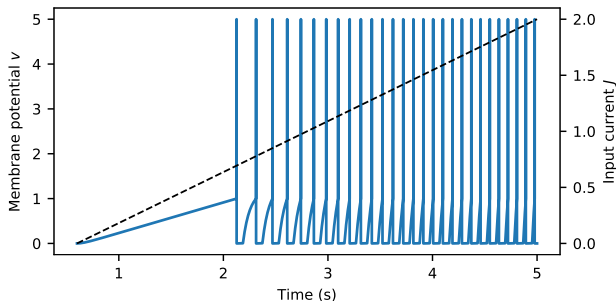UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

Reminder: The LIF Neuron

$$\frac{\mathrm{d}}{\mathrm{d}t}v(t) = -\frac{1}{\tau_{\mathrm{RC}}}\big(v(t) - J\big)\,, \qquad \text{if } v(t) < 1\,,$$
$$v(t) \leftarrow \delta(t - t_{\mathrm{th}})\,, \qquad\qquad \text{if } t = t_{\mathrm{th}}\,,$$
$$v(t) \leftarrow 0\,, \qquad\qquad\qquad \text{if } t > t_{\mathrm{th}} \text{ and } t \geq t_{\mathrm{th}} + \tau_{\mathrm{ref}}\,,$$
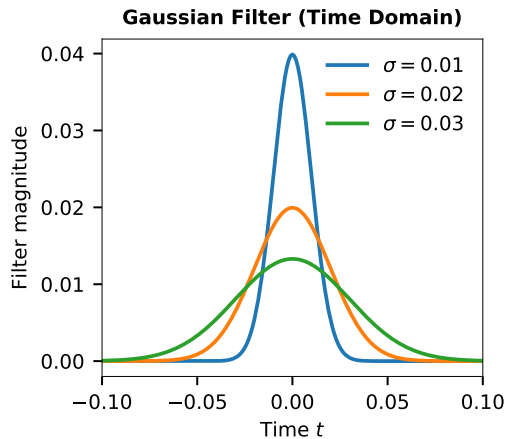
Temporal Decoding of Two Neurons
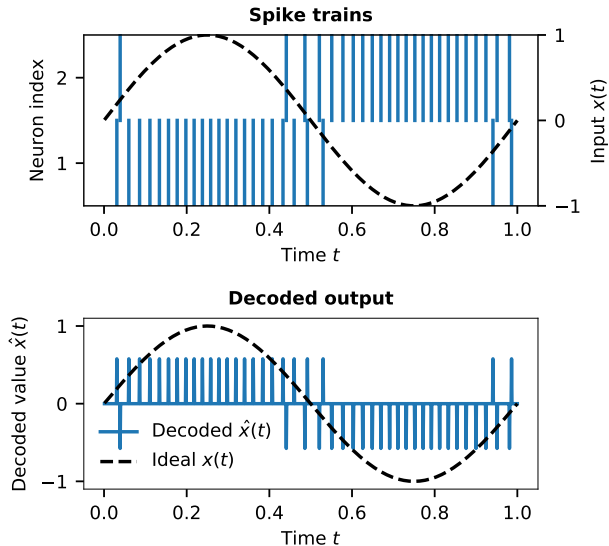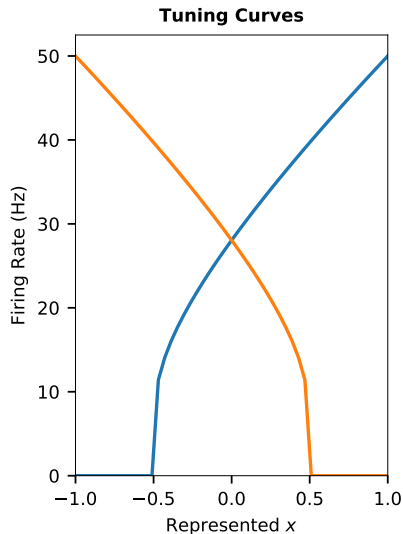
Temporal Decoding of One Hundred Neurons
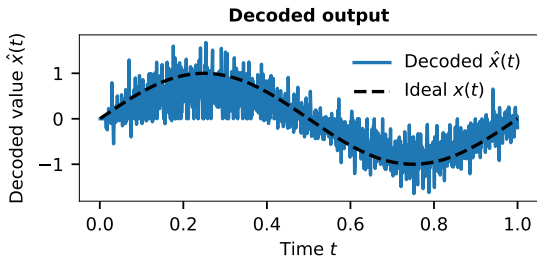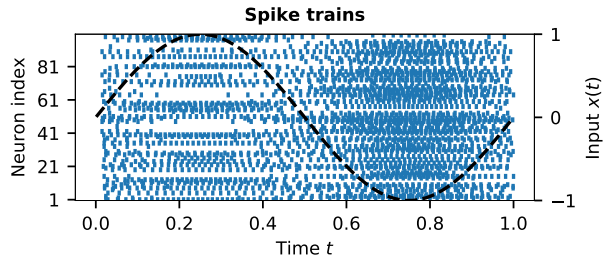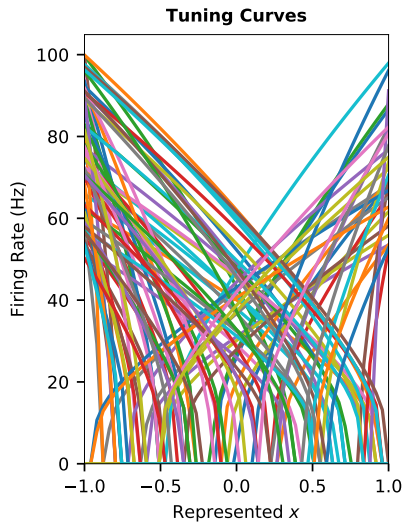
Filtering by Convolution

