Polytechnique Montréal
Département de Mathématiques et de Génie Industriel

MTH1102D - Calcul II
Été 2023

Devoir 2

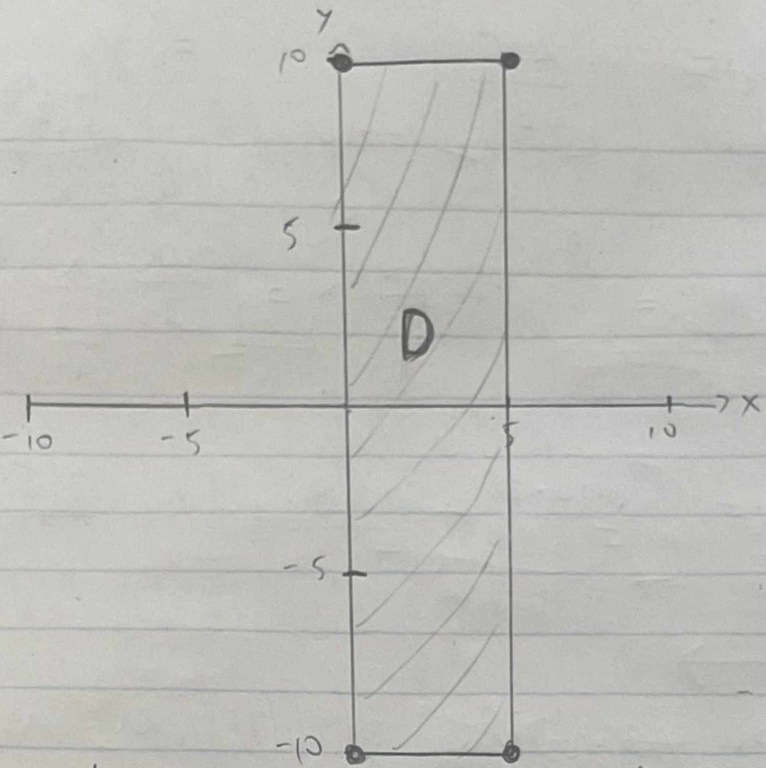
Nom : ______________________ **Prénom :** ______________________

Matricule : ____________________ **Groupe :** 01

Question corrigée	Autres questions	Total
4,5	4	8.5 /10

#1

a)



Équations des droites qui bornent D :
Passant par (0, 10) (0, -10) :

Verticale : $x = 0$

Passant par (5, -10) (5, 10) :
Verticale : $x = 5$

Passant par (0, 10) (5, 10) :
Horizontale : $y = 10$

Passant par (0, -10) (5, -10) :
Horizontale : $y = -10$

Le domaine D :
$0 \leq x \leq 5 \qquad -10 \leq y \leq 10$

Type 1

$$\int_0^5 \int_{-10}^{10} \left(xy^2 + \frac{x^2 y}{\sqrt{10 + x^2 y^2}} \right) dy\,dx$$

$$= \underbrace{\int_0^5 \int_{-10}^{10} xy^2 \, dy\,dx}_{1} + \underbrace{\int_0^5 \int_{-10}^{10} \frac{x^2 y}{\sqrt{10 + x^2 y^2}} \, dy\,dx}_{2}$$

① $$\int_0^5 \int_{-10}^{10} x y^2 \, dy \, dx = \int_0^5 \left[\frac{x y^3}{3} \right]_{y=-10}^{y=10} dx = \int_0^5 \left[\left(\frac{x \cdot 10^3}{3} \right) - \left(\frac{x \cdot -10^3}{3} \right) \right] dx$$

$$= \int_0^5 \left(\frac{1000x}{3} + \frac{1000x}{3} \right) dx = \int_0^5 \frac{2000x}{3} \, dx = \left[\frac{2000}{3} \; \frac{x^2}{2} \right]_{x=0}^{x=5}$$

$$\left[\frac{2000 \, x^2}{6} \right]_{x=0}^{x=5} = \left[\frac{2000 \cdot 5^2}{6} - \frac{2000 \cdot 0^2}{6} \right] = \frac{25\,000}{3}$$

