SYDE 556/750

Simulating Neurobiological Systems
Lecture 6: Recurrent Dynamics

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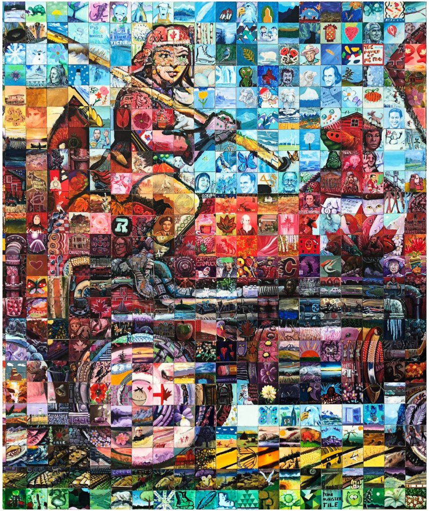
February 4 & 6 & 11, 2020



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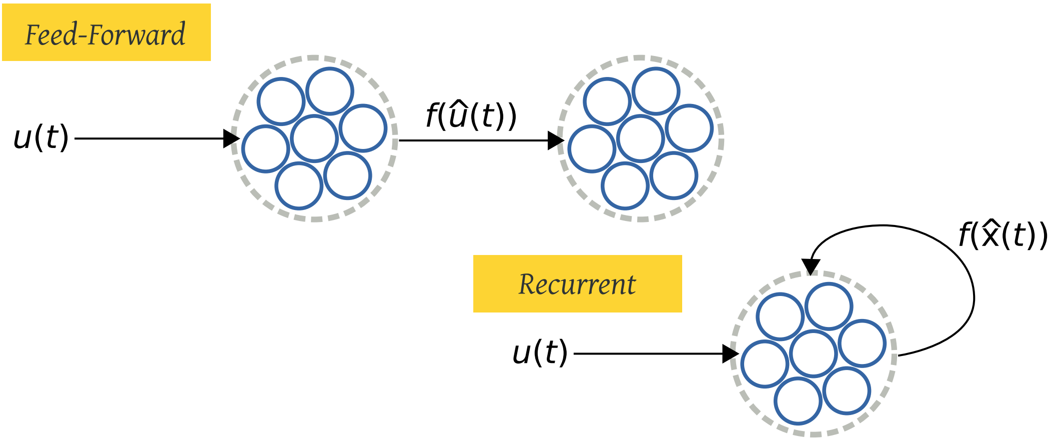
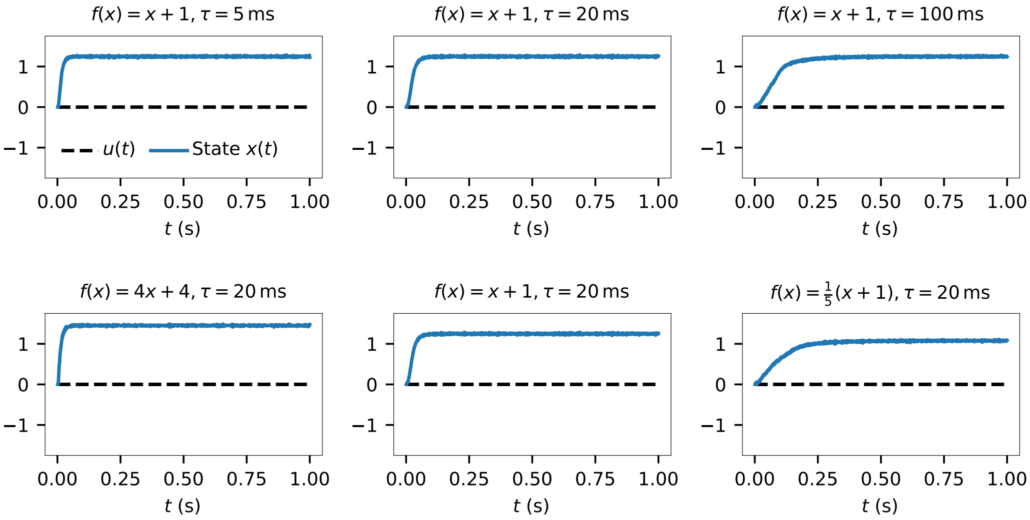
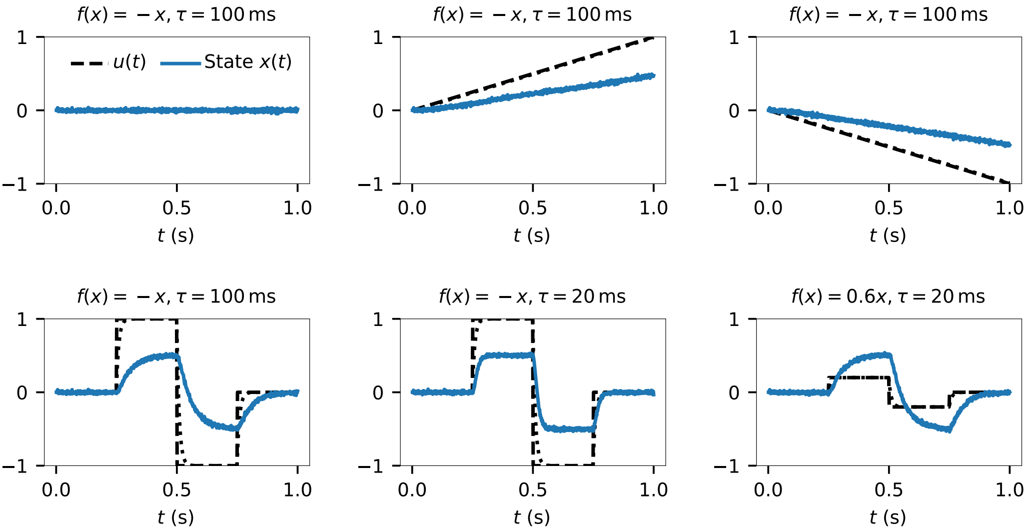
Feed Forward vs. Recurrent Connections



