SYDE 556/750

Simulating Neurobiological Systems
Lecture 2: Neurons

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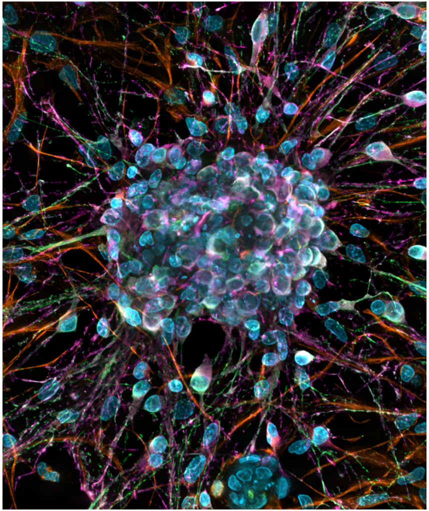
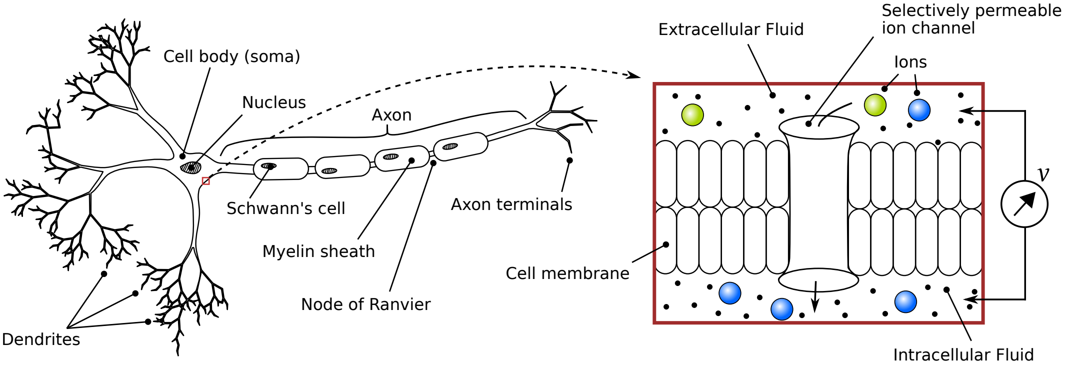
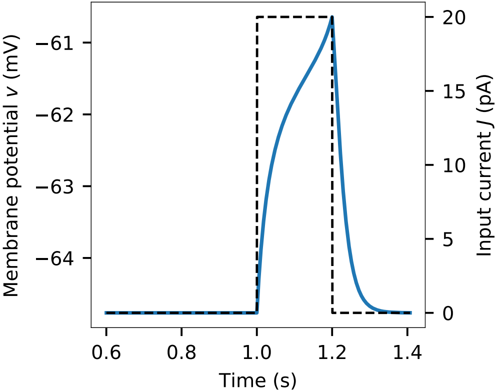


Textbook Neuron and Cell Membrane



Injecting a Current Into a Detailed Neuron Model



Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

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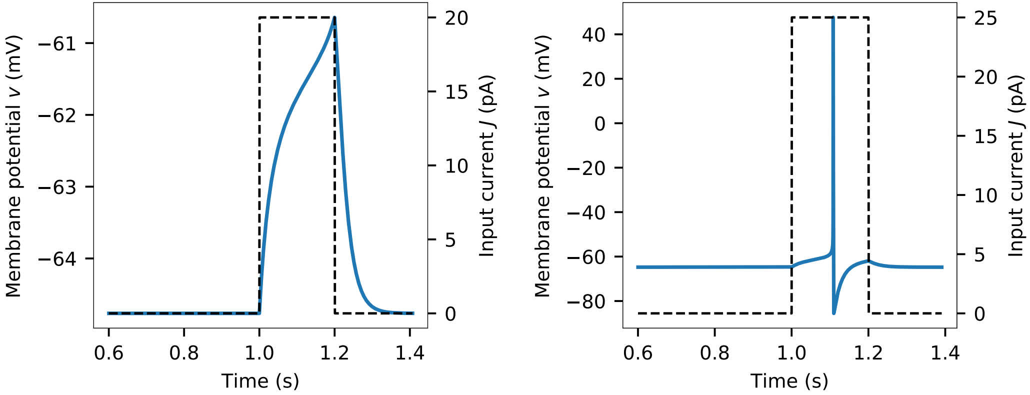
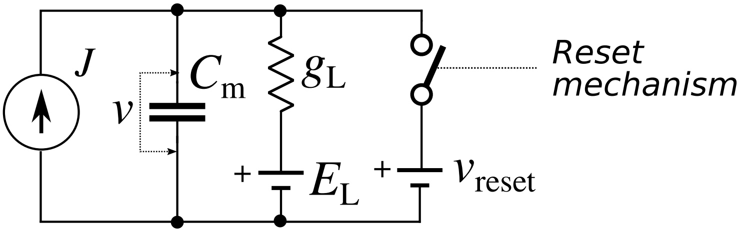


Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

Basic High-Level Details (Lapicque, 1907)

1. The cell acts like a *capacitor*, i.e., the voltage increases while we're injecting a current.
2. The capacitor is *leaky*. As soon as we stop injecting a current, the voltage collapses back to the resting potential E_{L}.
3. As soon as the voltage surpasses a certain value, the *threshold potential* v_{th}, the cell will generate a spike.
4. Shortly after the spike has been produced, the voltage drops below the resting potential. During this period, the *refractory period* of length τ_{ref}, we cannot get the neuron to spike again, even if we apply relatively large input currents J.

The Leaky Integrate-and-Fire Equivalent Circuit

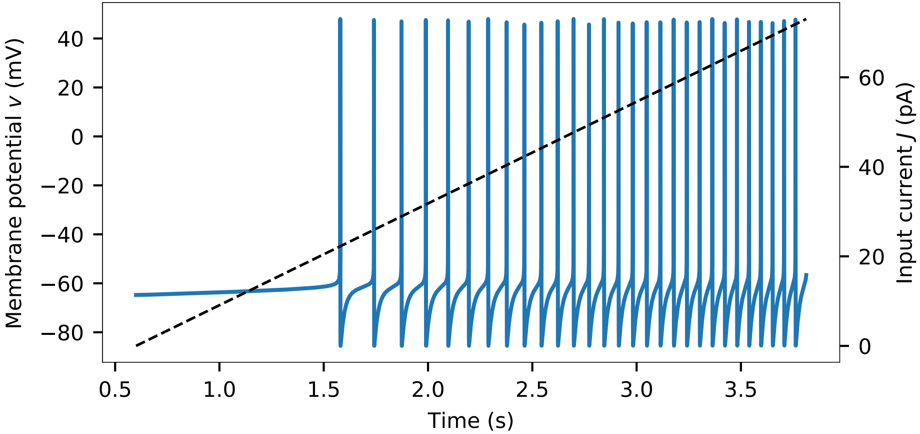


$$\frac{\mathrm{d}}{\mathrm{d}t} v(t) = \frac{1}{C_\mathrm{m}} \left(g_\mathrm{L}(E_\mathrm{L} - v(t)) + J \right), \quad \text{if } v(t) < v_\mathrm{th} \,. \tag{1}$$

if $v(t) = v_\mathrm{th}$ at $t = t_\mathrm{th}$, output a spike $\left(\delta(t - t_\mathrm{th})\right)$ and:

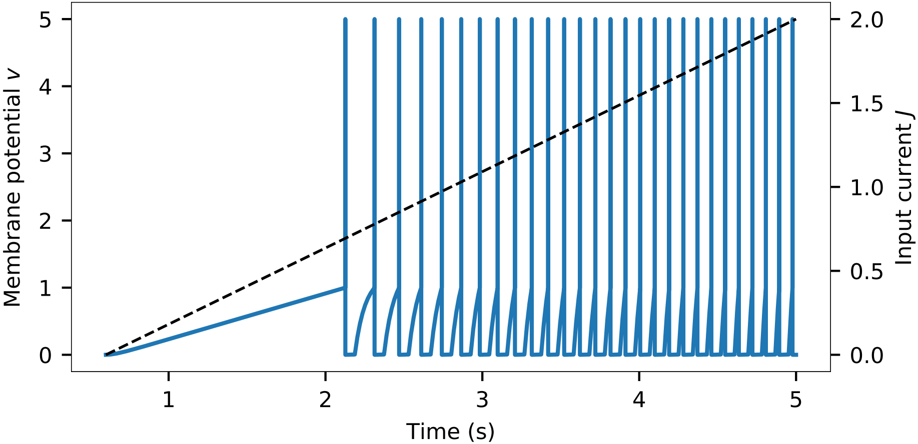
$$v(t) \leftarrow v_\mathrm{reset} \,, \quad \text{if } t_\mathrm{th} < t \leq t_\mathrm{th} + \tau_\mathrm{ref} \,, \tag{2}$$