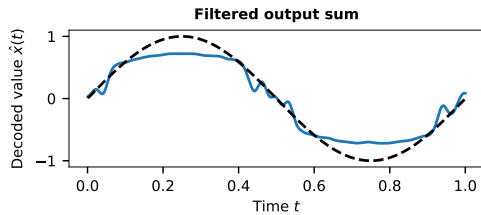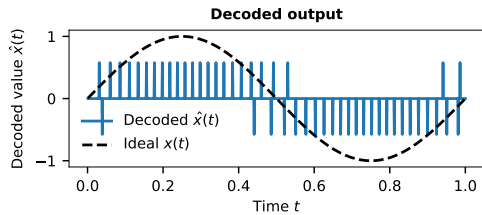**Gaussian Filter (Time Domain)**

Gaussian Filter
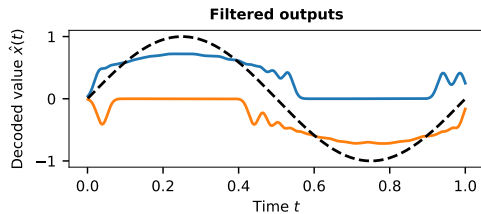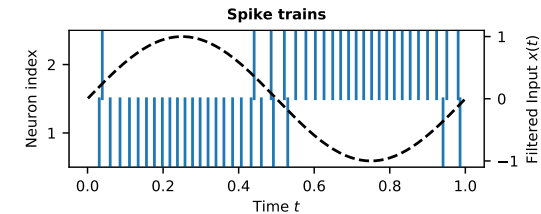
$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

where c chosen s.t. $\int_{-\infty}^{\infty} h(t)\, \mathrm{d}t = 1$

Convolution

$$\big(f * g\big)(t) = \int_{-\infty}^{\infty} f(t - \tau)g(\tau)\, \mathrm{d}\tau$$

Filtering a Spike Train

Random Signals
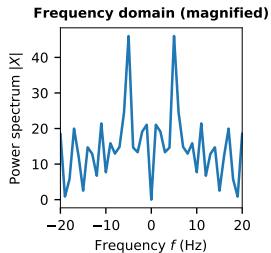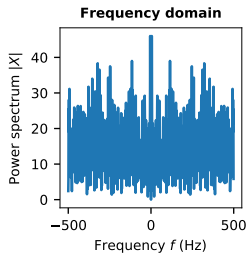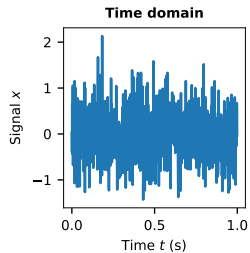
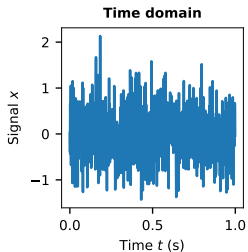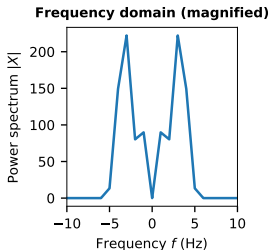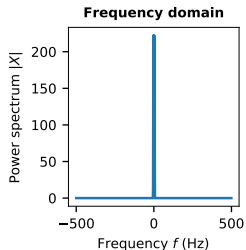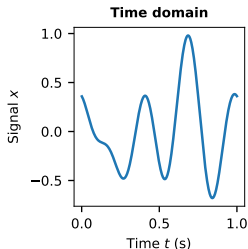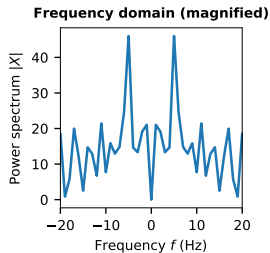**White Noise**
(zero mean)

Random Signals
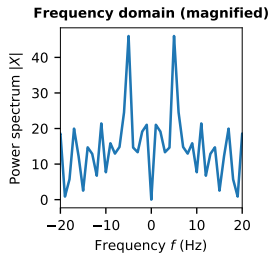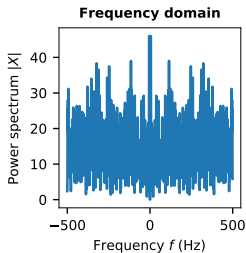
White Noise
(zero mean)

