

SYDE 556/750

Simulating Neurobiological Systems
Lecture 4: Temporal Representations

Andreas Stöckel

January 22 & 28, 2020

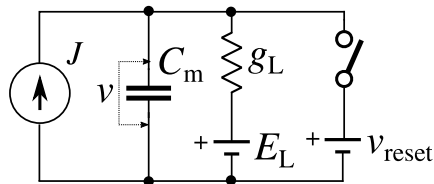
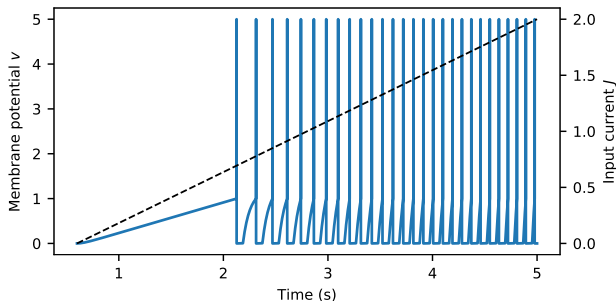


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Reminder: The LIF Neuron



$$\frac{d}{dt}v(t) = -\frac{1}{\tau_{RC}}(v(t) - J),$$

$$v(t) \leftarrow \delta(t - t_{th}),$$

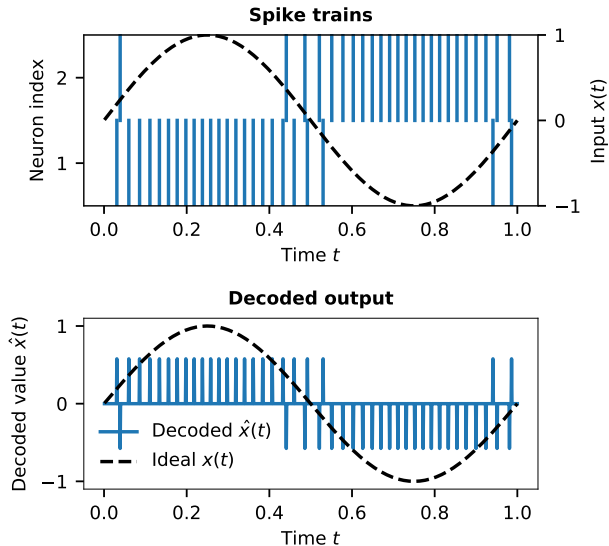
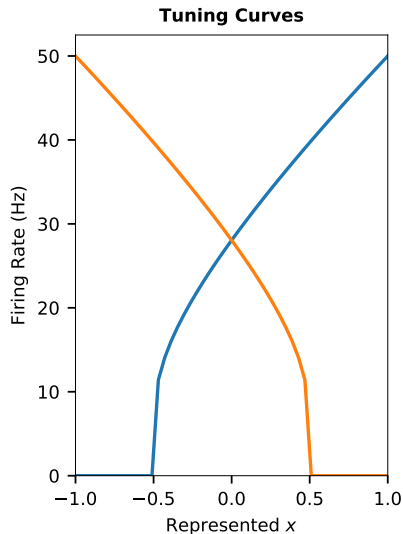
$$v(t) \leftarrow 0,$$

$$\text{if } v(t) < 1,$$

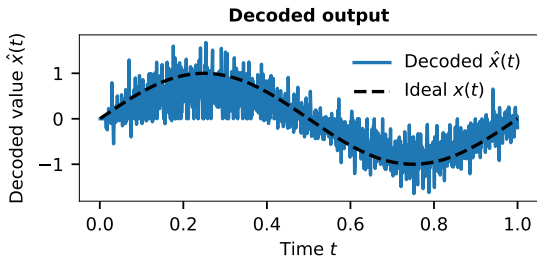
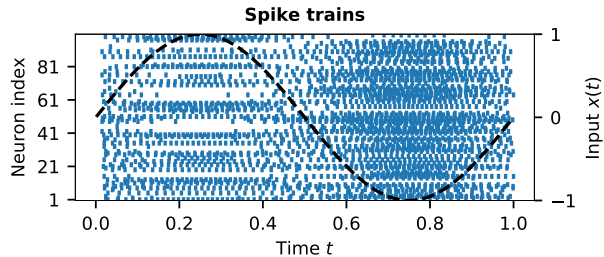
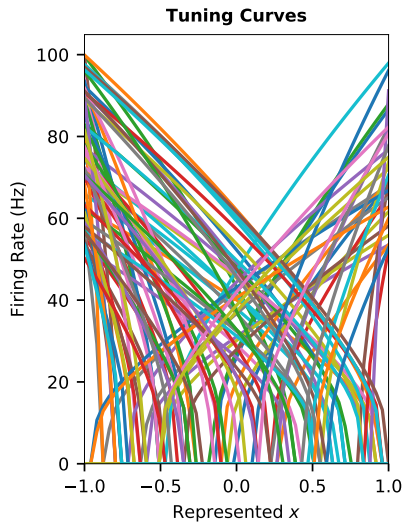
$$\text{if } t = t_{th},$$

