POLYTECHNIQUE
MONTRÉAL

PHS 4700
Physique pour les applications multimédia

Résumé

D.Seddaoui

Cinématique

$$\frac{d\vec{v}_c(t)}{dt} = \vec{a}_c(t, \vec{v}_c, \vec{r}_c, \vec{\omega}(t), \vec{\Omega}(t))$$

$$\frac{d\vec{r}_c(t)}{dt} = \vec{v}_c(t, \vec{v}_c, \vec{r}_c, \vec{\omega}(t), \vec{\Omega}(t))$$

$$\frac{d\vec{\omega}(t)}{dt} = \vec{\alpha}(t, \vec{v}_c, \vec{r}_c, \vec{\omega}(t), \vec{\Omega}(t))$$

$$\frac{d\vec{\Omega}(t)}{dt} = \vec{\omega}(t, \vec{v}_c, \vec{r}_c, \vec{\omega}(t), \vec{\Omega}(t))$$

Cinématique

$$\boldsymbol{R}^{G\leftarrow L} = \boldsymbol{R}_z(\theta_z)\boldsymbol{R}_y(\theta_y)\boldsymbol{R}_x(\theta_x)$$

$$\boldsymbol{R}_x(\theta_x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{pmatrix}$$

$$\boldsymbol{R}_y(\theta_y) = \begin{pmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{pmatrix}$$

$$\boldsymbol{R}_z(\theta_z) = \begin{pmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{R}^{G\leftarrow L} = \begin{pmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{pmatrix} = \begin{pmatrix} \vec{e}_x^{\,L} & \vec{e}_y^{\,L} & \vec{e}_z^{\,L} \end{pmatrix}$$

$$\boldsymbol{R}(t) = \begin{pmatrix} u_x^2 + (u_y^2 + u_z^2)\cos\theta & u_x u_y(1-\cos\theta) - u_z\sin\theta & u_x u_z(1-\cos\theta) + u_y\sin\theta \\ u_x u_y(1-\cos\theta) + u_z\sin\theta & u_y^2 + (u_z^2 + u_x^2)\cos\theta & u_y u_z(1-\cos\theta) - u_x\sin\theta \\ u_x u_z(1-\cos\theta) - u_y\sin\theta & u_y u_z(1-\cos\theta) + u_x\sin\theta & u_z^2 + (u_x^2 + u_y^2)\cos\theta \end{pmatrix}$$

Cinématique

$$\frac{d\boldsymbol{R}(t)}{dt} = \tilde{\boldsymbol{\omega}}(t)\boldsymbol{R}(t)$$

$$\tilde{\boldsymbol{\omega}}(t) = \begin{pmatrix} 0 & -\omega_z(t) & \omega_y(t) \\ \omega_z(t) & 0 & -\omega_x(t) \\ -\omega_y(t) & \omega_x(t) & 0 \end{pmatrix}$$

Cinématique

$$\vec{r}_i(t) = \vec{r}_c(t) + \boldsymbol{R}(t)\vec{r}_{i,c}(t_0)$$
$$\vec{v}_i(t) = \vec{v}_c(t) + \vec{\omega}(t) \times \left(\boldsymbol{R}(t)\vec{r}_{i,c}(t_0)\right)$$

$$\vec{v}^G = \boldsymbol{R}^{G \leftarrow L}\vec{v}^L$$

$$\boldsymbol{M}^G = \boldsymbol{R}^{G \leftarrow L}\boldsymbol{M}^L\left(\boldsymbol{R}^{G \leftarrow L}\right)^{-1}$$

Cinématique

$$\vec{\vec{r}} = (\cos(\theta/2), \sin(\theta/2)\vec{u}^T)^T$$

$$\vec{\vec{q}}\,' = (0, (\vec{v}')^T)^T = \vec{\vec{r}}\,(0, \vec{v}^T)^T\,\vec{\vec{r}}^* = \vec{\vec{r}}\,\vec{\vec{q}}\,\vec{\vec{r}}^*$$

$$\frac{d\vec{\vec{R}}(t)}{dt} = \frac{1}{2}\vec{\vec{R}}(t)\vec{\vec{\omega}}(t)$$

$$\vec{\vec{\omega}}(t) = (0, \vec{\omega}(t))^T$$

$$\vec{\vec{q}} = q_0 + iq_x + jq_y + kq_z$$

$$i^2 = j^2 = k^2 = -1$$

$$ij = k = -ji$$

$$jk = i = -kj$$

$$ki = j = -ik$$

$$\begin{aligned}\vec{\vec{q}}^1\,\vec{\vec{q}}^2 = (&q_0^1 q_0^2 - q_x^1 q_x^2 - q_y^1 q_y^2 - q_z^1 q_z^2,\\ &q_0^1 q_x^2 + q_x^1 q_0^2 + q_y^1 q_z^2 - q_z^1 q_y^2,\\ &q_0^1 q_y^2 + q_y^1 q_0^2 + q_z^1 q_x^2 - q_x^1 q_z^2,\\ &q_0^1 q_z^2 + q_z^1 q_0^2 + q_x^1 q_y^2 - q_y^1 q_x^2)^T\end{aligned}$$

Dynamique

Translation

$$\vec{p}(t) = m\vec{v}(t)$$

$$\vec{F}(t) = \frac{d\vec{p}(t)}{dt}$$

Masse constante $\vec{a}(t) = \frac{\vec{F}(t)}{m}$

$$\vec{F}_a = -\vec{F}_r$$

$$E = \int_0^L \vec{F}_a \cdot d\vec{l}$$

Rotation

$$\vec{L}(t) = \boldsymbol{I}\vec{\omega}(t)$$

$$\vec{\tau}(t) = \frac{d\vec{L}(t)}{dt}$$

$$\vec{\tau}(t) = \frac{d(\boldsymbol{I}\vec{\omega}(t))}{dt} = \boldsymbol{I}\vec{\alpha} + \frac{d\boldsymbol{I}}{dt}\vec{\omega}$$

$$\vec{\tau}(t) = (\vec{r}(t) - \vec{r}_c) \times \vec{F}(t)$$

$$\vec{\alpha}(t) = \frac{d\vec{\omega}(t)}{dt} = (\boldsymbol{I}(t))^{-1}\left(\vec{\tau}(t) + \vec{L}(t) \times \vec{\omega}(t)\right)$$