Bandlimited
White Noise
(zero mean,
5 Hz bandwidth)

Random Signals
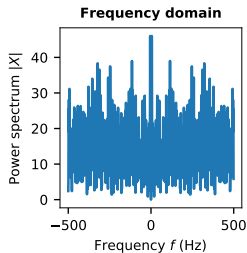
White Noise
(zero mean)

Bandlimited
White Noise
(zero mean,
10 Hz bandwidth)

Filtering a Spike Train for a Random Signal

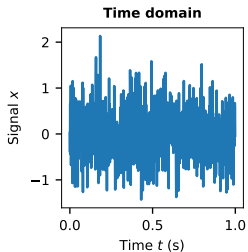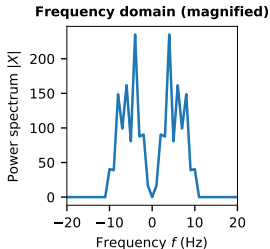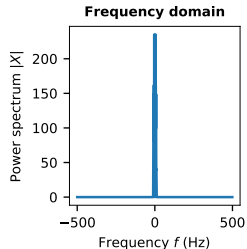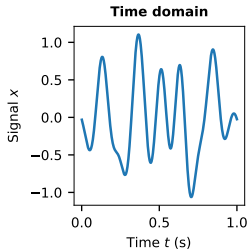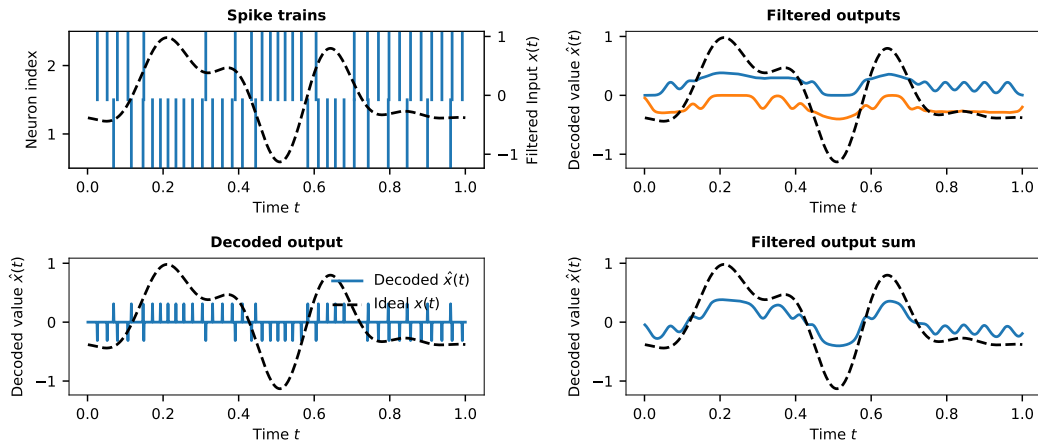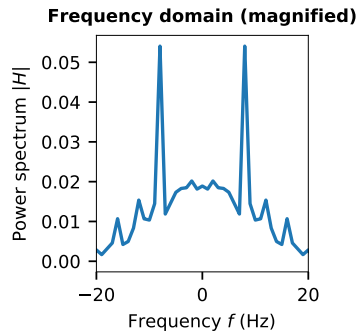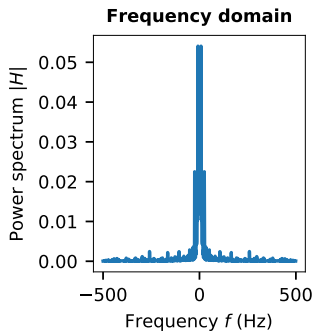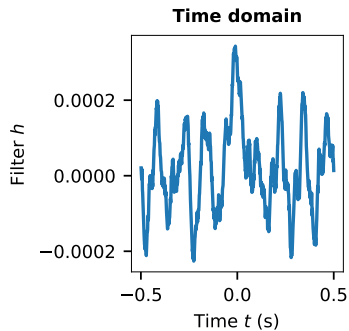# Optimal Filter

