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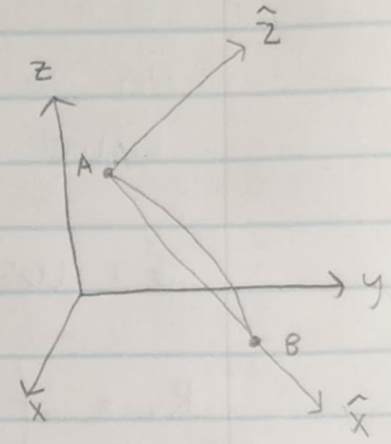
Q1 a) Position lorsque $z = 0$?



$$0 = 5 - 0 \cdot \Delta t - \frac{1}{2} \cdot 10 \cdot \Delta t^2$$

$$5 = \frac{1}{2} \cdot 10 \cdot \Delta t^2 \quad \Rightarrow \Delta t = 1\,s$$

$$r_x = 1 + 1 \cdot \Delta t = 1 + 1 \cdot 1 = 2\,m$$

$$r_y = 1 + 2 \cdot \Delta t = 1 + 2 \cdot 1 = 3\,m$$

$$\vec{r}_b = (2, 3, 0)^T$$

b) $$\vec{X} = \vec{r}_b - \vec{r}_a = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

$$\hat{X} = \frac{\begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}}{\sqrt{1^2+2^2+5^2}} = \begin{pmatrix} 1/\sqrt{30} \\ 2/\sqrt{30} \\ -5/\sqrt{30} \end{pmatrix}$$

$\hookrightarrow \sqrt{30}$

$$\hat{x} \cdot \hat{y} = 0 \qquad \begin{pmatrix} 1/\sqrt{30} \\ 2/\sqrt{30} \\ -5/\sqrt{30} \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ 0 \end{pmatrix} = 0$$

$$\frac{1}{\sqrt{30}} y_1 + \frac{2}{\sqrt{30}} y_2 = 0$$

$$y_1 = -\frac{2}{\cancel{\sqrt{30}}} y_2 \cdot \cancel{\sqrt{30}}$$

$$y_1 = -2y_2$$

$$\vec{y} = \begin{pmatrix} -2y_2 \\ y_2 \\ 0 \end{pmatrix} \qquad 1 = \sqrt{4y_2^2 + y_2^2} \quad \Rightarrow y_2 = \frac{\sqrt{5}}{5}$$

$$\hat{y} = \begin{pmatrix} -2\sqrt{5}/5 \\ \sqrt{5}/5 \\ 0 \end{pmatrix}$$

$$\hat{z} = \hat{x} \times \hat{y} = \begin{pmatrix} \sqrt{6}/6 \\ \sqrt{6}/3 \\ \sqrt{6}/6 \end{pmatrix}$$

$$\begin{matrix} \frac{1}{\sqrt{30}} & \frac{2}{\sqrt{30}} & \frac{-5}{\sqrt{30}} \\ \frac{-2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \end{matrix}$$

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c) $$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{\underbrace{\|\vec{A}\|\cdot\|\vec{B}\|}_{=1}}\right)$$ =1 pour nous

On va trouver tous les angles entre les nouveaux axes et les anciens

$$\theta = \cos^{-1}\left(\begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}1/\sqrt{30}\\2/\sqrt{30}\\-5/\sqrt{30}\end{pmatrix}\right) = \cos^{-1}(1/\sqrt{30}) = 79{,}5°$$

$$R_x = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos 79.5 & -\sin 79.5\\ 0 & \sin 79.5 & \cos 79.5\end{pmatrix}$$

$$\text{nouveau } \hat{y}' = R_x\,\hat{y} \rightarrow \begin{pmatrix}-2\sqrt{5}/5\\ \sqrt{5}/5\\ 0\end{pmatrix}$$

$$= \begin{pmatrix}-2\sqrt{5}/5\\ 0{,}0815\\ 0{,}43972\end{pmatrix}$$

$$\theta = \cos^{-1}\left(\begin{pmatrix}0\\1\\0\end{pmatrix}\cdot\begin{pmatrix}-2\sqrt{5}/5\\ 0{,}0815\\ 0{,}43972\end{pmatrix}\right) = \cos^{-1}(0{,}0815) = 85{,}33°$$

$$R_y = \begin{pmatrix}\cos 85.33 & 0 & \sin 85.33\\ 0 & 1 & 0\\ -\sin 85.33 & 0 & \cos 85.33\end{pmatrix}$$

$$\text{nouveau } \hat{z}' = R_y R_x\,\hat{z} = R_y\begin{pmatrix}\sqrt{6}/6\\ -0{,}252\\ 0{,}8772\end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix}\sqrt{6}/6\\ \sqrt{6}/3\\ \sqrt{6}/6\end{pmatrix} = \begin{pmatrix}0{,}9075\\ -0{,}252\\ -0{,}833\end{pmatrix}$$

$$\theta = \cos^{-1}\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\begin{pmatrix}0{,}9075\\ 0{,}252\\ -0{,}833\end{pmatrix}\right) = \cos^{-1}(-0{,}833) = 146{,}4°$$

$$R_z = \begin{pmatrix}\cos 146{,}4 & -\sin 146{,}4 & 0\\ \sin 146{,}4 & \cos 146{,}4 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$R = R_z R_y R_x = \ldots$$

