SYDE 556/750

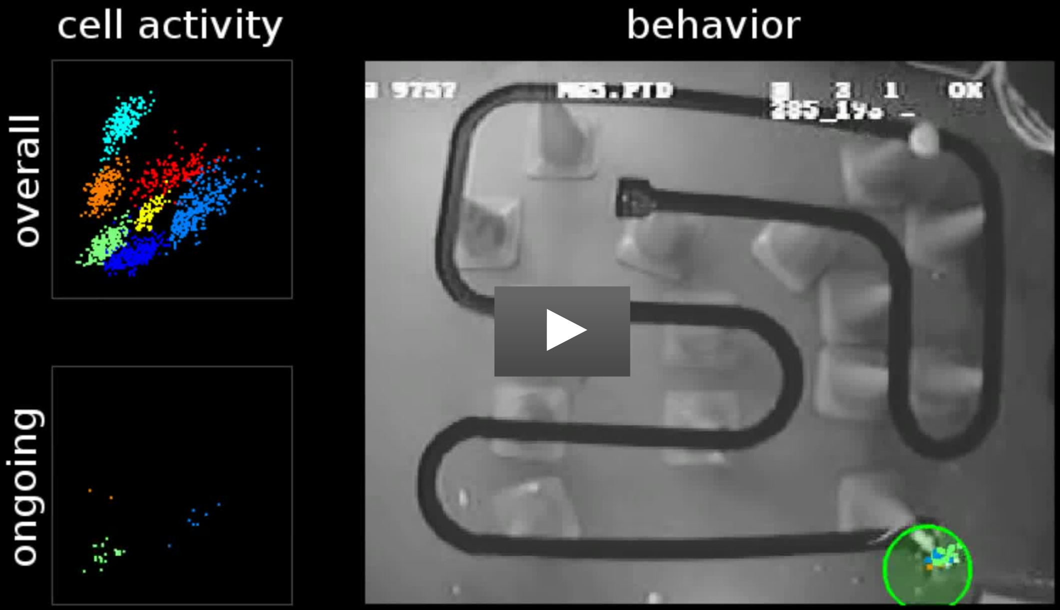
Simulating Neurobiological Systems
Lecture 3: Representations

Andreas Stöckel

January 14 & 16, 2020

UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

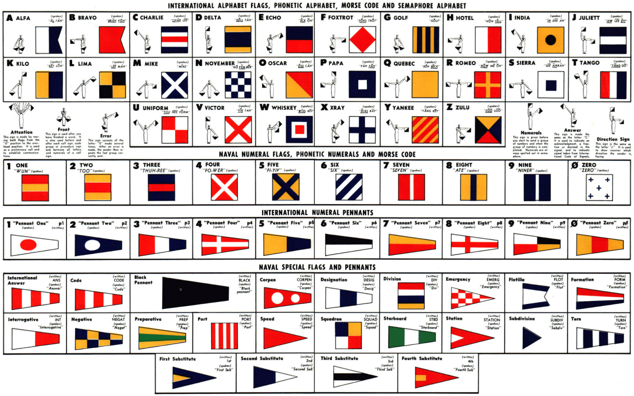

Visual Cortex
Mapping receptive fields


cell activity
behavior
overall
ongoing

NEF Principle 1: Representation

NEF Principle 1 – Representation
Groups (“populations”, or “ensembles”) of neurons *represent* represent values via nonlinear encoding and linear decoding.

Lossless Codes



A • ▬
B ▬ • • •
C ▬ • ▬ •
D ▬ • •
E •
F • • ▬ •
G ▬ ▬ •
H • • • •
I • •
J • ▬ ▬ ▬
K ▬ • ▬
L • ▬ • •
M ▬ ▬
N ▬ •
O ▬ ▬ ▬
P • ▬ ▬ •
Q ▬ ▬ • ▬
R • ▬ •
S • • •
T ▬

U • • ▬
V • • • ▬
W • ▬ ▬
X ▬ • • ▬
Y ▬ • ▬ ▬
Z ▬ ▬ • •

1 • ▬ ▬ ▬ ▬
2 • • ▬ ▬ ▬
3 • • • ▬ ▬
4 • • • • ▬
5 • • • • •
6 ▬ • • • •
7 ▬ ▬ • • •
8 ▬ ▬ ▬ • •
9 ▬ ▬ ▬ ▬ •
0 ▬ ▬ ▬ ▬ ▬

Encoding: $\mathbf{a} = f(\mathbf{x})$ Decoding: $\mathbf{x} = f^{-1}(\mathbf{a})$

Binary numbers: Nonlinear encoding, linear decoding

- Represent a natural number between 0 and $2^n - 1$ as n binary digits.

