

**SYDE 556/750**

**Simulating Neurobiological Systems**  
**Lecture 6: Recurrent Dynamics**

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February 4 & 6 & 11, 2020

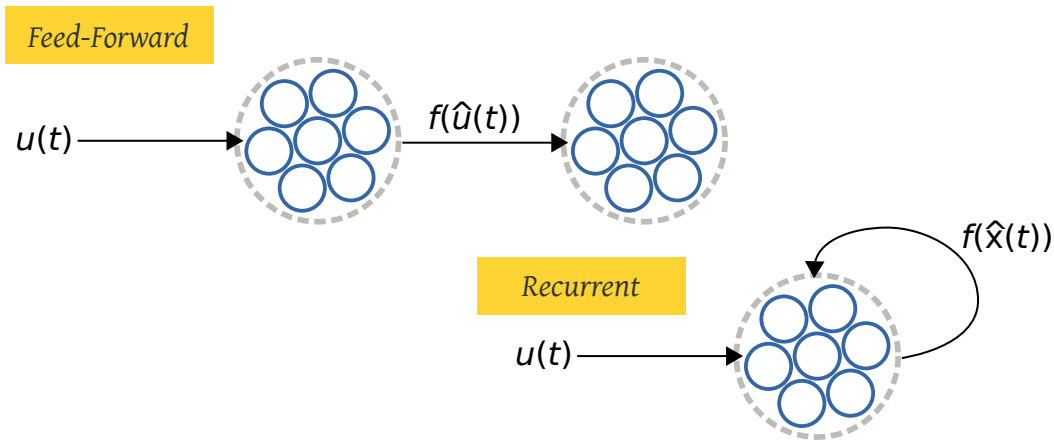


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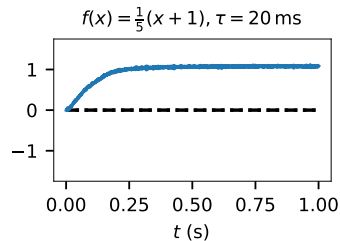
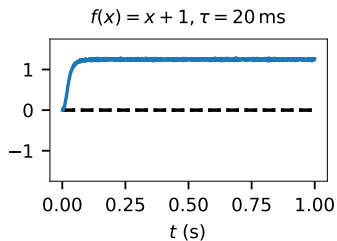
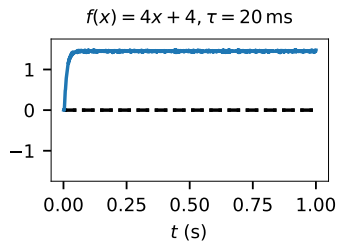
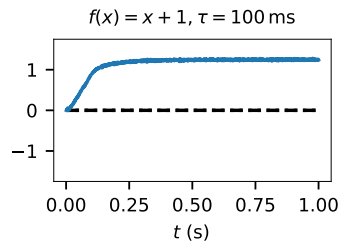
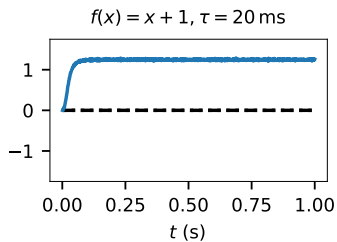
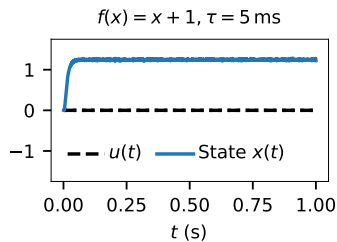
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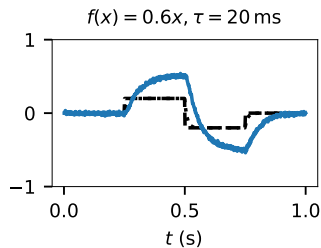
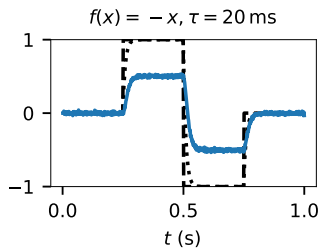
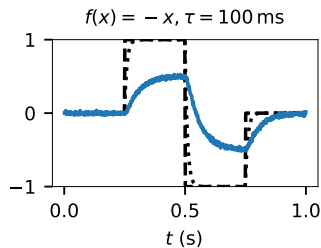
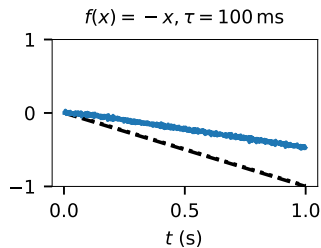
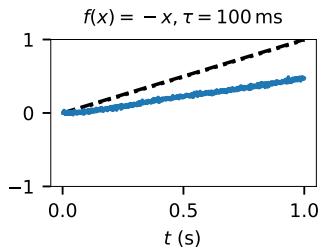
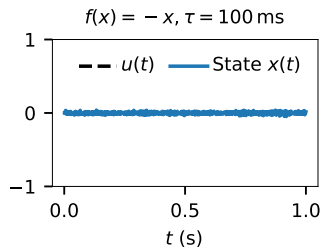
## Feed Forward vs. Recurrent Connections