Dynamique

$$\vec{r}_c = \frac{1}{m}\sum_{n=1}^{N}\int_{\mathcal{V}_n} \vec{r}\rho_n(\vec{r})d^3r$$

$$\boldsymbol{I}_c = \int_{\mathcal{V}} \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -yx & x^2+z^2 & -yz \\ -zx & -zy & x^2+y^2 \end{pmatrix} \rho(\vec{r})d^3r$$

$$m = \sum_{n=1}^{N}\int_{\mathcal{V}_n} \rho_n(\vec{r})d^3r$$

$$\boldsymbol{I}_d = \boldsymbol{I}_c + m\begin{pmatrix} (d_{c,y}^2+d_{c,z}^2) & -d_{c,x}d_{c,y} & -d_{c,x}d_{c,z} \\ -d_{c,y}d_{c,x} & (d_{c,x}^2+d_{c,z}^2) & -d_{c,y}d_{c,z} \\ -d_{c,z}d_{c,x} & -d_{c,z}d_{c,y} & (d_{c,x}^2+d_{c,y}^2) \end{pmatrix}$$

$$\boldsymbol{I}_c = \sum_i \boldsymbol{I}_{i,c}^{RT}$$

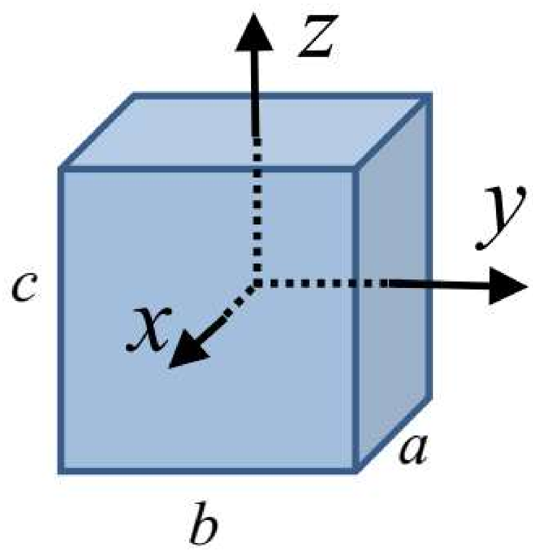
Moments d'inertie de quelques géométries simples:

Parallélépipède dont les axes sont parallèles aux axes x (longueur a), y (largeur b) et z (hauteur c) par rapport à son centre de masse

$$I_{c,xx} = \frac{m}{12}(b^2 + c^2)$$

$$I_{c,yy} = \frac{m}{12}(a^2 + c^2)$$

$$I_{c,zz} = \frac{m}{12}(a^2 + b^2)$$



Les termes non diagonaux sontnuls.

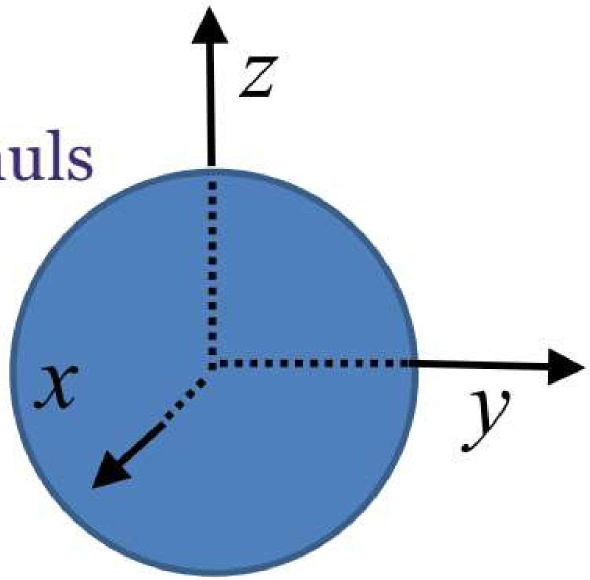
Dynamique

Sphères de rayon r par rapport à son centre de masse

- Pleine

$$I_{c,xx} = I_{c,yy} = I_{c,zz} = \frac{2m}{5} r^2$$

Les termes hors de la diagonale sont nuls



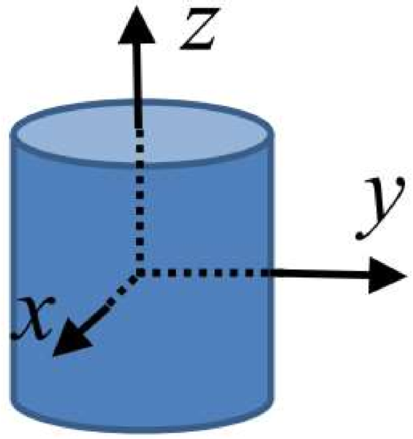
- Creux

$$I_{c,xx} = I_{c,yy} = I_{c,zz} = \frac{2m}{3} r^2$$

Les termes hors de la diagonale sont nuls.

Dynamique

Cylindre de rayon r et de longueur l aligné avec l'axe des z par rapport à son centre demasse



- Plein

$$I_{c,zz} = \frac{m}{2} r^2$$

$$I_{c,xx} = I_{c,yy} = \frac{m}{4} r^2 + \frac{m}{12} l^2$$

Les termes hors de la diagonale sont nuls.

- Creux

$$I_{c,zz} = m r^2$$

$$I_{c,xx} = I_{c,yy} = \frac{m}{2} r^2 + \frac{m}{12} l^2$$

Les termes hors de la diagonale sont nuls.

