



**POLYTECHNIQUE
MONTRÉAL**

PHS 4700

Physique pour les applications multimédia

Résumé

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$$\frac{d\vec{v}_c(t)}{dt} = \vec{a}_c(t, \vec{v}_c, \vec{r}_c, \vec{\omega}(t), \vec{\Omega}(t))$$

$$\frac{d\vec{r}_c(t)}{dt} = \vec{v}_c(t, \vec{v}_c, \vec{r}_c, \vec{\omega}(t), \vec{\Omega}(t))$$

$$\frac{d\vec{\omega}(t)}{dt} = \vec{\alpha}(t, \vec{v}_c, \vec{r}_c, \vec{\omega}(t), \vec{\Omega}(t))$$

$$\frac{d\vec{\Omega}(t)}{dt} = \vec{\omega}(t, \vec{v}_c, \vec{r}_c, \vec{\omega}(t), \vec{\Omega}(t))$$

Cinématique

$$\mathbf{R}^{G \leftarrow L} = \mathbf{R}_z(\theta_z) \mathbf{R}_y(\theta_y) \mathbf{R}_x(\theta_x)$$

$$\mathbf{R}_x(\theta_x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta_x & \cos \theta_x \end{pmatrix}$$

$$\mathbf{R}_y(\theta_y) = \begin{pmatrix} \cos \theta_y & 0 & \sin \theta_y \\ 0 & 1 & 0 \\ -\sin \theta_y & 0 & \cos \theta_y \end{pmatrix}$$

$$\mathbf{R}_z(\theta_z) = \begin{pmatrix} \cos \theta_z & -\sin \theta_z & 0 \\ \sin \theta_z & \cos \theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R}^{G \leftarrow L} = \begin{pmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{pmatrix} = (\vec{e}_x^L \quad \vec{e}_y^L \quad \vec{e}_z^L)$$

$$\mathbf{R}(t) = \begin{pmatrix} u_x^2 + (u_y^2 + u_z^2) \cos \theta & u_x u_y (1 - \cos \theta) - u_z \sin \theta & u_x u_z (1 - \cos \theta) + u_y \sin \theta \\ u_x u_y (1 - \cos \theta) + u_z \sin \theta & u_y^2 + (u_z^2 + u_x^2) \cos \theta & u_y u_z (1 - \cos \theta) - u_x \sin \theta \\ u_x u_z (1 - \cos \theta) - u_y \sin \theta & u_y u_z (1 - \cos \theta) + u_x \sin \theta & u_z^2 + (u_x^2 + u_y^2) \cos \theta \end{pmatrix}$$

$$\frac{d\mathbf{R}(t)}{dt} = \tilde{\boldsymbol{\omega}}(t)\mathbf{R}(t)$$

$$\tilde{\boldsymbol{\omega}}(t) = \begin{pmatrix} 0 & -\omega_z(t) & \omega_y(t) \\ \omega_z(t) & 0 & -\omega_x(t) \\ -\omega_y(t) & \omega_x(t) & 0 \end{pmatrix}$$

$$\vec{r}_i(t) = \vec{r}_c(t) + \mathbf{R}(t)\vec{r}_{i,c}(t_0)$$

$$\vec{v}_i(t) = \vec{v}_c(t) + \vec{\omega}(t) \times (\mathbf{R}(t)\vec{r}_{i,c}(t_0))$$

$$\vec{v}^G = \mathbf{R}^{G \leftarrow L} \vec{v}^L$$

$$\mathbf{M}^G = \mathbf{R}^{G \leftarrow L} \mathbf{M}^L (\mathbf{R}^{G \leftarrow L})^{-1}$$

$$\vec{r} = (\cos(\theta/2), \sin(\theta/2) \vec{u}^T)^T$$

$$\vec{q}' = (0, (\vec{v}')^T)^T = \vec{r} (0, \vec{v}^T)^T \vec{r}^* = \vec{r} \vec{q} \vec{r}^*$$

$$\frac{d\vec{R}(t)}{dt} = \frac{1}{2} \vec{R}(t) \vec{\omega}(t)$$

$$\vec{\omega}(t) = (0, \vec{\omega}(t))^T$$

$$\vec{q} = q_0 + iq_x + jq_y + kq_z$$

$$i^2 = j^2 = k^2 = -1$$

$$ij = k = -ji$$

$$jk = i = -kj$$

$$ki = j = -ik$$

$$\begin{aligned} \vec{q}^1 \vec{q}^2 = & (q_0^1 q_0^2 - q_x^1 q_x^2 - q_y^1 q_y^2 - q_z^1 q_z^2, \\ & q_0^1 q_x^2 + q_x^1 q_0^2 + q_y^1 q_z^2 - q_z^1 q_y^2, \\ & q_0^1 q_y^2 + q_y^1 q_0^2 + q_z^1 q_x^2 - q_x^1 q_z^2, \\ & q_0^1 q_z^2 + q_z^1 q_0^2 + q_x^1 q_y^2 - q_y^1 q_x^2)^T \end{aligned}$$