Injecting a Current Ramp into a Detailed Neuron Model



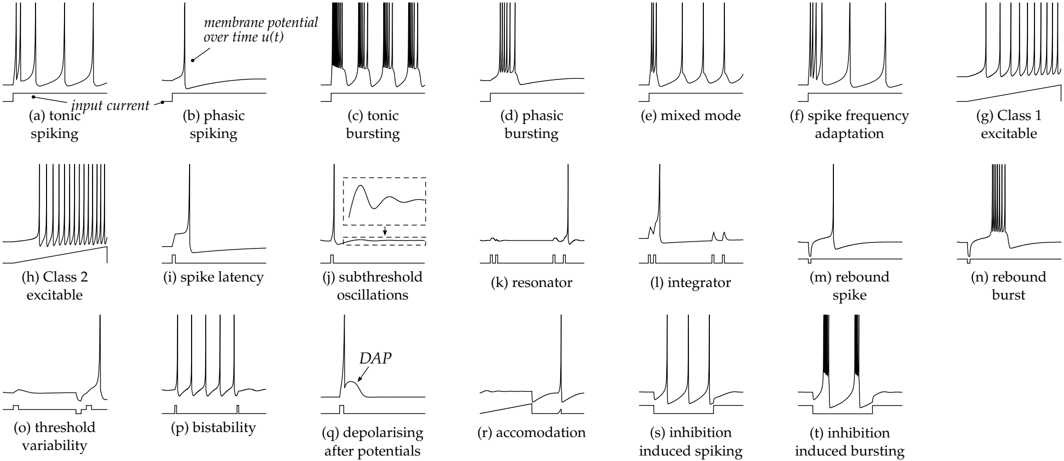
Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

Injecting a Current Ramp into a LIF Neuron Model



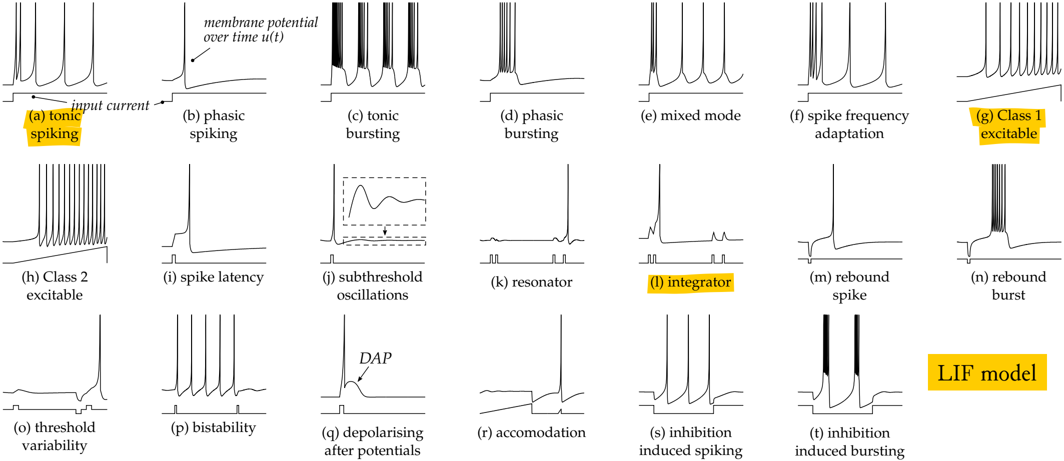
(note normalization to $v_{th} = 1$, $v_{reset} = E_L = 0$)

Limitations of the LIF Neuron Model



(a) tonic spiking
(b) phasic spiking
(c) tonic bursting
(d) phasic bursting
(e) mixed mode
(f) spike frequency adaptation
(g) Class 1 excitable
(h) Class 2 excitable
(i) spike latency
(j) subthreshold oscillations
(k) resonator
(l) integrator
(m) rebound spike
(n) rebound burst
(o) threshold variability
(p) bistability
(q) depolarising after potentials
(r) accomodation
(s) inhibition induced spiking
(t) inhibition induced bursting

Limitations of the LIF Neuron Model



LIF model

LIF Rate Approximation

- need to compute t_{th} (the time $v(t_{\mathrm{th}}) = v_{th}$)
- assume: J is constant and $v(0) = 0$.