$$\text{if } t > t_{th} \text{ and } t \geq t_{th} + \tau_{ref},$$

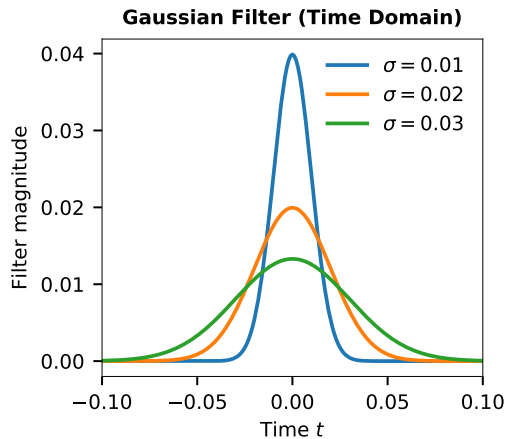
Temporal Decoding of Two Neurons



Temporal Decoding of One Hundred Neurons



Filtering by Convolution



Gaussian Filter

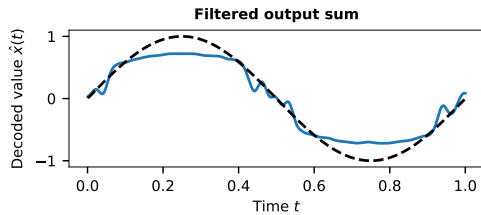
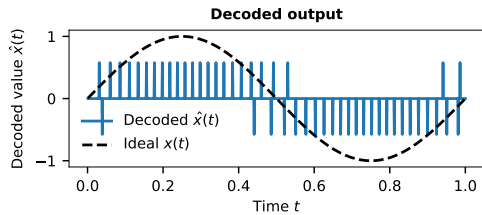
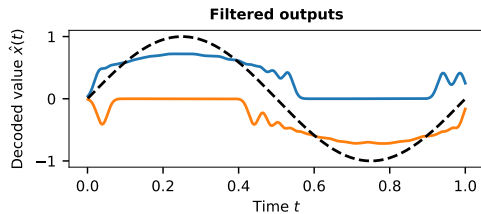
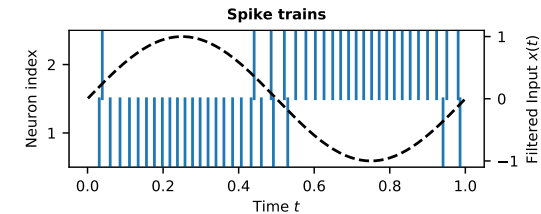
$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

where c chosen s.t. $\int_{-\infty}^{\infty} h(t) dt = 1$

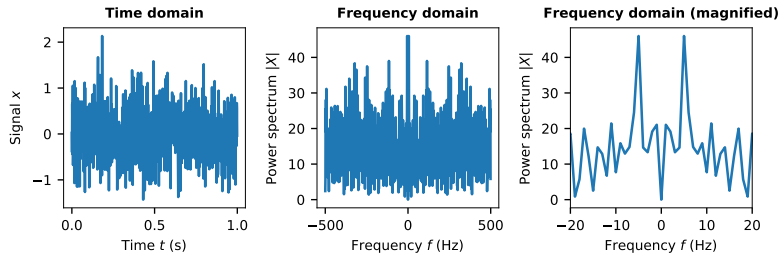
Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - \tau)g(\tau) d\tau$$

Filtering a Spike Train

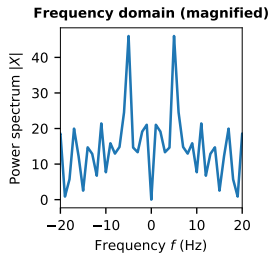
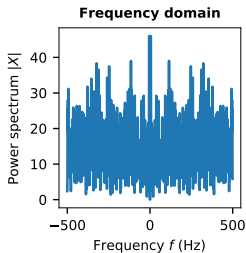
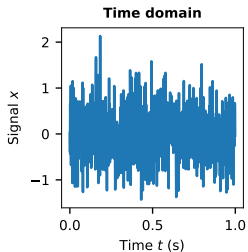


Random Signals

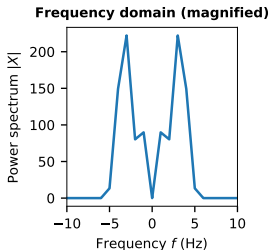
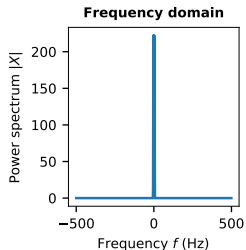
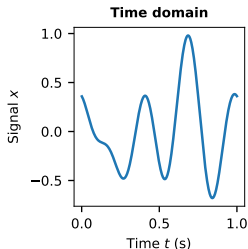


White Noise
(zero mean)