Translation

$$\vec{p}(t) = m\vec{v}(t)$$

$$\vec{F}(t) = \frac{d\vec{p}(t)}{dt}$$

Masse constante $\vec{a}(t) = \frac{\vec{F}(t)}{m}$

$$\vec{F}_a = -\vec{F}_r$$

$$E = \int_0^L \vec{F}_a \cdot d\vec{l}$$

Rotation

$$\vec{L}(t) = I\vec{\omega}(t)$$

$$\vec{\tau}(t) = \frac{d\vec{L}(t)}{dt}$$

$$\vec{\tau}(t) = \frac{d(I\vec{\omega}(t))}{dt} = I\vec{\alpha} + \frac{dI}{dt}\vec{\omega}$$

$$\vec{\tau}(t) = (\vec{r}(t) - \vec{r}_c) \times \vec{F}(t)$$

$$\vec{\alpha}(t) = \frac{d\vec{\omega}(t)}{dt} = (I(t))^{-1} \left(\vec{\tau}(t) + \vec{L}(t) \times \vec{\omega}(t) \right)$$

$$\vec{r}_c = \frac{1}{m} \sum_{n=1}^N \int_{\mathcal{V}_n} \vec{r} \rho_n(\vec{r}) d^3 r \quad \mathbf{I}_c = \int_{\mathcal{V}} \begin{pmatrix} y^2 + z^2 & -xy & -xz \\ -yx & x^2 + z^2 & -yz \\ -zx & -zy & x^2 + y^2 \end{pmatrix} \rho(\vec{r}) d^3 r$$

$$m = \sum_{n=1}^N \int_{\mathcal{V}_n} \rho_n(\vec{r}) d^3 r \quad \mathbf{I}_d = \mathbf{I}_c + m \begin{pmatrix} (d_{c,y}^2 + d_{c,z}^2) & -d_{c,x}d_{c,y} & -d_{c,x}d_{c,z} \\ -d_{c,y}d_{c,x} & (d_{c,x}^2 + d_{c,z}^2) & -d_{c,y}d_{c,z} \\ -d_{c,z}d_{c,x} & -d_{c,z}d_{c,y} & (d_{c,x}^2 + d_{c,y}^2) \end{pmatrix}$$

$$\mathbf{I}_c = \sum_i \mathbf{I}_{i,c}^{RT}$$

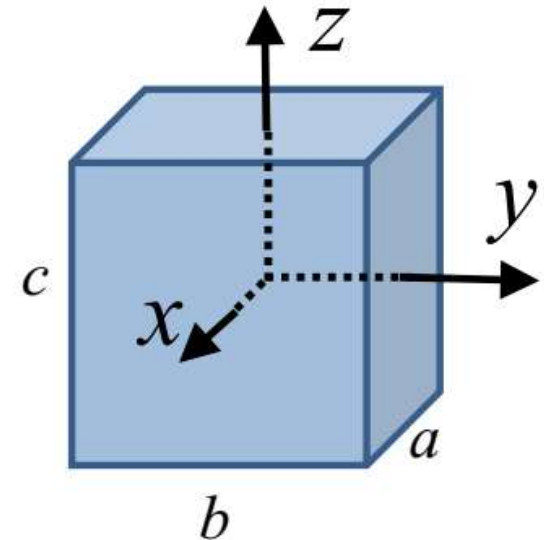
Moments d'inertie de quelques géométries simples:

Parallélépipède dont les axes sont parallèles aux axes x (longueur a), y (largeur b) et z (hauteur c) par rapport à son centre de masse

$$I_{c,xx} = \frac{m}{12}(b^2 + c^2)$$

$$I_{c,yy} = \frac{m}{12}(a^2 + c^2)$$

$$I_{c,zz} = \frac{m}{12}(a^2 + b^2)$$



Les termes non diagonaux sont nuls.

Sphères de rayon r par rapport à son centre de masse

□ Pleine

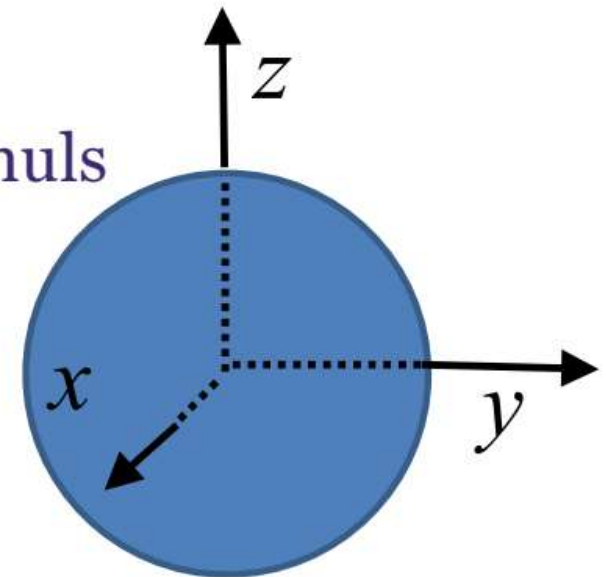
$$I_{c,xx} = I_{c,yy} = I_{c,zz} = \frac{2m}{5}r^2$$

Les termes hors de la diagonale sont nuls

□ Creux

$$I_{c,xx} = I_{c,yy} = I_{c,zz} = \frac{2m}{3}r^2$$

Les termes hors de la diagonale sont nuls.



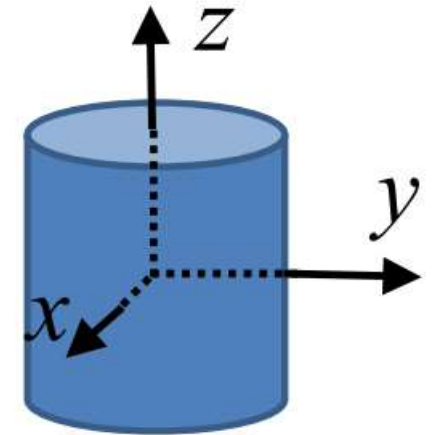
Dynamique

Cylindre de rayon r et de longueur l aligné avec l'axe des z par rapport à son centre de masse

□ Plein

$$I_{c,zz} = \frac{m}{2} r^2$$

$$I_{c,xx} = I_{c,yy} = \frac{m}{4} r^2 + \frac{m}{12} l^2$$



Les termes hors de la diagonale sont nuls.