$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

Filtering a Spike Train for a Random Signal (Optimal Filter)
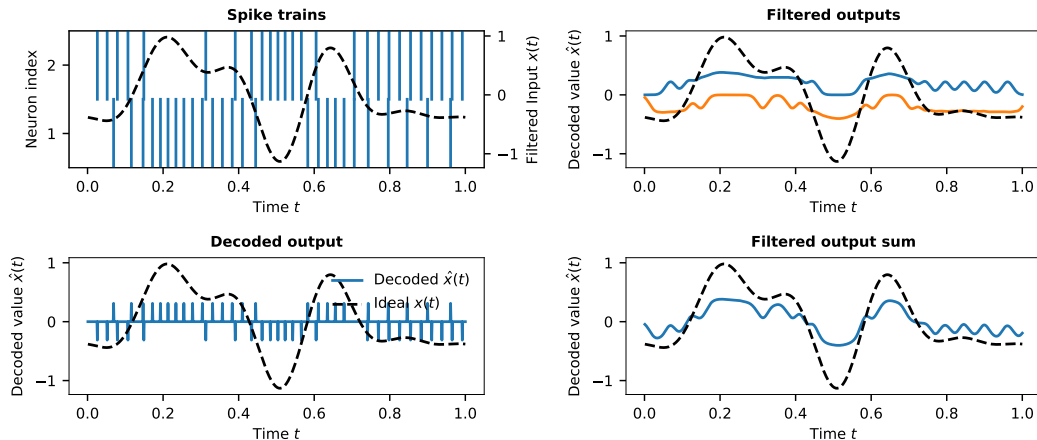
Filtering a Spike Train for a Random Signal (Optimal Filter)
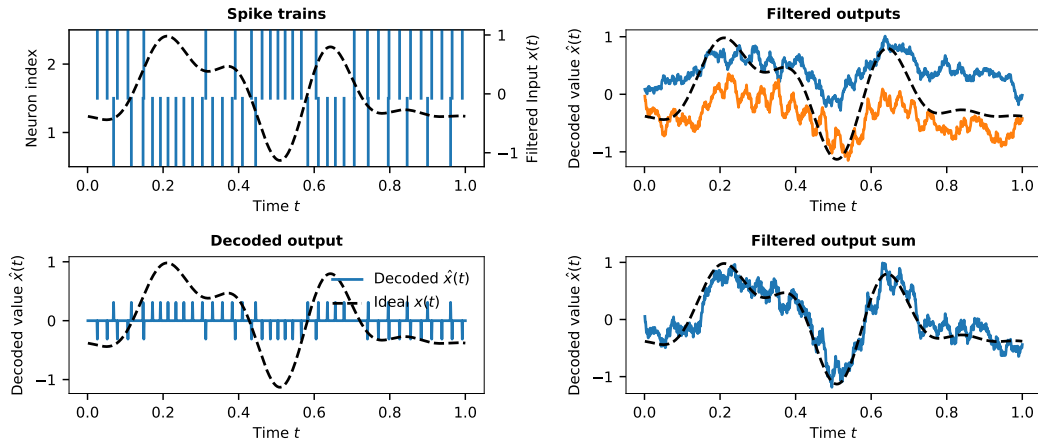
Filtering a Spike Train for a Random Signal (Optimal Filter)

Filtering a Spike Train for a Random Signal (Optimal Filter)

Optimal Filter (Improved)