Random Signals

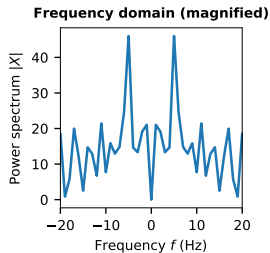
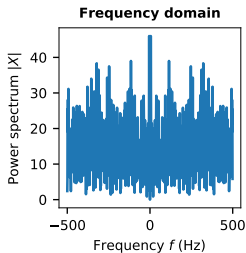
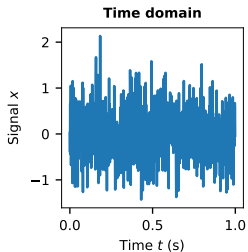


White Noise
(zero mean)

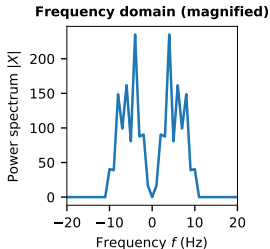
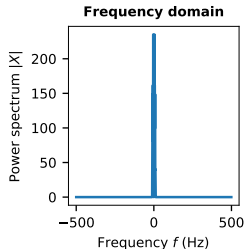
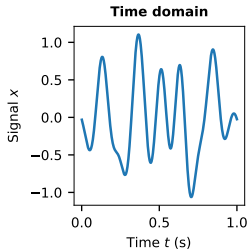


Bandlimited
White Noise
(zero mean,
5 Hz bandwidth)

Random Signals

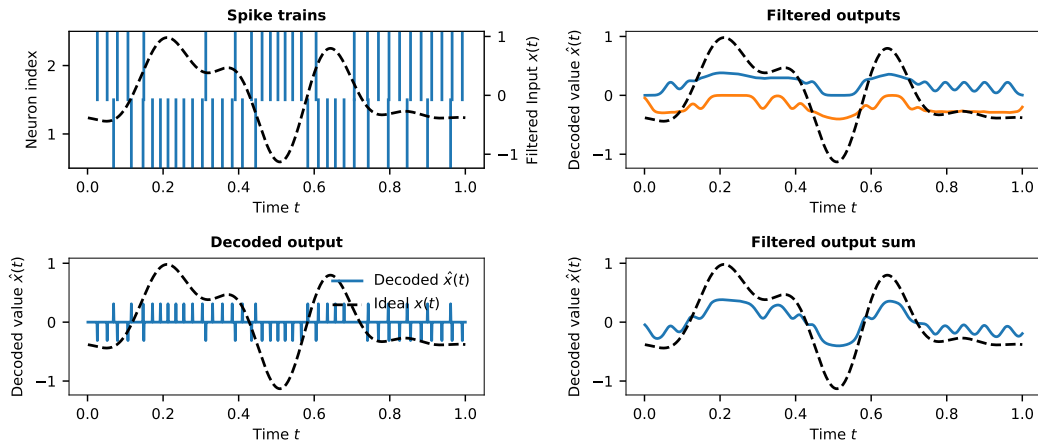


White Noise
(zero mean)

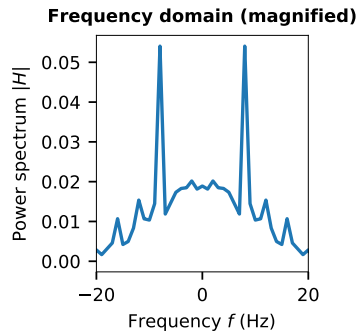
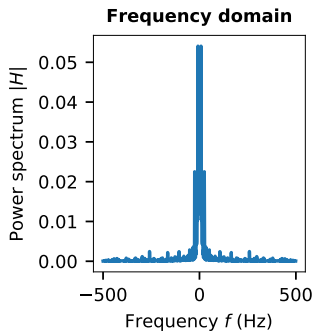
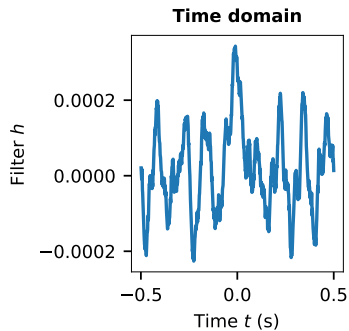


Bandlimited
White Noise
(zero mean,
10 Hz bandwidth)

