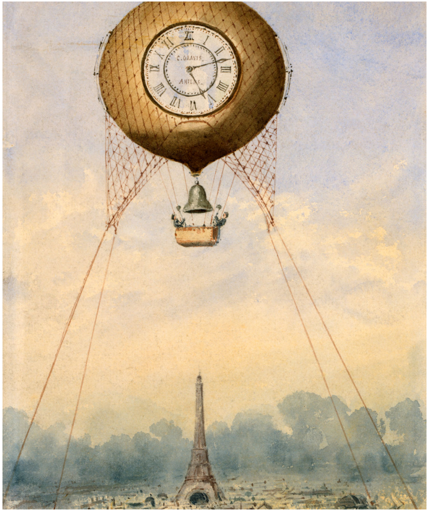
SYDE 556/750

Simulating Neurobiological Systems
Lecture 4: Temporal Representations

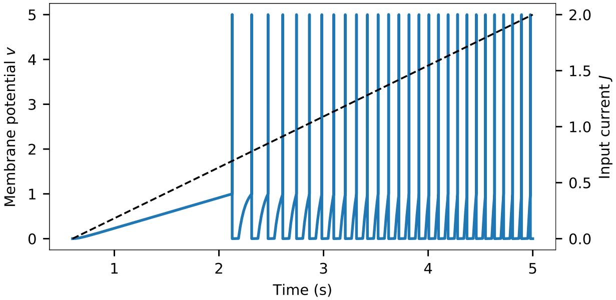


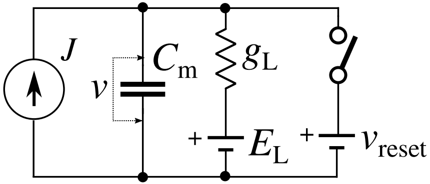
Andreas Stöckel

January 22 & 28, 2020

UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

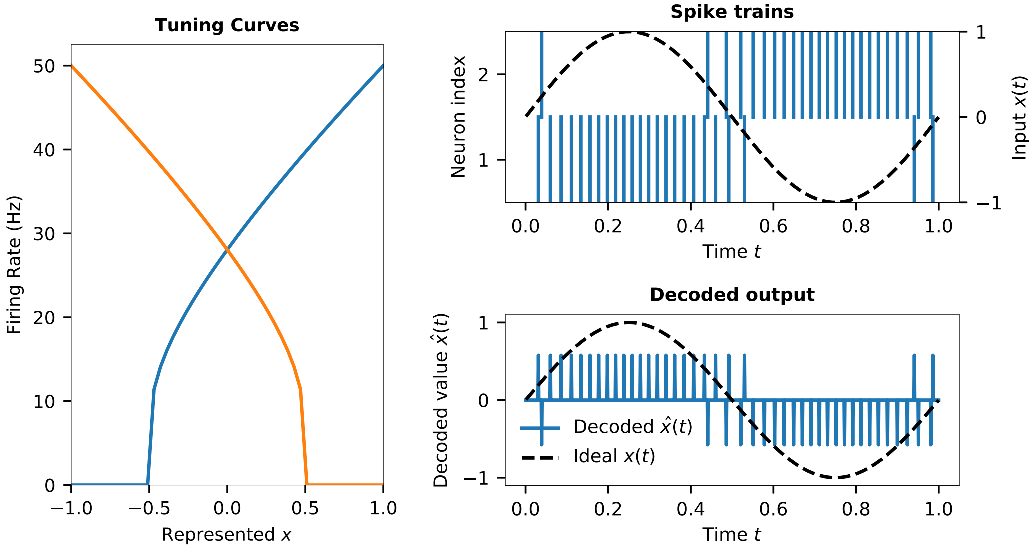
Reminder: The LIF Neuron



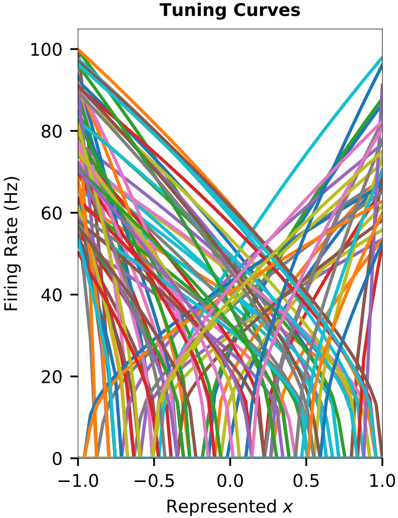


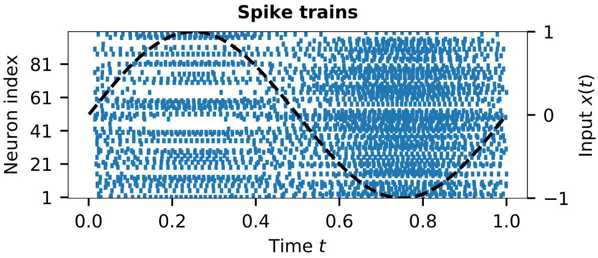
$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}v(t) &= -\frac{1}{\tau_\mathrm{RC}}\big(v(t) - J\big)\,, && \text{if } v(t) < 1\,,\\
