SYDE 556/750

# Simulating Neurobiological Systems Lecture 10: Symbols and Symbol-like Representations

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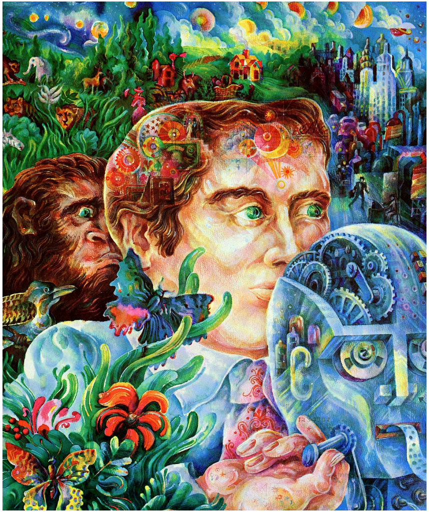


# Classical Representation of Knowledge

- "The number eight comes after the number nine":

$$\mathbf{isSucc}(\mathtt{EIGHT}, \mathtt{NINE}) \,.$$

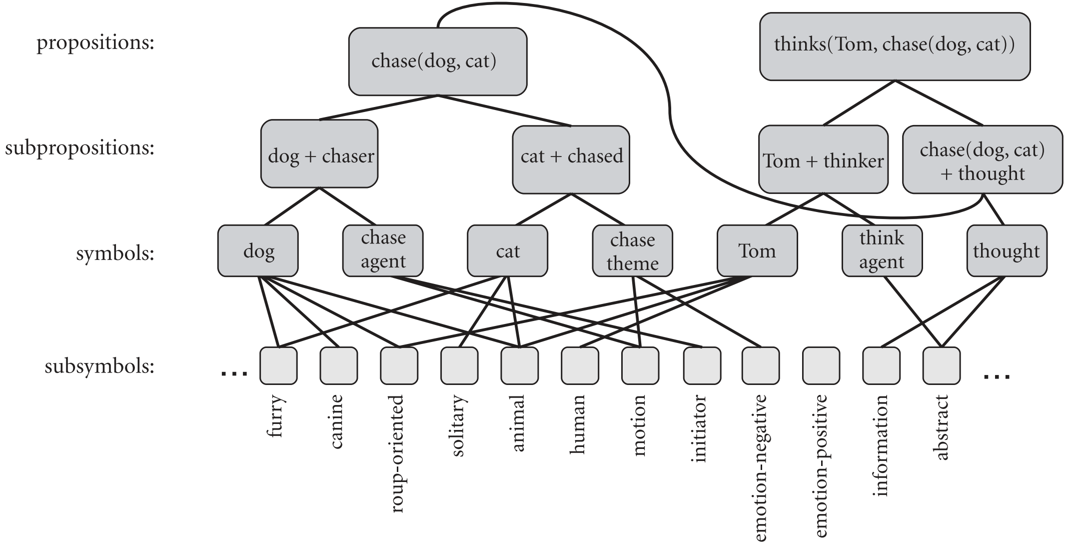
- "All dogs chase cats":

$$\forall x \forall y \left(\mathbf{isDog}(x) \wedge \mathbf{isCat}(y)\right) \rightarrow \mathbf{doesChase}(x, y) \,.$$

- "Anne knows that Bill thinks that Charlie likes Dave":

$$\mathbf{knows}\Big(\mathtt{ANNE}, \text{``}\mathbf{thinks}(\mathtt{BILL}, \text{`}\mathbf{likes}(\mathtt{CHARLIE}, \mathtt{DAVE})\text{'})\text{''}\Big) \,.$$