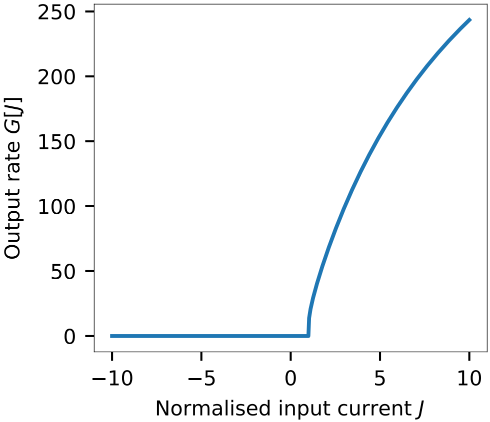
$$v(t) = -\int_0^t \frac{1}{\tau_{\mathrm{RC}}}\big(v(t') - RJ\big)\,\mathrm{d}t' = RJ\left(1 - e^{-\frac{t}{\tau_{\mathrm{RC}}}}\right) .$$

$$v_{\mathrm{th}} = RJ\left(1 - e^{-\frac{t_{\mathrm{th}}}{\tau_{\mathrm{RC}}}}\right) \Leftrightarrow 1 - \frac{v_{\mathrm{th}}}{RJ} = e^{-\frac{t_{\mathrm{th}}}{\tau_{\mathrm{RC}}}} ,$$

$$t_{\mathrm{th}} = -\tau_{\mathrm{RC}} \log\left(1 - \frac{v_{\mathrm{th}}}{RJ}\right)$$

$$G[J] = \begin{cases} \frac{1}{\tau_{\mathrm{ref}} - \tau_{\mathrm{RC}} \log\left(1 - \frac{v_{\mathrm{th}}}{RJ}\right)} & \text{if } 1 - \frac{v_{\mathrm{th}}}{RJ} > 0 , \\ 0 & \text{otherwise} . \end{cases}$$

Artifical Rate Neurons: LIF

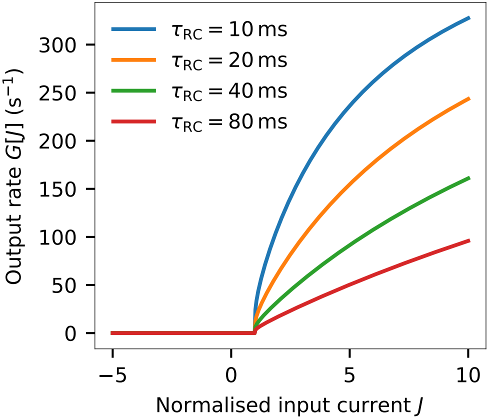


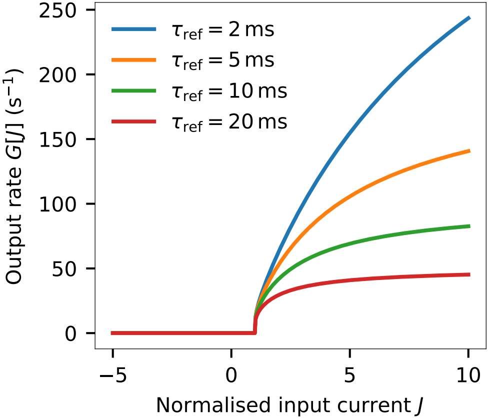
$$G[J] = \frac{1}{\tau_{\mathrm{ref}} - \tau_{\mathrm{RC}} \log\left(1 - \frac{1}{J}\right)}$$

Usefulness to neurobiological systems modellers:

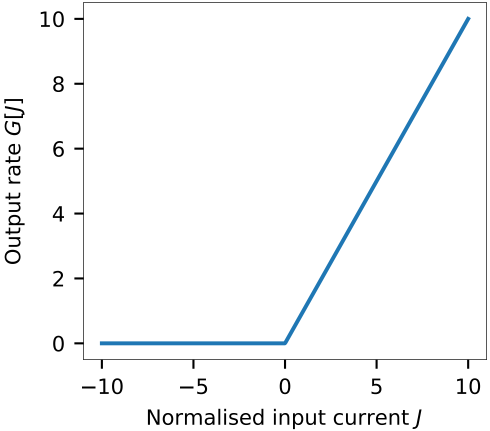
- ⊕ Biologically motivated
- ⊕ Captures saturation effects
- ● Relatively slow to evaluate numerically (for machine-learning people)
- ⊖ Spike onset is smooth in noisy systems

Exploring the LIF Rate Approximation





Artifical Rate Neurons: ReLU

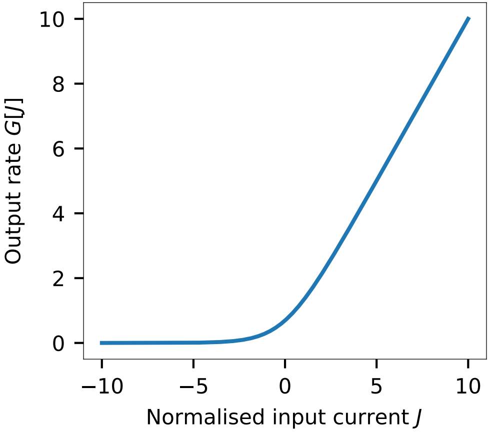


$$G[J] = \max\{0, J\}$$

Usefulness to neurobiological systems modellers:

- Fast to evaluate
- Rough approximation of the LIF response curve
- Does not capture saturation effects
- Spike onset is smooth in noisy systems

Artifical Rate Neurons: Smooth ReLU (Softplus)

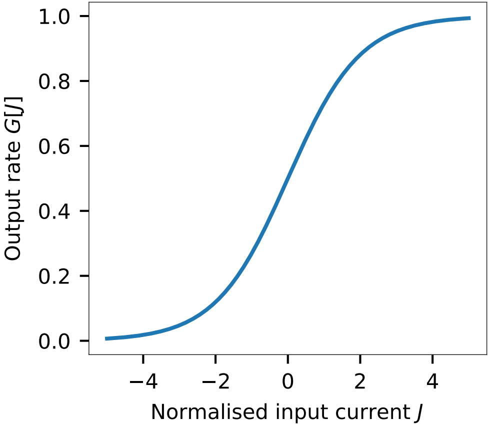


$$G[J] = \log(1 + \exp(J))$$

Usefulness to neurobiological systems modellers:

- ⊕ Models smooth spike onset
- ● Rough approximation of the LIF response curve
- ⊖ Does not capture saturation effects

Artifical Rate Neurons: Logistic Function

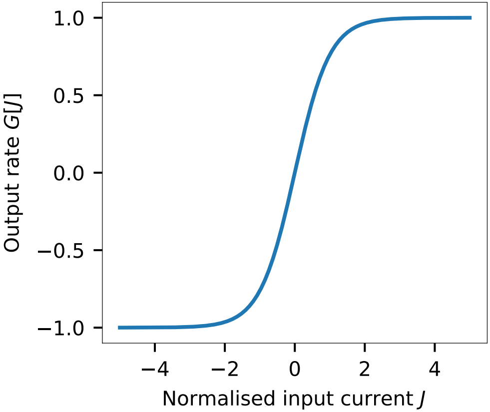


$$G[J] = \frac{1}{1 + e^{-J}}$$

Usefulness to neurobiological systems modellers:

- Models smooth spike onset and saturation (?)

Artifical Rate Neurons: Hyperbolic Tangent



$$G[J] = \tanh(J) = \frac{e^J - e^{-J}}{e^J + e^{-J}}$$

Usefulness to neurobiological systems modellers:

- Models smooth spike onset and saturation (?)
- Negative rates

Image sources

Title slide
Image of rat primary cortical neurons in culture.
Author: ZEISS Microscopy, `http://www.zeiss.com/celldiscoverer`.
From Wikimedia.