v(t) &\leftarrow \delta(t - t_\mathrm{th})\,, && \text{if } t = t_\mathrm{th}\,,\\
v(t) &\leftarrow 0\,, && \text{if } t > t_\mathrm{th} \text{ and } t \geq t_\mathrm{th} + \tau_\mathrm{ref}\,,
\end{aligned}
$$

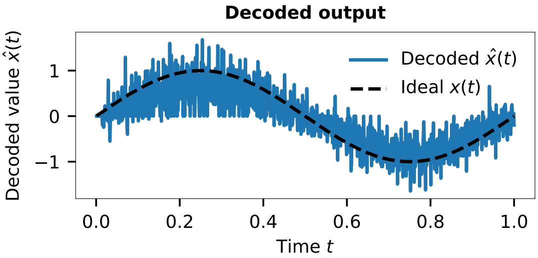
Temporal Decoding of Two Neurons



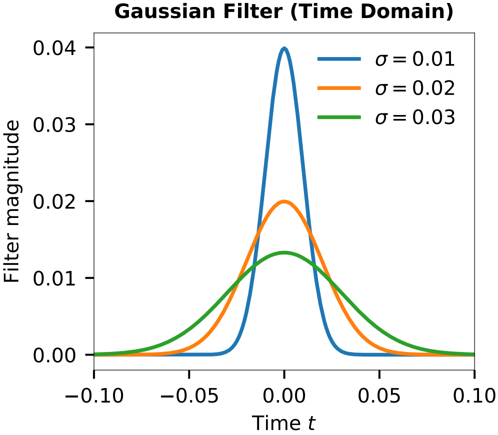
Temporal Decoding of One Hundred Neurons







Filtering by Convolution



Gaussian Filter

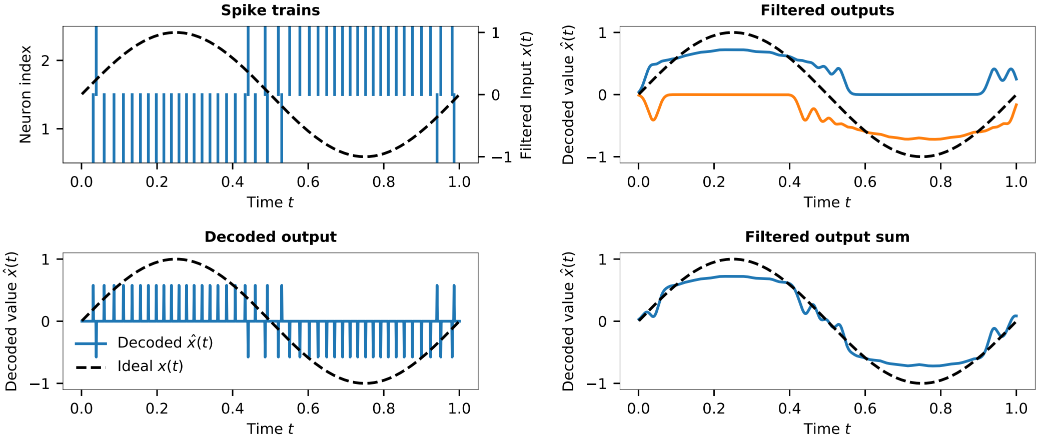
$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