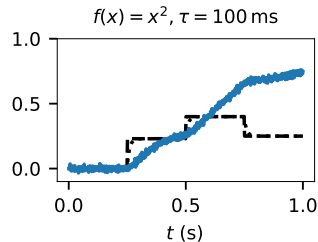
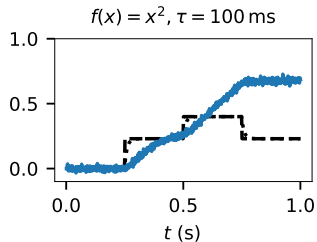
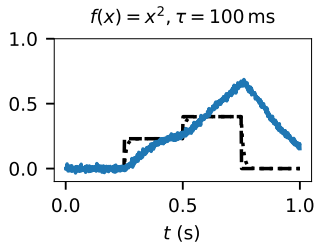
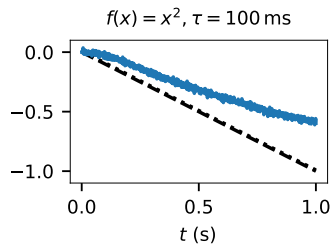
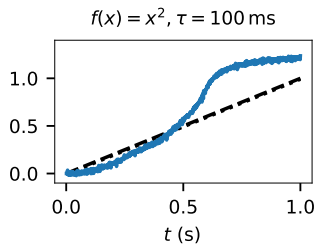
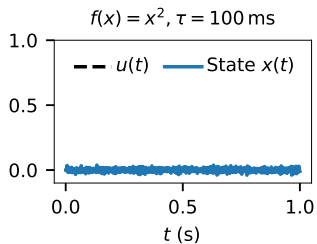
# Recurrence Experiments (I)