Filtering a Spike Train for a Random Signal

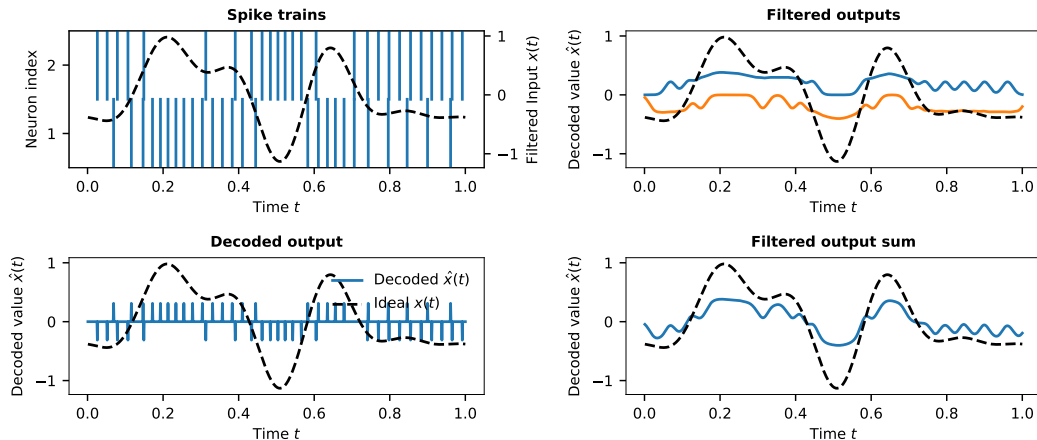


Optimal Filter

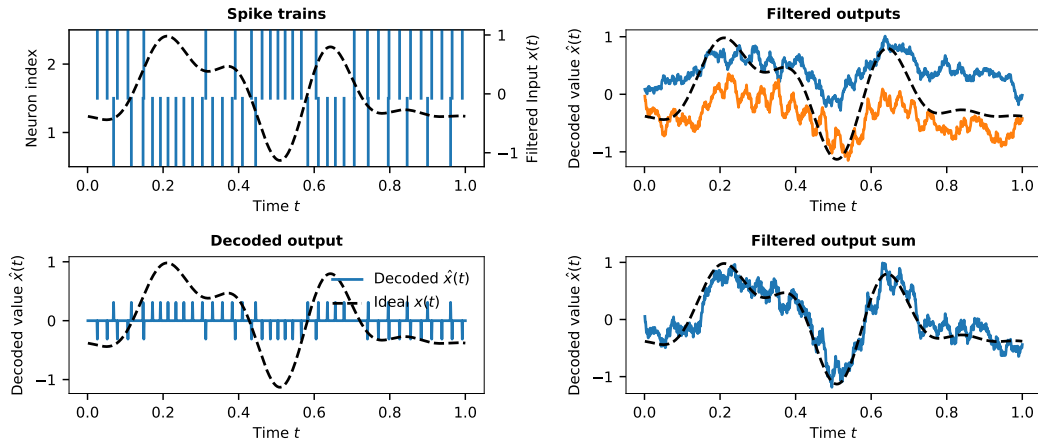


$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

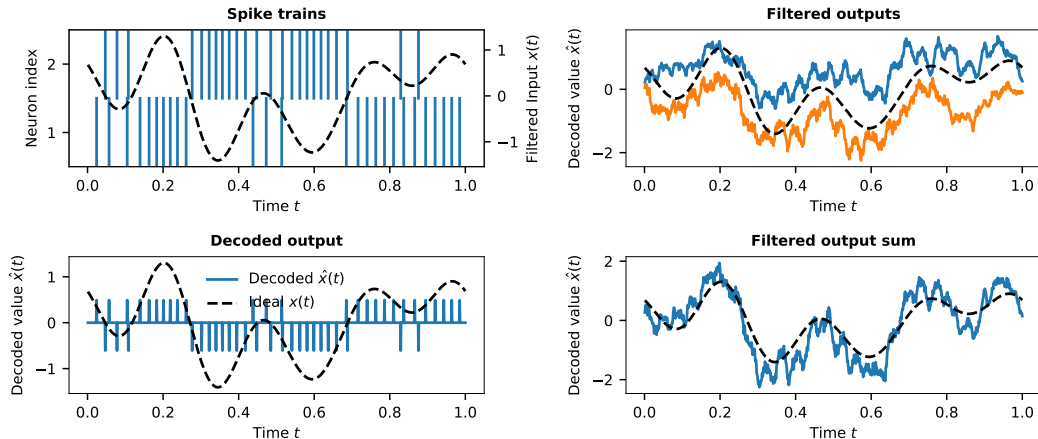
Filtering a Spike Train for a Random Signal (Optimal Filter)



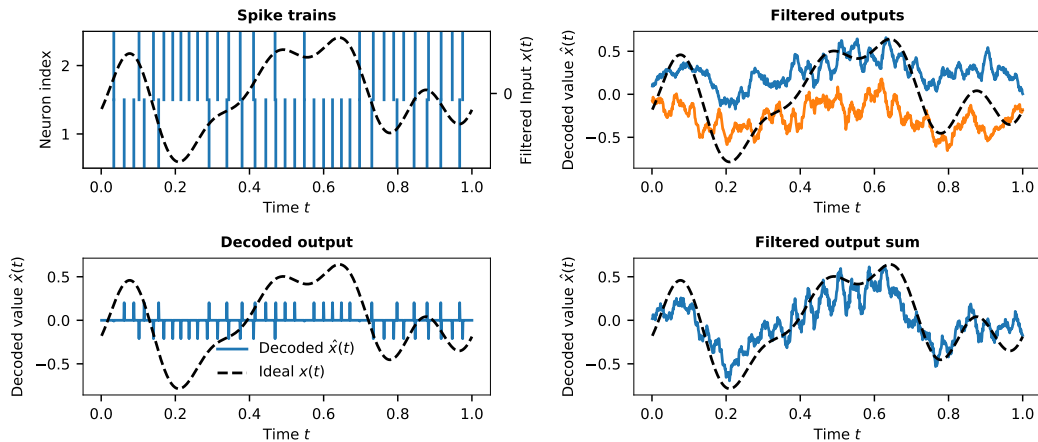
Filtering a Spike Train for a Random Signal (Optimal Filter)



