

**SYDE 556/750**

**Simulating Neurobiological Systems**  
**Lecture 9: Analysing Representations**

Andreas Stöckel

March 5, 2020



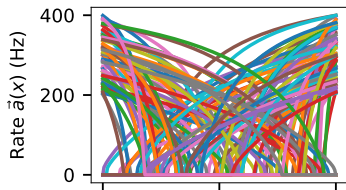
UNIVERSITY OF  
**WATERLOO**

FACULTY OF  
ENGINEERING

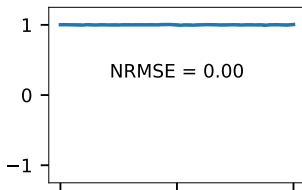


# Decoding Polynomials

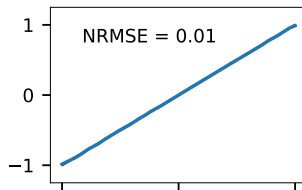
**Tuning curves ( $n = 128$ )**



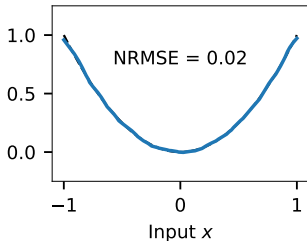
$f(x) = 1$



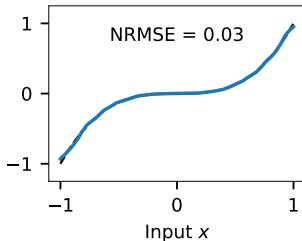
$f(x) = x$



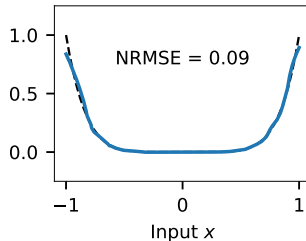
$f(x) = x^2$



$f(x) = x^3$

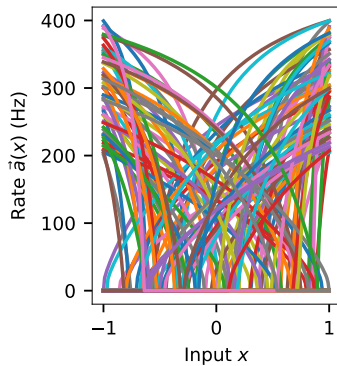


$f(x) = x^6$

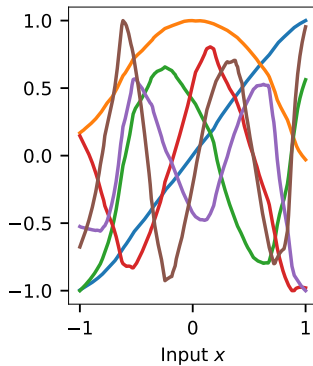


# LIF Tuning Curve Principal Components

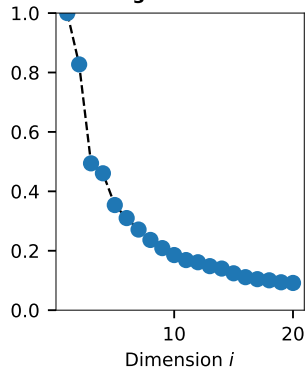
**Tuning curves ( $n = 128$ )**



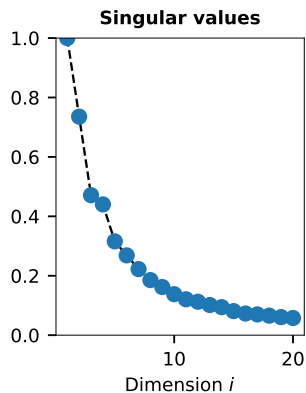
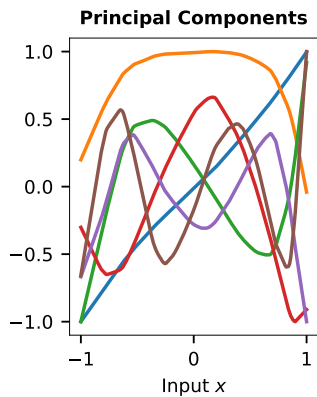
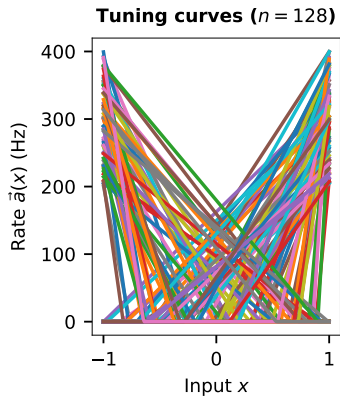
**Principal Components**