# Solution Attempt 1: Neural Synchrony (I)

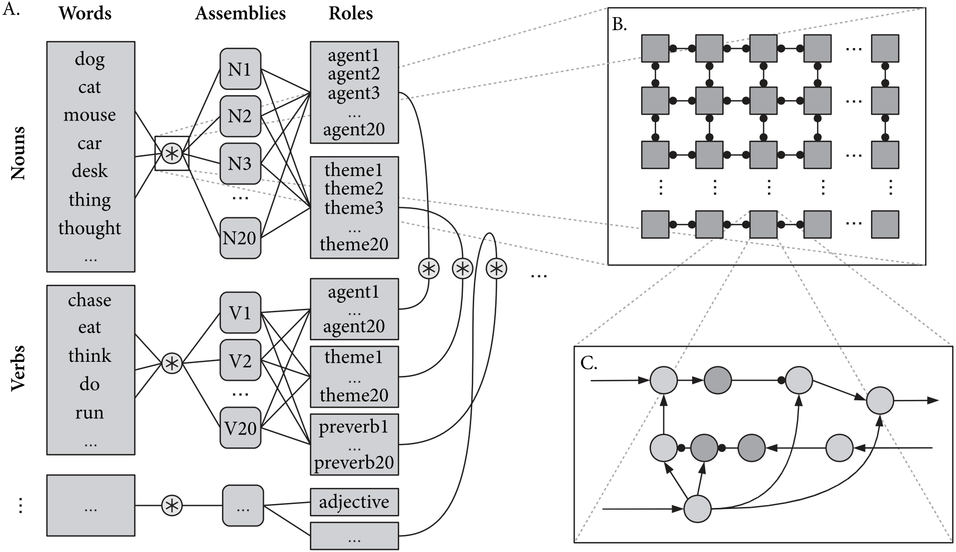


# Solution Attempt 1: Neural Synchrony (II)

- ⊕ Solves the binding problem
- ● Localist representation
- ● Unclear how to solve problems 1 to 3

- ⊖ Unclear how these oscillations are generated and controlled
- ⊖ Unclear how the representations are processed
- ⊖ Exponential explosion of neurons required to represent concepts

# Solution Attempt 2: Neural Blackboard Architecture (I)



## Solution Attempt 2: Neural Blackboard Architecture (II)

- ⊕ Fewer resources than LISA
- ⊕ Solves all four of Jackendoffs challenges (according to the authors)
- ⊕ Explains limitations of human sentence representation
- ● (At least partially) localist representation

- ⊖ Particular structure; does not match biology
- ⊖ Large number of neurons; about $500 \times 10^6$ to represent sentences
- ⊖ Only considers *representation*, no control structures

# Solution Attempt 3: Vector Operators

**Idea:** High-dimensional vectors $\mathbf{x} \in \mathbb{R}^d$ represent symbols; bind using tensor product

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{pmatrix} \qquad \text{(Outer product)}$$

$$\begin{aligned} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} &= \begin{pmatrix} a_{11} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} & a_{12} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \\ a_{21} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} & a_{22} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \end{pmatrix} \qquad \text{(Tensor product)} \\ &= \begin{pmatrix} a_{11} b_{11} & a_{11} b_{12} & a_{12} b_{11} & a_{12} b_{12} \\ a_{11} b_{21} & a_{11} b_{22} & a_{12} b_{21} & a_{12} b_{22} \\ a_{21} b_{11} & a_{21} b_{12} & a_{22} b_{11} & a_{22} b_{12} \\ a_{21} b_{21} & a_{21} b_{22} & a_{22} b_{21} & a_{22} b_{22} \end{pmatrix} \end{aligned}$$

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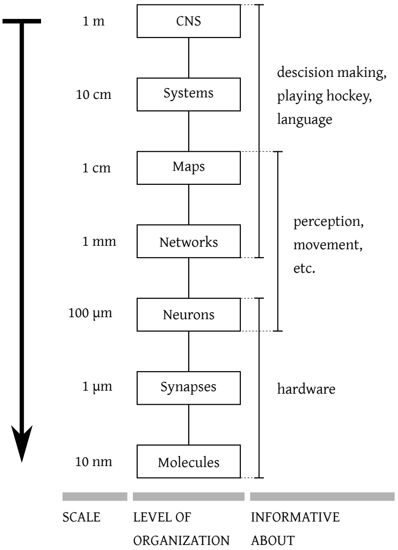
⊖ Scales extremely poorly $d^n$ for $n$ binding operations

# A Deeper Problem: Cognitive Science vs. Neuroscience

- Trying very hard to map purely symbolic architectures onto neurons.
- Neural aspects are treated as *mere implementation details*.
- Instance of **top-down modelling**:
  High-level cognitive architectures are mapped onto biology.
- Hope of many cognitive scientists:
  If successful, **neurons do not matter**.