where c chosen s.t. $\int_{-\infty}^{\infty} h(t)\,\mathrm{d}t = 1$

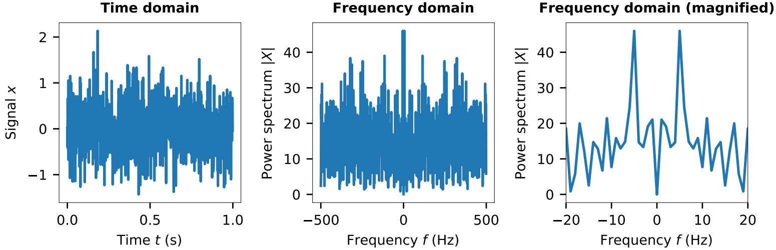
Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,\mathrm{d}\tau$$

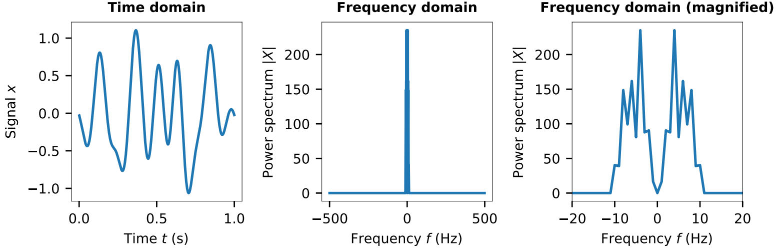
Filtering a Spike Train



Random Signals

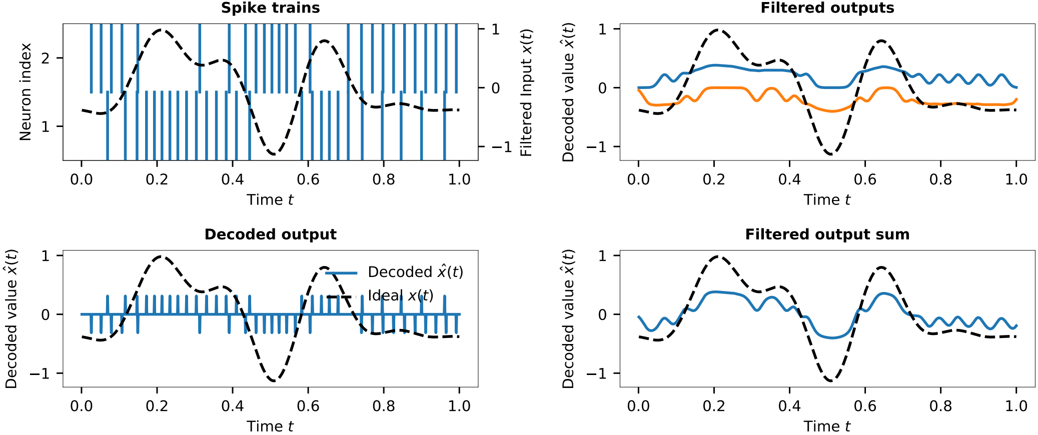


White Noise
(zero mean)

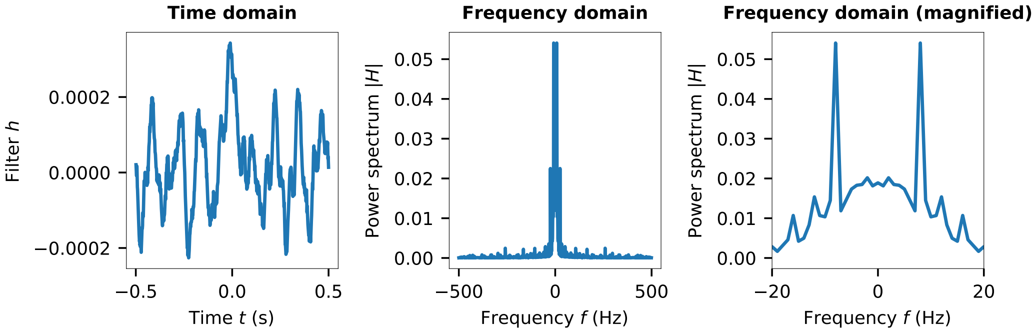


Bandlimited
White Noise
(zero mean,
10 Hz bandwidth)