Dynamique

Exemple de forces indépendantes de la rotation des solides

électrique
$$\vec{F}^G_{1,2} = k\frac{q_1 q_2(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

gravitationnelle
$$\vec{F}^G_{1,2} = -G\frac{m_1 m_2(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

Amortissement
$$\vec{F}^a = -k_a\left[(\vec{v} - \vec{v}_a)\cdot\hat{u}\right]\hat{u}$$

Frottement sec
$$\vec{F}^f = -\mu_s N\frac{\vec{F}}{|\vec{F}|}$$

Viscosité
$$\vec{F}^{\text{vis}} = \begin{cases} -k\eta\vec{v} & |\vec{v}| > 0 \\ 0 & |\vec{v}| = 0 \end{cases}$$

Portance
$$|\vec{F}^P| = \frac{1}{2}\rho|\vec{v}|^2 A C_P$$

Dynamique

Exemple de forces liées à la rotation des solides

Condition de roulement

$$\vec{v}_r = -\vec{\omega} \times \vec{r}_{p,c}$$

Frottement de roulement

$$\vec{F}^{\text{roulement}} = -\mu_d m g \hat{z}$$

$$\tau^{\text{roulement}} = \vec{r}_{d,c} \times \vec{F}^{\text{roulement}}$$

Frottement de glissement

$$\vec{F}^f = -\mu_c N \frac{\vec{v}_{\text{glissement}}}{|\vec{v}_{\text{glissement}}|}$$

$$\vec{\tau}^{\text{glissement}} = (\vec{r}_p - \vec{r}_c) \times \vec{F}^f = \vec{r}_{p,c} \times \vec{F}^f$$

Force de Magnus

$$\vec{F}^M = \frac{1}{2}\rho|\vec{v}|^2 A C_M(|\vec{\omega}|r/2|\vec{v}|)\frac{(\vec{\omega} \times \vec{v})}{|(\vec{\omega} \times \vec{v})|}$$

Résolution numérique des EDO

Pour les solides, la forme générale de $\vec{q}(t)$ et $\vec{g}[\vec{q}(t), t]$ est :

$$\vec{q}(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \\ q_5(t) \\ q_6(t) \\ q_7(t) \\ q_8(t) \\ q_9(t) \\ q_{10}(t) \\ q_{11}(t) \\ q_{12}(t) \\ q_{13}(t) \\ q_{14}(t) \\ q_{15}(t) \\ q_{16}(t) \\ q_{17}(t) \\ q_{18}(t) \end{pmatrix} = \begin{pmatrix} \left.\begin{matrix} v_{c,x}(t) \\ v_{c,y}(t) \\ v_{c,z}(t) \end{matrix}\right\} \vec{v}_c(t) \\ \left.\begin{matrix} x_c(t) \\ y_c(t) \\ z_c(t) \end{matrix}\right\} \vec{r}_c(t) \\ \left.\begin{matrix} \omega_x(t) \\ \omega_y(t) \\ \omega_z(t) \end{matrix}\right\} \vec{\omega}(t) \\ \left.\begin{matrix} \boldsymbol{R}_{xx}(t) \\ \boldsymbol{R}_{xy}(t) \\ \boldsymbol{R}_{xz}(t) \\ \boldsymbol{R}_{yx}(t) \\ \boldsymbol{R}_{yy}(t) \\ \boldsymbol{R}_{yz}(t) \\ \boldsymbol{R}_{zx}(t) \\ \boldsymbol{R}_{zy}(t) \\ \boldsymbol{R}_{zz}(t) \end{matrix}\right\} \boldsymbol{R}(t) \end{pmatrix}$$

$$\vec{g}(\vec{q}, t) = \begin{pmatrix} \vec{a}_c(t) \left\{\begin{matrix} a_{c,x}(\vec{q}, t) \\ a_{c,y}(\vec{q}, t) \\ a_{c,z}(\vec{q}, t) \end{matrix}\right. \\ \vec{v}_c(t) \left\{\begin{matrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{matrix}\right. \\ \vec{\alpha}(t) \left\{\begin{matrix} \alpha_x(\vec{q}, t) \\ \alpha_y(\vec{q}, t) \\ \alpha_z(\vec{q}, t) \end{matrix}\right. \\ \tilde{\boldsymbol{\omega}}\boldsymbol{R}(t) \left\{\begin{matrix} q_8(t)q_{16}(t) - q_9(t)q_{13}(t) \\ q_8(t)q_{17}(t) - q_9(t)q_{14}(t) \\ q_8(t)q_{18}(t) - q_9(t)q_{15}(t) \\ q_9(t)q_{10}(t) - q_7(t)q_{16}(t) \\ q_9(t)q_{11}(t) - q_7(t)q_{17}(t) \\ q_9(t)q_{12}(t) - q_7(t)q_{18}(t) \\ q_7(t)q_{13}(t) - q_8(t)q_{10}(t) \\ q_7(t)q_{14}(t) - q_8(t)q_{11}(t) \\ q_7(t)q_{15}(t) - q_8(t)q_{12}(t) \end{matrix}\right. \end{pmatrix}$$

Résolution numérique des EDO

Si on décide d'utiliser les quaternions de rotation au lieu des matrices de rotation, on utilisera

$$\vec{q}(t) = \begin{pmatrix} v_{c,x}(t) \\ v_{c,y}(t) \\ v_{c,z}(t) \\ x_c(t) \\ y_c(t) \\ z_c(t) \\ \omega_x(t) \\ \omega_y(t) \\ \omega_z(t) \\ \vec{\vec{R}}(t)\left\{\begin{matrix} \boldsymbol{R}_0(t) \\ \boldsymbol{R}_1(t) \\ \boldsymbol{R}_2(t) \\ \boldsymbol{R}_3(t) \end{matrix}\right. \end{pmatrix} \qquad \vec{\mathrm{g}}(\vec{q},t) = \begin{pmatrix} a_{c,x}(\vec{q},t) \\ a_{c,y}(\vec{q},t) \\ a_{c,z}(\vec{q},t) \\ q_1(t) \\ q_2(t) \\ q_3(t) \\ \alpha_x(\vec{q},t) \\ \alpha_y(\vec{q},t) \\ \alpha_z(\vec{q},t) \\ \frac{1}{2}\vec{\vec{R}}(t)\vec{\vec{\omega}}(t)\left\{\begin{matrix} (-q_{11}(t)q_7(t) - q_{12}(t)q_8(t) - q_{13}(t)q_9(t))/2 \\ (q_{10}(t)q_7(t) + q_{12}(t)q_9(t) - q_{13}(t)q_8(t))/2 \\ (q_{10}(t)q_8(t) + q_{13}(t)q_7(t) - q_{11}(t)q_9(t))/2 \\ (q_{10}(t)q_9(t) + q_{11}(t)q_8(t) - q_{12}(t)q_7(t))/2 \end{matrix}\right. \end{pmatrix}$$