□ Creux

$$I_{c,zz} = m r^2$$

$$I_{c,xx} = I_{c,yy} = \frac{m}{2} r^2 + \frac{m}{12} l^2$$

Les termes hors de la diagonale sont nuls.

Exemple de forces indépendantes de la rotation des solides

électrique $\vec{F}_{1,2}^G = k \frac{q_1 q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$

gravitationnelle $\vec{F}_{1,2}^G = -G \frac{m_1 m_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$

Amortissement $\vec{F}^a = -k_a [(\vec{v} - \vec{v}_a) \cdot \hat{u}] \hat{u}$

Frottement sec $\vec{F}^f = -\mu_s N \frac{\vec{F}}{|\vec{F}|}$

Viscosité $\vec{F}^{\text{vis}} = \begin{cases} -k\eta \vec{v} & |\vec{v}| > 0 \\ 0 & |\vec{v}| = 0 \end{cases}$

Portance $|\vec{F}^P| = \frac{1}{2} \rho |\vec{v}|^2 A C_P$

Exemple de forces liées à la rotation des solides

Condition de roulement $\vec{v}_r = -\vec{\omega} \times \vec{r}_{p,c}$

Frottement de roulement $\vec{F}^{\text{roulement}} = -\mu_d m g \hat{z}$
 $\vec{\tau}^{\text{roulement}} = \vec{r}_{d,c} \times \vec{F}^{\text{roulement}}$

Frottement de glissement $\vec{F}^f = -\mu_c N \frac{\vec{v}_{\text{glissement}}}{|\vec{v}_{\text{glissement}}|}$
 $\vec{\tau}^{\text{glissement}} = (\vec{r}_p - \vec{r}_c) \times \vec{F}^f = \vec{r}_{p,c} \times \vec{F}^f$

Force de Magnus $\vec{F}^M = \frac{1}{2} \rho |\vec{v}|^2 A C_M (|\vec{\omega}| r / 2 |\vec{v}|) \frac{(\vec{\omega} \times \vec{v})}{|(\vec{\omega} \times \vec{v})|}$

Résolution numérique des EDO

Pour les solides, la forme générale de $\vec{q}(t)$ et $\vec{g}[\vec{q}(t), t]$ est :

$$\begin{aligned}
 \vec{q}(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \\ q_5(t) \\ q_6(t) \\ q_7(t) \\ q_8(t) \\ q_9(t) \\ q_{10}(t) \\ q_{11}(t) \\ q_{12}(t) \\ q_{13}(t) \\ q_{14}(t) \\ q_{15}(t) \\ q_{16}(t) \\ q_{17}(t) \\ q_{18}(t) \end{pmatrix} &= \begin{pmatrix} v_{c,x}(t) \\ v_{c,y}(t) \\ v_{c,z}(t) \\ x_c(t) \\ y_c(t) \\ z_c(t) \\ \omega_x(t) \\ \omega_y(t) \\ \omega_z(t) \\ \mathbf{R}_{xx}(t) \\ \mathbf{R}_{xy}(t) \\ \mathbf{R}_{xz}(t) \\ \mathbf{R}_{yx}(t) \\ \mathbf{R}_{yy}(t) \\ \mathbf{R}_{yz}(t) \\ \mathbf{R}_{zx}(t) \\ \mathbf{R}_{zy}(t) \\ \mathbf{R}_{zz}(t) \end{pmatrix} = \begin{pmatrix} \vec{v}_c(t) \\ \vec{r}_c(t) \\ \vec{\omega}(t) \\ \mathbf{R}(t) \end{pmatrix}
 \end{aligned}$$

$$\vec{g}(\vec{q}, t) = \begin{pmatrix} \vec{a}_c(t) \\ \vec{v}_c(t) \\ \vec{\alpha}(t) \\ \tilde{\omega}\mathbf{R}(t) \end{pmatrix} = \begin{pmatrix} a_{c,x}(\vec{q}, t) \\ a_{c,y}(\vec{q}, t) \\ a_{c,z}(\vec{q}, t) \\ q_1(t) \\ q_2(t) \\ q_3(t) \\ \alpha_x(\vec{q}, t) \\ \alpha_y(\vec{q}, t) \\ \alpha_z(\vec{q}, t) \\ q_8(t)q_{16}(t) - q_9(t)q_{13}(t) \\ q_8(t)q_{17}(t) - q_9(t)q_{14}(t) \\ q_8(t)q_{18}(t) - q_9(t)q_{15}(t) \\ q_9(t)q_{10}(t) - q_7(t)q_{16}(t) \\ q_9(t)q_{11}(t) - q_7(t)q_{17}(t) \\ q_9(t)q_{12}(t) - q_7(t)q_{18}(t) \\ q_7(t)q_{13}(t) - q_8(t)q_{10}(t) \\ q_7(t)q_{14}(t) - q_8(t)q_{11}(t) \\ q_7(t)q_{15}(t) - q_8(t)q_{12}(t) \end{pmatrix}$$

