Devoir 1

Q1	Q2	Total
4	3	6

#1 ✓

a) $J_1 = \iint_R x\cos(xy)\, dA \qquad R = [0,1]\times[0,\frac{\pi}{4}]$

$$\boxed{\int_0^1 \int_0^{\frac{\pi}{4}} x\cos(xy)\, dy\, dx}$$

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① $\int_0^{\frac{\pi}{4}} x\cos(xy)\, dy = x\int_0^{\frac{\pi}{4}} \cos(xy)\, dy$

Intégration par changement de variable

$$\boxed{\begin{aligned} u &= xy \\ du &= x\,dy \\ \frac{du}{x} &= dy \end{aligned}}$$

$$= x\int_0^{\frac{\pi}{4}} \cos(u)\cdot\frac{du}{x}$$

$$= x\int_0^{\frac{\pi}{4}} \frac{\cos(u)}{x}\, du$$

$$= \cancel{x}\cdot\frac{1}{\cancel{x}}\int_0^{\frac{\pi}{4}} \cos(u)\, du = \Big[\sin(u)\Big]_0^{\frac{\pi}{4}} = \Big[\sin(xy)\Big]_{y=0}^{y=\frac{\pi}{4}}$$

$$= \sin\left(\frac{\pi}{4}x\right) - \sin(0)$$

$$= \sin\left(\frac{\pi x}{4}\right)$$

② $\int_0^1 \sin\left(\frac{\pi x}{4}\right) dx$

$$\boxed{\begin{aligned} u &= \frac{\pi x}{4} \\ du &= \frac{\pi}{4}\, dx \\ \frac{4}{\pi}\, du &= dx \end{aligned}}$$

$\boxed{x=0 \quad u=0}$

$\frac{\pi\cdot 0}{4} = 0$

$\boxed{x=1 \quad u=\frac{\pi}{4}}$

$\frac{\pi\cdot 1}{4} = \frac{\pi}{4}$

$$\int_0^1 \sin(u)\cdot\frac{4}{\pi}\, du$$

$$= \frac{4}{\pi}\int_0^{\frac{\pi}{4}} \sin(u)\, du = \frac{4}{\pi}\Big[-\cos(u)\Big]_0^{\frac{\pi}{4}}$$

✓

$$= \frac{4}{\pi}\left[-\cos\left(\frac{\pi x}{4}\right)\right]_0^{\frac{\pi}{4}} = \frac{4}{\pi}\left[\left(-\cos\left(\frac{\pi \cdot \frac{\pi}{4}}{4}\right) - \left(-\cos(0)\right)\right)\right]$$

$$= \frac{4}{\pi}\left[-\frac{1}{\sqrt{2}} + 1\right] = \frac{4 - 2\sqrt{2}}{\pi}$$

symbolab

$$\boxed{\int_0^1 \int_0^{\frac{\pi}{4}} x \cos(xy)\, dx\, dy}$$

$u = x \quad dv = \cos(xy)$

$du = dx \quad v = \sin(xy)$

① $\int u\, dv = uv - \int v\, du$

$$\Rightarrow \cancel{\int} u\, dv = x\sin(xy) - \int_0^{\frac{\pi}{4}} \sin(xy)\, dx \Rightarrow \int_0^1 x\sin(xy)\Big|_0^{\frac{\pi}{4}} + \cos(xy)\Big|_0^{\frac{\pi}{4}} dy$$

$$\Rightarrow \int_0^1 \frac{\pi}{4}\sin\left(\frac{\pi y}{4}\right) + \cos\left(\frac{\pi y}{4}\right) - 1\, dy$$

$$\Rightarrow -\frac{\pi}{4}\cos\left(\frac{\pi y}{4}\right)\Big|_0^1 + \sin\left(\frac{\pi}{4}y\right)\Big|_0^1 - 1$$

$$\Rightarrow -\frac{\pi}{4}\left(\frac{1}{\sqrt{2}} - 1\right) + \frac{1}{\sqrt{2}} - 1$$

$$\Rightarrow \frac{4 - 2\sqrt{2}}{\pi}$$

b) $R = [\frac{1}{2}, 1] \times [\frac{1}{2}, 1]$ $f(x,y) = (x^4+y^4)^{\frac{3}{4}}$

i) $L(x,y)$ au point milieu du carré

Taylor degré 1 : $f(x) \approx L(x) = f(a) + f'(a)(x-a)$

1) Trouver le point milieu du carré R :

$$(x_0, y_0) = \left(\frac{3}{4}, \frac{3}{4}\right)$$

2) Dérivées partielles de $f(x,y)$

- Par rapport à x :

$$\frac{\partial f}{\partial x} = \frac{3}{\cancel{4}} \cdot \cancel{4} x^3 \cdot (x^4+y^4)^{-1/4} = 3x^3 (x^4+y^4)^{-1/4}$$

- Par rapport à y :

$$\frac{\partial f}{\partial y} = \frac{3}{\cancel{4}} \cdot \cancel{4} y^3 \cdot (x^4+y^4)^{-1/4}$$

$$= 3y^3 (x^4+y^4)^{-1/4}$$

3) Évaluer dérivées partielles au point milieu

- $\frac{\partial f}{\partial x}$ au point milieu $(x_0, y_0) = \left(\frac{3}{4}, \frac{3}{4}\right)$

$$\frac{\partial f}{\partial x} = 3 \cdot \left(\frac{3}{4}\right)^3 \cdot \left(\left(\frac{3}{4}\right)^4 + \left(\left(\frac{3}{4}\right)^4\right)^{-1/4}\right) = \frac{81}{256}$$

