EduElevators
with
Standard Bank

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 23 pages./
Hierdie nasienriglyne bestaan uit 23 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede)**
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is)**
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

1.1	$a = -23{,}846\ldots$ $b = 0{,}227\ldots$ $\hat{y} = -23{,}85 + 0{,}23x$	✓ $a = -23{,}846\ldots$ ✓ $b = 0{,}227\ldots$ ✓ equation (3)
1.2	$\hat{y} = -23{,}85 + 0{,}23(550)$ $y = 102{,}65$ **OR** $y = 101{,}02$	✓ substitution of 550 ✓ answer (2) ✓✓ $y = 101{,}02$ (calculator) (2)
1.3	$r = 0{,}98$	✓ $r = 0{,}98$ (1)
1.4	Very strong positive correlation	✓ strong positive (1)

50	100	130	150	180	190	200	200

1.5.1	$\bar{x} = \frac{1200}{8}$ $\bar{x} = 150$ **OR** $\bar{x} = 150$	✓ 1200 ✓ answer (2) ✓✓ $\bar{x} = 150$ (2)
1.5.2	$\sigma = 50{,}50$	✓ $\sigma = 50{,}50$ (1)
1.5.3	$\bar{x} - \sigma$ $= 150 - 50{,}50$ $= 99{,}50$ $\therefore$ 1 stop	✓ calculation of $\bar{x} - \sigma$ ✓ answer (2)
		[12]

QUESTION/*VRAAG* 2

<table>
<tr><td>2.1</td><td>

Number of glasses of water per day	Number of staff members	Cumulative frequency
$0 \leq x < 2$	5	5
$2 \leq x < 4$	15	20
$4 \leq x < 6$	13	33
$6 \leq x < 8$	5	38
$8 \leq x < 10$	2	40

</td><td>✓ 5; 20
✓ 40
(2)</td></tr>
<tr><td>2.2</td><td>40 staff members</td><td>✓ answer
(1)</td></tr>
<tr><td>2.3</td><td>33 staff members</td><td>✓ answer
(1)</td></tr>
<tr><td>2.4</td><td>

$$\bar{x} = \frac{\left(1\times\left(5+\frac{k}{2}\right)\right)+(3\times15)+\left(5\times\left(13+\frac{k}{2}\right)\right)+(7\times5)+(9\times2)}{40+k}=4$$

$$5+\frac{k}{2}+45+65+\frac{5k}{2}+35+18=160+4k$$

$$3k+168=160+4k$$

$$k=8$$

OR

$$\bar{x} = \frac{(1\times5)+(15\times3)+(13\times5)+(5\times7)+(2\times9)}{40}$$