Résolution numérique des EDO

Si on décide d'utiliser les quaternions de rotation au lieu des matrices de rotation, on utilisera

$$\vec{q}(t) = \begin{pmatrix} v_{c,x}(t) \\ v_{c,y}(t) \\ v_{c,z}(t) \\ x_c(t) \\ y_c(t) \\ z_c(t) \\ \omega_x(t) \\ \omega_y(t) \\ \omega_z(t) \\ \vec{R}(t) \begin{cases} R_0(t) \\ R_1(t) \\ R_2(t) \\ R_3(t) \end{cases} \end{pmatrix} \quad \vec{g}(\vec{q}, t) = \begin{pmatrix} a_{c,x}(\vec{q}, t) \\ a_{c,y}(\vec{q}, t) \\ a_{c,z}(\vec{q}, t) \\ q_1(t) \\ q_2(t) \\ q_3(t) \\ \alpha_x(\vec{q}, t) \\ \alpha_y(\vec{q}, t) \\ \alpha_z(\vec{q}, t) \\ \frac{1}{2} \vec{R}(t) \vec{\omega}(t) \begin{cases} (-q_{11}(t)q_7(t) - q_{12}(t)q_8(t) - q_{13}(t)q_9(t))/2 \\ (q_{10}(t)q_7(t) + q_{12}(t)q_9(t) - q_{13}(t)q_8(t))/2 \\ (q_{10}(t)q_8(t) + q_{13}(t)q_7(t) - q_{11}(t)q_9(t))/2 \\ (q_{10}(t)q_9(t) + q_{11}(t)q_8(t) - q_{12}(t)q_7(t))/2 \end{cases} \end{pmatrix}$$

Résolution numérique des EDO

$$\frac{d\vec{q}(t)}{dt} = \vec{g}(\vec{q}(t), t) \quad \vec{q}(t_0) = \vec{q}_0$$

Euler. $\vec{q}(t_n) = \vec{q}(t_{n-1}) + \vec{g}(\vec{q}(t_{n-1}), t_{n-1})\Delta t + O(\Delta t^2)$

Runge-Kutta $\vec{q}(t_n) = \vec{q}(t_{n-1}) + \frac{\Delta t}{6} [\vec{k}_1 + 2\vec{k}_2 + 2\vec{k}_3 + \vec{k}_4] + O(\Delta t^5)$

$$\vec{k}_1 = \vec{g}(\vec{q}(t_{n-1}), t_{n-1})$$

$$\vec{k}_2 = \vec{g}\left(\vec{q}(t_{n-1}) + \frac{\Delta t}{2}\vec{k}_1, t_{n-1} + \frac{\Delta t}{2}\right)$$

$$\vec{k}_3 = \vec{g}\left(\vec{q}(t_{n-1}) + \frac{\Delta t}{2}\vec{k}_2, t_{n-1} + \frac{\Delta t}{2}\right)$$

$$\vec{k}_4 = \vec{g}(\vec{q}(t_{n-1}) + \Delta t\vec{k}_3, t_{n-1} + \Delta t)$$

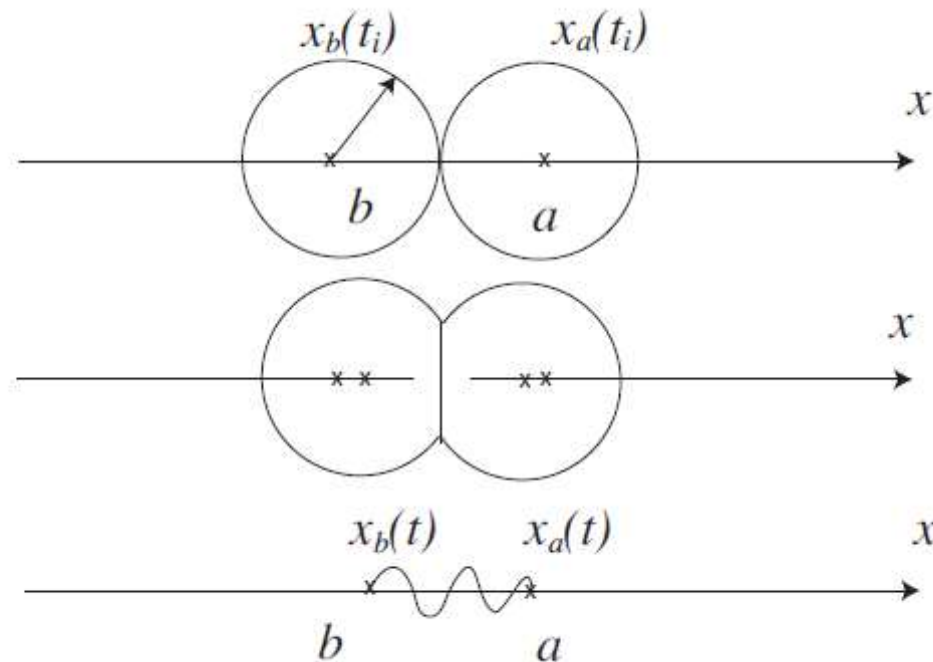
Dynamique des collisions

Méthode des forces

$$F_{a/b}(t) = -k_r(L_r(t) - L_r(t_i)) \frac{x_{a/b}(t) - x_{b/a}(t)}{|x_{a/b}(t) - x_{b/a}(t)|}$$

$$F_{a/b}(t) = -k_a(v_{x,a/b}(t) - v_{x,b/a}(t))$$

$$L_r(t) = |x_a(t) - x_b(t)|$$



Dynamique des collisions

Méthode des conditions initiales

$$\vec{v}_a(t_f) = \vec{v}_a(t_i) + \frac{\vec{J}}{m_a}$$

$$\vec{v}_b(t_f) = \vec{v}_b(t_i) - \frac{\vec{J}}{m_b}$$

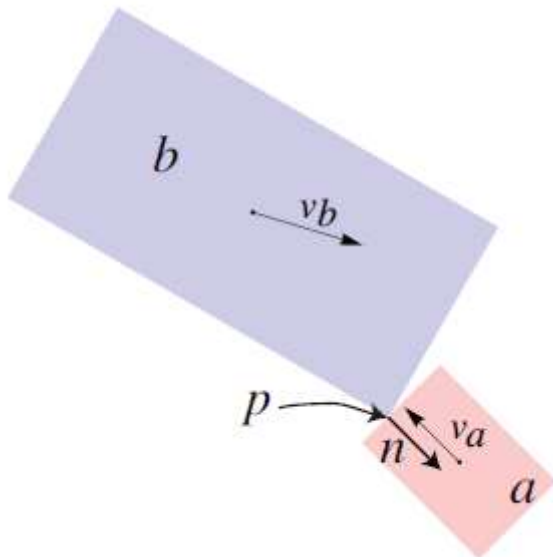
$$\vec{J} = \hat{n} j + \hat{t} j_t$$

$$\vec{\omega}_a(t_f) = \vec{\omega}_a(t_i) + \mathbf{I}_a^{-1} (\vec{r}_{a,p} \times \vec{J})$$

