

SYDE 556/750

Simulating Neurobiological Systems
Lecture 9: Analysing Representations

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March 5, 2020



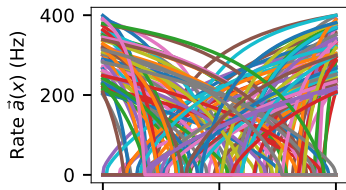
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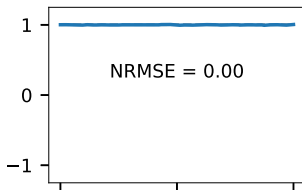


Decoding Polynomials

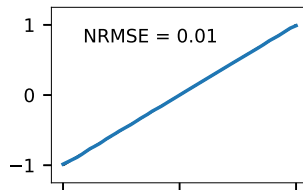
Tuning curves ($n = 128$)



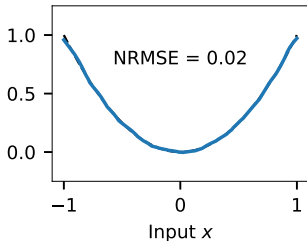
$f(x) = 1$



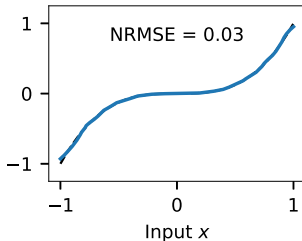
$f(x) = x$



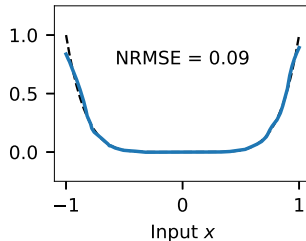
$f(x) = x^2$



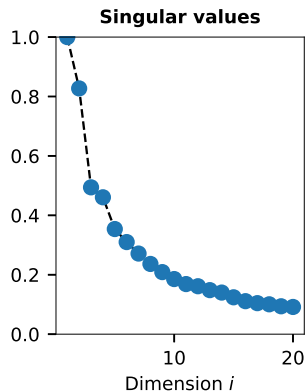
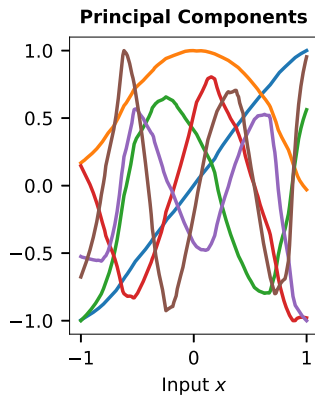
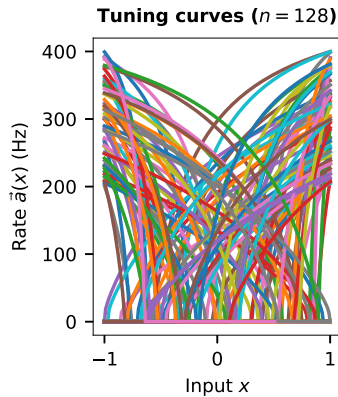
$f(x) = x^3$



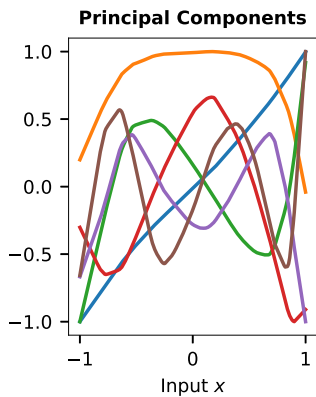
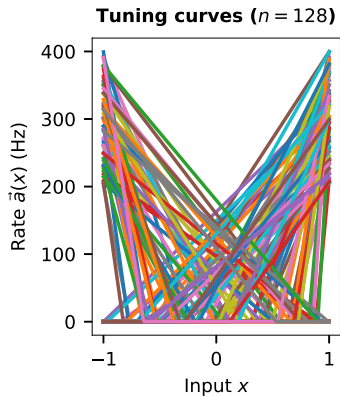
$f(x) = x^6$