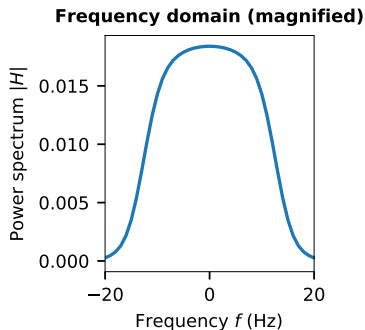
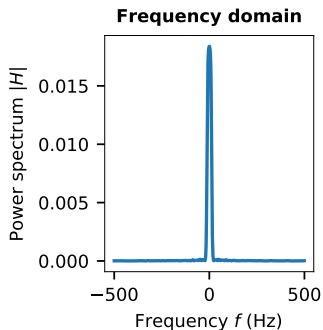
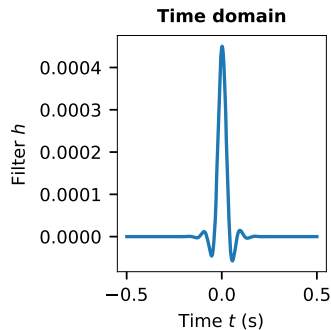
Filtering a Spike Train for a Random Signal (Optimal Filter)



Filtering a Spike Train for a Random Signal (Optimal Filter)

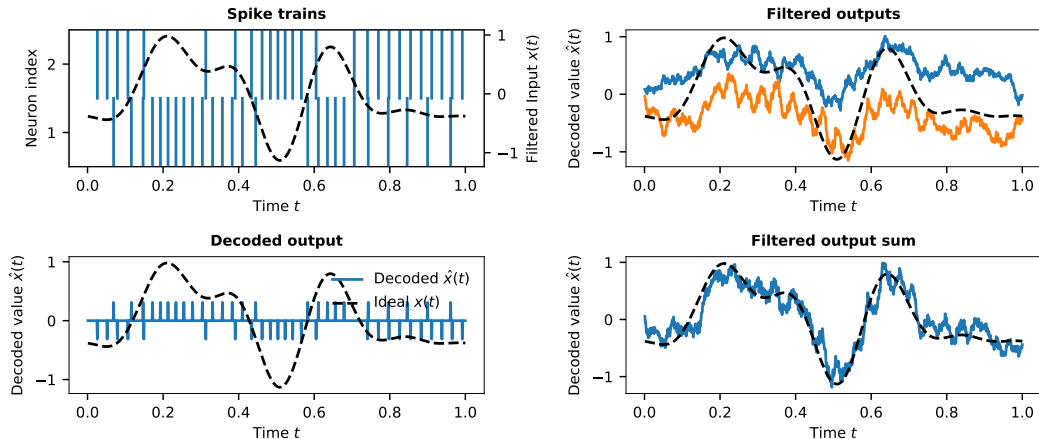


Optimal Filter (Improved)