$$\vec{\omega}_b(t_f) = \vec{\omega}_b(t_i) - \mathbf{I}_b^{-1} (\vec{r}_{b,p} \times \vec{J})$$

$$\vec{v}_r^- = \vec{v}_{a,p}(t_i) - \vec{v}_{b,p}(t_i)$$

$$v_r^- = \hat{n} \cdot \vec{v}_r^-$$



$$\hat{u} = \frac{\vec{v}_r^- \times \hat{n}}{|\vec{v}_r^- \times \hat{n}|}$$

$$\hat{t} = \hat{n} \times \hat{u}$$

Dynamique des collisions

Méthode des conditions initiales

$$\begin{aligned}\vec{v}_a(t_f) &= \vec{v}_a(t_i) + \frac{\vec{J}}{m_a} \\ \vec{v}_b(t_f) &= \vec{v}_b(t_i) - \frac{\vec{J}}{m_b} \\ \vec{J} &= \hat{n} j + \hat{t} j_t\end{aligned}$$

$$\begin{aligned}\vec{\omega}_a(t_f) &= \vec{\omega}_a(t_i) + \mathbf{I}_a^{-1} (\vec{r}_{a,p} \times \vec{J}) \\ \vec{\omega}_b(t_f) &= \vec{\omega}_b(t_i) - \mathbf{I}_b^{-1} (\vec{r}_{b,p} \times \vec{J}) \\ \vec{v}_r^- &= \vec{v}_{a,p}(t_i) - \vec{v}_{b,p}(t_i) \\ v_r^- &= \hat{n} \cdot \vec{v}_r^-\end{aligned}$$

Avec frottement

Sans frottement

$$\begin{aligned}j &= -\alpha(1 + \epsilon) v_r^- \\ \alpha &= \frac{1}{\frac{1}{m_a} + \frac{1}{m_b} + G_a + G_b} \\ G_a &= \hat{n} \cdot [(\mathbf{I}_a^{-1} (\vec{r}_{a,p} \times \hat{n})) \times \vec{r}_{a,p}] \\ G_b &= \hat{n} \cdot [(\mathbf{I}_b^{-1} (\vec{r}_{b,p} \times \hat{n})) \times \vec{r}_{b,p}]\end{aligned}$$

$$\begin{aligned}j_t &= \begin{cases} \alpha_t \mu_c (1 + \epsilon) v_r^- & \text{si } \mu_s (1 + \epsilon) |v_r^-| < |\hat{t} \cdot \vec{v}_r^-| \\ -\alpha_t |\hat{t} \cdot \vec{v}_r^-| & \text{sinon} \end{cases} \\ \alpha_t &= \frac{1}{\frac{1}{m_a} + \frac{1}{m_b} + G_{a,t} + G_{b,t}} \\ G_{a,t} &= \hat{t} \cdot [(\mathbf{I}_a^{-1} (\vec{r}_{a,p} \times \hat{t})) \times \vec{r}_{a,p}] \\ G_{b,t} &= \hat{t} \cdot [(\mathbf{I}_b^{-1} (\vec{r}_{b,p} \times \hat{t})) \times \vec{r}_{b,p}]\end{aligned}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$n_m = \frac{c}{c_m} = \sqrt{\epsilon_r \mu_r}$$

$$v\lambda = c$$

$$c = 299792458 \text{ m/s}$$

$$\rho(r) \propto \frac{1}{4\pi r^2}$$

Optique

équations pour une ligne en 3D

$$\vec{r}(s) = \vec{r}(0) + s\hat{u}$$

$$\hat{u}_i = \frac{\vec{r}_m - \vec{r}_s}{|\vec{r}_m - \vec{r}_s|} \quad \hat{j} = \frac{\hat{u}_i \times \hat{i}}{|\hat{u}_i \times \hat{i}|} \quad \hat{k} = \hat{i} \times \hat{j}$$

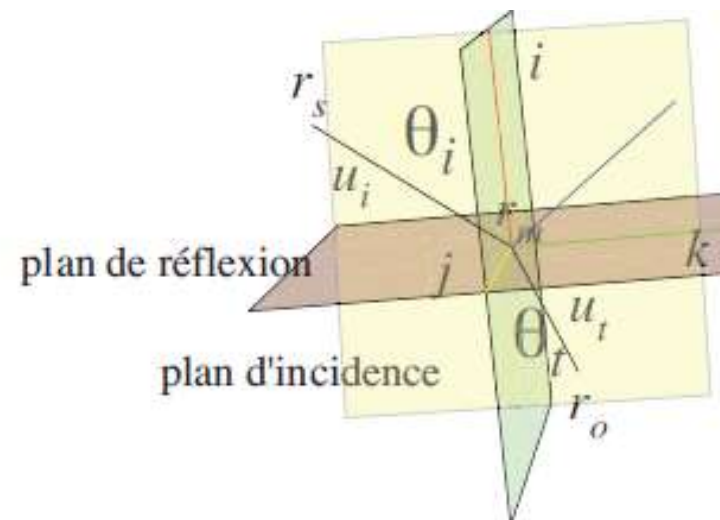
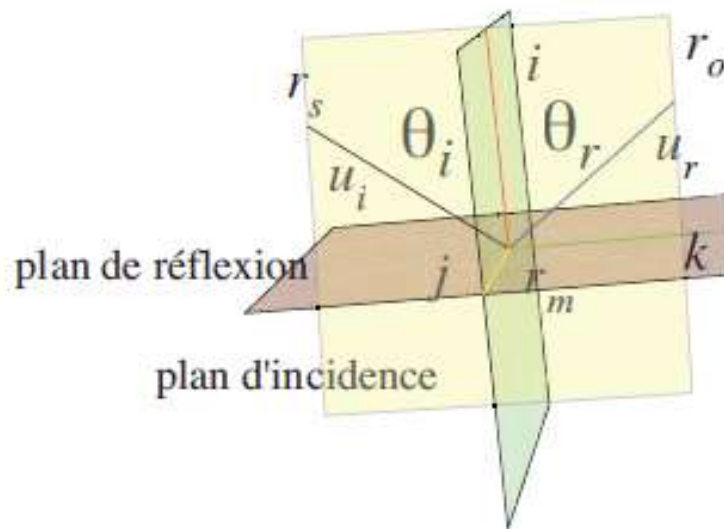
$$\sin \theta_r = \sin \theta_i$$