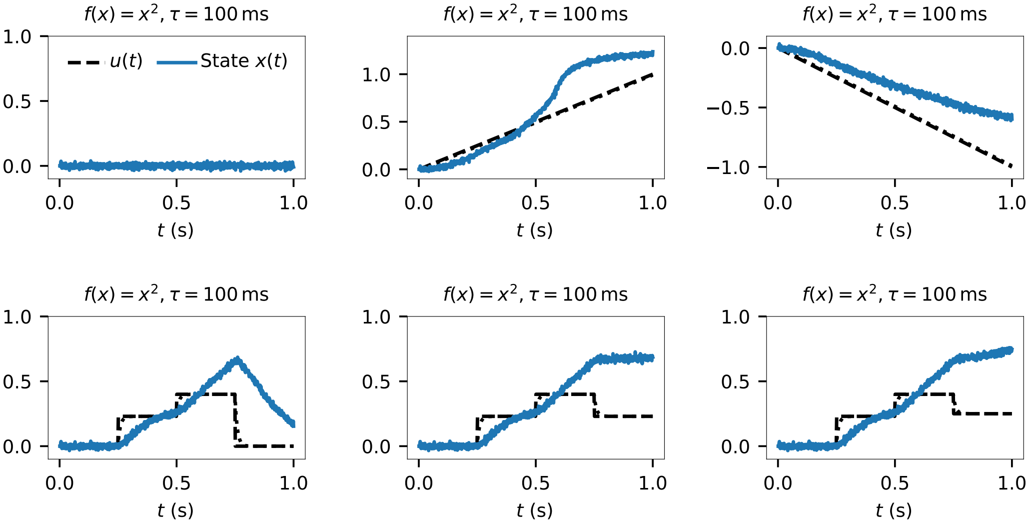
Recurrence Experiments (I)