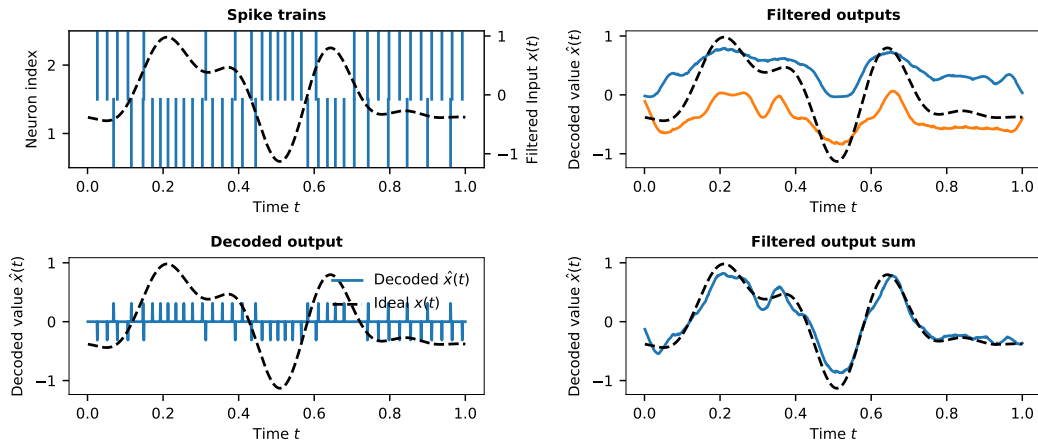


$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$

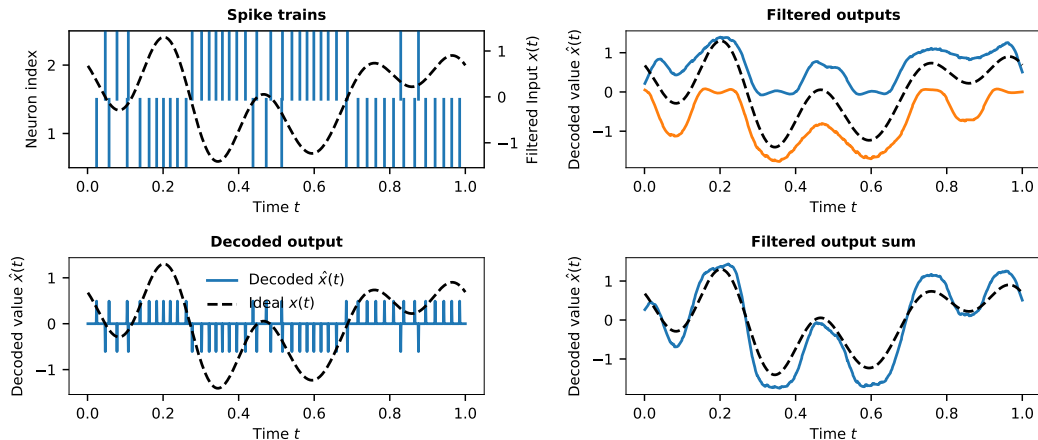
Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



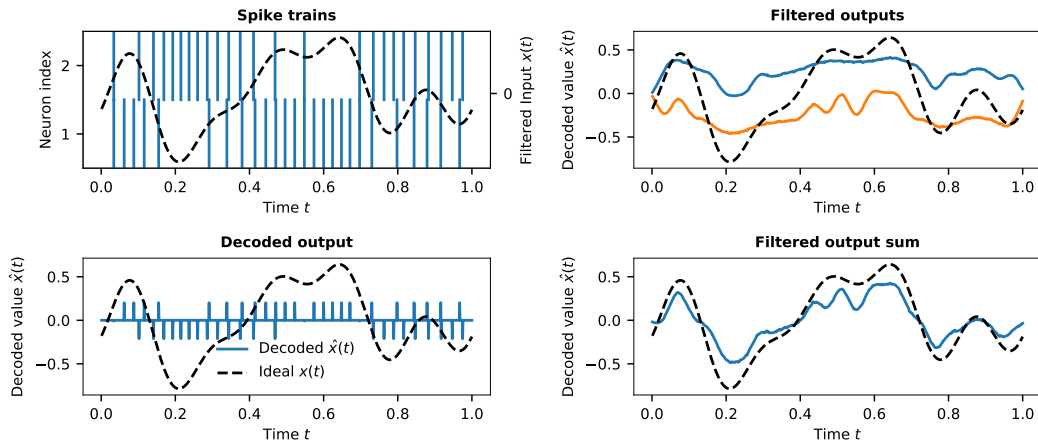
Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



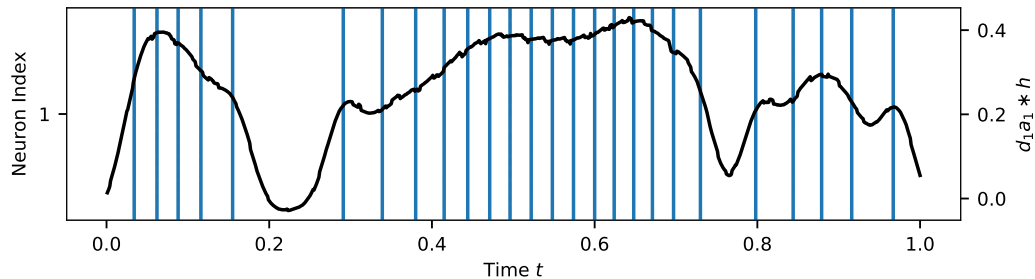
Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



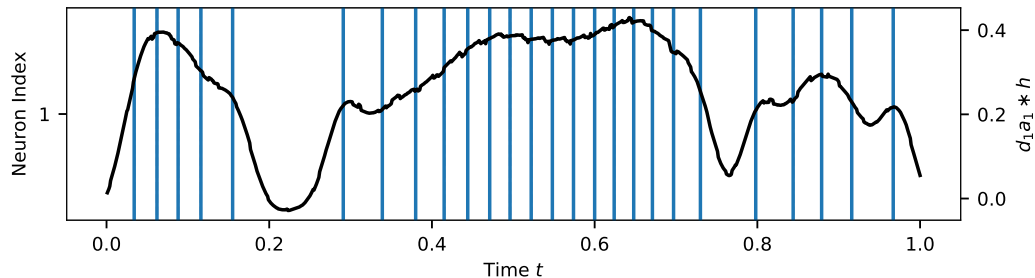
Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