$$\hat{u}_r = \hat{u}_i - 2\hat{i}(\hat{u}_i \cdot \hat{i}) = \cos \theta_i \hat{i} + \sin \theta_i \hat{k}$$

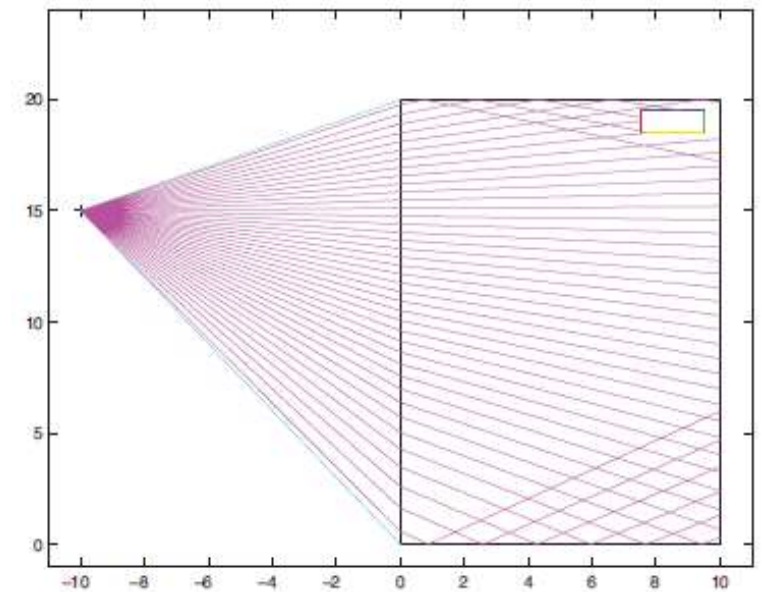
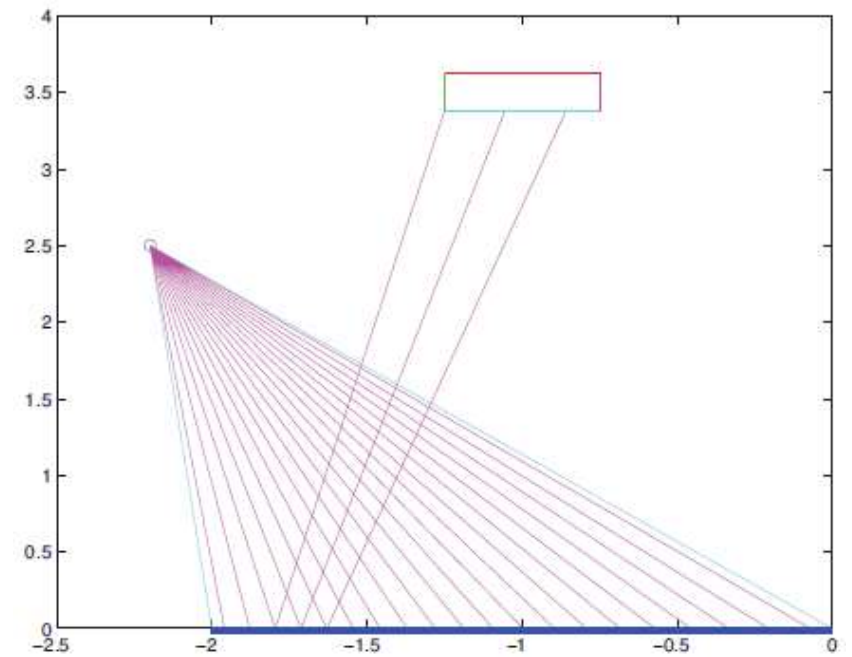
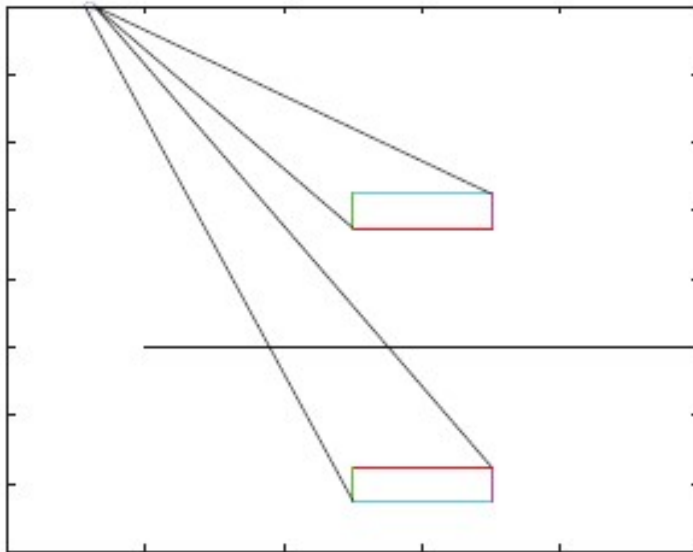
$$n_i \sin \theta_i = n_t \sin \theta_t$$

$$\frac{\sin(\theta_i)}{c_i} = \frac{\sin(\theta_t)}{c_t}$$

$$\hat{u}_t = -\hat{i} \cos \theta_t + \hat{k} \sin \theta_t$$



- méthode des images virtuelles
- méthode des rayons.