$= 4,2$

$\bar{x}_{\text{old}} - \bar{x}_{\text{current}} = 4,2-4$

$= 0,2$

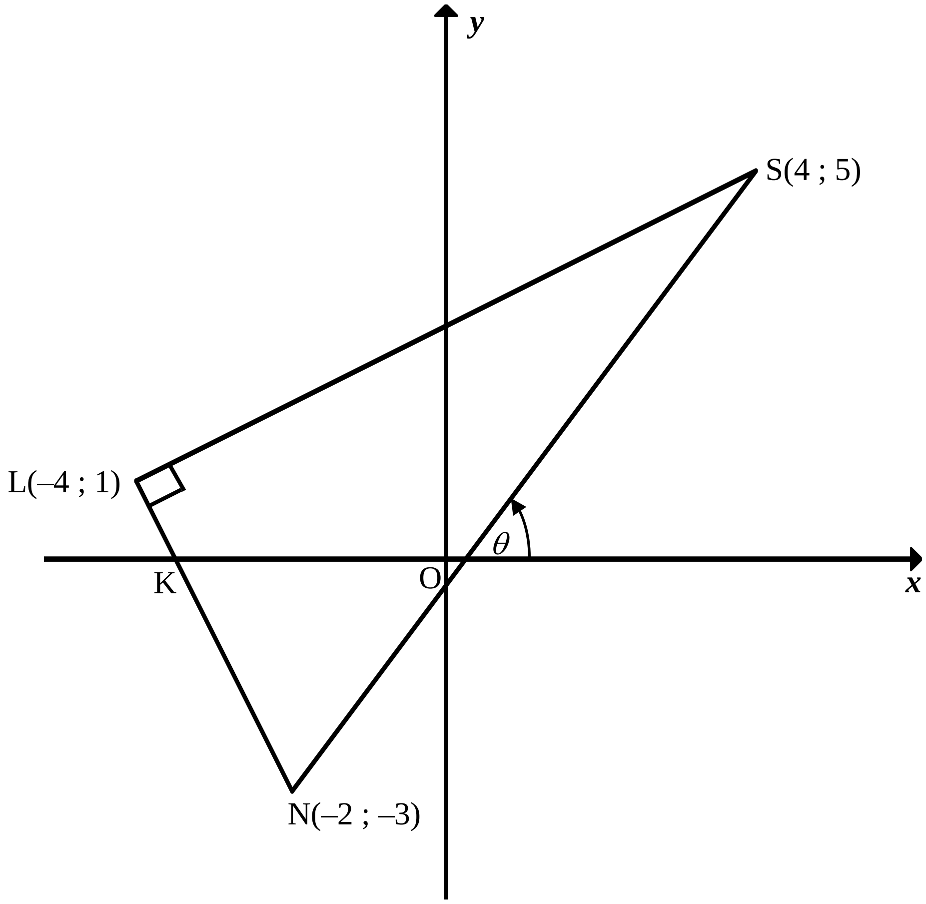
$\therefore 0,2\times 40$

$= 8$ teachers

</td><td>✓ answer from Q2.2 + k
✓ $\left(1\times\left(5+\frac{k}{2}\right)\right)$
✓ $\left(5\times\left(13+\frac{k}{2}\right)\right)$
✓ answer
(4)

✓ 4,2
✓ $\bar{x}_{\text{old}}-4$
✓ difference
✓ answer
(4)</td></tr>
<tr><td></td><td></td><td>**[8]**</td></tr>
</table>