Recurrence Experiments (II)



Recurrence Experiments (III)



NEF Principle 3: Dynamics

Time-Invariant Dynamical System

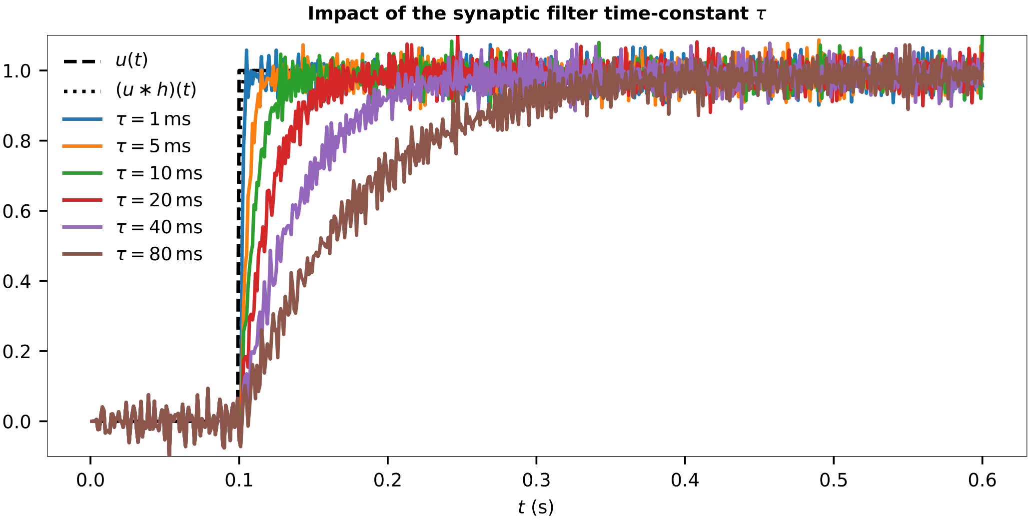
$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = f(\mathbf{x}(t), \mathbf{u}(t))$$

Linear Time-Invariant (LTI) Dynamical System

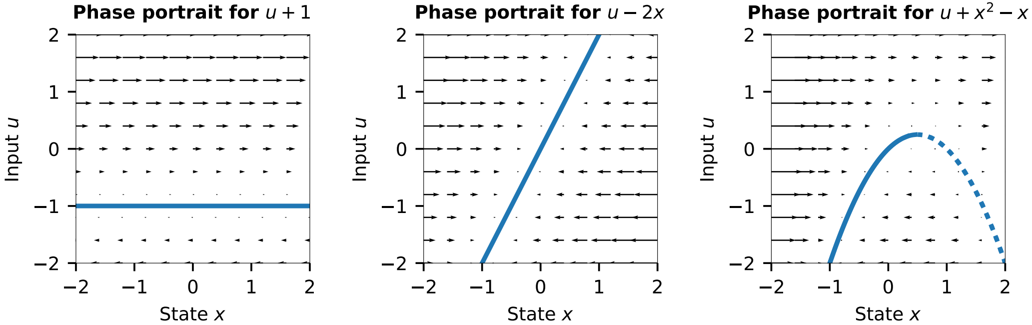
$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

NEF Principle 3 – Dynamics
Neural dynamics are characterized by considering neural representations as control theoretic state variables. We can use control theory (and dynamical systems theory) to analyse and construct these systems.