- $\frac{\partial f}{\partial y}$ au point milieu $(x_0, y_0) = \left(\frac{3}{4}, \frac{3}{4}\right)$

$$\frac{\partial f}{\partial y} = 3 \cdot \left(\frac{3}{4}\right)^3 \cdot \left(\left(\frac{3}{4}\right)^4 + \left(\left(\frac{3}{4}\right)^4\right)^{-1/4}\right) = \frac{81}{256}$$

Polynôme Taylor degré 1 :

$$L(x,y) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)$$

<u>Substituer</u>

$$L(x,y) = f\left(\frac{3}{4},\frac{3}{4}\right) + \cancel{\frac{81}{256}}\left(x-\frac{3}{4}\right) + \cancel{\frac{81}{256}}\left(y-\frac{3}{4}\right)$$

Avec :

$$f\left(\frac{3}{4},\frac{3}{4}\right) = \left[\left(\frac{3}{4}\right)^4 + \left(\frac{3}{4}\right)^4\right]^{3/4} = \left(2\cdot\left(\frac{3}{4}\right)^4\right)^{3/4} = \left(2\cdot\left(\frac{81}{256}\right)\right)^{3/4} = \left(\frac{162}{256}\right)^{3/4}$$

<u>Réponse</u>

$$L(x,y) = \left(\frac{162}{256}\right)^{\frac{3}{4}} + \frac{81}{256}\left(x-\frac{3}{4}\right) + \frac{81}{256}\left(y-\frac{3}{4}\right)$$

ii) Calculer estimation :

$$\iint_D f(x,y)\, dA \approx \iint_D L(x,y)\, dA$$

La région R est définie par $R = [\frac{1}{2}, 1] \times [\frac{1}{2}, 1]$
et il faut intégrer $L(x,y)$ sur cette région

Par rapport y

$$\iint_D L(x,y)\, dA = \int_{1/2}^{1}\int_{1/2}^{1} \left(\frac{162}{256}\right)^{3/4} + \frac{81}{256}\left(x - \frac{3}{4}\right) + \frac{81}{256}\left(y - \frac{3}{4}\right) dy\,dx$$

① $\displaystyle\int_{1/2}^{1} \left(\frac{162}{256}\right)^{3/4} y + \frac{81}{256}\left(y - \frac{3}{4}\right) dy$

$$\left[\left(\frac{162}{256}\right)^{3/4} \frac{y^2}{2} + \frac{81}{256}\left(\frac{y^2}{2} - \frac{3}{4}y\right)\right]_{1/2}^{1}$$ Symbolab

$$= \left(\frac{162}{256}\right)^{\frac{15}{32}}$$

Par rapport x

② $\displaystyle\int_{1/2}^{1} \left(\frac{162}{256}\right)^{\frac{15}{32}} dx = \left[\left(\frac{162}{256}\right)^{\frac{15}{32}} \cdot \left(x - \frac{1}{2}\right)\right]_{\frac{1}{2}}^{1}$ Symbolab

$$= \left(\frac{162}{256}\right)^{\frac{15}{32}}$$

$$\iint_D f(x,y)\, dA \approx \iint_D L(x,y)\, dA \approx \left(\frac{162}{256}\right)^{\frac{15}{32}}$$

#2

a) 0/3

b) $$J_3 = \int_0^{16}\int_{\sqrt{y}}^{4} \frac{x^2}{1+y\sqrt{x}}\,dx\,dy$$ 3

$$= \int_0^4\int_0^{x^2} \frac{x^2}{1+y\sqrt{x}}\,dy\,dx = \int_0^{x^2} \frac{x^2}{1+y\sqrt{x}}\,dy$$

$$= x^2 \cdot \int_0^{x^2} \frac{1}{1+\sqrt{x}\,y}\,dy$$

$$= x^2 \int_1^{1+\sqrt{x}\,x^2} \frac{1}{\sqrt{x}\,u}\,du$$

$$= x^2 \frac{1}{\sqrt{x}} \cdot \int_1^{1+\sqrt{x}\,x^2} \frac{1}{u}\,du$$

$$= x^2 \frac{1}{\sqrt{x}} \left[\ln(u)\right]_1^{1+\sqrt{x}\,x^2}$$

$$= x^{\frac{3}{2}} \left[\ln(u)\right]_1^{x^2\sqrt{x}+1}$$

$$= x^{\frac{3}{2}} \ln\left(x^2\sqrt{x}+1\right)$$

$$= \int_0^4 x^{\frac{3}{2}} \ln\left(x^2\sqrt{x}+1\right)dx = \frac{2}{5}\left(33\ln(33)-32\right)$$

$$= \int_1^{33} \frac{2\ln(u)}{5}\,du = \frac{2}{5}\cdot\int_1^{33} \ln(u)\,du$$

✓

$$= \frac{2}{5}\left[u \ln(u) - \int 1\,du\right]_1^{33}$$

$$\int 1\,du = u \quad = \frac{2}{5}\left[u \ln(u) - u\right]_1^{33}$$

$$33 \ln(33) - 32$$

3/3

$$= \frac{2}{5}\left(33 \ln(33) - 32\right)$$

$$= \frac{2}{5}\left(33 \ln(33) - 32\right)$$

✓