② f fonction impaire (Utiliser symétrie)

$$f(-y) = -f(y)$$

$$\int_{-10}^{10} \frac{x^2 y}{\sqrt{10 + x^2 y^2}} \, dy$$

$$f(-y) : \quad \frac{x^2 \overset{(-y)}{y}}{\sqrt{10 + x^2 \underset{(-y)}{y^2}}} = \frac{-x^2 y}{\sqrt{10 + x^2 y^2}}$$

$$-f(y) : \quad \frac{-x^2 y}{\sqrt{10 + x^2 y^2}}$$

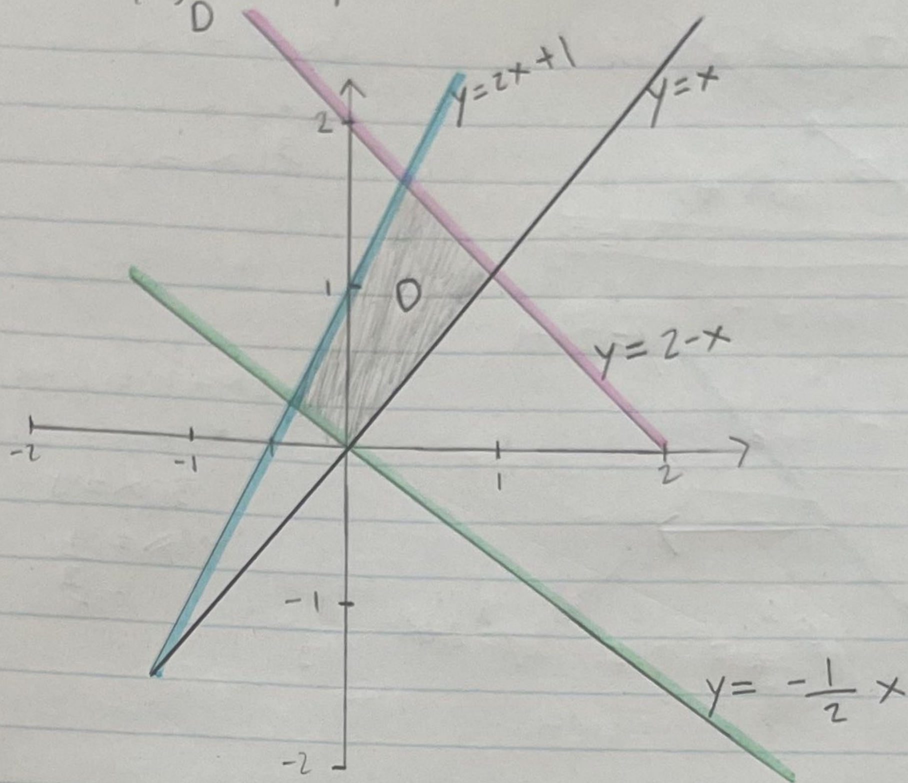
$$f(y) \neq f(-y)$$

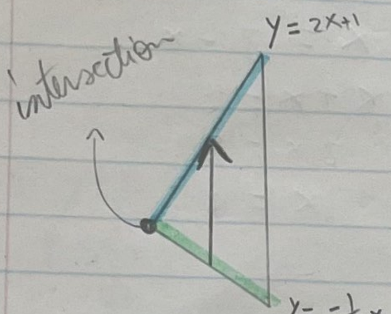
car

$$\frac{x^2 y}{\sqrt{10 + x^2 y^2}} \neq -\frac{x^2 y}{\sqrt{10 + x^2 y^2}} \Rightarrow 0$$

Rép : $\frac{25\,000}{3}$ ✓

b) $J_2 = \iint_D (x+y)\, dA$

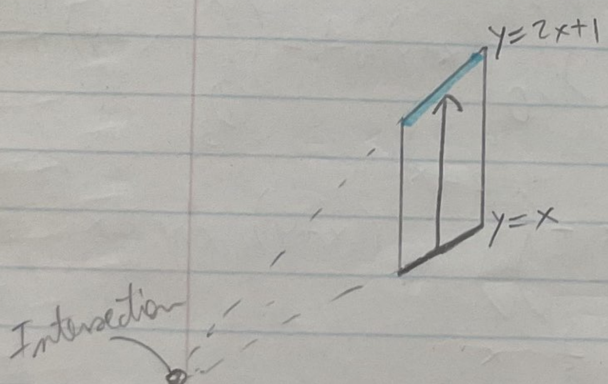




$$2x+1 = -\frac{1}{2}x$$
$$2x + \frac{1}{2}x = -1$$
$$2{,}5\,x = -1$$
$$x = -\frac{2}{5}$$

$$-\frac{2}{5} \leq x \leq 0$$

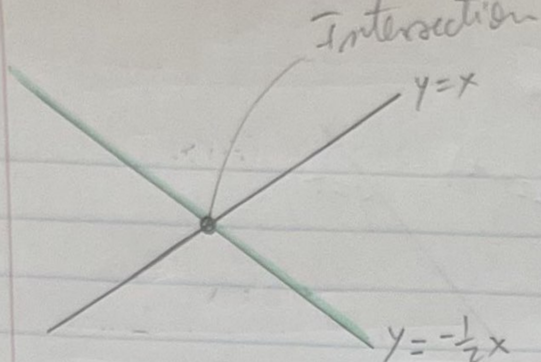
$$D_1 = \left\{ (x,y) \in \mathbb{R}^2 \mid -\frac{2}{5} \leq x \leq 0 \,,\ -\frac{1}{2}x \leq y \leq 2x+1 \right\}$$