Making Sense of Dynamics

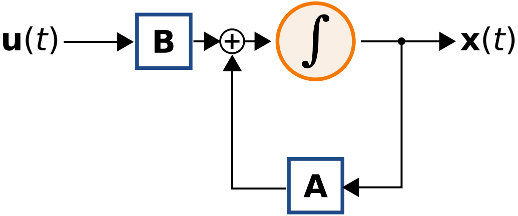


Phase Portraits

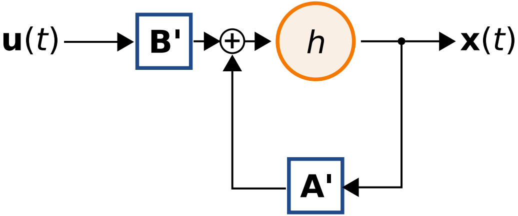


Implementing Dynamics using a Neural Ensemble

Evaluating an LTI
using an Integrator



Evaluating an LTI
using a Synaptic Filter



Implementing Dynamical Systems as a Neural Ensemble

LTI System

$$
\begin{aligned}
\varphi(\mathbf{u}, \mathbf{x}) &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\
\varphi'(\mathbf{u}, \mathbf{x}) &= \mathbf{A}'\mathbf{x} + \mathbf{B}'\mathbf{u} \\
\mathbf{A}' &= \tau\mathbf{A} + \mathbf{I} \\
\mathbf{B}' &= \tau\mathbf{B}\,.
\end{aligned}
$$

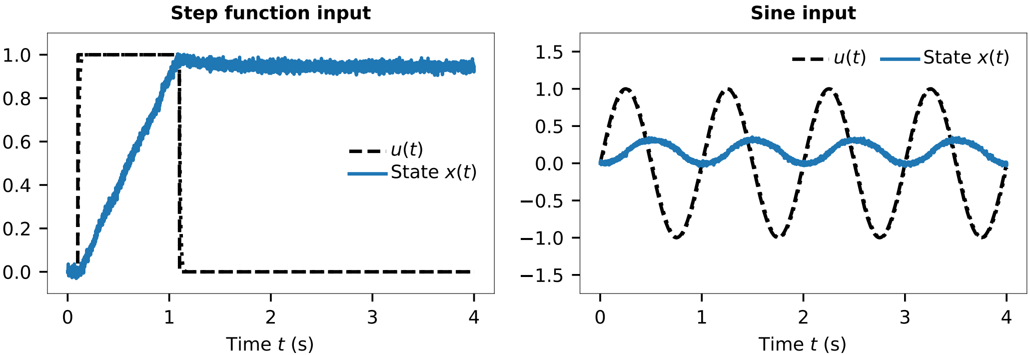
Additive Time-Invariant System

$$
\begin{aligned}
\varphi(\mathbf{u}, \mathbf{x}) &= f(\mathbf{x}) + g(\mathbf{u}) \\
\varphi'(\mathbf{u}, \mathbf{x}) &= f'(\mathbf{x}) + g'(\mathbf{u}) \\
f'(\mathbf{x}) &= \tau f(\mathbf{x}) + \mathbf{x} \\
g'(\mathbf{u}) &= \tau g(\mathbf{u})
\end{aligned}
$$