Filtering a Spike Train for a Random Signal

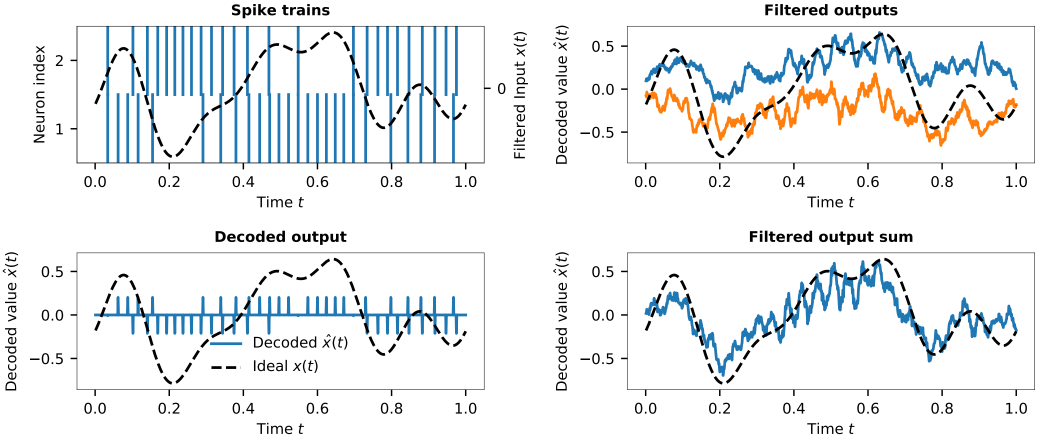


Optimal Filter

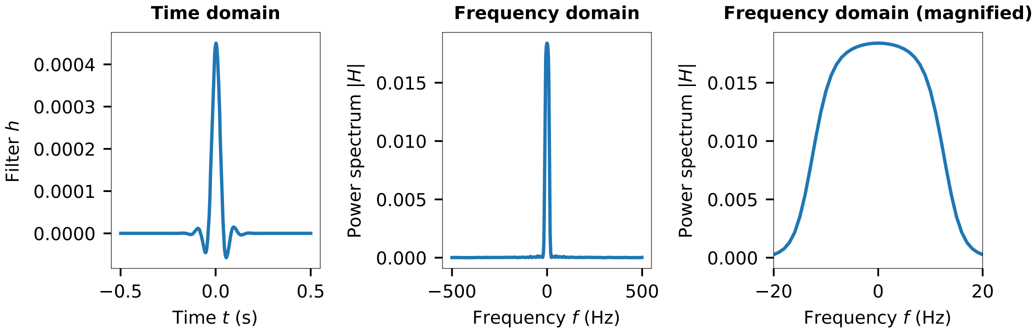


$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

Filtering a Spike Train for a Random Signal (Optimal Filter)

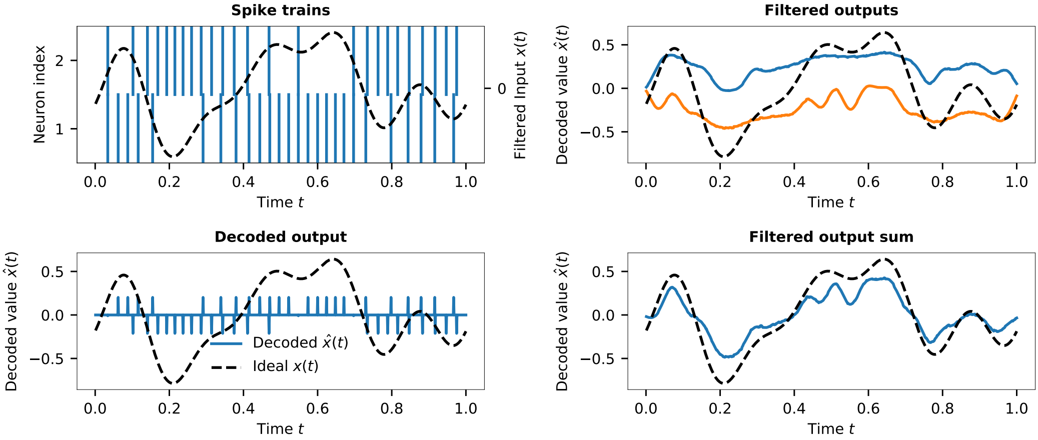


Optimal Filter (Improved)