# VSAs: Potential Binding Operators (I)

$$\begin{pmatrix}1\\0\\1\\0\end{pmatrix} \oplus \begin{pmatrix}1\\1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\1\\0\end{pmatrix}$$

*(XOR)*

$$\begin{pmatrix}A\\B\\C\\D\end{pmatrix} \odot \begin{pmatrix}E\\F\\G\\H\end{pmatrix} = \begin{pmatrix}AE\\BF\\CG\\DH\end{pmatrix}$$

*(Hadamard Product)*

# VSAs: Potential Binding Operators (II)

$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \circledast \begin{pmatrix} E \\ F \\ G \\ H \end{pmatrix} = \begin{pmatrix} AE & + & BH & + & CG & + & DF \\ AF & + & BE & + & CH & + & DG \\ AG & + & BF & + & CE & + & DH \\ AH & + & BG & + & CF & + & DE \end{pmatrix} \quad \textit{(Circular Convolution)}$$

Circular Convolution is a “compressed” outer product:

$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \otimes \begin{pmatrix} E \\ F \\ G \\ H \end{pmatrix} = \begin{pmatrix} AE & AF & AG & AH \\ BE & BF & BG & BH \\ CE & CF & CG & CH \\ DE & DF & DG & DH \end{pmatrix} \quad \textit{(Outer Product)}$$

# Sentence Encoding Revisited

- "The number eight comes after the number nine":

$$\mathtt{NUMBER} \circledast \mathtt{EIGHT} + \mathtt{SUCC} \circledast \mathtt{NINE}\,.$$

- "The dog chases the cat":

$$\mathtt{DOG} \circledast \mathtt{SUBJ} + \mathtt{CAT} \circledast \mathtt{OBJ} + \mathtt{CHASE} \circledast \mathtt{VERB}\,.$$

- "Anne knows that Bill thinks that Charlie likes Dave":

$$\begin{aligned}&\mathtt{SUBJ} \circledast \mathtt{ANNE} + \mathtt{ACT} \circledast \mathtt{KNOWS} + \mathtt{OBJ} \circledast \\ &\Big(\mathtt{SUBJ} \circledast \mathtt{BILL} + \mathtt{ACT} \circledast \mathtt{THINKS} + \mathtt{OBJ} \circledast \\ &\big(\mathtt{SUBJ} \circledast \mathtt{CHARLIE} + \mathtt{ACT} \circledast \mathtt{LIKES} + \mathtt{OBJ} \circledast \mathtt{DAVE}\big)\Big)\,.\end{aligned}$$

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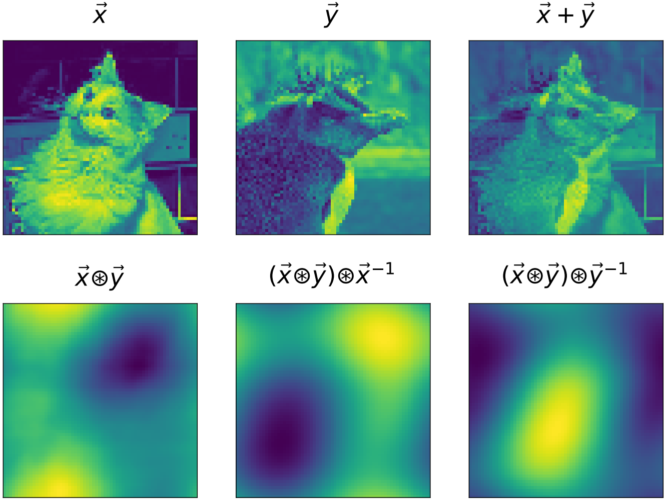
⚠ **Compression of information; graceful degradation**

# Circular Convolution: Dissimilarity and Reversibility



# Image sources

**Title slide**
Bell telephone magazine, 1922, American Telephone and Telegraph Company Wikimedia.