(le temps me manque mais multiplier les 3 donne notre R totale

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Q2 a) $F = m\vec{a}$

$\vec{a} = \dfrac{F}{m}$

$\dfrac{AH}{\cancel{3}} = \dfrac{Kh^2 \cancel{h}}{\cancel{3}}$

$A = Kh^2$

$0,01 = K \cdot 0,2^2$

$K = \dfrac{1}{4}$

$V = \dfrac{h^3}{12}$

$F_z = -mg + \rho g V_{imm}(z)$

$V_{imm}(z) = \underbrace{\dfrac{AH}{3}}_{\text{total}} - \underbrace{\dfrac{z^3}{12}}_{\text{volume sorti}}$

$z_c = z - \frac{3}{4}H$

$z = z_c + \frac{3}{4}H$

$F = -mg + \rho g \left(\dfrac{AH}{3} - \dfrac{z^3}{12}\right)$

$F = -mg + \rho g \left(\dfrac{AH}{3} - \dfrac{(z_c + \frac{3}{4}H)^3}{12}\right)$

$a = -g + \dfrac{\rho g}{m}\left(\dfrac{AH}{3} - \dfrac{(z_c + \frac{3}{4}H)^3}{12}\right)$

b) $\vec{q}(t) = \begin{pmatrix} v_z \\ z_c \end{pmatrix}$ $\qquad \vec{g}(t, \vec{q}(t)) = \begin{pmatrix} a_z \\ q_1 \end{pmatrix}$

$\vec{q}_0 \begin{pmatrix} 0 \\ -\frac{3}{4}H \end{pmatrix}$ $\qquad \vec{g}(t, \vec{q}(t)) = \begin{pmatrix} -g + \dfrac{\rho g}{m}\left(\dfrac{AH}{3} - \dfrac{(z_c + \frac{3}{4}H)^3}{12}\right) \\ q_1 \end{pmatrix}$

c) $\vec{q}_1(t = 0,001) = \vec{q}_0 + \vec{g}_0 \cdot \Delta t$

$= \begin{pmatrix} 0 \\ -\frac{3}{4}H \end{pmatrix} + \begin{pmatrix} -g + \dfrac{\rho g}{m}\left(\dfrac{AH}{3} - \cancel{\dfrac{(-\frac{3}{4}H + \frac{3}{4}H)^3}{12}}\right) \\ 0 \end{pmatrix} \cdot \Delta t$

0

$= \begin{pmatrix} \left(0 - g + \dfrac{\rho g A}{3m} H\right) \cdot 0,001 \\ -\frac{3}{4}H \end{pmatrix}$

$= \begin{pmatrix} 0,07186 \\ -0,15 \end{pmatrix}$

Q3 1956576

a) $$I = \begin{bmatrix} \frac{m}{4}r^2+\frac{m}{12}l^2 & & \\ & \frac{m}{4}r^2+\frac{m}{12}l^2 & \\ & & \frac{m}{2}r^2 \end{bmatrix}$$

$$\frac{m}{2}r^2 = \frac{m}{4}r^2+\frac{m}{12}l^2$$

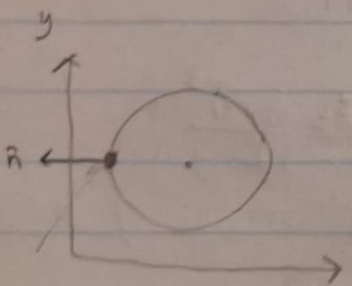
$$l^2 = \left(\frac{m}{2}r^2 - \frac{m}{4}r^2\right)\cdot\frac{12}{m}$$

$$l = \frac{\sqrt{3}}{25} \approx 0{,}0693 \text{ m}$$

b) $$\vec{r}_{c,p} = \vec{r}_c - \vec{r}_p = \begin{pmatrix} 2 \\ 2 \\ 3{,}5 \end{pmatrix} - \begin{pmatrix} 1{,}96 \\ 2 \\ 3{,}54 \end{pmatrix} = \begin{pmatrix} 0{,}04 \\ 0 \\ -0{,}04 \end{pmatrix}$$

c) $$\vec{v}_r^- = \vec{v}_a - \vec{v}_b = \begin{pmatrix} 5 \\ 0 \\ 10 \end{pmatrix} - \begin{pmatrix} 100 \\ 100 \\ 150 \end{pmatrix} = \begin{pmatrix} -95 \\ -100 \\ -140 \end{pmatrix}$$