Résolution numérique des EDO

$$\frac{d\vec{q}(t)}{dt} = \vec{g}(\vec{q}(t), t) \qquad \vec{q}(t_0) = \vec{q}_0$$

Euler $\vec{q}(t_n) = \vec{q}(t_{n-1}) + \vec{g}(\vec{q}(t_{n-1}), t_{n-1})\Delta t + O(\Delta t^2)$

Runge-Kutta $\vec{q}(t_n) = \vec{q}(t_{n-1}) + \frac{\Delta t}{6}\left[\vec{k}_1 + 2\vec{k}_2 + 2\vec{k}_3 + \vec{k}_4\right] + O(\Delta t^5)$

$$\vec{k}_1 = \vec{g}(\vec{q}(t_{n-1}), t_{n-1})$$

$$\vec{k}_2 = \vec{g}(\vec{q}(t_{n-1}) + \frac{\Delta t}{2}\vec{k}_1, t_{n-1} + \frac{\Delta t}{2})$$

$$\vec{k}_3 = \vec{g}(\vec{q}(t_{n-1}) + \frac{\Delta t}{2}\vec{k}_2, t_{n-1} + \frac{\Delta t}{2})$$

$$\vec{k}_4 = \vec{g}(\vec{q}(t_{n-1}) + \Delta t\vec{k}_3, t_{n-1} + \Delta t)$$

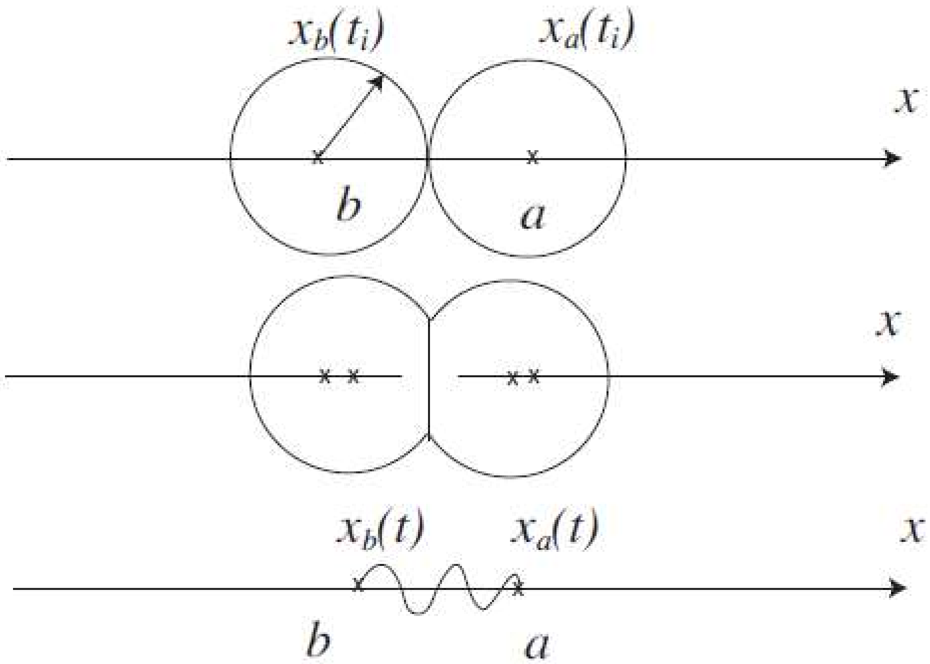
Dynamique des collisions

Méthode des forces

$$F_{a/b}(t) = -k_r(L_r(t) - L_r(t_i))\frac{x_{a/b}(t) - x_{b/a}(t)}{|x_{a/b}(t) - x_{b/a}(t)|}$$

$$F_{a/b}(t) = -k_a(v_{x,a/b}(t) - v_{x,b/a}(t))$$

$$L_r(t) = |x_a(t) - x_b(t)|$$



Dynamique des collisions

Méthode des conditions initiales

$$\vec{v}_a(t_f) = \vec{v}_a(t_i) + \frac{\vec{J}}{m_a}$$

$$\vec{v}_b(t_f) = \vec{v}_b(t_i) - \frac{\vec{J}}{m_b}$$

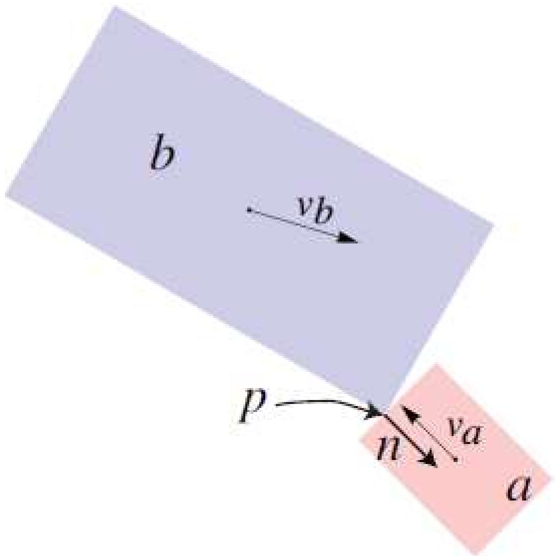
$$\vec{J} = \hat{n}\, j + \hat{t}\, j_t$$