Onde transverse électrique

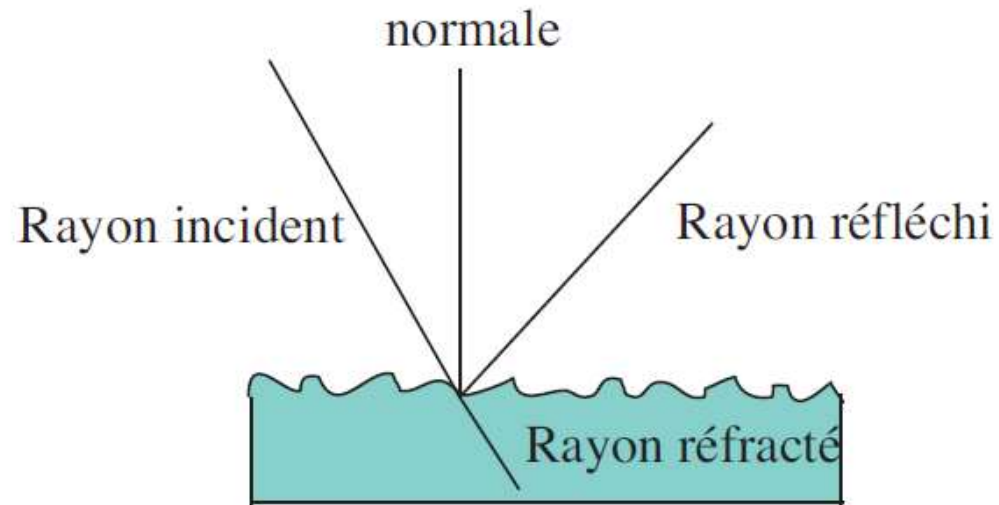
$$\left(\frac{R}{T} \right) = \left(\frac{\left[\frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)} \right]^2}{1 - R} \right)$$

Onde transverse magnétique

$$\left(\frac{R}{T} \right) = \left(\frac{\left[\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} \right]^2}{1 - R} \right)$$

Onde non polarisée

$$\left(\frac{R}{T} \right) = \left(\frac{\frac{1}{2} \left\{ \left[\frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)} \right]^2 + \left[\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} \right]^2 \right\}}{1 - R} \right)$$



Modèle de Phong

Modèle de la
réflexion physique

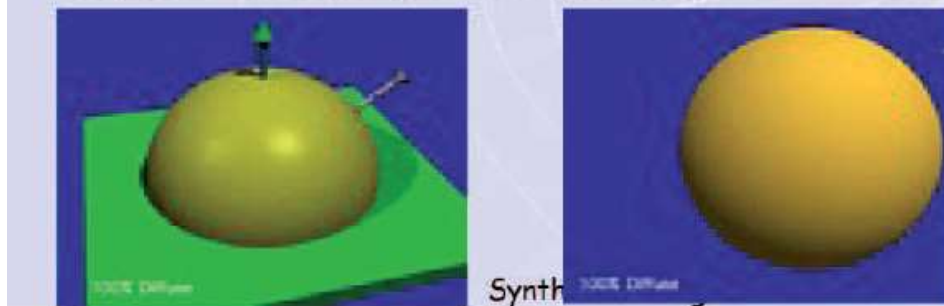
Modèle de Phong

$$I_{\text{Phong}} = k_{\text{diffuse}} I_{\text{diffuse}} + k_{\text{si}} I_{\text{si}}$$

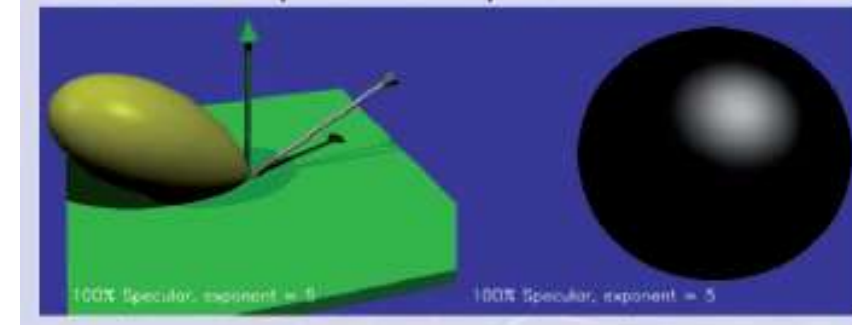
$$I_{\text{diffuse}} = k_{d,c} I_i (\vec{L} \cdot \vec{n}) = k_{d,c} I_i \cos \theta$$

$$I_{\text{si}} = k_{\text{si},c} I_i (\vec{R} \cdot \vec{V})^m$$

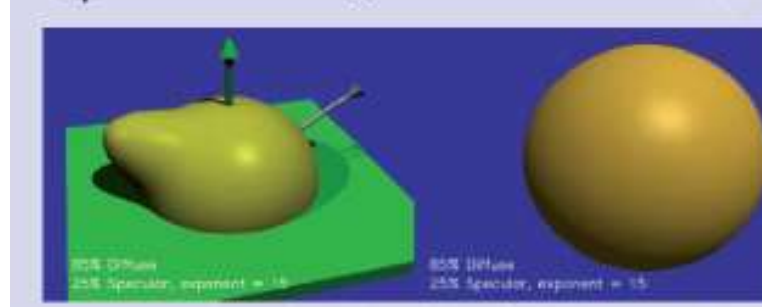
- Diffusion totale (loi de Lambert)