$a \to$ cyl
$b \to$ balle



d) $$\vec{n} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1{,}96 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0{,}04 \\ 0 \\ 0 \end{pmatrix} \to \hat{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

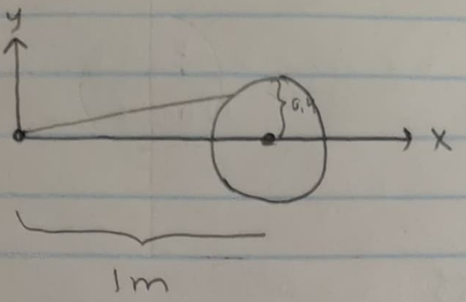
e) $$\vec{v}_b(t_f) = \vec{v}_b(t_i) - \frac{\hat{n}\, j}{m_b}$$

$$j = \frac{-(1+\varepsilon)\cdot v_r^-}{\frac{1}{m_a}+\frac{1}{m_b}} \qquad v_r^- = \hat{n}\cdot\vec{v}_{a-b} = -95$$

$$j = \frac{-(1+0{,}6)\cdot -95}{\frac{1}{0{,}5}+\frac{1}{0{,}05}} = 6{,}91$$

$$\vec{v}_b(t_f) = \begin{pmatrix} 100 \\ 100 \\ 150 \end{pmatrix} - \frac{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\cdot 6{,}91}{0{,}05} = \begin{pmatrix} -92{,}2 \\ 100 \\ 150 \end{pmatrix} \text{ m/s}$$

Q4 1956576



a) Pour être à la surface, on doit satisfaire

$(x-h)^2 + (y-k)^2 = R^2$

$(x-1)^2 + y^2 = 0,4^2$

On sait que $x = s \cdot \frac{3}{\sqrt{10}}$ et $y = s \cdot \frac{1}{\sqrt{10}}$

$$\left(\frac{3s}{\sqrt{10}} - 1\right)^2 + \frac{s^2}{10} = 0,4^2$$

$$\frac{9}{10}s^2 - \frac{6s}{\sqrt{10}} + 1 + \frac{s^2}{10} = 0,4^2$$

$$s^2 - \frac{6}{\sqrt{10}}s + 1 = 0,16$$

$$s^2 - \frac{6}{\sqrt{10}}s + 0,84 = 0 \Rightarrow s = 1,19$$

$s = 0,703$ → premier à arriver

Donc $x = 0,703 \cdot \frac{3}{\sqrt{10}} = 0,668$

et $y = 0,703 \cdot \frac{1}{\sqrt{10}} = 0,222$

$$\vec{r}_p = \begin{pmatrix} 0,668 \\ 0,222 \end{pmatrix}$$

b) $\vec{n} = \begin{pmatrix} 0,668 \\ 0,222 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -0,332 \\ 0,222 \end{pmatrix}$ $\|\vec{n}\| = 0,4$

$$\hat{n} = \frac{\vec{n}}{0,4} = \begin{pmatrix} -0,83 \\ 0,555 \end{pmatrix}$$

c) $\vec{k} = \vec{\imath} \times \vec{\jmath}$

$$\vec{j} = \begin{pmatrix} 3/\sqrt{10} \\ 1/\sqrt{10} \\ 0 \end{pmatrix} \times \begin{pmatrix} -0,83 \\ 0,555 \\ 0 \end{pmatrix}$$

$$\vec{j} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{k} = \begin{pmatrix} -0,83 \\ 0,555 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0,555 \\ 0,83 \\ 0 \end{pmatrix}$$

→ verso

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$$S_t = \vec{u}_t \cdot \vec{K}$$

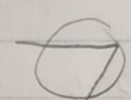
$$S_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0,555 \\ 0,83 \\ 0 \end{pmatrix} = 0,555$$

$$0,555 = \frac{1}{n_{boule}} \vec{u}_i \cdot \vec{K}$$

$$n_{boule} = \frac{\begin{pmatrix} 3/\sqrt{10} \\ 1/\sqrt{10} \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0,555 \\ 0,83 \\ 0 \end{pmatrix}}{0,555}$$

$$n_{boule} = \frac{\frac{3}{\sqrt{10}} \cdot 0,555 + \frac{1}{\sqrt{10}} \cdot 0,83}{0,555} = \boxed{1,42}$$

d) → Doit subir une réflexion totale interne



→ Possible car $n_i > n_t$

$$-\left|\arcsin\left(\frac{1}{1,42}\right)\right| \leq \theta_i \leq \left|\arcsin\left(\frac{1}{1,42}\right)\right|$$

$$-44,77 \leq \theta_i \leq 44,77$$

$S_i = \vec{u}_i \cdot \vec{K} = 0,555$ (voir c))

donc $\theta_i = \sin^{-1}(0,555) = 33,71°$

↳ fait partie de l'intervalle de réfraction permis, donc il ne sera pas réfléchi et ne croisera donc pas l'axe des x à nouveau