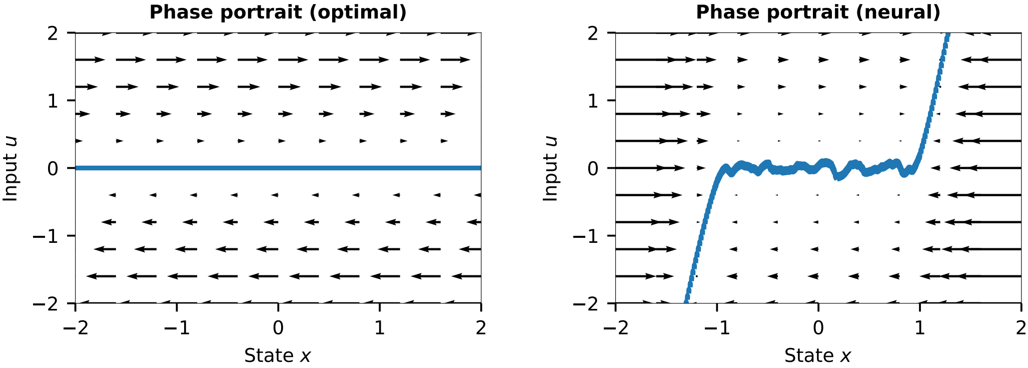
“General” Recipe

Scale the original dynamics by τ, add feedback $\mathbf{x}$

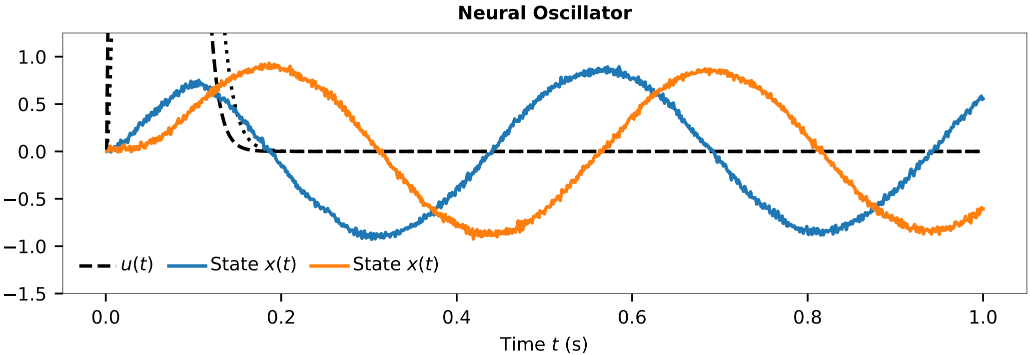
Integrator Example (I)



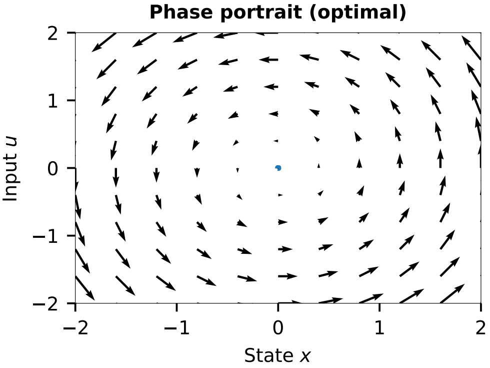
Integrator Example (II)

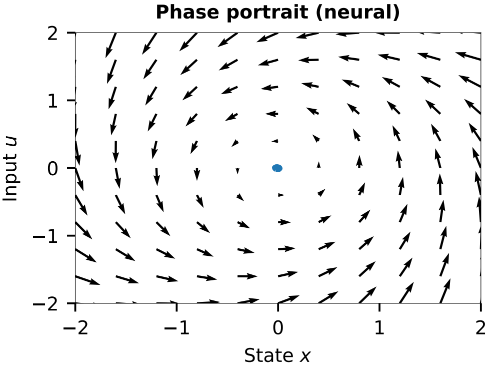


Oscillator Example (I)

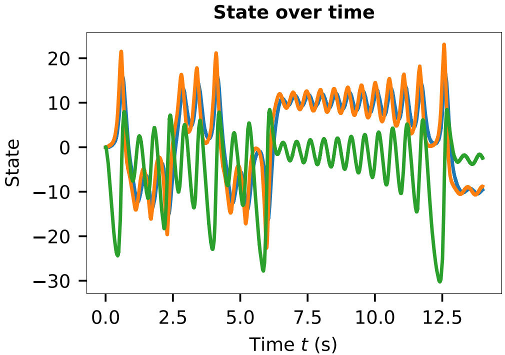


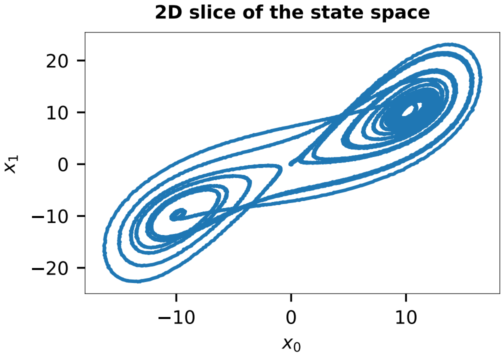
Oscillator Example (II)