$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)
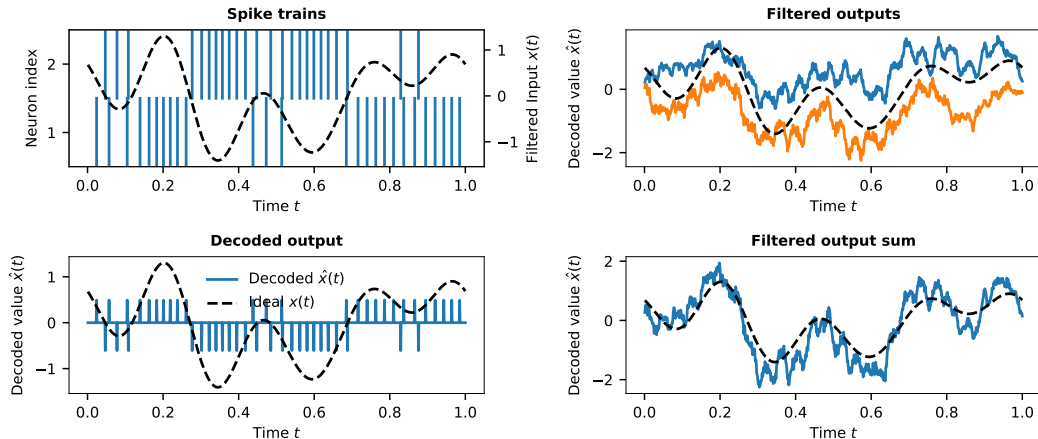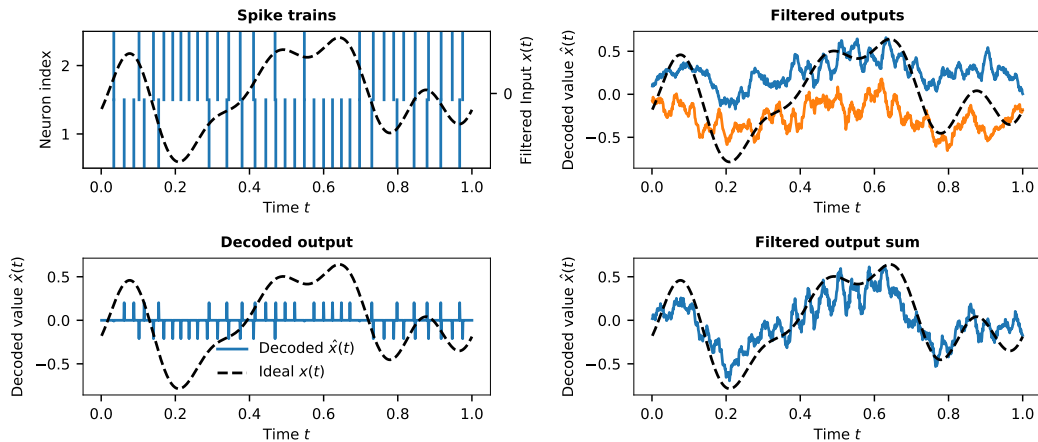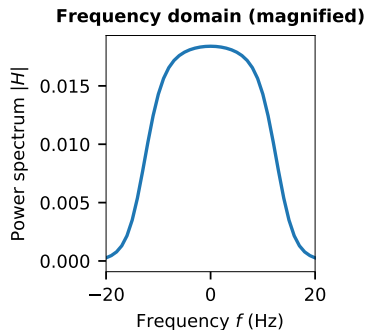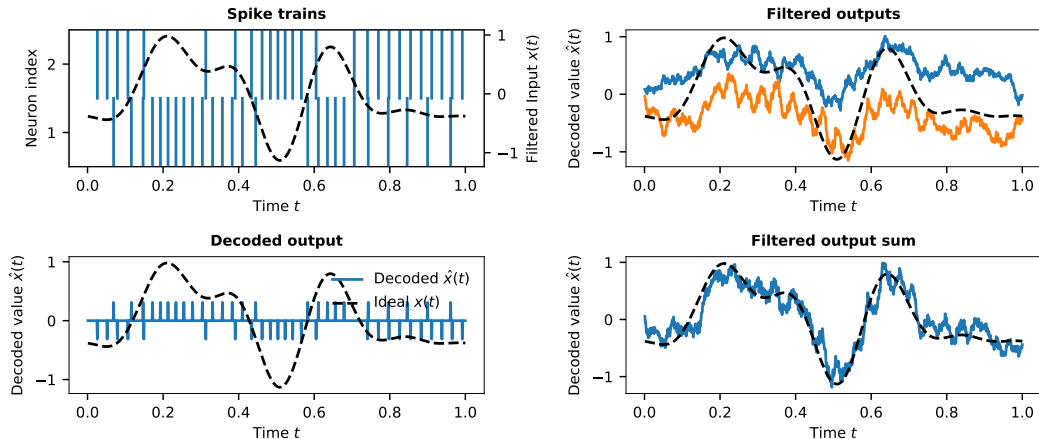
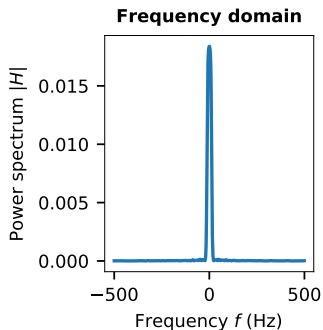# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)