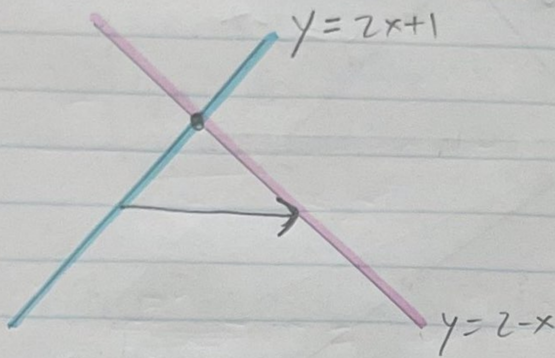
$$2x+1 = x$$
$$2x - x = -1$$
$$x = -1$$

$$-1 \leq x \leq 0$$

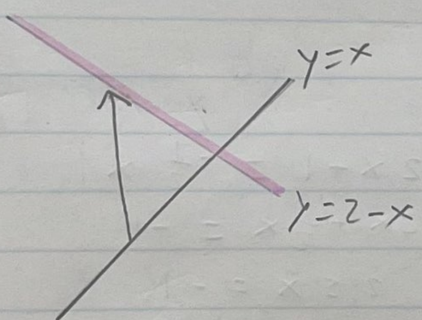
$$D_2 = \left\{ (x,y) \in \mathbb{R}^2 \mid 0 \leq x \leq \frac{1}{3},\ x \leq y \leq 2x+1 \right\}$$



$$x = -\frac{1}{2}x$$
$$x + \frac{1}{2}x = 0$$
$$\frac{3}{2}x = 0$$



$$2x+1 = 2-x$$
$$2x + x = 2-1$$
$$3x = 1$$
$$x = \frac{1}{3}$$



$$x = 2-x$$
$$x+x = 2$$
$$2x = 2$$
$$x = 1$$

$$\frac{1}{3} \le x \le 1$$

$$D_3 = \left\{ (x,y) \in \mathbb{R}^2 \mid \frac{1}{3} \le x \le 1,\ x \le y \le 2-x \right\}$$

$$\iint_D (x+y)\,dA = \iint_{D_1} (x+y)\,dA + \iint_{D_2} (x+y)\,dA + \iint_{D_3} (x+y)\,dA$$

$$= \int_{-\frac{2}{5}}^{0}\int_{-\frac{1}{2}x}^{2x+1} (x+y)\,dy\,dx + \int_{0}^{1/3}\int_{x}^{2x+1} (x+y)\,dy\,dx + \int_{\frac{1}{3}}^{1}\int_{x}^{2-x} (x+y)\,dy\,dx$$

Symbolab

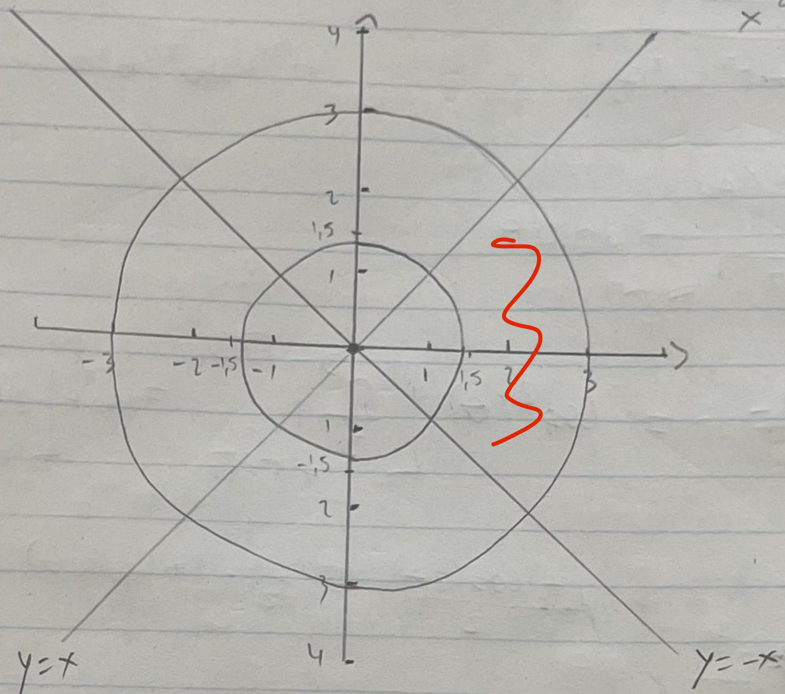
$$= \frac{4}{75} + \frac{59}{162} + \frac{56}{81} = \frac{499}{450}$$

