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Examen Final A20

Question 1 :

a) Nous avons :

$$\vec{a} = \vec{g} = \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix}$$

$$\vec{v}(t) = \vec{a}\,t + \vec{v}_A = \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix} t + \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -10t \end{pmatrix}$$

$$\vec{r} = \frac{1}{2}\vec{a}\,t^2 + \vec{v}_A t + \vec{r}_A = \begin{pmatrix} 0 \\ 0 \\ -5 \end{pmatrix} t^2 + \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} t + \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} t+1 \\ 2t+1 \\ -5t^2+5 \end{pmatrix}$$

intersection Plan OXY $\Rightarrow z = 0 \Rightarrow -5t^2+5 = 0 \Rightarrow t = \begin{cases} 1 \\ \cancel{-1} \end{cases}$

$$\boxed{t = 1s} \Rightarrow \vec{r}_B = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

b) $A\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$ et $B\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \Rightarrow \vec{BA}\begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$ $\quad BA = \sqrt{1^2+2^2+5^2} = \sqrt{30}$

referentiel local : $\Rightarrow A'\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ et $B'\begin{pmatrix} \sqrt{30} \\ 0 \\ 0 \end{pmatrix}$

Par conséquent :

$$\vec{A} = 1.\hat{X} + 1.\hat{Y} + 5\hat{Z} = 0.\hat{x} + 0.\hat{y} + 0.\hat{z}$$

$$\vec{B} = 2.\hat{X} + 3.\hat{Y} + 0\hat{Z} = \sqrt{30}\hat{x} + 0.\hat{y} + 0.\hat{z}$$

Soit $\vec{V} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z} = V_x\hat{X} + V_Y\hat{Y} + V_Z\hat{Z}$

Alors :

$$\hat{x} = e_{xX}\hat{X} + e_{xY}\hat{Y} + e_{xZ}\hat{Z}$$
$$\hat{y} = e_{yX}\hat{X} + e_{yY}\hat{Y} + e_{yZ}\hat{Z}$$
$$\hat{z} = e_{zX}\hat{X} + e_{zY}\hat{Y} + e_{zZ}\hat{Z}$$

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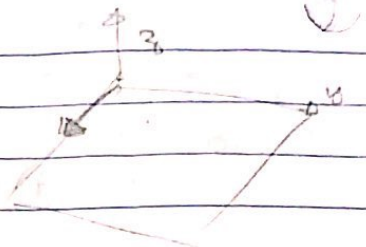
$$\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} e_{xX} & e_{yX} & e_{zX} \\ e_{xY} & e_{yY} & e_{zY} \\ e_{xZ} & e_{yZ} & e_{zZ} \end{pmatrix} \begin{pmatrix} \sqrt{30} \\ 0 \\ 0 \end{pmatrix}$$



$\Rightarrow e_{xX}\sqrt{30} = 2 \Rightarrow e_{xX} = \frac{2}{\sqrt{30}}$

$\Rightarrow e_{xY}\sqrt{30} = 3 \Rightarrow e_{xY} = \frac{3}{\sqrt{30}}$

$\Rightarrow e_{xZ}\sqrt{30} = 0 \Rightarrow e_{xZ} = 0$

$$\Rightarrow \boxed{\hat{x} = \frac{2}{\sqrt{30}}\hat{X} + \frac{3}{\sqrt{30}}\hat{Y}}$$

l'axe oy sera horizontale et contiendra la trajectoire de la projectile.

Prenons deux instant

$t = 0{,}5\,s \Rightarrow \begin{pmatrix} 1{,}5 \\ 2 \\ 3{,}75 \end{pmatrix}$

$t = 1 \Rightarrow \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1{,}5 \\ 2 \\ 3{,}75 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 11{,}25 \\ 7{,}5 \\ 0{,}5 \end{pmatrix} \Rightarrow$ représente la normale du plan $\hat{y}$.

et dan le référentiel: le plan oy à un normale de $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 11{,}25 \\ 7{,}5 \\ 0{,}5 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} e_{yX} = 11{,}25 \\ e_{yY} = 7{,}5 \\ e_{yZ} = 0{,}5 \end{cases}$

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$$\boxed{\hat{y} = 11{,}25\,\hat{X} + 7{,}5\,\hat{Y} + 0{,}5\,\hat{Z}}$$