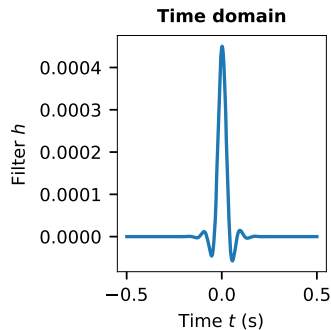
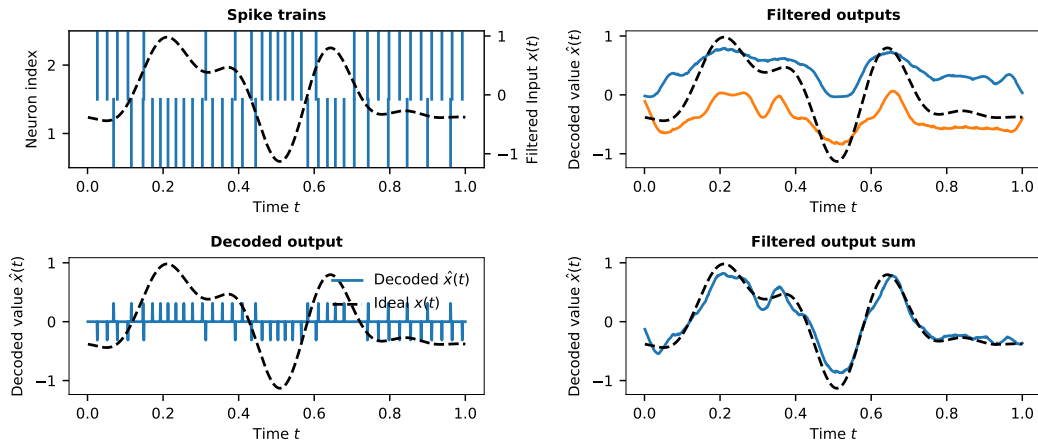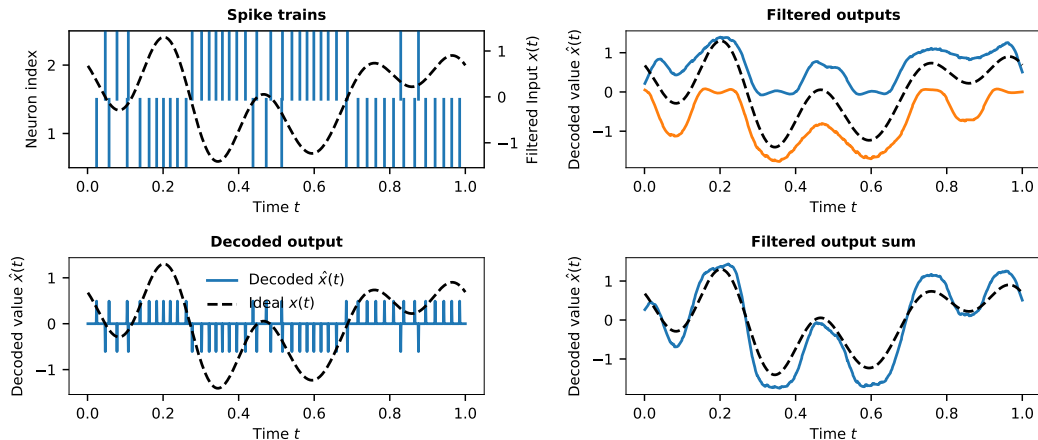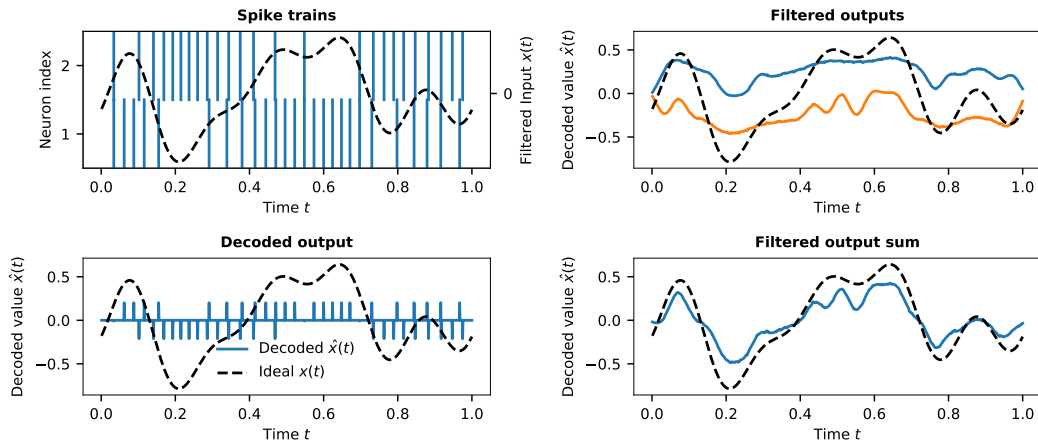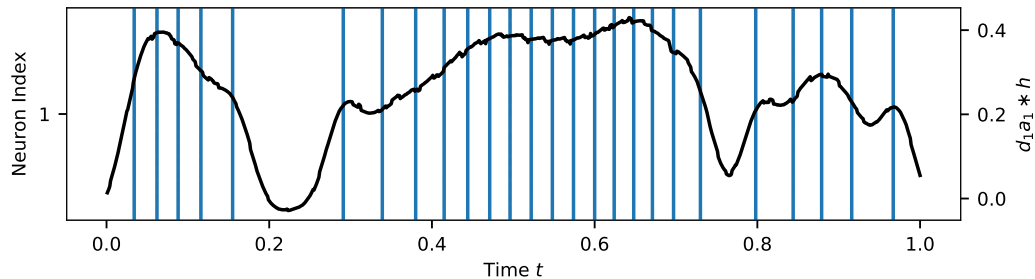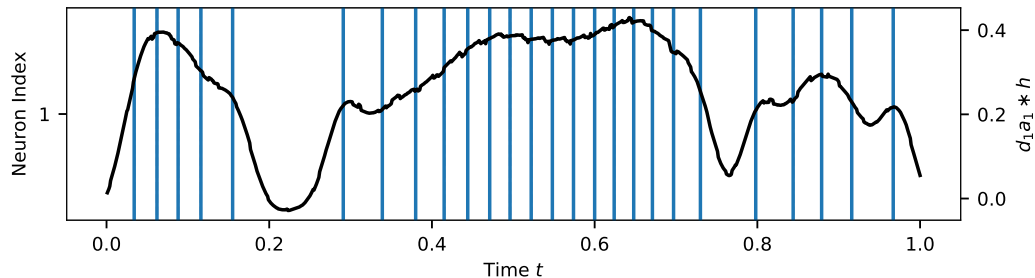# Pros and Cons of the Optimal Filter