## Recurrence Experiments (II)



# Recurrence Experiments (III)



## NEF Principle 3: Dynamics

### Time-Invariant Dynamical System

$$\frac{d\mathbf{x}(t)}{dt} = f(\mathbf{x}(t), \mathbf{u}(t))$$

### Linear Time-Invariant (LTI)

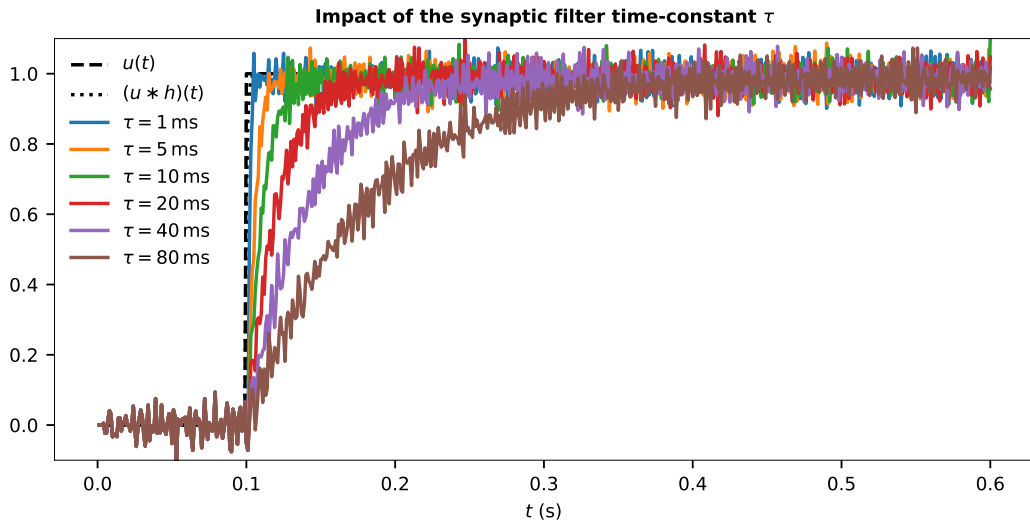
### Dynamical System

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

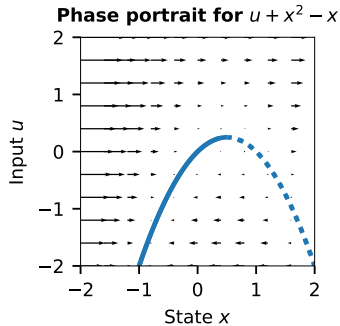
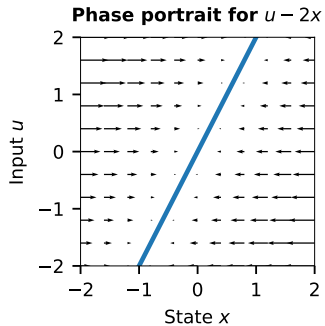
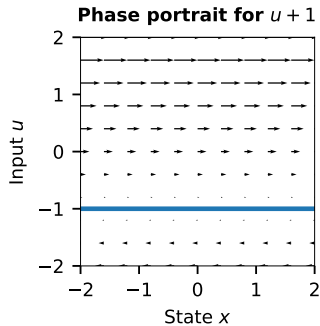
### NEF Principle 3 – Dynamics

Neural dynamics are characterized by considering neural representations as control theoretic state variables. We can use control theory (and dynamical systems theory) to analyse and construct these systems.

# Making Sense of Dynamics



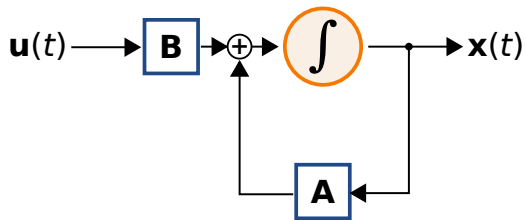
# Phase Portraits



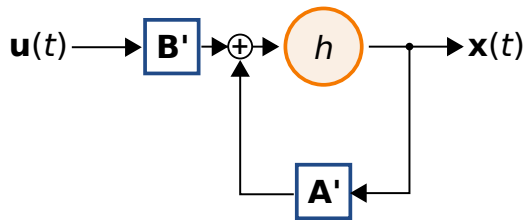


# Implementing Dynamics using a Neural Ensemble

Evaluating an LTI  
using an Integrator



Evaluating an LTI  
using a Synaptic Filter



# Implementing Dynamical Systems as a Neural Ensemble

## LTI System

$$\phi(\mathbf{u}, \mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\phi'(\mathbf{u}, \mathbf{x}) = \mathbf{A}'\mathbf{x} + \mathbf{B}'\mathbf{u}$$

$$\mathbf{A}' = \tau\mathbf{A} + \mathbf{I}$$

$$\mathbf{B}' = \tau\mathbf{B}.$$

## Additive Time-Invariant System

$$\phi(\mathbf{u}, \mathbf{x}) = f(\mathbf{x}) + g(\mathbf{u})$$

$$\phi'(\mathbf{u}, \mathbf{x}) = f'(\mathbf{x}) + g'(\mathbf{u})$$

$$f'(\mathbf{x}) = \tau f(\mathbf{x}) + \mathbf{x}$$

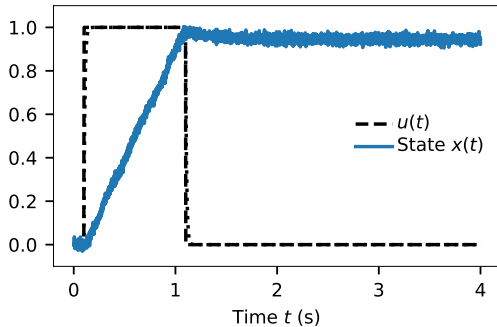
$$g'(\mathbf{u}) = \tau g(\mathbf{u})$$

## “General” Recipe

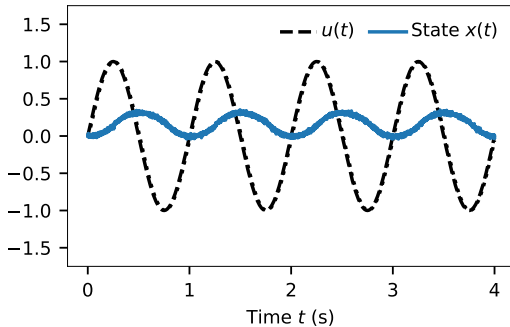
Scale the original dynamics by  $\tau$ , add feedback  $\mathbf{x}$

