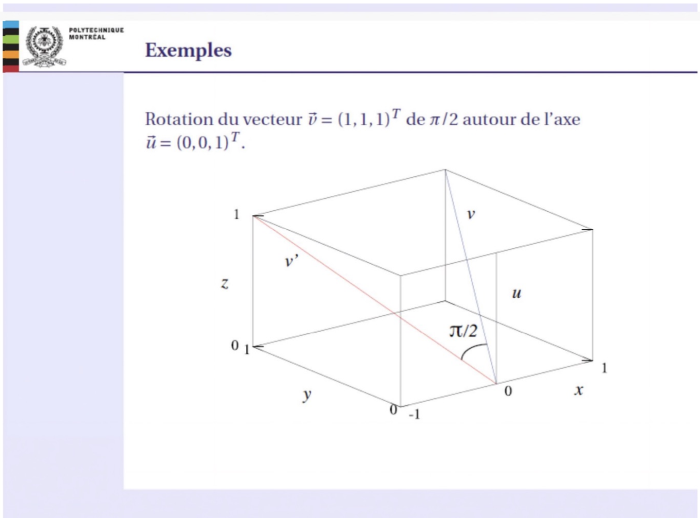
Cours 2 (9 Sept. 2021)

Exemple (chapitre 2, diapo 38)



Solution :

$$\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \longrightarrow \boxed{\vec{\vec{u}} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}}$$

$$\left.\begin{aligned} \theta &= \frac{\pi}{2} \\ \hat{u} &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{aligned}\right\}$$

$$\vec{\vec{r}} = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\, u_x \\ \sin\frac{\theta}{2}\, u_y \\ \sin\frac{\theta}{2}\, u_z \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{4} \\ 0 \\ 0 \\ \sin\frac{\pi}{4} \end{pmatrix} = \boxed{\frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}}$$

$$\vec{v}' = \vec{r}\,\vec{v}\,\vec{r}^*$$

$$\vec{v}'\,\vec{r}^* = \frac{\sqrt{2}}{2}\begin{pmatrix}0\\1\\1\\1\end{pmatrix}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix} = \frac{\sqrt{2}}{2}\begin{pmatrix}0-0-0+1\\0+1-1-0\\0+1+0+1\\0+1+0-0\end{pmatrix} =$$

$$= \frac{\sqrt{2}}{2}\begin{pmatrix}1\\0\\2\\1\end{pmatrix}$$

$$\vec{r}\,\vec{v}\,\vec{r}^* = \frac{\sqrt{2}}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\frac{\sqrt{2}}{2}\begin{pmatrix}1\\0\\2\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\begin{pmatrix}1\\0\\2\\1\end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix}1-0-0-1\\0+0+0-2\\2-0-0+0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\-2\\2\\2\end{pmatrix}$$

$$\boxed{\vec{v}' = \begin{pmatrix}0\\-1\\1\\1\end{pmatrix}}$$ ← Réponse finale

Cours 3 (14 Sept. 2021)

Exemple : Pyramide (cours 2, diapo 81)

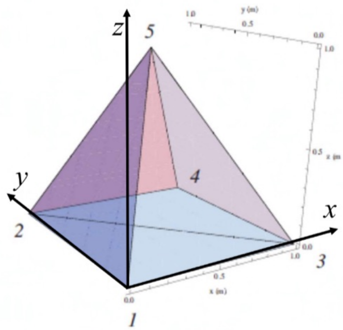
POLYTECHNIQUE MONTRÉAL

Centre de masse

Exemple:

Volume et centre de masse d'une pyramide déformée

Sommet	x (m)	y (m)	z (m)
1	0.0	0.0	0.0
2	0.0	1.0	0.0
3	1.0	0.0	0.0
4	1.0	1.0	0.0
5	0.2	0.2	1.0



2021-09-13

$$dV_i = \frac{1}{6}\left(a_i\left(b_i \times c_i\right)\right) \qquad V = \sum dV_i$$

$$d_i = \frac{1}{4}\left(\vec{a_i} + \vec{b_i} + \vec{c_i}\right) \qquad \vec{V_{cm}} = \frac{1}{V} dV_i \vec{d_i}$$

triangle	a_i	b_i	c_i	dV_i	$4d_i$
1	0, 0, 0	1, 0, 0	0.2, 0.2, 1	0	
2	1, 0, 0	1, 1, 0	0.2, 0.2, 1	$\frac{1}{6}$	
3	0, 1, 0	0.2, 0.2, 1	1, 1, 0	$\frac{1}{6}$	
4				0	
5				0	
6				0	

$4d_i$

2 [2.2, 1.2, 1]

3 [1.2; 2.2, 1]

Calculs :

$$dV_1 = \frac{1}{6}\left[\begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\end{pmatrix}\times\begin{pmatrix}0.2\\0.2\\1\end{pmatrix}\right] = 0$$

$$dV_2 = \frac{1}{6}\left[\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}\times\begin{pmatrix}0.2\\0.2\\1\end{pmatrix}\right] = \frac{1}{6}\left[\begin{pmatrix}0\\8\\8\end{pmatrix}\begin{pmatrix}1\\-1\\0\end{pmatrix}\right] = \frac{1}{6}$$

$$dV_3 = \frac{1}{6}\left[\begin{pmatrix}0\\1\\0\end{pmatrix}\cdot\begin{pmatrix}0.2\\0.2\\1\end{pmatrix}\times\begin{pmatrix}1\\0\\0\end{pmatrix}\right] = \frac{1}{6}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}-1\\1\\0\end{pmatrix} = \frac{1}{6}$$

$$\boxed{V = \frac{2}{6} = \frac{1}{3} \quad [m^3]}$$

$$\vec{r}_{cn} = \frac{3}{4} \times \frac{1}{8} \begin{pmatrix} 2.2 \\ 1.2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1.2 \\ 2.2 \\ 1 \end{pmatrix}$$

$$\boxed{\vec{r}_{cn} = \frac{1}{8} \begin{pmatrix} 3.4 \\ 3.4 \\ 2 \end{pmatrix} \quad m}$$