Pros and Cons of the Optimal Filter

➕ **Precise**
Good for analysing data after the fact

Pros and Cons of the Optimal Filter
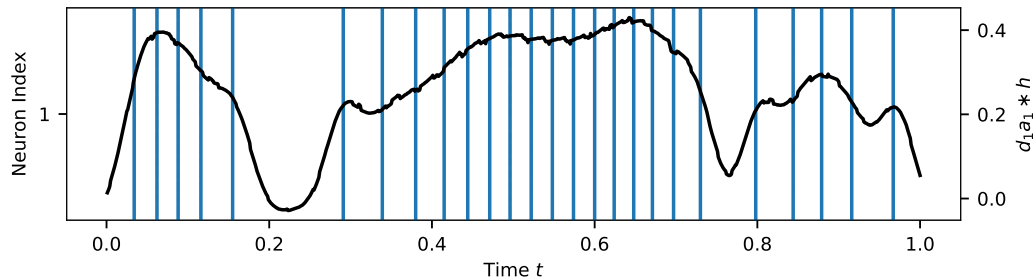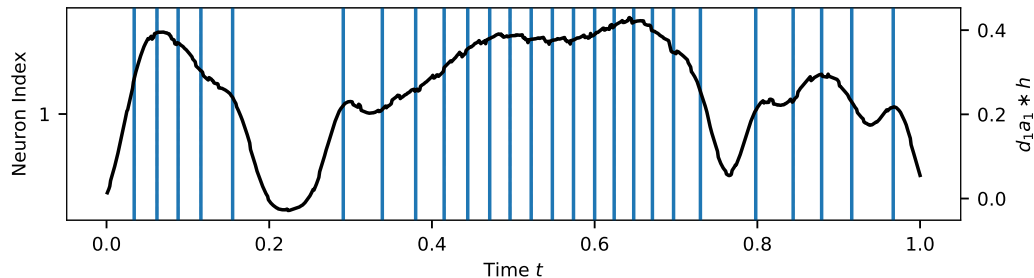

➕ Precise
Good for analysing data after the fact

➖ Non-causal
Does not describe a biological process

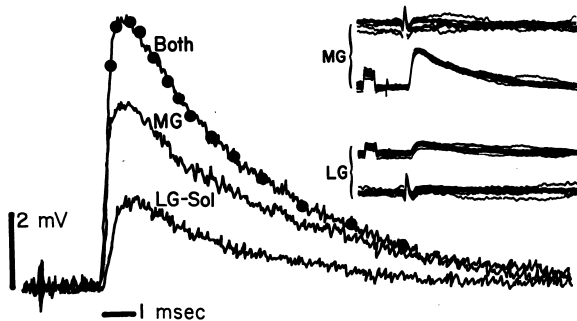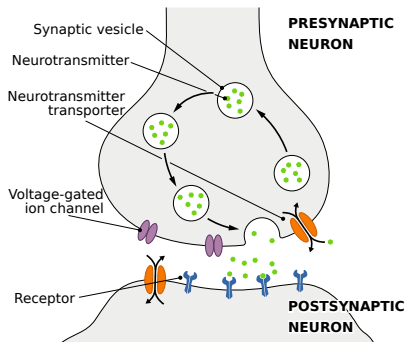# Pros and Cons of the Optimal Filter

⊕ Precise
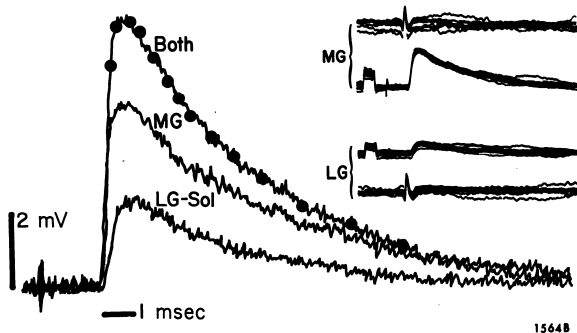Good for analysing data after the fact

⊖ Non-causal
Does not describe a biological process

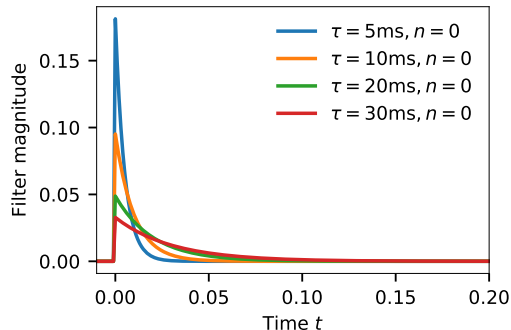We need to find a mechanism that low-pass filters spikes over time!

Synapses as Filters

Synapses as Filters
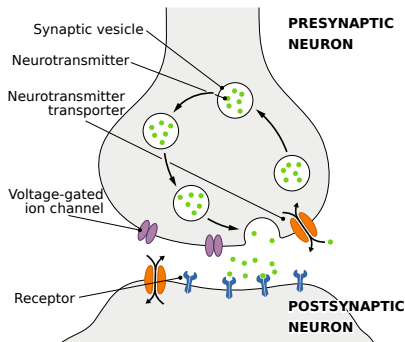
Post-synaptic currents (EPSCs, IPSCs) are low-pass filtered spike trains!