# Integrator Example (I)

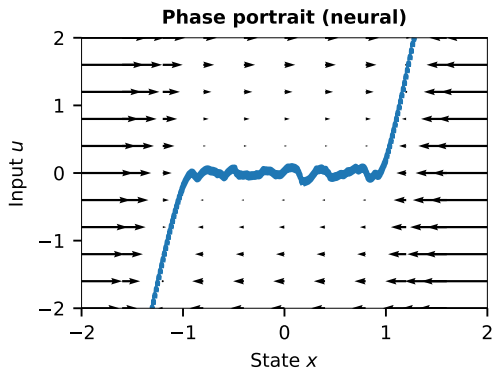
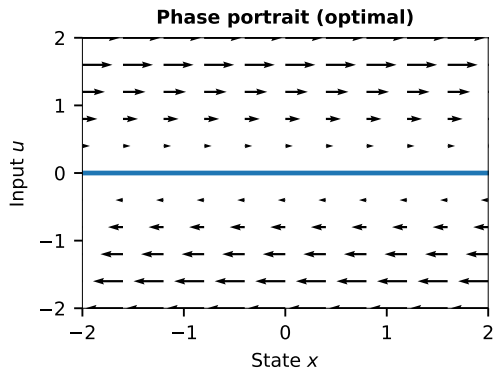
Step function input



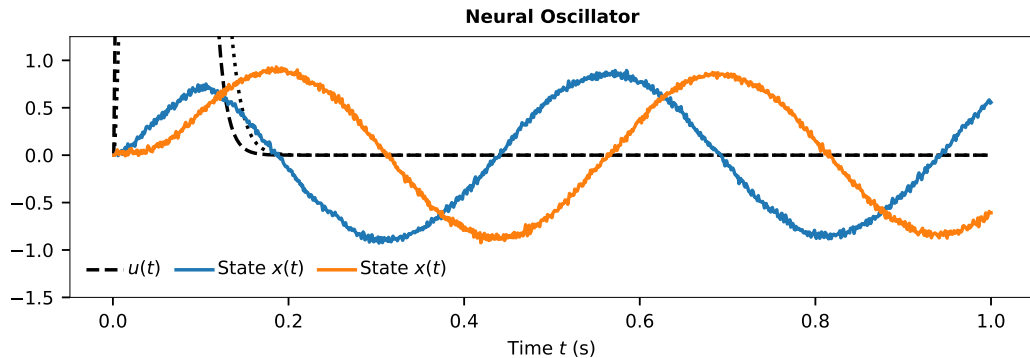
Sine input



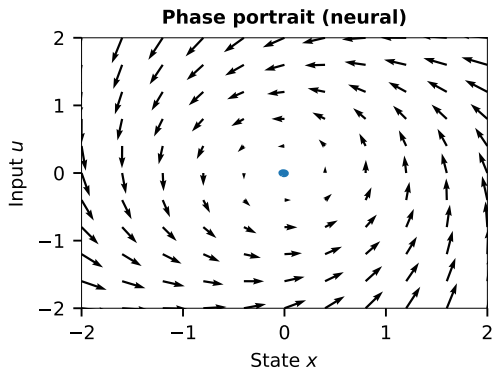
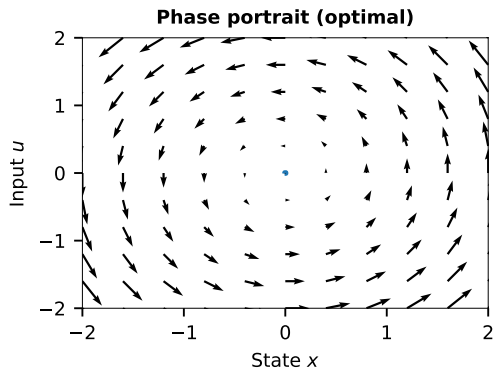
## Integrator Example (II)