Binary numbers: Nonlinear encoding, linear decoding

- Represent a natural number between 0 and $2^n - 1$ as n binary digits.
- **Nonlinear encoding**

$$a_i = \big(f(x)\big)_i = \begin{cases} 1 & \text{if } x - 2^i \left\lfloor \frac{x}{2^i} \right\rfloor > 2^{i-1}\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

Binary numbers: Nonlinear encoding, linear decoding

- Represent a natural number between 0 and $2^n - 1$ as n binary digits.
- **Nonlinear encoding**

$$a_i = \big(f(x)\big)_i = \begin{cases} 1 & \text{if } x - 2^i \left\lfloor \frac{x}{2^i} \right\rfloor > 2^{i-1}\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

- **Linear decoding**

$$x = f^{-1}(\mathbf{a}) = \sum_{i=0}^{n-1} 2^i a_i = \mathbf{F}\mathbf{a} = \begin{pmatrix} 1 & 2 & \dots & 2^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix}\,.$$

Binary numbers: Nonlinear encoding, linear decoding

- Represent a natural number between 0 and $2^n - 1$ as n binary digits.
- **Nonlinear encoding**

$$a_i = \left(f(x)\right)_i = \begin{cases} 1 & \text{if } x - 2^i \left\lfloor \frac{x}{2^i} \right\rfloor > 2^{i-1}, \\ 0 & \text{otherwise}. \end{cases}$$

- **Linear decoding**

$$x = f^{-1}(\mathbf{a}) = \sum_{i=0}^{n-1} 2^i a_i = \mathbf{F}\mathbf{a} = \begin{pmatrix} 1 & 2 & \dots & 2^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix}.$$

- This is a **distributed code**.

Binary numbers: Nonlinear encoding, linear decoding

- Represent a natural number between 0 and $2^n - 1$ as n binary digits.
- **Nonlinear encoding**

$$a_i = \big(f(x)\big)_i = \begin{cases} 1 & \text{if } x - 2^i \left\lfloor \frac{x}{2^i} \right\rfloor > 2^{i-1}\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

- **Linear decoding**

$$x = f^{-1}(\mathbf{a}) = \sum_{i=0}^{n-1} 2^i a_i = \mathbf{F}\mathbf{a} = \begin{pmatrix} 1 & 2 & \dots & 2^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix}\,.$$

- This is a **distributed code**. But, **not robust** against additive noise!

Lossy codes

- **Lossy code**
  Inverse f^{-1} does not exist, instead *approximate* the represented value

  Encoding: $\mathbf{a} = f(\mathbf{x})$ Decoding: $\mathbf{x} \approx g(\mathbf{a})$

Lossy codes

- **Lossy code**
  Inverse f^{-1} does not exist, instead *approximate* the represented value

$$\text{Encoding:} \quad \mathbf{a} = f(\mathbf{x}) \qquad\qquad \text{Decoding:} \quad \mathbf{x} \approx g(\mathbf{a})$$

- **Examples**
  - Audio, image, and video coding schemes
    (MP3, JPEG, H.264)
  - Basis transformation onto first n principal components (PCA)