Concernant l'axe oz il reste inchangé dans B et translaté en A ⇒ on peut le considéré comme une simple rotation autour $\hat{X}$

$$\sin\theta = \frac{5}{\sqrt{30}} \Rightarrow \theta = 73{,}23°$$

$$\vec{r} = \left(\cos\frac{73{,}23°}{2},\ \sin\left(\frac{73{,}23}{2}\right)\begin{pmatrix} 2/\sqrt{30} \\ 3/\sqrt{30} \\ 0 \end{pmatrix}\right)$$

$$\vec{r} = \left(0{,}84,\ 2\times 0{,}54/\sqrt{30},\ \frac{0{,}54\times 3}{\sqrt{30}},\ 0\right)$$

$$\vec{r} = (0{,}84,\ 0{,}197,\ 0{,}3,\ 0)$$

$$V' = \vec{r}\,(\vec{v}\ \vec{r}^*)$$

$$\vec{v}\ \vec{r}^* = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ -0{,}197 \\ -0{,}3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0{,}3 \\ -0{,}197 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0{,}84 \\ 0{,}197 \\ 0{,}3 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0{,}3 \\ 0{,}197 \\ 0 \end{pmatrix} = \begin{pmatrix} 0{,}1182 \\ 0{,}212 \\ 0{,}11 \\ -0{,}05 \end{pmatrix}$$

$$\hat{z} = 0{,}1182\,\hat{X} + 0{,}11\,\hat{Y} + 0{,}05\,\hat{Z}$$

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Matrice de rotation

$$\vec{V}(V_x, V_y, V_z)^t = R\vec{V}(V_x, V_y, V_z)$$

local $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\Downarrow$

Global $\begin{pmatrix} \frac{2}{\sqrt{30}} \\ \frac{3}{\sqrt{30}} \\ 0 \end{pmatrix}$ $\begin{pmatrix} 11,25 \\ 7,5 \\ 0,5 \end{pmatrix}$ $\begin{pmatrix} 0,212 \\ 0,11 \\ -0,05 \end{pmatrix}$

$\Downarrow$

$$R = \begin{pmatrix} \frac{2}{\sqrt{30}} & 11,25 & 0,2.52 \\ 3/\sqrt{30} & 7,5 & 0,11 \\ 0 & 0,5 & -0,05 \end{pmatrix}$$

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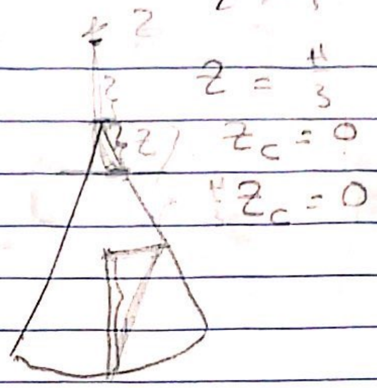

Question 2:

$\vec{v}_{c,0} = \begin{pmatrix} 0 \\ 0 \\ 0,2 \end{pmatrix}$

a) $\vec{a} = \frac{1}{m} \sum \vec{F} = \frac{1}{m}\left(-m_c g + \rho g V\right)\hat{z}$

$V = V_{tot} - \frac{A'}{3} z$ avec $V_{tot} = \frac{AH}{3}$ et

$A' = \pi r'^2 = \pi r'^2$

$$\frac{r}{H} = \frac{r'}{z} \Rightarrow \boxed{r' = \frac{z r}{H}}$$

$$A' = \frac{\pi r^2 z^2}{H^2}$$

$$V = \frac{AH}{3} - \frac{A}{H^2 3} z^3$$

aussi $\boxed{z = \frac{1}{4} H + z_c}$

$$\vec{a} = \frac{1}{m}\left(-m g + \rho g \left(\frac{AH}{3} - \frac{A}{H^2 3}\left(z_c + \frac{1}{4} H\right)^3\right)\right)\hat{z}$$

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b)

$$\vec{q}\begin{pmatrix} v_z \\ z_c \end{pmatrix} \quad \vec{q}_0\begin{pmatrix} 0,2 \\ -0,05 \end{pmatrix} \quad \vec{g}\begin{pmatrix} \frac{1}{m}\left(-mg+\rho g\left(\frac{AH}{3}-\frac{A}{H^2 3}\left(q_2+\frac{1}{4}H\right)^3\right)\right) \\ q_1 \end{pmatrix}$$

c) $\Delta t = 0,001\,s$ $\quad t_0 = 0$ et $t_1 = 0,001$.

$$q(t_1) = \begin{pmatrix} 0,2 \\ -0,05 \end{pmatrix} + g\left(\begin{pmatrix} 0,2 \\ -0,05 \end{pmatrix}, 0\right) \times 0,001.$$

$$= \begin{pmatrix} 0,2 \\ -0,05 \end{pmatrix} + \begin{pmatrix} 12,5(-0,08 \times 9,81 \ldots \\ 0,2 \end{pmatrix} \times 0,001 = \begin{pmatrix} 0,2 + 0,001 \times 71,99 \\ -0,05 + 0,2 \times 0,001 \end{pmatrix}$$

$$= \begin{pmatrix} 0,27194 \\ -0,0498 \end{pmatrix}$$

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Questions 3:

$m_c = 95$ $\vec{r}_c \begin{pmatrix} 2 \\ 2 \\ 3,5 \end{pmatrix}$ $\vec{v}_c \begin{pmatrix} 5 \\ 0 \\ 10 \end{pmatrix}$ $\vec{u}_c \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ $w_c = 0$ $R = 4$

$m_b = 50 g$ $\vec{v}_b = \begin{pmatrix} 100 \\ 100 \\ 150 \end{pmatrix}$

$\vec{r}_p = \begin{pmatrix} 1,96 \\ 2 \\ 3,54 \end{pmatrix}$

① On sait que : $I_c = \begin{bmatrix} I_{cxx} & 0 & 0 \\ 0 & I_{cyy} & 0 \\ 0 & 0 & I_{czz} \end{bmatrix} = K \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\frac{m}{12} \ell^2 + \frac{m R^2}{4} = K \Rightarrow \ell = \sqrt{\left(K - \frac{mR^2}{4}\right) \times \frac{12}{m}}$

$\ell = \sqrt{24 \cancel{K} - 4,8 \times 10^{-3}}$ m

② $\vec{r}_{c,p} = \vec{r}_c - \vec{r}_p = \begin{pmatrix} 2 - 1,96 \\ 2 - 2 \\ 3,5 - 3,54 \end{pmatrix} = \begin{pmatrix} 0,04 \\ 0 \\ -0,04 \end{pmatrix}$

③ $\vec{V}_r = \hat{n} \left(v_{b,p}(t_i) - v_{c,p}(t_i) \right)$

$\vec{V}_r = \hat{n} \left[\begin{pmatrix} 100 \\ 100 \\ 150 \end{pmatrix} - \begin{pmatrix} 5 \\ 0 \\ 10 \end{pmatrix} \right] = \begin{pmatrix} 95 \\ 100 \\ 140 \end{pmatrix} \hat{n}$

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d) $\vec{r}_{P,C} = \begin{pmatrix} 2 \\ 2 \\ 3,5 \end{pmatrix} + \frac{e}{2} \Rightarrow$

$$\frac{\vec{r}_{P,C} \times \vec{v}_{r^-}}{|\vec{r}_{P,C} \times \vec{v}_{r^-}|} = \hat{n}$$

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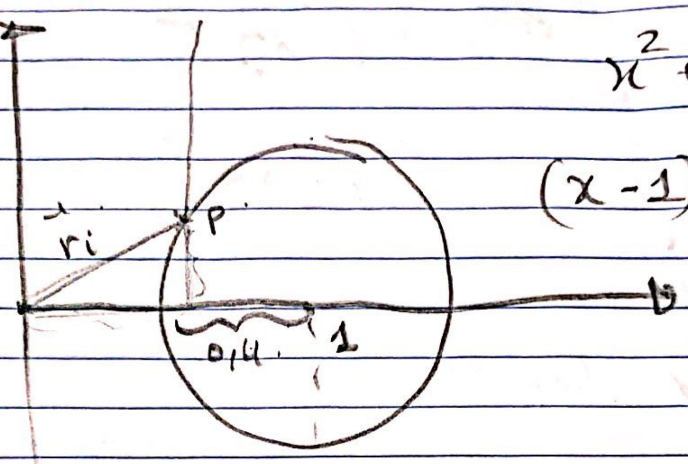
Exo04: Optiques



x^2+y^2

$(x-1)^2+y^2=(0,4)^2$

$\vec{r}_0=\begin{pmatrix}1\\0\end{pmatrix}$ $\vec{u}_i\begin{pmatrix}3/\sqrt{10}\\1/\sqrt{10}\end{pmatrix}$

$\vec{u}_i=\dfrac{\vec{r}_p-\vec{r}_i}{\|\vec{r}_p-\vec{r}_i\|}$

a) $\left(\frac{3}{\sqrt{10}}t-1\right)^2+\left(\frac{1}{\sqrt{10}}t\right)^2=0,4^2$

$\frac{9}{10}t^2-\frac{6}{\sqrt{10}}t+1+\frac{t^2}{10}=0,4^2$

$t^2-\frac{6}{\sqrt{10}}t+0,84=0$

$\boxed{\Delta=0,24}$ $t_1=\dfrac{\frac{6}{\sqrt{10}}-\sqrt{0,24}}{2}$ et $t_2=\dfrac{\frac{6}{\sqrt{10}}+\sqrt{0,24}}{2}$