# Oscillator Example (I)

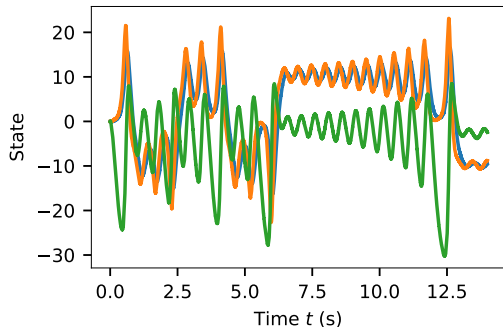


## Oscillator Example (II)

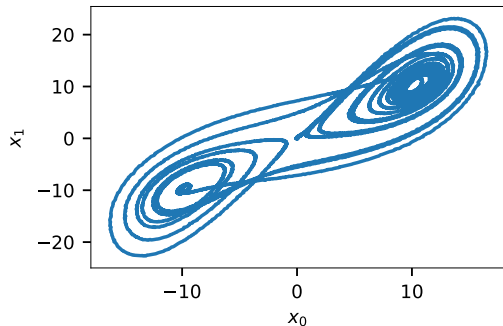


# Lorentz Attractor

State over time



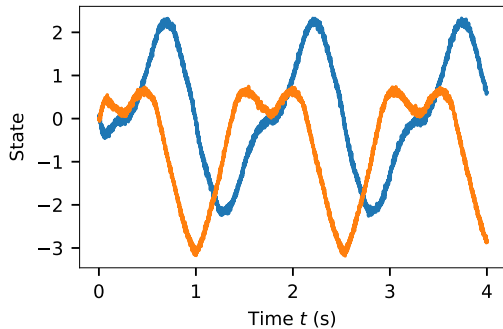
2D slice of the state space



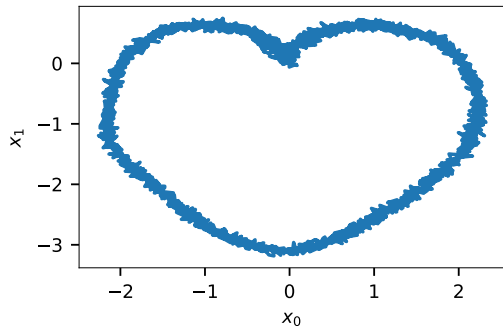
$$\frac{d\mathbf{x}(t)}{dt} = \begin{pmatrix} 10x_2(t) - 10x_1(t) \\ -x_1(t)x_3(t) - x_2(t) \\ x_1(t)x_2(t) - \frac{8}{3}(x_3(t) + 28) - 28 \end{pmatrix}$$

# Heart Shape

State over time

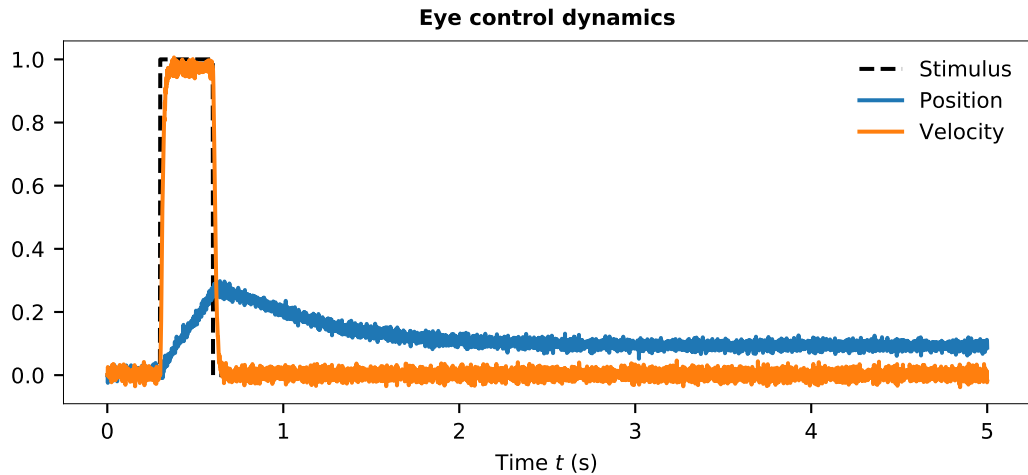


2D slice of the state space





# Horizontal Eye Control



# Image sources

## **Title slide**

“The Canada 150 Mosaic Mural”

Author: Mosaic Canada Murals.

From Wikimedia.