QUESTION/*VRAAG* 3

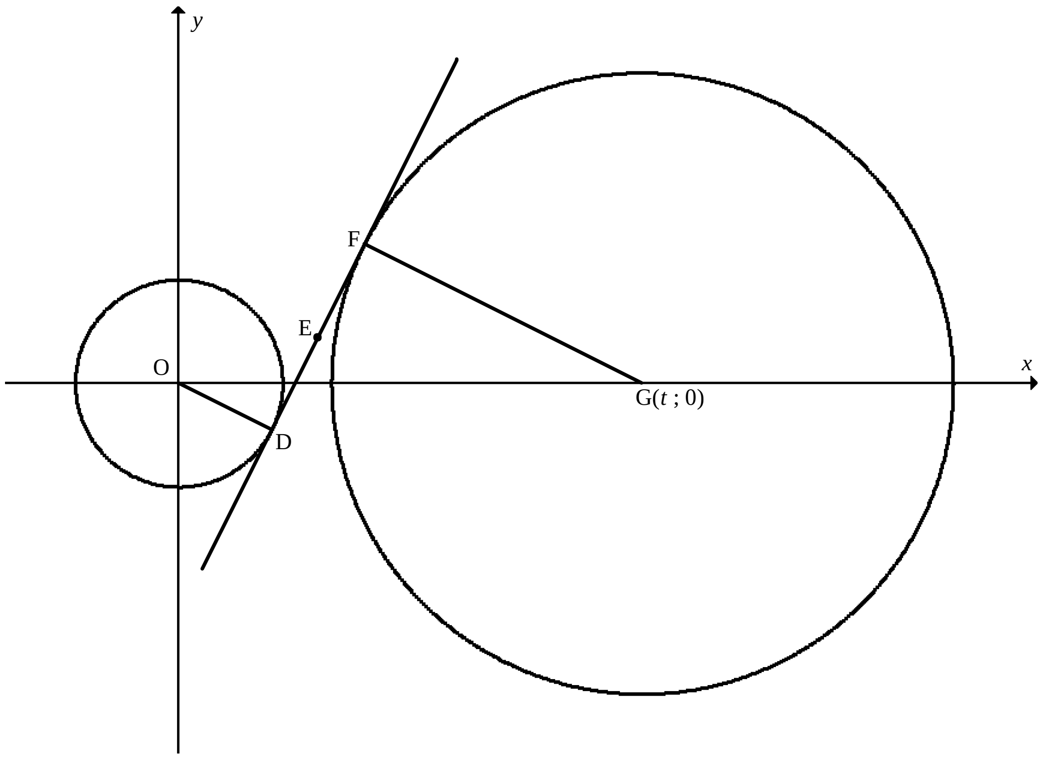


3.1	$SL = \sqrt{(x_S - x_L)^2 + (y_S - y_L)^2}$ $SL = \sqrt{(4-(-4))^2 + (5-1)^2}$ $SL = \sqrt{80} = 4\sqrt{5} = 8,94$ units	✓ substitution of S and L into correct formula ✓ answer (2)
3.2	$m_{SN} = \dfrac{5-(-3)}{4-(-2)}$ $m_{SN} = \dfrac{4}{3}$	✓ substitution of S and N into correct formula ✓ answer (2)
3.3	$m = \tan\theta = \dfrac{4}{3}$ $\theta = 53,13°$	✓ $\tan\theta = m_{SN}$ ✓ answer (2)
3.4	$m_{LN} = \dfrac{1-(-3)}{-4-(-2)}$ $m_{LN} = -2$ $L\hat{K}O = 116,565...°$ $L\hat{N}S = 116,565...° - 53,13°$ $L\hat{N}S = 63,44°$	✓ $m_{LN} = -2$ ✓ size of $L\hat{K}O$ ✓ answer (3)

	OR SN = 10 units $\sin L\hat{N}S = \dfrac{4\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$	✓ SN = 10 units ✓ correct trig ratio ✓ answer (3)
	OR $LN = 2\sqrt{5}$ units $\tan L\hat{N}S = \dfrac{4\sqrt{5}}{2\sqrt{5}}$ $L\hat{N}S = 63{,}44°$	✓ $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
	OR SN = 10 units $LN = 2\sqrt{5}$ units $\cos L\hat{N}S = \dfrac{2\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$	✓ SN = 10 units and $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
3.5	$m = \dfrac{4}{3}$ $1 = \dfrac{4}{3}(-4) + c$ **OR** $y - 1 = \dfrac{4}{3}(x - (-4))$ $c = \dfrac{19}{3}$ $y - 1 = \dfrac{4}{3}x + \dfrac{16}{3}$ $y = \dfrac{4}{3}x + \dfrac{19}{3}$ $y = \dfrac{4}{3}x + \dfrac{19}{3}$	✓ m_{SN} ✓ substitution of m_{SN} & L ✓ equation (3)
3.6	$SL = 4\sqrt{5}$ $LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \dfrac{1}{2}(4\sqrt{5})(2\sqrt{5})$ $= 20$ units^2 **OR**	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)