#2

a) $J_3 = \iint_D \frac{x^2}{\sqrt{x^2+y^2}}\, dA$

$y = x$
$y = -x$
$x^2 + y^2 = 2$
$x^2 + y^2 = 9$

4.5



$\boxed{\begin{aligned} x &= r\cos(\theta) \\ y &= r\sin(\theta) \end{aligned}}$

- $y = x$

$r\sin(\theta) = r\cos(\theta)$

$\tan(\theta) = 1 \qquad \theta = \frac{\pi}{4}$

- $y = -x$

$r\sin(\theta) = -r\cos(\theta)$

$\tan(\theta) = -1 \qquad \theta = \frac{5\pi}{4}$

- $x^2 + y^2 = 2$

$(r\cos(\theta))^2 + (r\sin(\theta))^2 = 2$

$r^2\cos^2(\theta) + r^2\sin^2(\theta) = 2$

$r^2(\cos^2(\theta) + \sin^2(\theta)) = 2 \longrightarrow = 1$

$r^2 = 2 \Rightarrow r = \sqrt{2}$

- $x^2 + y^2 = 9$

$r^2 = 9 \qquad r = 3$

$\theta : \frac{\pi}{4}$ à $\frac{5\pi}{4}$

$r : \sqrt{2}$ à 3

$$\int_{-\frac{\pi}{4}}^{\cancel{\frac{5\pi}{4}}} \int_{\sqrt{2}}^{3} \frac{(r\cos(\theta))^2 \checkmark}{\sqrt{(r\cos(\theta))^2+(r\sin(\theta))^2}} \, \overset{\checkmark}{r} \, dr \, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_{\sqrt{2}}^{3} \frac{r^3\cos^2(\theta)}{\sqrt{r^2\cos^2(\theta)+r^2\sin^2(\theta)}} \, dr \, d\theta \quad \longrightarrow \quad r^2\big(\underbrace{\cos^2(\theta)+\sin^2(\theta)}_{=1}\big)$$

$$= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_{\sqrt{2}}^{3} \frac{r^{\cancel{3}}\cos^2(\theta)}{\sqrt{r^2} = \cancel{r}} \, dr \, d\theta$$

① $$\cos^2(\theta)\int_{\sqrt{2}}^{3} r^2 \, dr = \cos^2(\theta)\left[\frac{r^3}{3}\right]_{r=\sqrt{2}}^{r=3} = \cos^2\theta\left[\overbrace{\frac{27-2\sqrt{2}}{3}}^{\text{symbolab}}\right]$$

② $$\left(\frac{27-2\sqrt{2}}{3}\right)\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \cos^2\theta \, d\theta \quad \longrightarrow \quad \frac{1+\cos(2\theta)}{2} = \frac{1}{2}\cdot 1+\cos(2\theta)$$

$$= \left(\frac{27-2\sqrt{2}}{3}\right)\cdot\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} 1+\cos(2\theta) \, d\theta$$

2/3

~~Symbolab~~ intégrale facile !