$$\vec{\omega}_a(t_f) = \vec{\omega}_a(t_i) + \mathbf{I}_a^{-1}\left(\vec{r}_{a,p} \times \vec{J}\right)$$

$$\vec{\omega}_b(t_f) = \vec{\omega}_b(t_i) - \mathbf{I}_b^{-1}\left(\vec{r}_{b,p} \times \vec{J}\right)$$

$$\vec{v}_r^{\,-} = \vec{v}_{a,p}(t_i) - \vec{v}_{b,p}(t_i)$$

$$v_r^- = \hat{n}.\vec{v}_r^{\,-}$$



$$\hat{u} = \frac{\vec{v}_r^{\,-} \times \hat{n}}{|\vec{v}_r^{\,-} \times \hat{n}|}$$

$$\hat{t} = \hat{n} \times \hat{u}$$

Dynamique des collisions

Méthode des conditions initiales

$$\vec{v}_a(t_f) = \vec{v}_a(t_i) + \frac{\vec{J}}{m_a}$$

$$\vec{v}_b(t_f) = \vec{v}_b(t_i) - \frac{\vec{J}}{m_b}$$

$$\vec{J} = \hat{n}\, j + \hat{t}\, j_t$$

$$\vec{\omega}_a(t_f) = \vec{\omega}_a(t_i) + \mathbf{I}_a^{-1}\left(\vec{r}_{a,p} \times \vec{J}\right)$$

$$\vec{\omega}_b(t_f) = \vec{\omega}_b(t_i) - \mathbf{I}_b^{-1}\left(\vec{r}_{b,p} \times \vec{J}\right)$$

$$\vec{v}_r^{\,-} = \vec{v}_{a,p}(t_i) - \vec{v}_{b,p}(t_i)$$

$$v_r^- = \hat{n}.\vec{v}_r^{\,-}$$

Sans frottement

$$j = -\alpha(1+\epsilon)\, v_-^r$$

$$\alpha = \frac{1}{\frac{1}{m_a} + \frac{1}{m_b} + G_a + G_b}$$

$$G_a = \hat{n} \cdot \left[\left(\mathbf{I}_a^{-1}(\vec{r}_{a,p} \times \hat{n})\right) \times \vec{r}_{a,p}\right]$$

$$G_b = \hat{n} \cdot \left[\left(\mathbf{I}_b^{-1}(\vec{r}_{b,p} \times \hat{n})\right) \times \vec{r}_{b,p}\right]$$

Avec frottement

$$j_t = \begin{cases} \alpha_t \mu_c (1+\epsilon) v_r^- & \text{si } \mu_s(1+\epsilon)|v_r^-| < |\hat{t} \cdot \vec{v}_r^{\,-}| \\ -\alpha_t |\hat{t} \cdot \vec{v}_r^{\,-}| & \text{sinon} \end{cases}$$

$$\alpha_t = \frac{1}{\frac{1}{m_a} + \frac{1}{m_b} + G_{a,t} + G_{b,t}}$$

$$G_{a,t} = \hat{t} \cdot \left[\left(\mathbf{I}_a^{-1}(\vec{r}_{a,p} \times \hat{t})\right) \times \vec{r}_{a,p}\right]$$

$$G_{b,t} = \hat{t} \cdot \left[\left(\mathbf{I}_b^{-1}(\vec{r}_{b,p} \times \hat{t})\right) \times \vec{r}_{b,p}\right]$$

Optique

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad n_m = \frac{c}{c_m} = \sqrt{\varepsilon_r \mu_r} \qquad \nu\lambda = c$$

$$c = 299792458 \text{ m/s} \qquad \rho(r) \propto \frac{1}{4\pi r^2}$$

Optique

équations pour une ligne en 3D

$$\vec{r}(s) = \vec{r}(0) + s\hat{u}$$

$$\hat{u}_i = \frac{\vec{r}_m - \vec{r}_s}{|\vec{r}_m - \vec{r}_s|} \qquad \hat{j} = \frac{\hat{u}_i \times \hat{i}}{|\hat{u}_i \times \hat{i}|} \qquad \hat{k} = \hat{i} \times \hat{j}$$

