Modeling and Optimization with OPL

6 Simple techniques of stochastic optimization

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Inhalt

6.1 Scenario method

6.2 Chance constrainend programming

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Example: Relieve Ethiopia

Long term warehouse capacity: 8 houses

Warehouse demand:

Resource	Scenario I (30%)	Scenario II (30%)	Scenario III (25%)	Scenario IV (15%)
Food	4	2	3	4
Dringking water	3	5	3	5
Medication	3	3	1	4

Short term warehouse costs

- 3500$ for a food warehouse
- 1600$ for a drinking water warehouse
- 5200$ for a medication warehouse

6.1 Scenario method
6.2 Chance constrainend programming

Two stage stochastic optimization

Scenario method

- Special case of two stage stochastic optimization
- Random event = occurance of one out of a finite number of scenarios
- Stochastic objective function often replaced by expected value

6.1 Scenario method
6.2 Chance constrainend programming

Equivalent deterministic model using scenario method

- Index set I of scenarios
- Parameter p_i: probability of scenario $i \in I$
- Scenario independent parameters and here-and-now-decision-variables have no scenario index
- Scenario dependent parameters and wait-and-see-decision-variables have a scenario index
- With a finite number of scenarios, the expected value becomes a convex combination and is therefore linear

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6.1 Scenario method
6.2 Chance constrainend programming

Model: stochastic resource planning

Index sets:

I set of scenarios
R set of resources

Parameters:

p_i probability of scenario $i \in I$
c_r cost for additional short term capacity of resource $r \in R$
d_{ri} demand of resource $r \in R$ in scenario $i \in I$
k capacity for long term resource allocation

Decision variables:

x_r long term capacity allocated to resource $r \in R$
y_{ri} short term capacity bought for resource $r \in R$ in scenario $i \in I$

Model description:

$$
\begin{aligned}
\min \quad & \sum_{i \in I} p_i \cdot \left(\sum_{r \in R} c_r \cdot y_{ri} \right) & & \\
s.t. \quad & \sum_{r \in R} x_r \leq k & & \text{(I)} \\
& x_r + y_{ri} \geq d_{ri} \qquad \forall r \in R, i \in I & & \text{(II)} \\
& x_r, y_{ri} \in \mathbb{Z}^+ \qquad \forall r \in R, i \in I & &
\end{aligned}
$$

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Example: Lewig Pattming

Sales prices and capacity

	I_1	I_2	I_3	capacity
R_1	3.2	2.4	2.6	600
R_2	6.5	8.3	7.8	800
SP	1.4	1.6	1.5	

Demand estimator

$$\left.\begin{array}{l} D_{I_1} \sim N(25,10) \\ D_{I_2} \sim N(10,5) \\ D_{I_3} \sim N(18,7) \end{array}\right\} \text{Probability of a stock out shall be } < 5\%$$

6.1 Scenario method
6.2 Chance constrainend programming

Model: Stochastic production problem (Alternative 1)

Index sets:

I	set of products
R	set of resources

Parameters:

v_i	sales price of product $i \in I$
c_r	capacity of resource $r \in R$
a_{ri}	capacity consumption of product $i \in I$ on resource $r \in R$
D_i	demand of product $i \in I$ (random variable)
α	α service level

Decision variables:

x_i	production quantity of product $i \in I$

Model description:

$$
\begin{aligned}
\min \quad & \sum_{i \in I} v_i \cdot x_i \\
s.t. \quad & \sum_{r \in R} a_{ri} x_i \leq c_r && \forall r \in R \qquad \text{(I)} \\
& P\left(D_i \leq x_i\right) \geq \alpha && \forall i \in I \qquad \text{(II)} \\
& x_i \geq 0 && \forall i \in I
\end{aligned}
$$

Equivalent deterministic model employing chance constrainend programming

with the example of the stochastic production problem:

- $P(D_i \leq x_i) = F_{D_i}(x_i)$ *(distribution function)*
- $P\left(D_i \leq x_i\right) \geq \alpha \iff x_i \geq F_{D_i}^{-1}(\alpha)$ *(constant)*

Precalculated constants for the example

i	l_1	l_2	l_3
$F_{D_i}^{-1}(0{,}95)$	41,4	18,2	29,5

6.1 Scenario method
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Model: Stochastic production problem (Alternative 2)

Index sets:

I	set of products
R	set of resources

Parameters:

v_i	sales price of product $i \in I$
c_r	capacity of resource $r \in R$
a_{ri}	capacity consumption of product $i \in I$ on resource $r \in R$
$F_{D_i}^{-1}(\alpha)$	α quantile of demand for product $i \in I$

Decision variables:

x_i	production quantity of product $i \in I$

Model description:

$$
\begin{aligned}
\min \quad & \sum_{i \in I} v_i \cdot x_i \\
s.t. \quad & \sum_{r \in R} a_{ri} x_i \leq c_r \qquad && \forall r \in R \qquad \text{(I)} \\
& x_i \geq F_{D_i}^{-1}(\alpha) && \forall i \in I \qquad \text{(II)} \\
& x_i \geq 0 && \forall i \in I
\end{aligned}
$$

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