- Réflecteur spéculaire imparfait



- Spécularité et diffusion



Ondes sonores

Équation d'onde sonore

$$\frac{\partial^2 \tilde{p}}{\partial t^2} = \gamma \frac{\partial^2 \tilde{p}}{\partial x^2} = c^2 \frac{\partial^2 \tilde{p}}{\partial x^2}$$

$$\frac{\partial^2 v}{\partial t^2} = \gamma \frac{\partial^2 v}{\partial x^2} = c^2 \frac{\partial^2 v}{\partial x^2}$$

Vitesse d'onde sonore

$$c_{\text{gaz}} = \sqrt{\frac{c_p \cdot p}{c_v \rho}} = \sqrt{\frac{\beta \cdot p}{\rho}} \approx \sqrt{\frac{\beta \cdot p_0}{\rho_0}}$$

$$c_{\text{air}}(T) = c_{\text{air}}(\Theta) = (331.3 + 0.606\Theta) \text{ m/s}$$

Intensité sonore

$$I = \frac{P}{S} = \frac{P}{4\pi r^2}$$

$$I = v \tilde{p}$$

$$I \approx \frac{\tilde{p}^2}{\rho c}$$

Ondes sonores

puissance sonore en dB

$$L_p = 10 \log \left(\frac{P}{10^{-12}} \right) \text{ dB}$$

(P en watt)

Intensité sonore en dB

$$L_I = 10 \log \left(\frac{I}{10^{-12}} \right) \text{ dB}$$

(I en $\frac{\text{W}}{\text{m}^2}$)

$$L_I(r) = L_P - 10 \log \left(\frac{4\pi r^2}{1} \right) \quad (r \text{ en m})$$

Atténuation des ondes sonores

Dispersion

$$L_I(r) = L_I(r_0) - 20 \log \left(\frac{r}{r_0} \right) \text{ dB}$$

Absorption

$$L_I(r) = L_I(r_0) - A(r - r_0) \text{ dB}$$

$$L_I(r) = L_I(r_0) - 20 \log \left(\frac{r}{r_0} \right) - \alpha(r - r_0) \text{ dB}$$

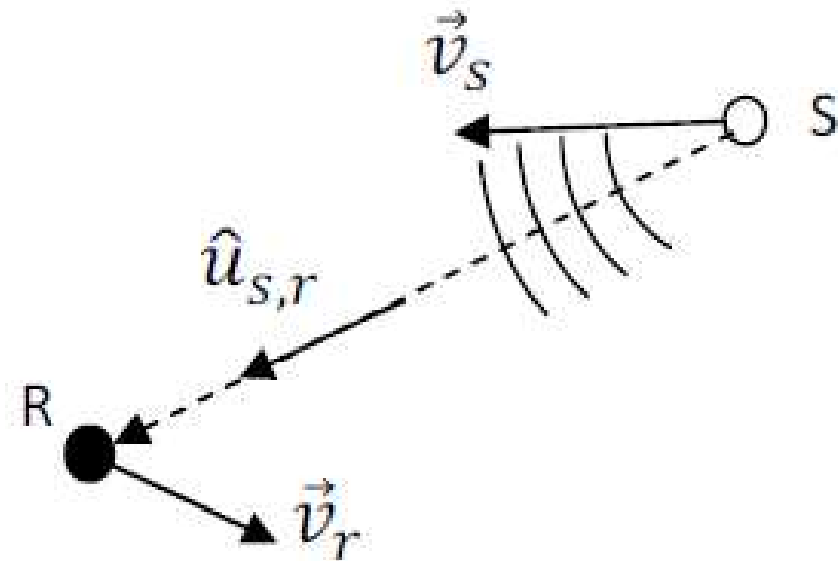
Effet Doppler

$$\vec{u}_{s,r} = \frac{\vec{r}_r(t) - \vec{r}_s(t)}{|\vec{r}_r(t) - \vec{r}_s(t)|}$$

$$v_r = \frac{c - (\vec{v}_r \cdot \vec{u}_{s,r})}{c - (\vec{v}_s \cdot \vec{u}_{s,r})} v_s = \frac{1 - \beta_r}{1 - \beta_s} v_s$$

$$\beta_r = \frac{(\vec{v}_r \cdot \vec{u}_{s,r})}{c} = \frac{|\vec{v}_r| \cos(\theta)}{c}$$

$$\beta_s = \frac{(\vec{v}_s \cdot \vec{u}_{s,r})}{c} = \frac{|\vec{v}_s| \cos(\theta')}{c}$$



Réflexion/réfraction des ondes sonores

$$\sin \theta_r = \sin \theta_i$$

$$\sin \theta_t = \left(\frac{c_t}{c_i} \right) \sin \theta_i$$

Réflexion/réfraction des ondes sonores

Impédance acoustique

Le coefficient de réflexion d'une onde acoustique à l'interface entre deux milieux d'impédance différentes est donné par:

$$R = \left(\frac{Z_i - Z_t}{Z_i + Z_t} \right)^2$$

avec $Z_i = \rho_i c_i$ et $Z_t = \rho_t c_t$

Le coefficient de transmission est donné par:

$$T = 1 - R = \frac{4 Z_i Z_t}{(Z_i + Z_t)^2}$$