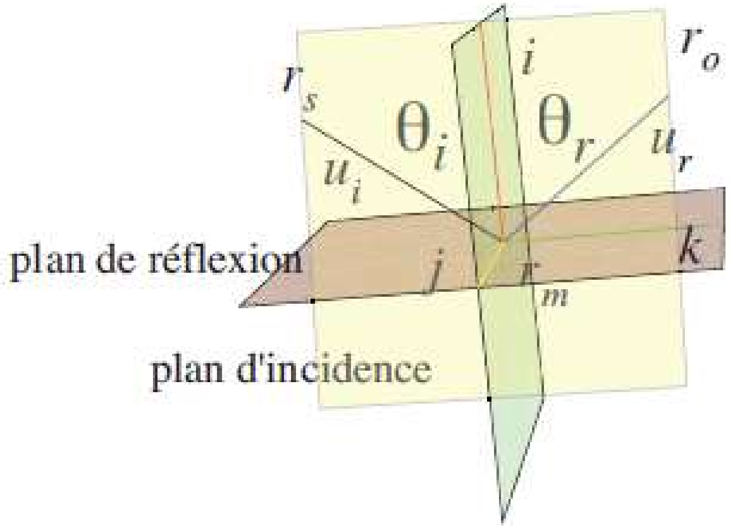
$$n_i \sin\theta_i = n_t \sin\theta_t$$

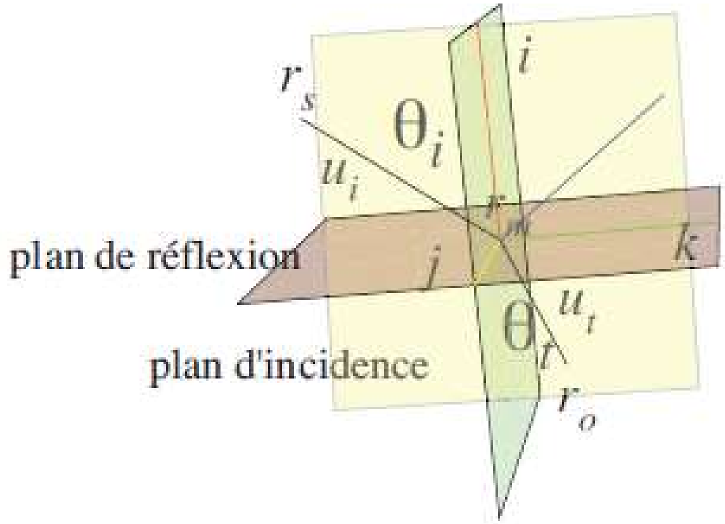
$$\frac{\sin(\theta_i)}{c_i} = \frac{\sin(\theta_t)}{c_t}$$

$$\sin\theta_r = \sin\theta_i$$

$$\hat{u}_r = \hat{u}_i - 2\hat{i}(\hat{u}_i \cdot \hat{i}) = \cos\theta_i \hat{i} + \sin\theta_i \hat{k}$$

$$\hat{u}_t = -\hat{i}\cos\theta_t + \hat{k}\sin\theta_t$$

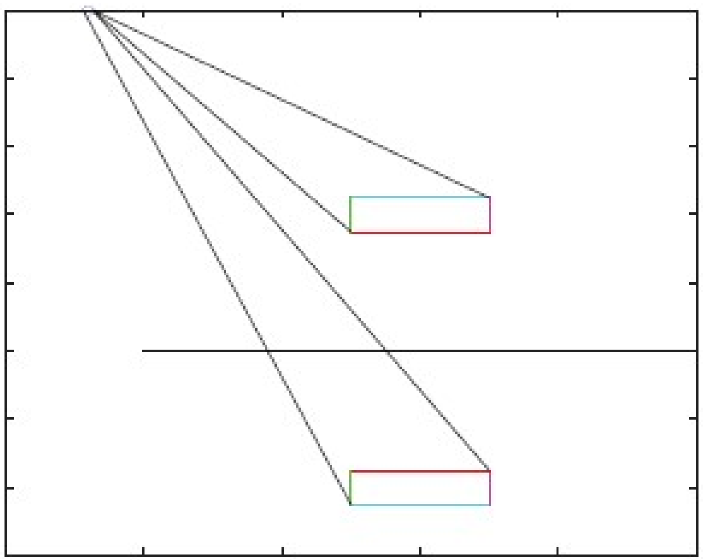


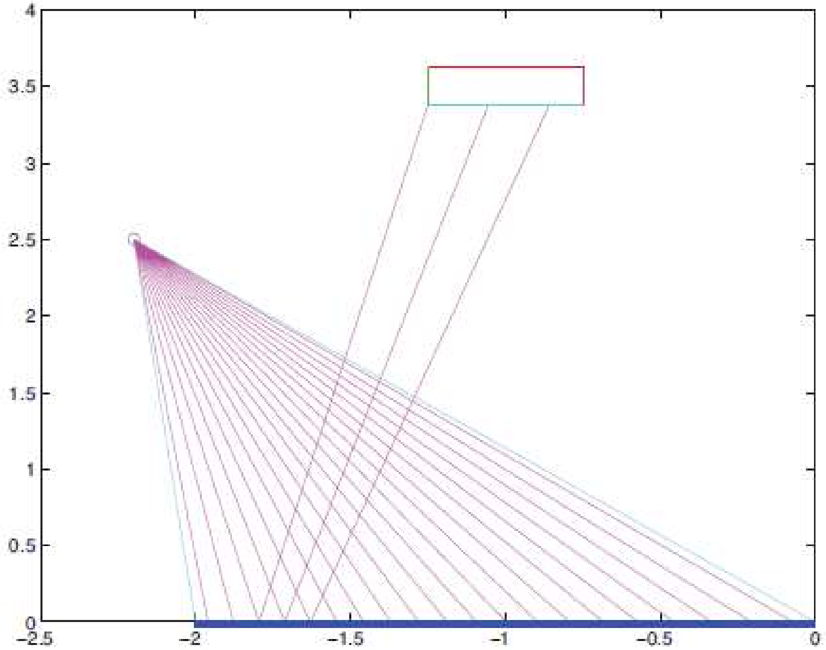


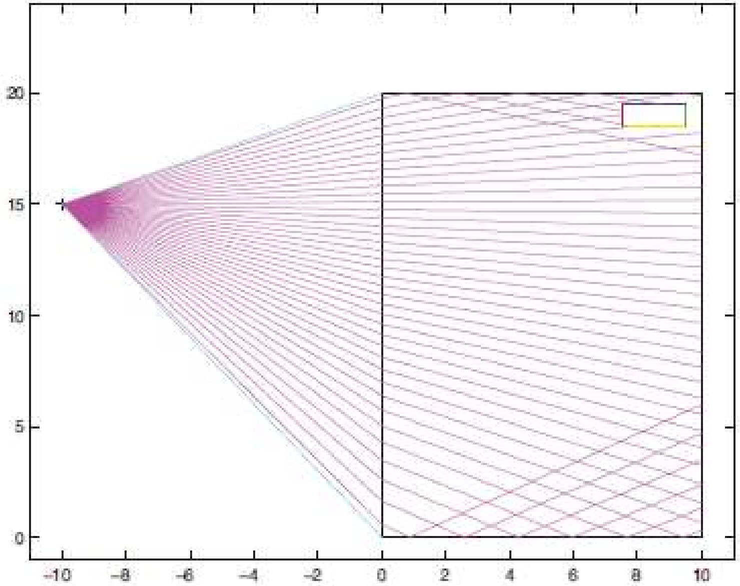
POLYTECHNIQUE
MONTRÉAL

Optique

- méthode des images virtuelles
- méthode des rayons.