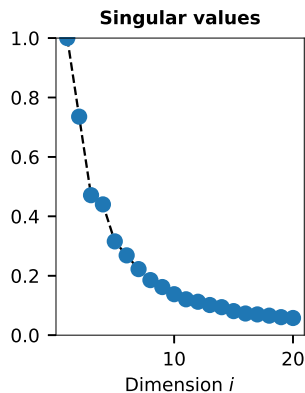
LIF Tuning Curve Principal Components



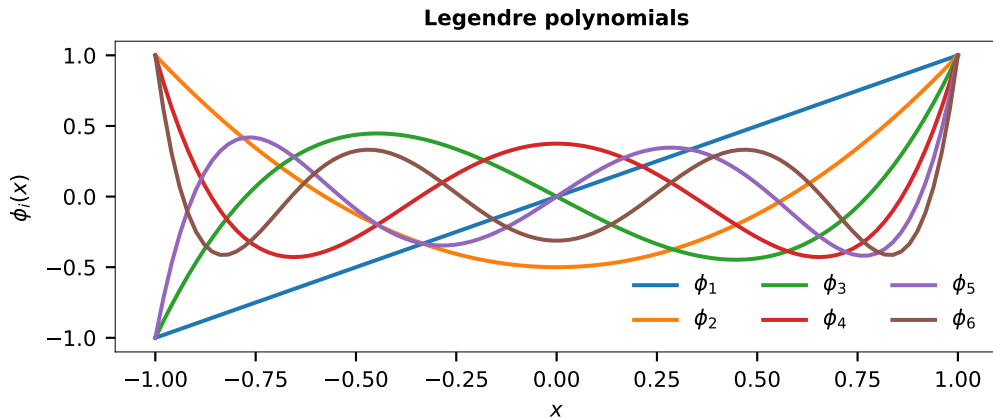
ReLU Tuning Curve Principal Components



\approx Legendre Basis



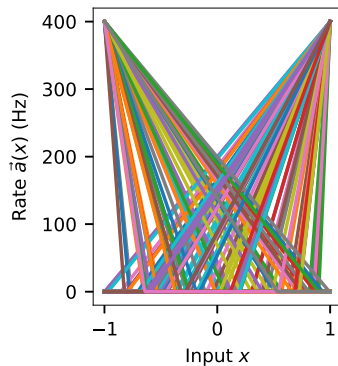
Reminder: Legendre Polynomials



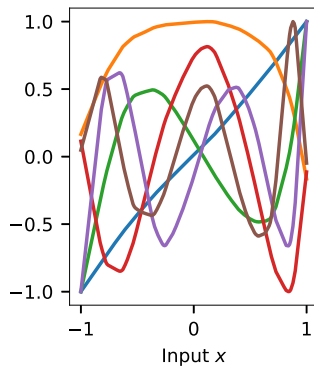
$$\varphi_i(x) = \frac{1}{2^i} \sum_{k=0}^i \binom{i}{k}^2 (x-1)^{i-k} (x+1)^k$$

Modifying the Basis – Same Maximum Rate (I)

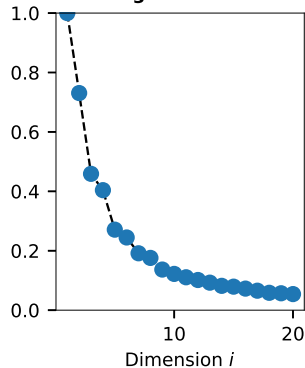
Tuning curves ($n = 128$)



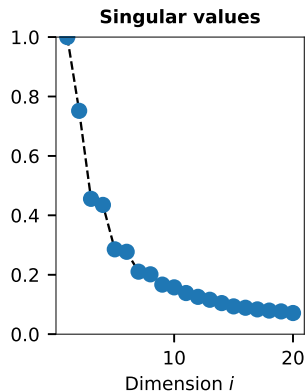
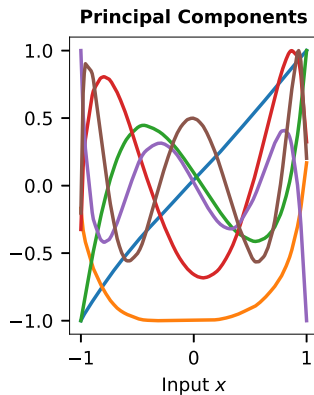
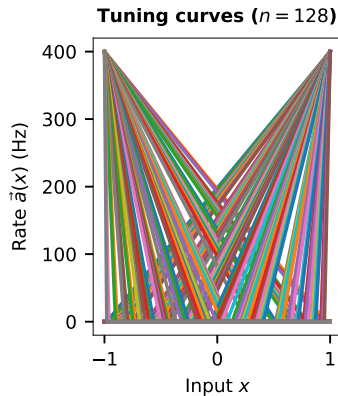
Principal Components