**Singular values**

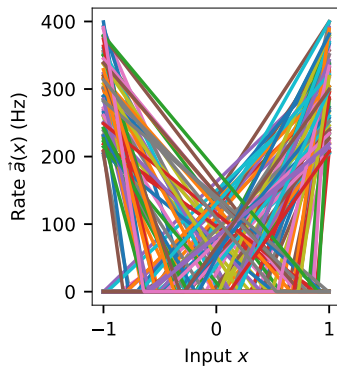


# ReLU Tuning Curve Principal Components

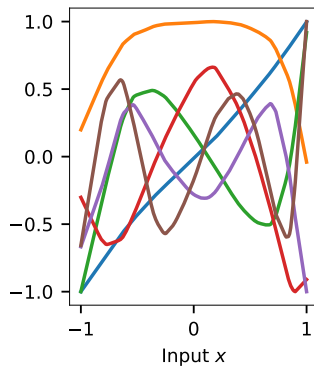


# ReLU Tuning Curve Principal Components

**Tuning curves ( $n = 128$ )**

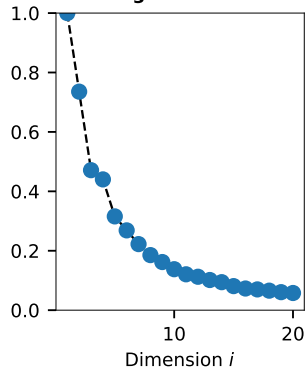


**Principal Components**

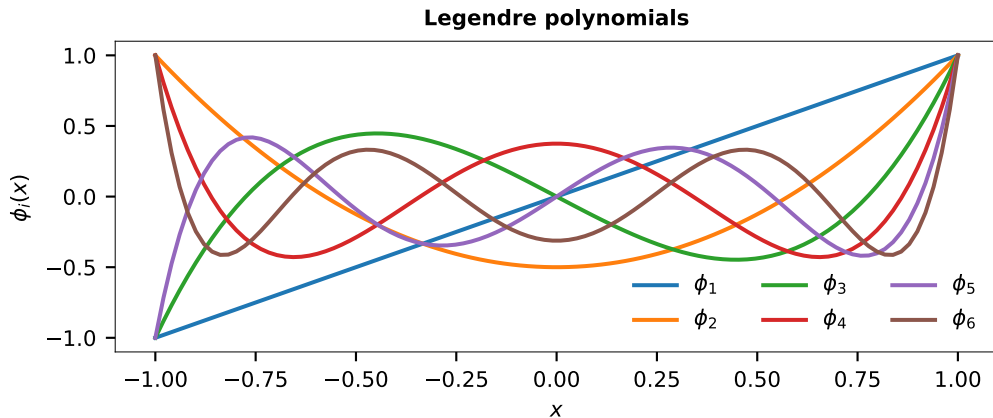


$\approx$  Legendre Basis

**Singular values**



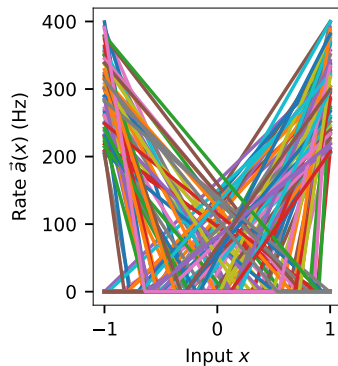
## Reminder: Legendre Polynomials



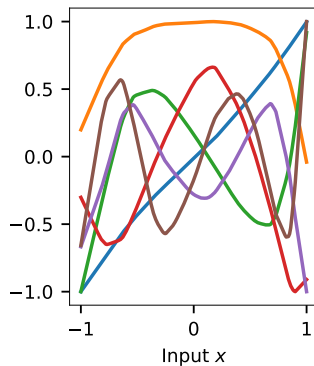
$$\varphi_i(x) = \frac{1}{2^i} \sum_{k=0}^i \binom{i}{k}^2 (x-1)^{i-k} (x+1)^k$$