Pros and Cons of the Optimal Filter



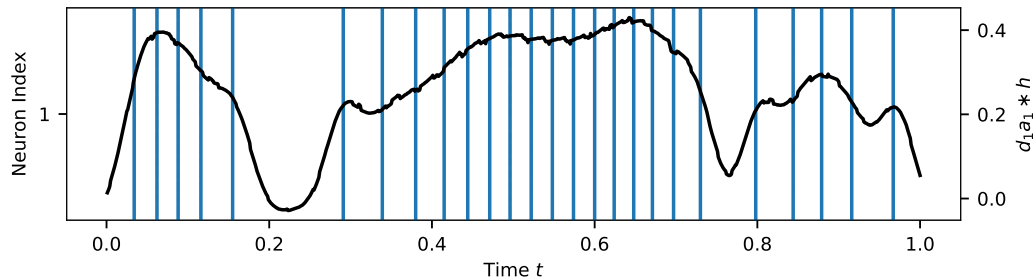
Pros and Cons of the Optimal Filter



+ Precise

Good for analysing data after the fact

Pros and Cons of the Optimal Filter



+ Precise

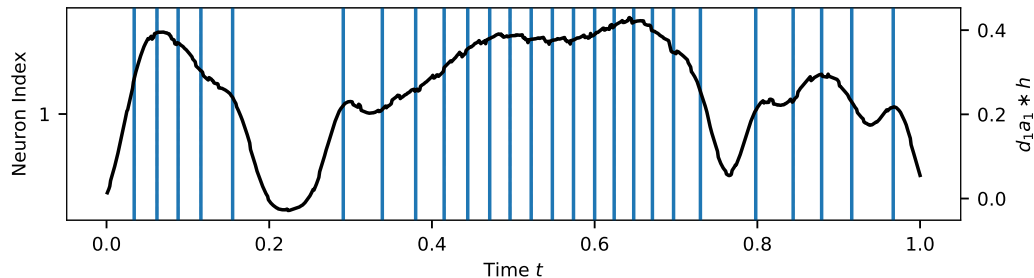
Good for analysing data after the fact



Non-causal

Does not describe a biological process

Pros and Cons of the Optimal Filter



+ Precise

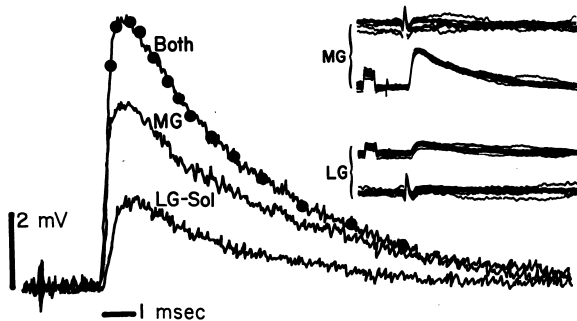
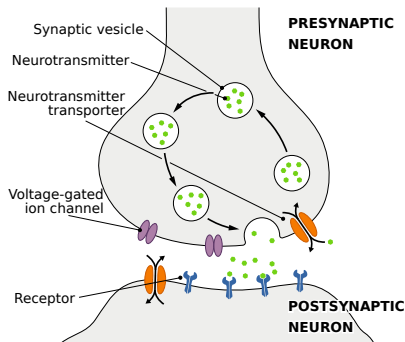
Good for analysing data after the fact

- Non-causal

Does not describe a biological process

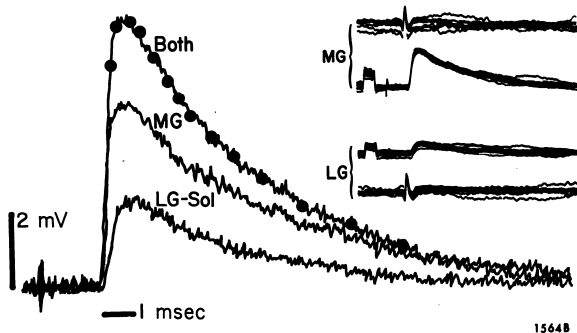
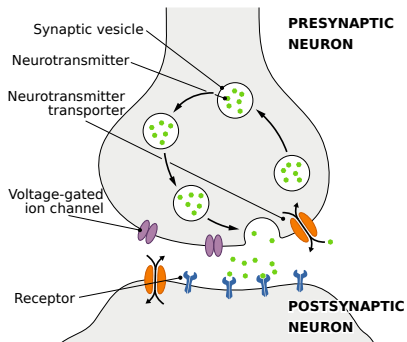
We need to find a mechanism that low-pass filters spikes over time!

Synapses as Filters



15648

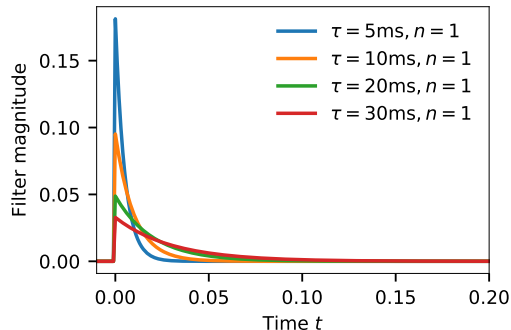
Synapses as Filters



Post-synaptic currents (EPSPs, IPSPs) are low-pass filtered spike trains!