Singular values

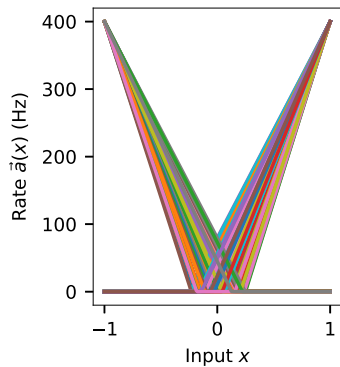


Modifying the Basis – Equidistant x -Intercepts (II)

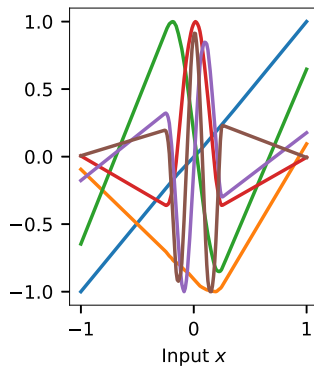


Modifying the Basis – Limited x-Intercepts (III)

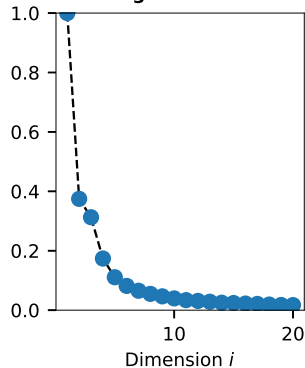
Tuning curves ($n = 128$)



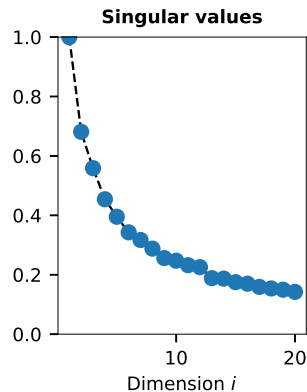
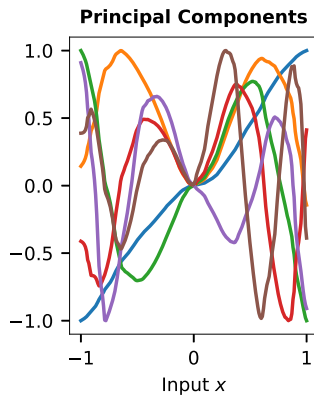
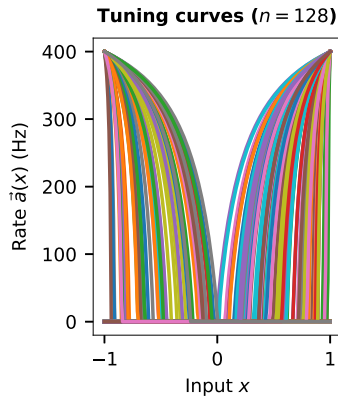
Principal Components



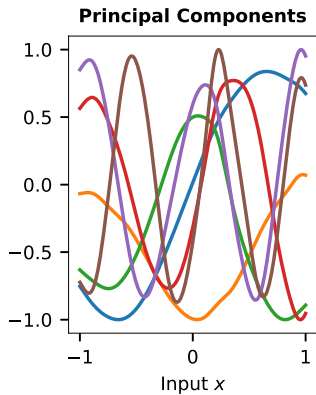
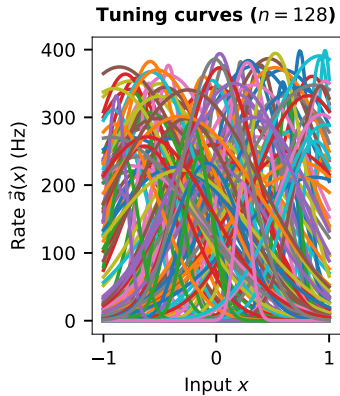
Singular values