## Modifying the Basis – Same Maximum Rate (I)

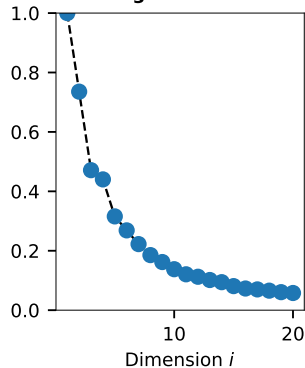
**Tuning curves ( $n = 128$ )**



**Principal Components**

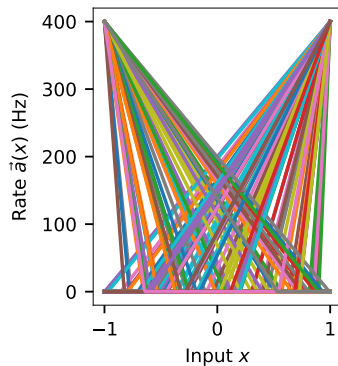


**Singular values**

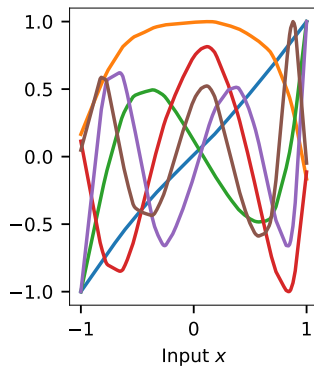


# Modifying the Basis – Same Maximum Rate (I)

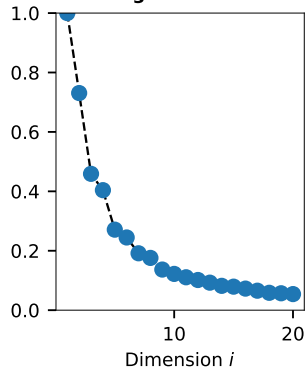
**Tuning curves ( $n = 128$ )**



**Principal Components**

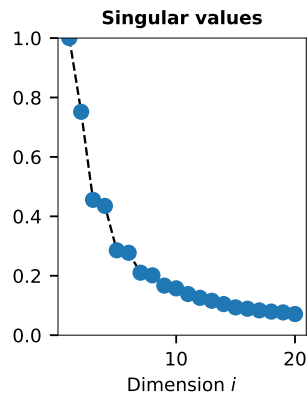
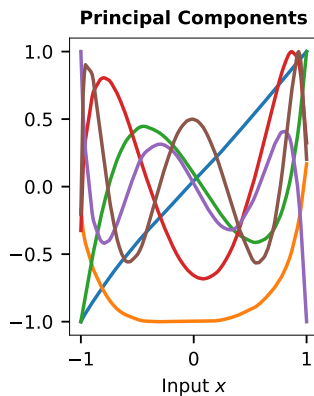
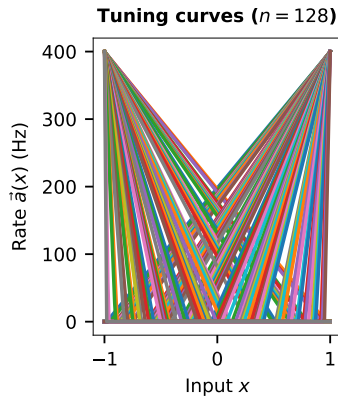