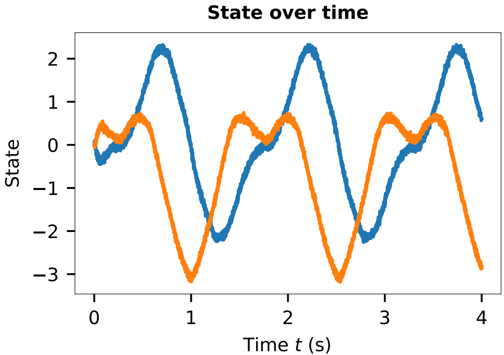
Lorentz Attractor



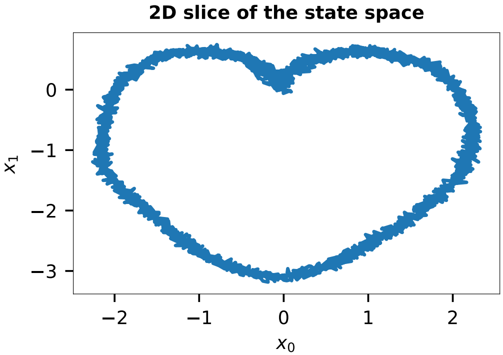


$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \begin{pmatrix} 10x_2(t) - 10x_1(t) \\ -x_1(t)x_3(t) - x_2(t) \\ x_1(t)x_2(t) - \frac{8}{3}(x_3(t) + 28) - 28 \end{pmatrix}$$

Heart Shape

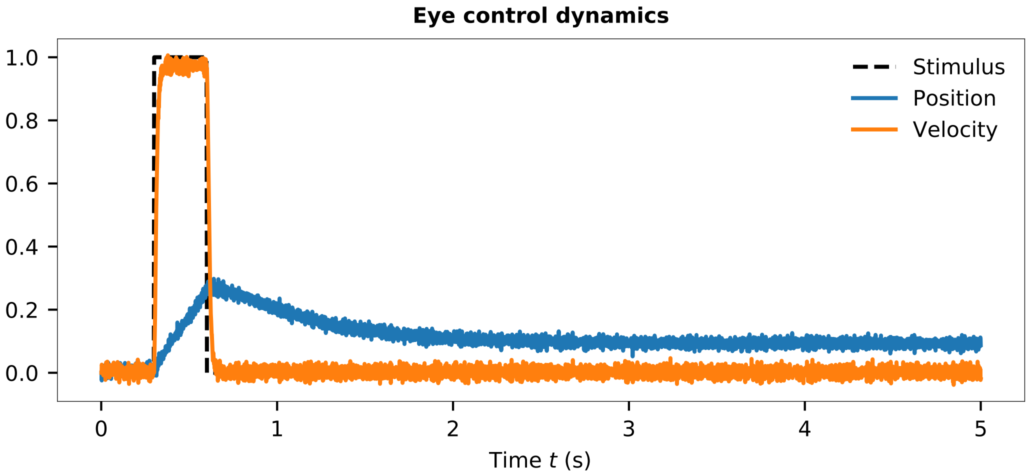


State over time
2
1
0
−1
−2
−3
State
0
1
2
3
4
Time t (s)



2D slice of the state space
0
−1
−2
−3
x_1
−2
−1
0
1
2
x_0

Horizontal Eye Control



Administrative Remarks

- **Project proposals** due Friday, February 14
  `http://compneuro.uwaterloo.ca/courses/syde-750/syde-556-possible-projects.html`
- **Assignment 2** due Thursday, February 13
- **Assignment 3** will be released later today, February 11 (due in three weeks)
- Some adjustments to the schedule

Image sources

Title slide
"The Canada 150 Mosaic Mural"
Author: Mosaic Canada Murals.
From Wikimedia.