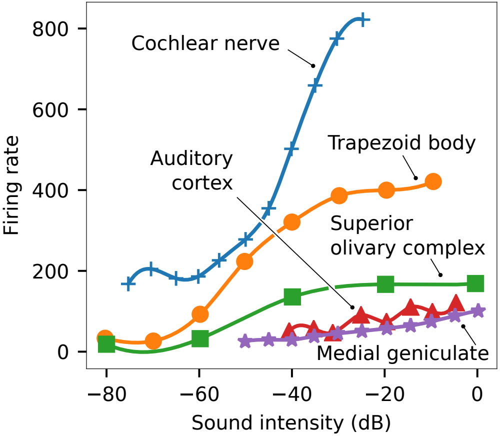
Lossy codes

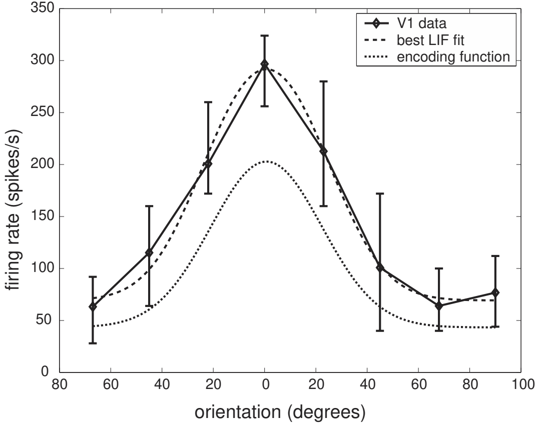
- **Lossy code**
  Inverse f^{-1} does not exist, instead *approximate* the represented value

$$\text{Encoding:} \quad \mathbf{a} = f(\mathbf{x}) \qquad\qquad \text{Decoding:} \quad \mathbf{x} \approx g(\mathbf{a})$$

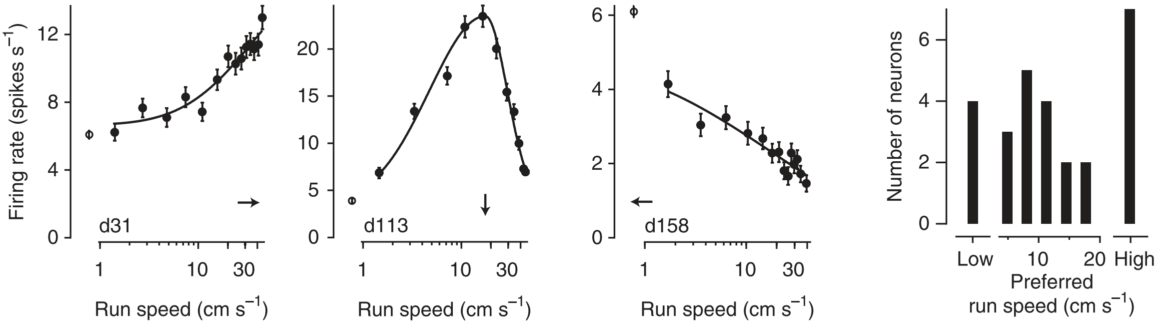
- **Examples**
  - Audio, image, and video coding schemes
    (MP3, JPEG, H.264)
  - Basis transformation onto first n principal components (PCA)
  - **Neural Representations**

Tuning curves (I)

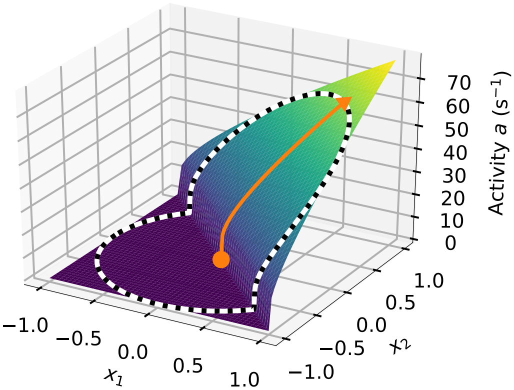




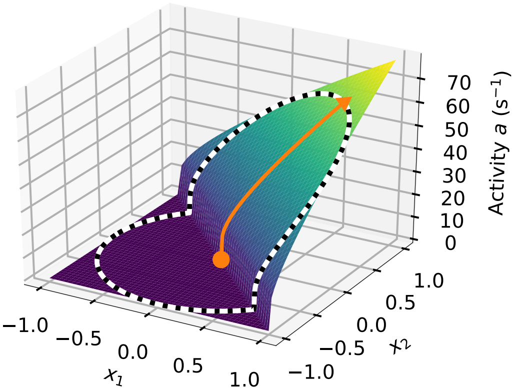
Tuning curves (II)