Optique

Onde transverse électrique

$$\begin{pmatrix} R \\ T \end{pmatrix} = \begin{pmatrix} \left[\dfrac{\sin(\theta_t-\theta_i)}{\sin(\theta_t+\theta_i)}\right]^2 \\ 1-R \end{pmatrix}$$

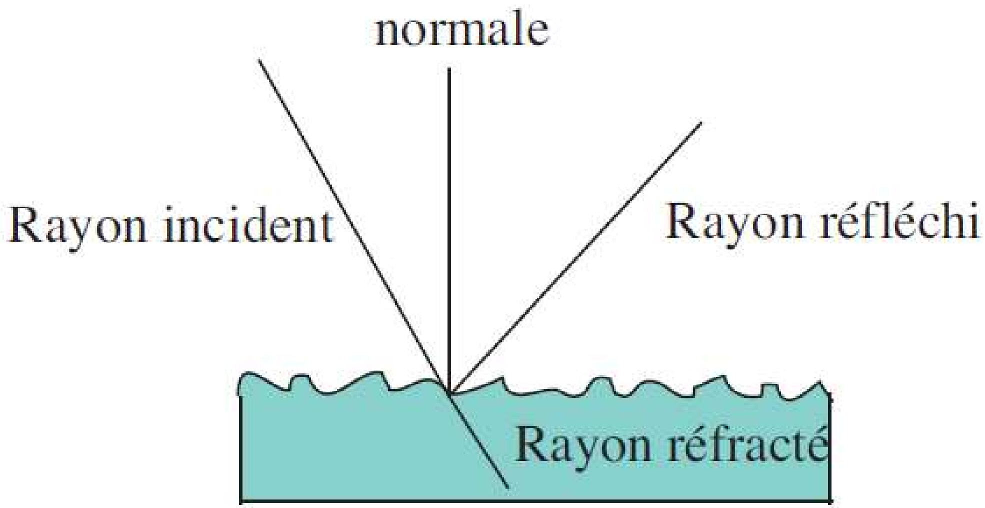
Onde transverse magnétique

$$\begin{pmatrix} R \\ T \end{pmatrix} = \begin{pmatrix} \left[\dfrac{\tan(\theta_t-\theta_i)}{\tan(\theta_t+\theta_i)}\right]^2 \\ 1-R \end{pmatrix}$$

Onde non polarisée

$$\begin{pmatrix} R \\ T \end{pmatrix} = \begin{pmatrix} \dfrac{1}{2}\left\{\left[\dfrac{\sin(\theta_t-\theta_i)}{\sin(\theta_t+\theta_i)}\right]^2 + \left[\dfrac{\tan(\theta_t-\theta_i)}{\tan(\theta_t+\theta_i)}\right]^2\right\} \\ 1-R \end{pmatrix}$$

Optique



Modèle de Phong

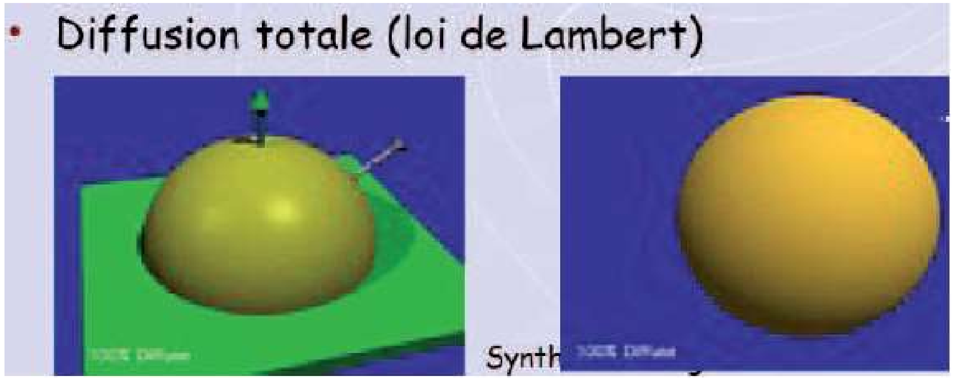
Modèle de la réflexion physique

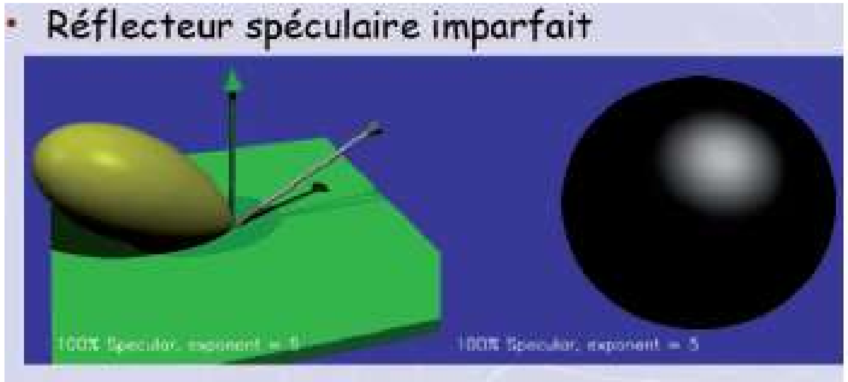
Modèle de Phong

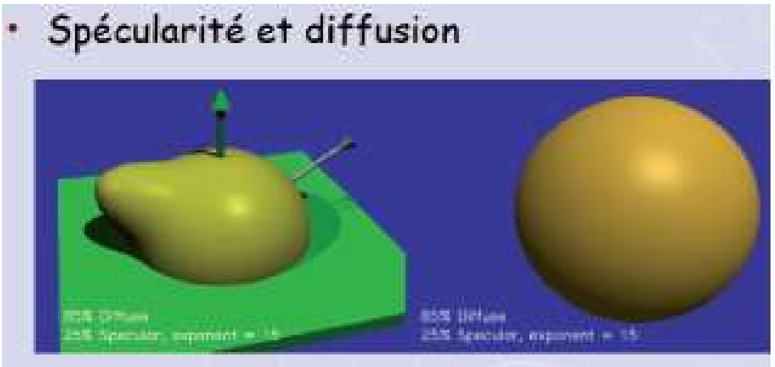
$$I_{\text{Phong}} = k_{\text{diffuse}} I_{\text{diffuse}} + k_{\text{si}} I_{\text{si}}$$