Exponential Low-Pass Filter (I)
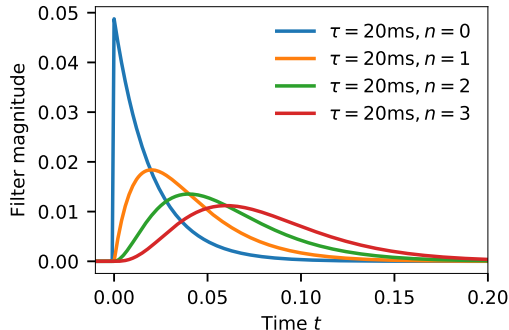
$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0, \\ 0 & \text{otherwise}, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t.$$
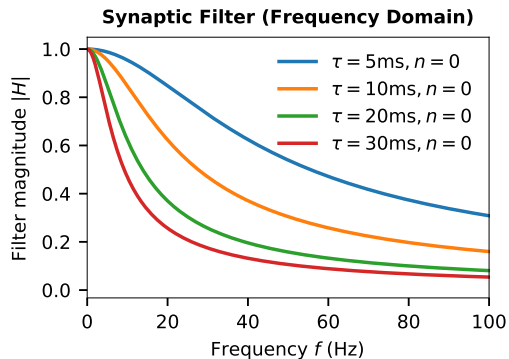
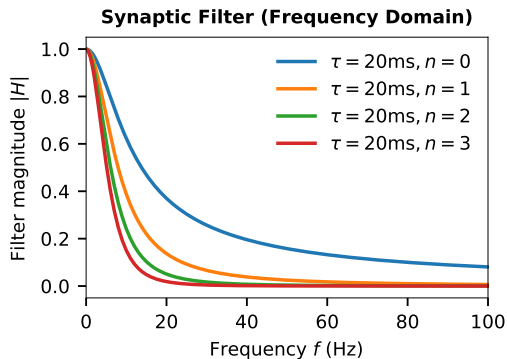# Exponential Low-Pass Filter (II)

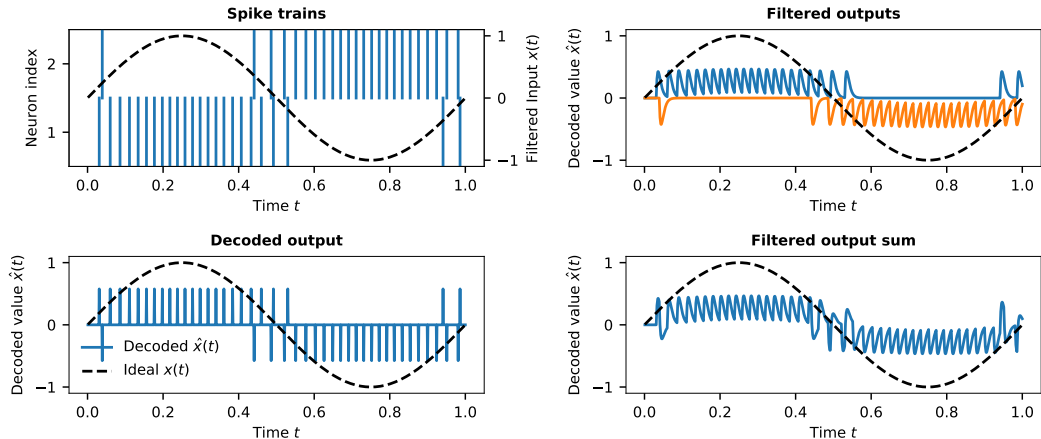
Synaptic Filter (Frequency Domain)

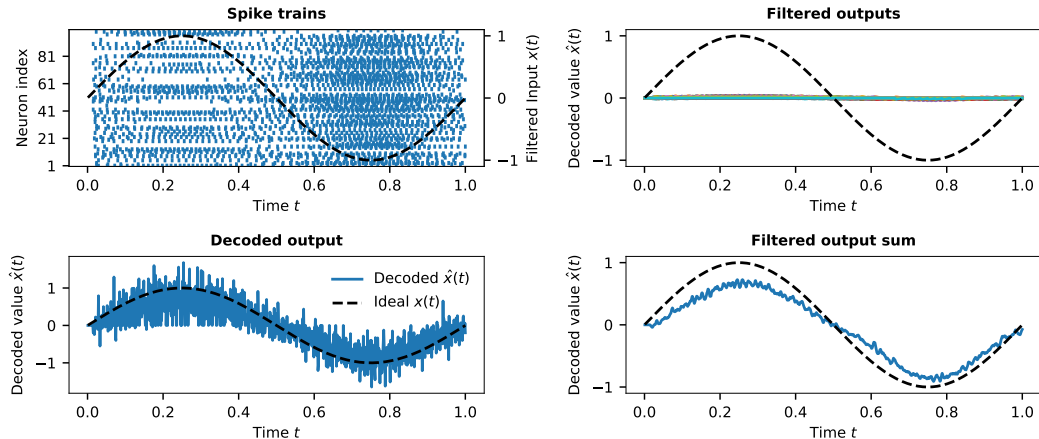$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0\,, \\ 0 & \text{otherwise}\,, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t\,.$$

Example: Synaptic Filter for Two Neurons

$$\tau = 5\,\text{ms},\, n = 1$$

Example: Synaptic Filter for One Hundred Neurons

$\tau = 5\,\text{ms}, n = 1$

Image sources

Title slide
"Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France."
Author: Camille Grávis, between 1889 and 1900.
From Wikimedia.