**Singular values**



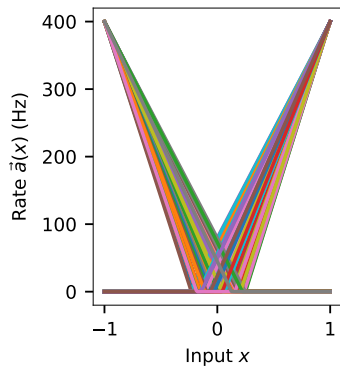


## Modifying the Basis – Equidistant $x$ -Intercepts (II)

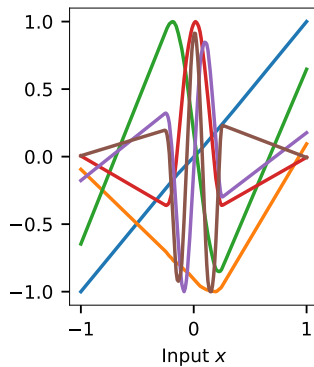


## Modifying the Basis – Limited x-Intercepts (III)

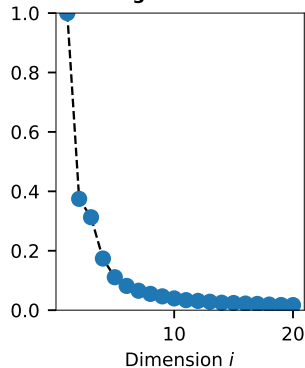
**Tuning curves ( $n = 128$ )**



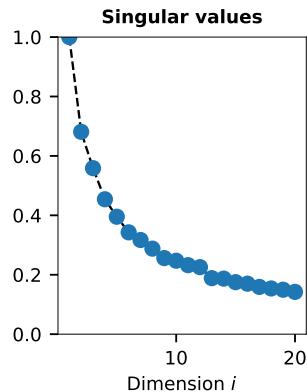
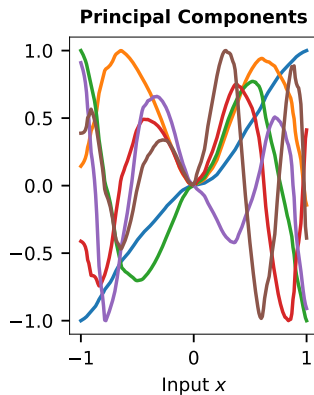
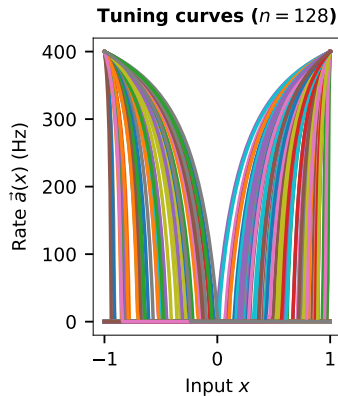
**Principal Components**



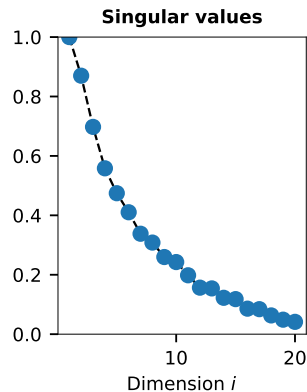
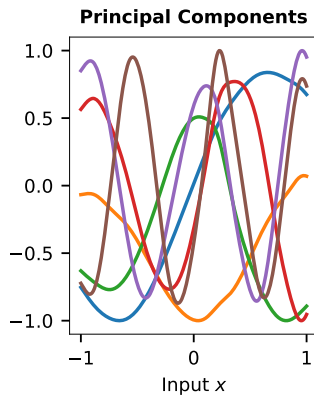
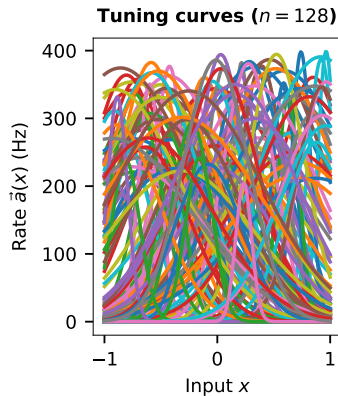
**Singular values**



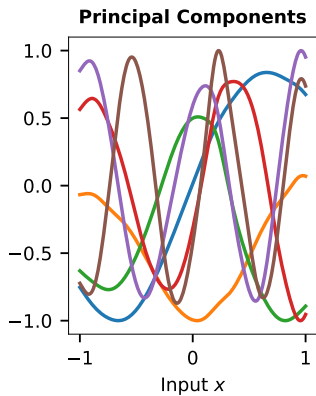
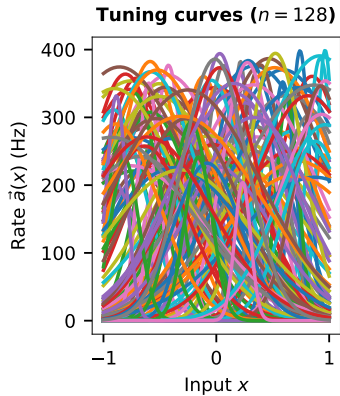
## Modifying the Basis – Symmetric Tuning Curves (IV)