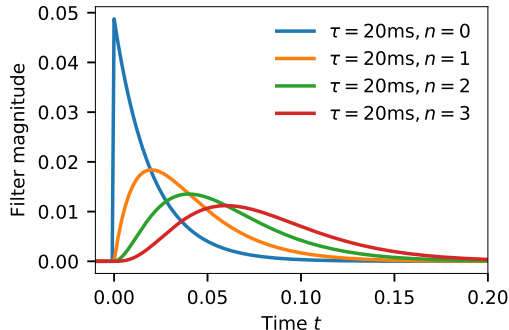
Exponential Low-Pass Filter (I)

Synaptic Filter (Time Domain)



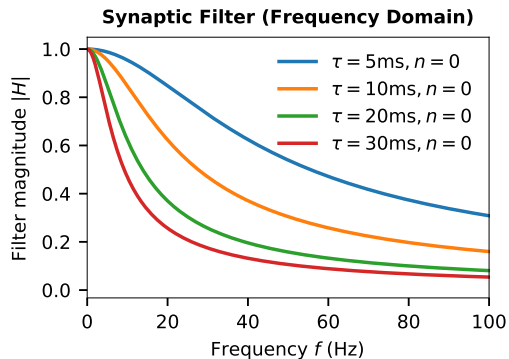
$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

Synaptic Filter (Time Domain)

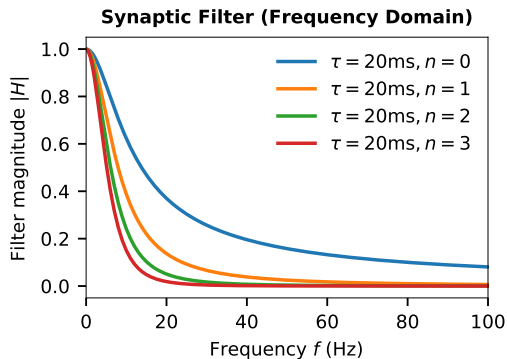


$$\text{where } c = \int_0^{\infty} t^n \exp^{-t/\tau} dt.$$

Exponential Low-Pass Filter (II)

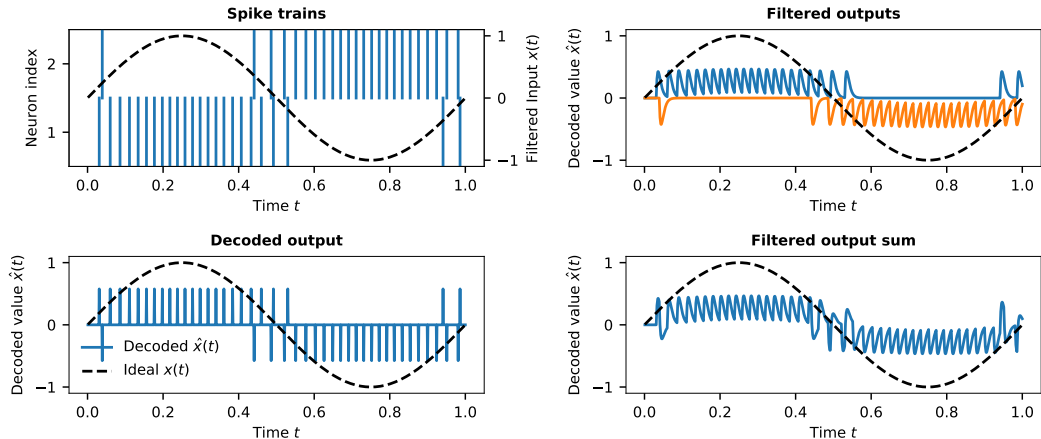


$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$



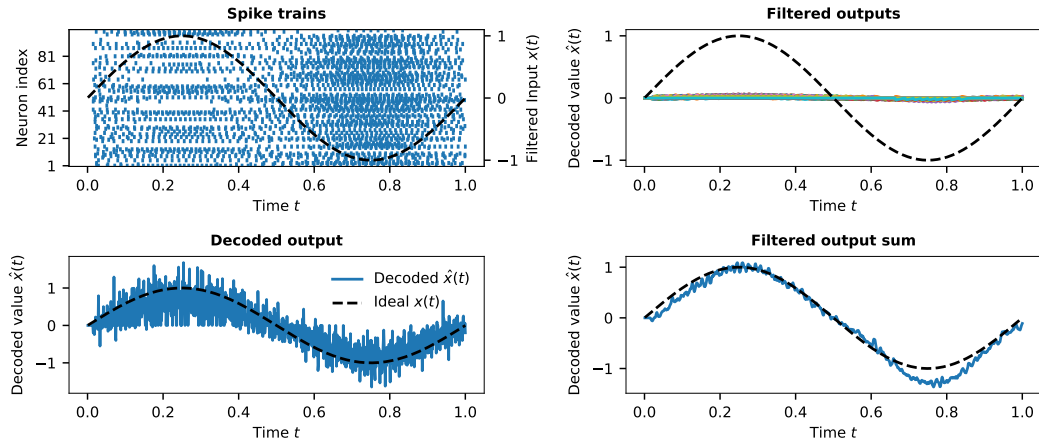
$$\text{where } c = \int_0^{\infty} t^n \exp^{-t/\tau} dt.$$

Example: Synaptic Filter for Two Neurons



$$\tau = 5 \text{ ms}, n = 1$$

Example: Synaptic Filter for One Hundred Neurons



$$\tau = 5 \text{ ms}, n = 1$$

Image sources

Title slide

“Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France.”

Author: Camille Grávis, between 1889 and 1900.

From Wikimedia.