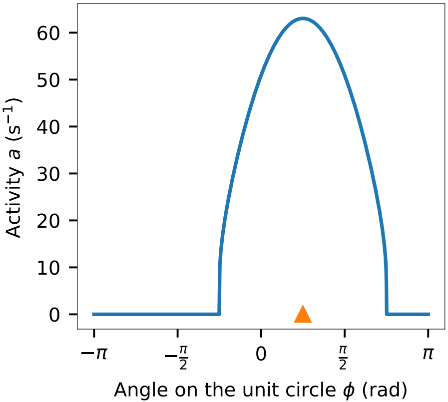


Preferred Directions in Higher Dimensions: Representing 2D Values

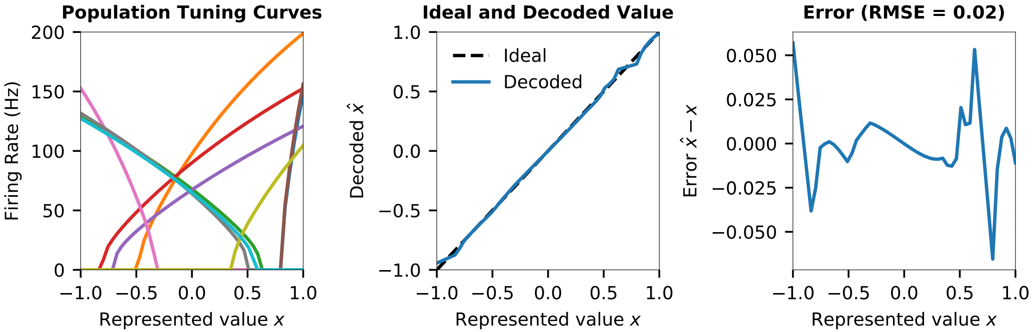


Preferred Directions in Higher Dimensions: Representing 2D Values

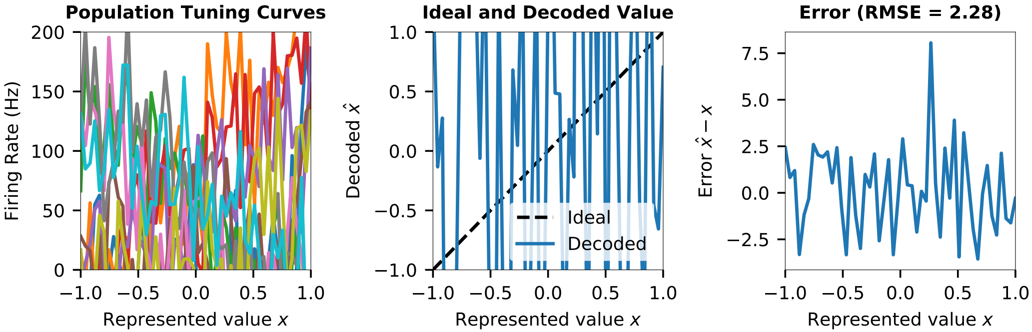




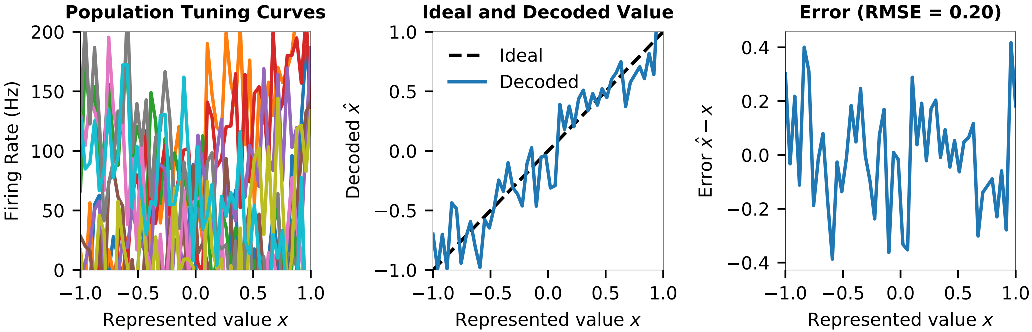
Decoding Without Taking Noise Into Account



Decoding Noisy $\mathbf{A}$ Without Taking Noise Into Account



Decoding Noisy $\mathbf{A}$ Accounting for Noise



Administration

- **Assignment 1 has been released.**

  The due date has been adjusted to January, 30.

- Some new potential times for office hours

  `Mon 15:30-16:30`, `Mon 16:30-17:30`, `Tue 15:00-16:00`,

  `Thu 11:30-12:30` (current slot),`Thu 12:30-13:30`

Image sources

Title slide
"The Ultimate painting."
Author: Clark Richert.
From Wikimedia.