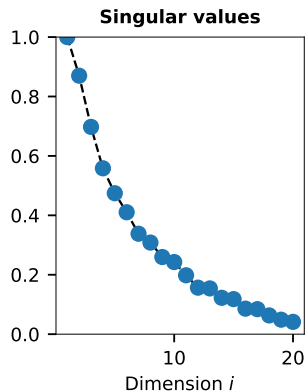
# Gaussian Tuning Curve Principal Components



# Gaussian Tuning Curve Principal Components

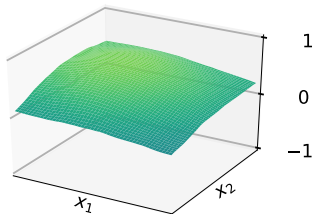


$\approx$  Fourier Basis

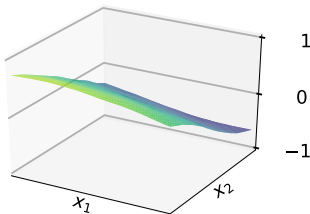


# PCA of 2D Tuning Curves

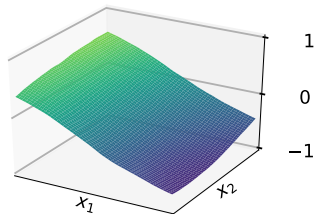
**Principal component 1**



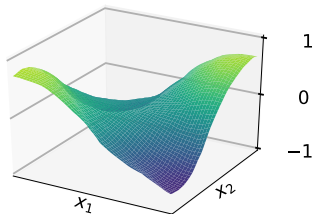
**Principal component 2**



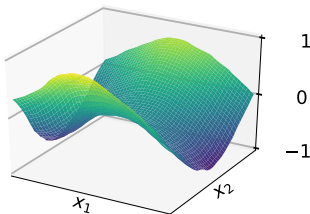
**Principal component 3**



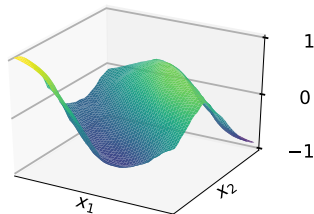
**Principal component 4**



**Principal component 5**

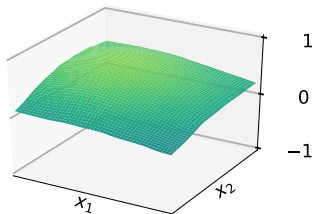


**Principal component 6**

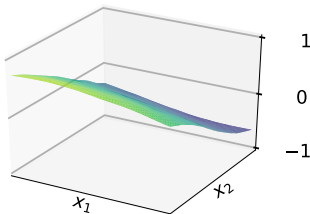


# PCA of 2D Tuning Curves

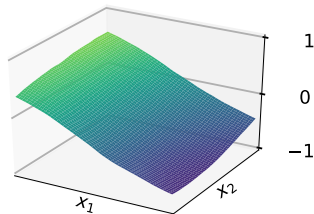
Principal component 1



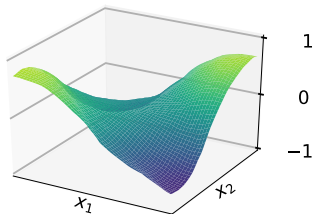
Principal component 2



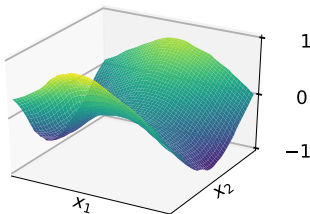
Principal component 3



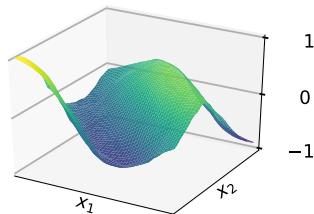
Principal component 4



Principal component 5



Principal component 6



Combination of 2D Polynomials

# Conclusions

- ▶ Can use **PCA** to find the basis functions underlying neural representations
- ▶ **Singular values** inversely proportional to noise
- ▶ **Basis function shape** depends on
  - ▶ x-intercept distributions
  - ▶ Neuron response curve  $G[J]$
- ▶ Finding optimal tuning curves for representations  
⇒ Full network optimization (must use gradient descent)



# Image sources

## **Title slide**

Maurice Denis: Homage to Cézanne, 1900  
From Wikimedia.