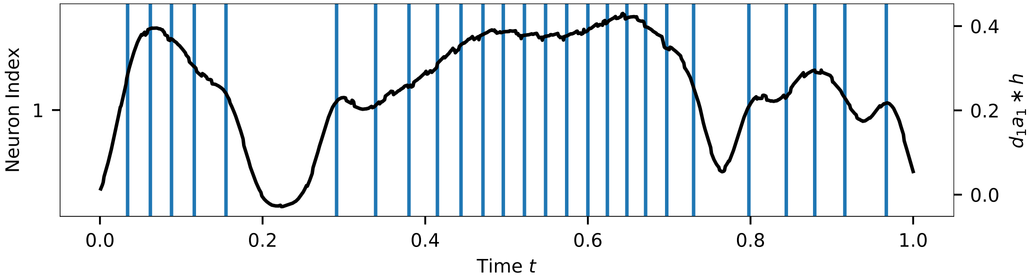


$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



Pros and Cons of the Optimal Filter

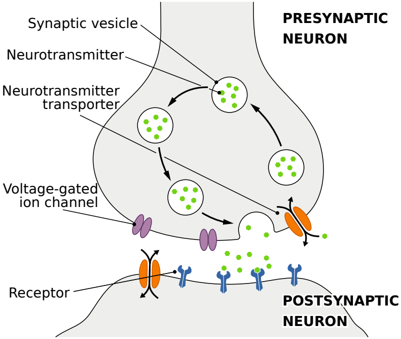


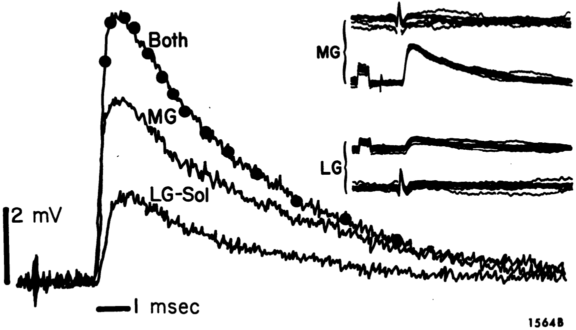
Precise
Good for analysing data after the fact

Non-causal
Does not describe a biological process

We need to find a mechanism that low-pass filters spikes over time!

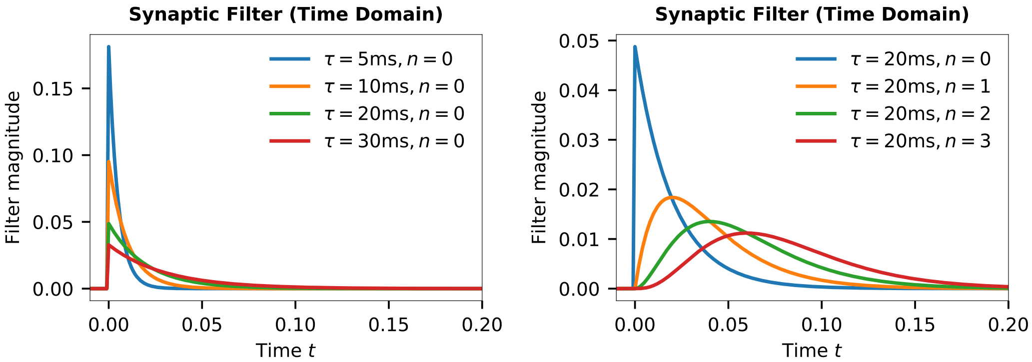
Synapses as Filters





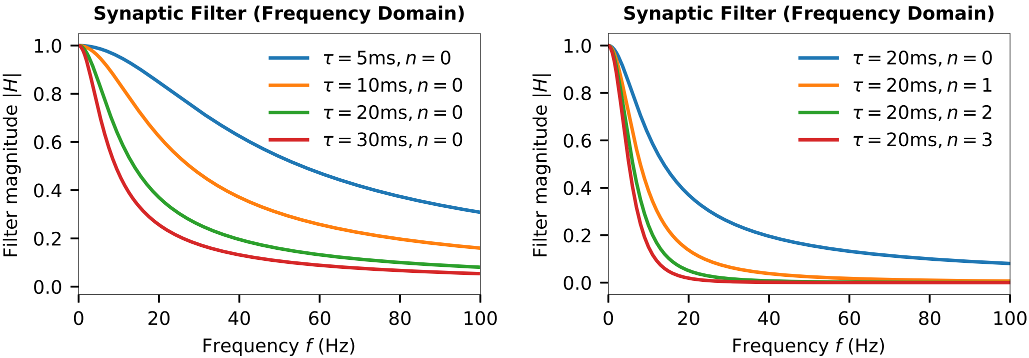
Post-synaptic currents (EPSCs, IPSCs) are low-pass filtered spike trains!