$$= \frac{27-2\sqrt{2}}{3}\cdot\frac{1}{2}(\pi+0)$$

$$= \frac{\pi(27-2\sqrt{2})}{6} \times$$

b) $z = x^2$

$z = (r\cos(\theta))^2$

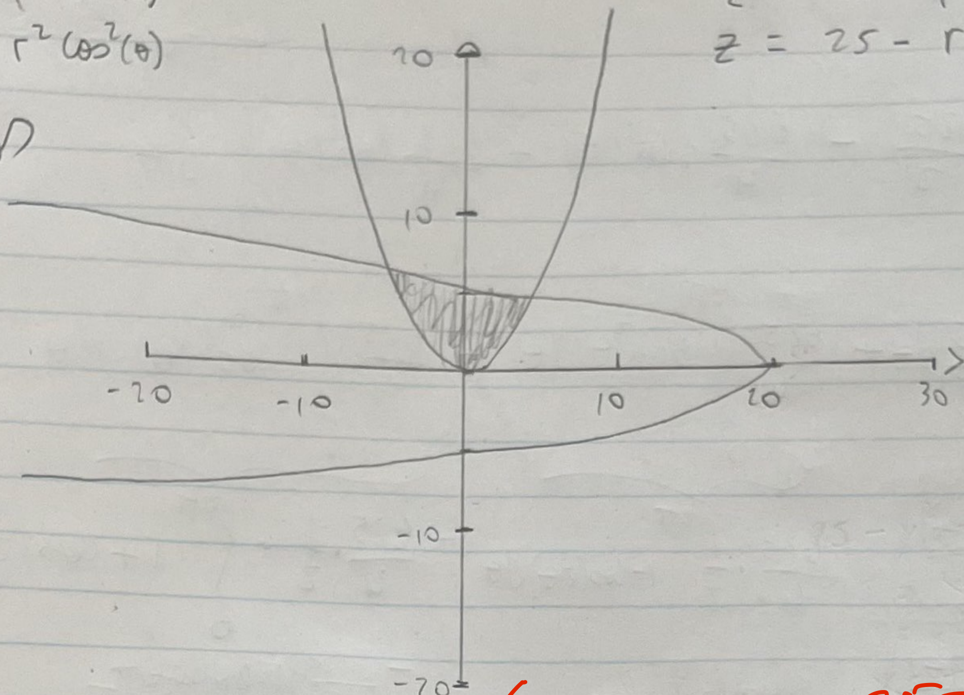
$z = r^2\cos^2(\theta)$

$z = 25 - y^2$

$z = 25 - (r\sin(\theta))^2$

$z = 25 - r^2\sin^2(\theta)$

Desmos



$$Vol(E) = \iint_D [(25 - y^2) - (x^2)]\, dA \checkmark$$

$$= \iint_D [25 - y^2 - x^2]\, dA$$

$25 - x^2 = y^2$

$\Rightarrow x^2 + y^2 = 25$

$r^2\cos^2(\theta) = 25 - r^2\sin^2(\theta)$

$r^2 = \frac{25}{(1+\tan^2(\theta))}$

justifier r

$r : 0 \leq r \leq 5$ ~~symbolab~~

$\theta : 0 \leq \theta \leq 2\pi$

$$Vol(E) = \iint_D [(25 - y^2) - (x^2)]\, dA$$

$$= \iint_D [25 - y^2 - x^2]\, dA$$

$$Vol(E) = \int_0^{2\pi}\int_0^5 [25 - (r\sin(\theta))^2 - (r\cos\theta)^2]\, r\, dr\, d\theta \checkmark$$

$$= \int_0^{2\pi}\int_0^5 [25 - r^2\sin^2(\theta) - r^2\cos^2(\theta)]\, r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^5 [25r - r^3\sin^2(\theta) - r^3\cos^2(\theta)]\, dr\, d\theta$$

$$= \int_0^{2\pi} \left[25\frac{r^2}{2} - \frac{r^4}{4}\sin^2(\theta) - \frac{r^4}{4}\cos^2(\theta) \right]_{r=0}^{r=5} d\theta$$

$$= \int_0^{2\pi} \left[\frac{625}{2} - \frac{625}{4}\sin^2(\theta) - \frac{625}{4}\cos^2(\theta) \right] d\theta$$

$$= \int_0^{2\pi} \frac{625}{2}\,d\theta - \underbrace{\frac{625}{4}\int_0^{2\pi} \sin^2(\theta)\,d\theta}_{} - \underbrace{\frac{625}{4}\int_0^{2\pi} \cos^2(\theta)\,d\theta}_{}$$

$$\boxed{\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}}$$

$$\boxed{\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}}$$

$$-\frac{625}{4}\cdot\frac{1}{2}\int_0^{2\pi} 1-\cos(2\theta)\,d\theta \qquad -\frac{625}{4}\cdot\frac{1}{2}\cdot\int_0^{2\pi} 1+\cos(2\theta)\,d\theta$$

~~symbolab~~ ~~symbolab~~

$$-\frac{625\pi}{4} \qquad -\frac{625\pi}{4}$$

$$\frac{625}{2}\int_0^{2\pi} d\theta$$

$$\frac{625}{\cancel{2}}\cdot \cancel{2}\pi = 625\pi$$

2.5/3

$$= 625\pi - \frac{625\pi}{4} - \frac{625\pi}{4}$$

simplifiez

$$= 625\pi - \frac{625\pi}{2} = \frac{625}{2}\pi$$