$$I_{\text{diffuse}} = k_{d,c} I_i (\vec{L} \cdot \vec{n}) = k_{d,c} I_i \cos\theta$$

$$I_{\text{si}} = k_{\text{si},c} I_i (\vec{R} \cdot \vec{V})^m$$







Ondes sonores

Équation d'onde sonore

$$\frac{\partial^2 \tilde{p}}{\partial t^2} = \gamma \frac{\partial^2 \tilde{p}}{\partial x^2} = c^2 \frac{\partial^2 \tilde{p}}{\partial x^2}$$

$$\frac{\partial^2 v}{\partial t^2} = \gamma \frac{\partial^2 v}{\partial x^2} = c^2 \frac{\partial^2 v}{\partial x^2}$$

Vitesse d'onde sonore

$$c_{\text{gaz}} = \sqrt{\frac{c_p \cdot p}{c_v \rho}} = \sqrt{\frac{\beta \cdot p}{\rho}} \approx \sqrt{\frac{\beta \cdot p_0}{\rho_0}}$$

$$c_{\text{air}}(T) = c_{\text{air}}(\Theta) = (331.3 + 0.606\Theta)\ \text{m/s}$$

Intensité sonore

$$I = \frac{P}{S} = \frac{P}{4\pi r^2}$$

$$I = v\tilde{p}$$

$$I \approx \frac{\tilde{p}^2}{\rho c}$$

Ondes sonores

puissance sonore en dB

$$L_p = 10 \log \left(\frac{P}{10^{-12}}\right) \text{ dB}$$

$$(P \text{ en watt})$$

Intensité sonore en dB

$$L_I = 10 \log \left(\frac{I}{10^{-12}}\right) \text{ dB}$$

$$\left(I \text{ en } \frac{\text{W}}{\text{m}^2}\right)$$

$$L_I(r) = L_P - 10 \log \left(\frac{4\pi r^2}{1}\right) \quad (r \; en \; \text{m})$$

Atténuation des ondes sonores

Dispersion

$$L_I(r) = L_I(r_0) - 20 \log \left(\frac{r}{r_0}\right) \text{ dB}$$

Absorption

$$L_I(r) = L_I(r_0) - A(r - r_0) \text{ dB}$$

$$L_I(r) = L_I(r_0) - 20 \log \left(\frac{r}{r_0}\right) - \alpha(r - r_0) \text{ dB}$$

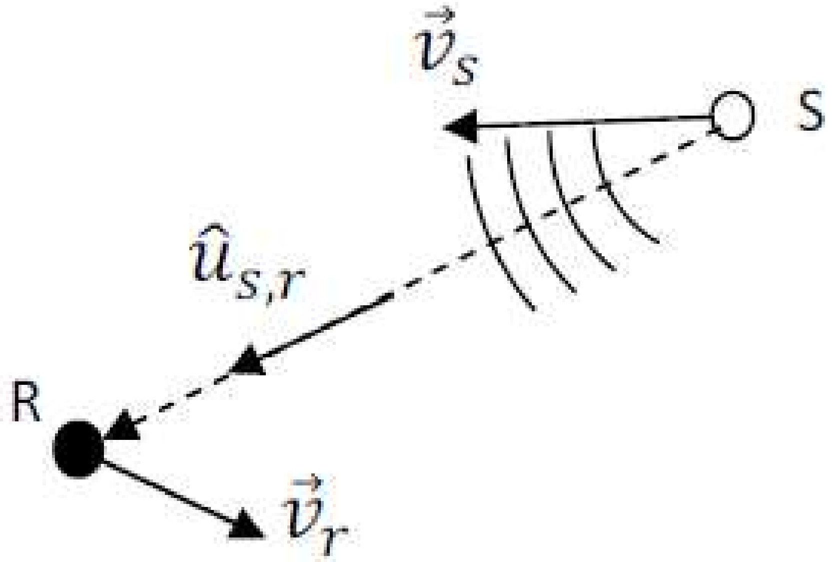
Effet Doppler

$$\vec{u}_{s,r} = \frac{\vec{r}_r(t) - \vec{r}_s(t)}{|\vec{r}_r(t) - \vec{r}_s(t)|}$$

$$\nu_r = \frac{c - (\vec{v}_r \cdot \vec{u}_{s,r})}{c - (\vec{v}_s \cdot \vec{u}_{s,r})}\nu_s = \frac{1 - \beta_r}{1 - \beta_s}\nu_s$$

$$\beta_r = \frac{(\vec{v}_r \cdot \vec{u}_{s,r})}{c} = \frac{|\vec{v}_r|\cos(\theta)}{c}$$

$$\beta_s = \frac{(\vec{v}_s \cdot \vec{u}_{s,r})}{c} = \frac{|\vec{v}_s|\cos(\theta')}{c}$$



Réflexion/réfraction des ondes sonores

$$\sin\theta_r = \sin\theta_i$$

$$\sin\theta_t = \left(\frac{c_t}{c_i}\right)\sin\theta_i$$

Réflexion/réfraction des ondes sonores

Impédance acoustique

Le coefficient de réflexion d'une onde acoustique à l'interface entre deux milieux d'impédance différentes est donné par:

$$R = \left(\frac{Z_i - Z_t}{Z_i + Z_t}\right)^2$$

avec $Z_i = \rho_i c_i$ et $Z_t = \rho_t c_t$

Le coefficient de transmission est donné par:

$$T = 1 - R = \frac{4\, Z_i\, Z_t}{(Z_i + Z_t)^2}$$