Exponential Low-Pass Filter (I)



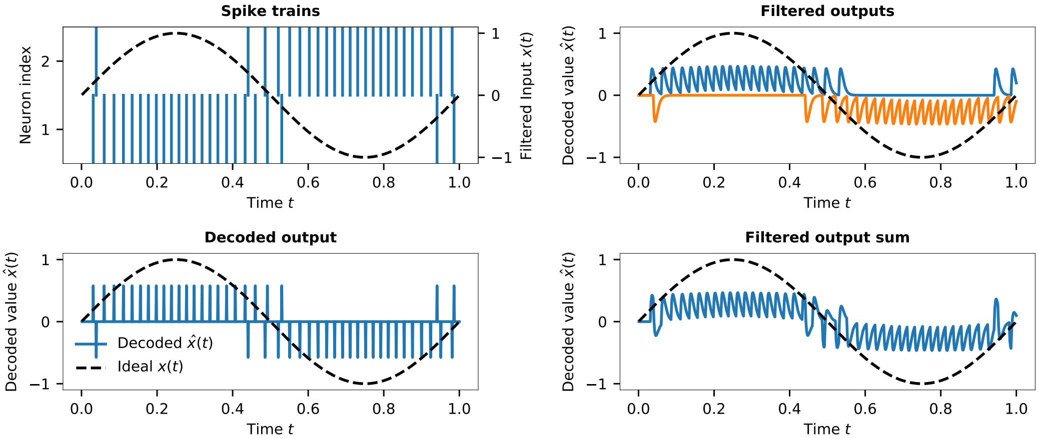
$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0\,, \\ 0 & \text{otherwise}\,, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t\,.$$

Exponential Low-Pass Filter (II)



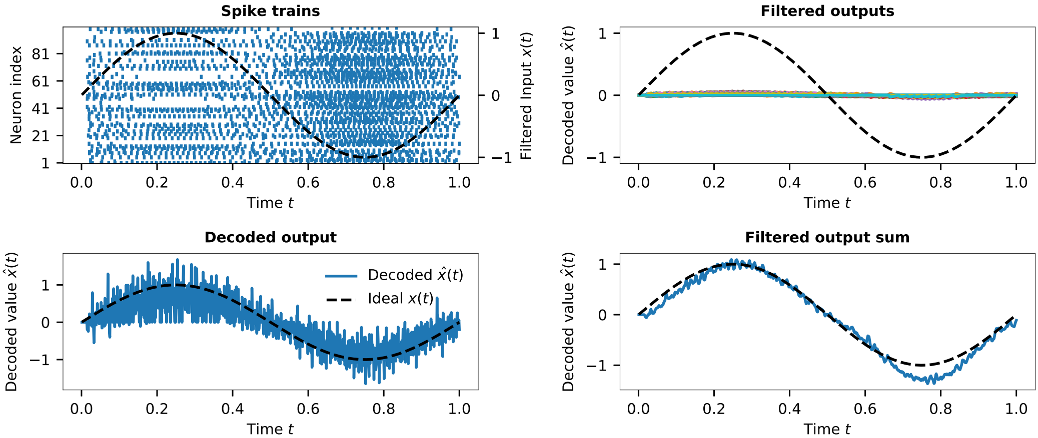
$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0\,, \\ 0 & \text{otherwise}\,, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t\,.$$

Example: Synaptic Filter for Two Neurons



$$\tau = 5\,\text{ms}, n = 1$$

Example: Synaptic Filter for One Hundred Neurons



$$\tau = 5\,\mathrm{ms},\ n = 1$$

Image sources

Title slide
"Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France."
Author: Camille Grávis, between 1889 and 1900.
From Wikimedia.