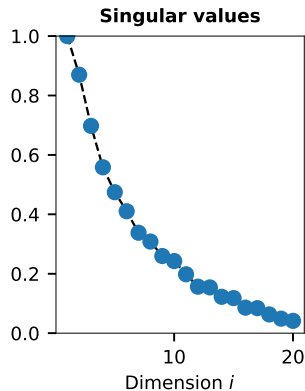
Modifying the Basis – Symmetric Tuning Curves (IV)



Gaussian Tuning Curve Principal Components

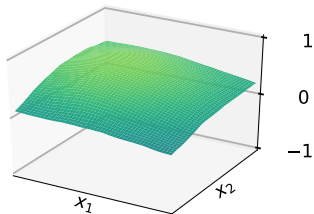


\approx Fourier Basis

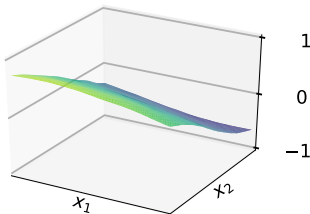


PCA of 2D Tuning Curves

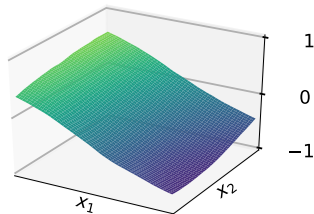
Principal component 1



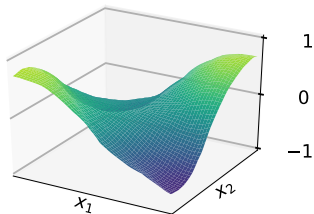
Principal component 2



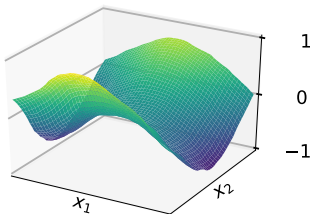
Principal component 3



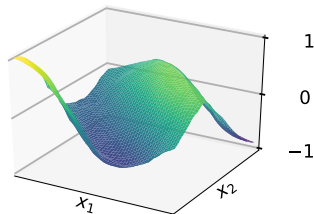
Principal component 4



Principal component 5

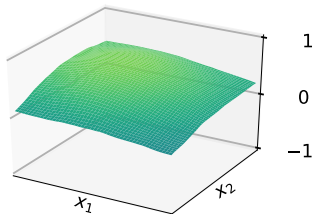


Principal component 6

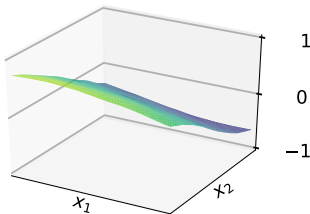


PCA of 2D Tuning Curves

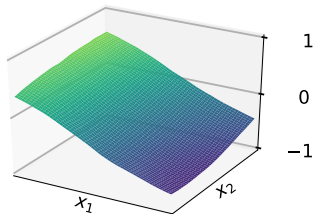
Principal component 1



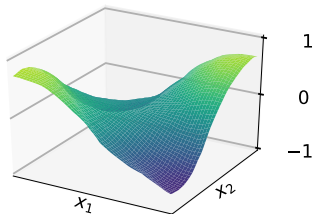
Principal component 2



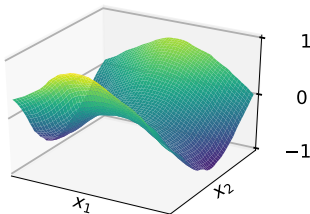
Principal component 3



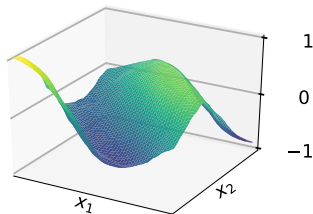
Principal component 4



Principal component 5



Principal component 6



Combination of 2D Polynomials

Conclusions

- ▶ Can use **PCA** to find the basis functions underlying neural representations
- ▶ **Singular values** inversely proportional to noise
- ▶ **Basis function shape** depends on
 - ▶ x-intercept distributions
 - ▶ Neuron response curve $G[J]$
- ▶ Finding optimal tuning curves for representations
⇒ Full network optimization (must use gradient descent)

Image sources

Title slide

Maurice Denis: Homage to Cézanne, 1900
From Wikimedia.