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$$\boxed{t = 0{,}7}$$

$$\vec{r}_p = \begin{pmatrix} 3/\sqrt{10}\, t \\ 1/\sqrt{10}\, t \end{pmatrix}$$

$$\begin{pmatrix} 0{,}66 \\ 0{,}22 \end{pmatrix}$$

2)

$$\hat{n} = \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|} =$$

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$$\boxed{t = 0{,}7}$$

$$\vec{r}_p = \begin{pmatrix} 3/\sqrt{10}\, t \\ 1/\sqrt{10}\, t \end{pmatrix}$$

$$\boxed{\vec{r}_p = \begin{pmatrix} 0{,}66 \\ 0{,}22 \end{pmatrix} \times \frac{1}{\sqrt{0{,}66^2 + 0{,}22^2}}} = \begin{pmatrix} 0{,}94 \\ 0{,}32 \end{pmatrix}$$

2)

$$\hat{n} = \frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|} = \begin{pmatrix} 0{,}66 - 1 \\ 0{,}22 - 0 \end{pmatrix}$$

$$= \begin{pmatrix} -0{,}34 \\ 0{,}22 \end{pmatrix} \times \frac{1}{\sqrt{0{,}34^2 + 0{,}22^2}}$$

$$\boxed{\vec{n} = \begin{pmatrix} -0{,}85 \\ 0{,}55 \end{pmatrix}}$$

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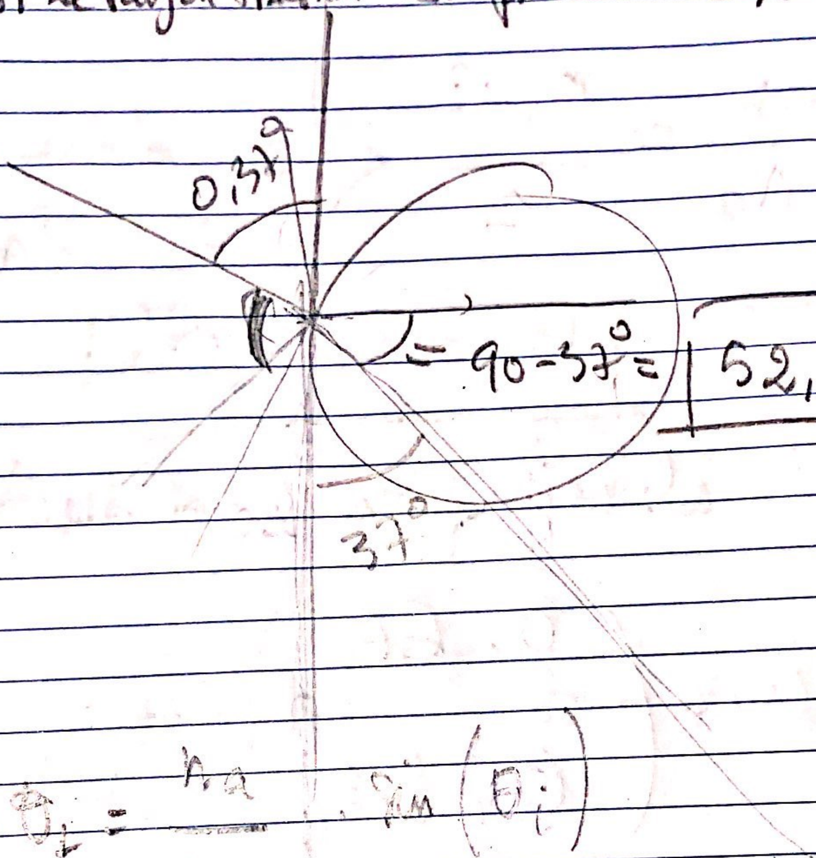
c) si le rayon transmis est parallèle à O_x



$= 90 - 37° = \boxed{52,93°}$

$$\sin \theta_t = \frac{n_a}{n_b} \cdot \sin(\theta_i)$$

$$n_b = n_a \frac{\sin(\theta_i)}{\sin \theta_t} = 1 \times \frac{0,32}{0,74}$$

$$\boxed{n_b = 0,43}$$

d) nous avons:

$$\frac{n_b}{n_a} = \frac{0,43}{1} < 1$$ donc elle se reflecte à l'interieur

avec $\boxed{\theta_r = \theta_i = 0}$

donc il refrappera le pt du début

$$\vec{a}\begin{pmatrix}0,66\\0,22\end{pmatrix}$$