	SN = 10 units $LN = \sqrt{(-4-(-2))^2+(1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \frac{1}{2}(10)(2\sqrt{5})\sin 63,44°$ $= 20 \text{ units}^2$	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)
3.7	$\hat{L} = 90°$ SN is a diameter of circle S, L, N [chord subtends 90° **OR** converse ∠ in semi-circle] Centre of circle = $P\left(\frac{4+(-2)}{2};\frac{5+(-3)}{2}\right)$ $= P(1;1)$ **OR** Let the coordinates of P be $(a\,;b)$. Then, PL = PN: $(-4-a)^2+(1-b)^2=(-2-a)^2+(-3-b)^2$ $a-2b=-1$ …..equation 1 If PS = PN, then: $4a+2b=6$ ………….. equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1\,;1)$ **OR** If PL = PN, then: $a-2b=-1$ …..equation 1 If PS = PL, then: $2a+b=3$.....equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1\,;1)$	✓ SN is a diameter of circle S, L, N ✓ *x*-value ✓ *y*-value (3) ✓ 2 correct linear equations ✓ *x*-value ✓ *y*-value (3) ✓ 2 correct linear equations ✓ *x*-value ✓ *y*-value (3)
3.8	$L\hat{P}N = \theta = 53{,}13°$ [alt ∠s; LP ‖ x –axis] $\therefore L\hat{P}S = 126{,}87°$ **OR** $L\hat{N}S = 63{,}44°$ $\therefore L\hat{P}S = 126{,}88°$ [∠ at centre = 2 ×∠ at circumference] **OR** $L\hat{S}N = 26{,}56°$ [sum of ∠s in Δ] $S\hat{L}P = 26{,}56°$ [∠s opp equal radii] $\therefore L\hat{P}S = 126{,}88°$ [sum of ∠s in Δ] **OR** $(4\sqrt{5})^2 = 5^2+5^2-2(5)(5)\cos L\hat{P}S$ $\cos L\hat{P}S = -\frac{3}{5}$ $\therefore L\hat{P}S = 126{,}87°$	✓ $L\hat{P}N$ ✓ answer (2) ✓ $L\hat{N}S$ ✓ answer (2) ✓ $L\hat{S}N$ ✓ answer (2) ✓ correct substitution into cosine formula ✓ answer (2)
		[20]

QUESTION/*VRAAG* 4

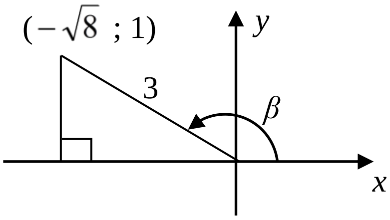


4.1	D(p ; -2) $x^2+y^2=20$ $p^2+(-2)^2=20$ $p^2=16$ $p=\pm4$ $p=4$	✓ substitution of point $D(p\,;-2)$ ✓ $p^2=16$ (2)
4.2	$\frac{4+x_F}{2}=6$ $\frac{-2+y_F}{2}=2$ $x_F=8$ $y_F=6$ $F(8;6)$ **OR** $x_E-x_D=6-4$ $y_E-y_D=2-(-2)$ $=2$ $=4$ $x_F=6+2=8$ $y_F=2+4=6$ $F(8;6)$	✓ method ✓ x-value ✓ y-value (3) ✓ method ✓ x-value ✓ y-value (3)

4.3	$m_{DE} = \frac{-2-2}{4-6}$ $m_{DE} = 2$ $-2 = 2(4) + c$ **OR** $y-(-2) = 2(x-4)$ $c = -10$ $y + 2 = 2x - 8$ $y = 2x - 10$ $y = 2x - 10$ **OR** $m_{OD} = -\frac{2}{4} = -\frac{1}{2}$ $\therefore m_{DE} = 2$ [tan ⊥ radius] $-2 = 2(4) + c$ **OR** $y-(-2) = 2(x-4)$ $c = -10$ $y + 2 = 2x - 8$ $y = 2x - 10$ $y = 2x - 10$	✓ correct substitution ✓ gradient of DE, DF or EF ✓ substitution of point D(4 ; –2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4) ✓ correct gradient of OD ✓ gradient of DE ✓ substitution of point D(4 ; –2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
4.4	$m_{DE} = 2$ $\therefore m_{GF} = -\frac{1}{2}$ [tan ⊥ radius] $\frac{0-6}{t-8} = -\frac{1}{2}$ $-(t-8) = 2(-6)$ $t = 20$ **OR** $y = 2x - 10$ $0 = 2x - 10$ $x = 5$ A(5 ; 0) In ΔAFG: FA ⊥ FG $FA^2 = (6-0)^2 + (8-5)^2 = 45$ $FG^2 = (t-8)^2 + (0-6)^2$ $= t^2 - 16t + 100$ $GA^2 = (t-5)^2$ $= t^2 - 10t + 25$ $\therefore GA^2 = GF^2 + FA^2$ $t^2 - 10t + 25 = t^2 - 16t + 100 + 45$ $6t = 120$ $t = 20$	 ✓ correct gradient of GF ✓ substitution of F ✓ answer (3) ✓ x-intercept of DF ✓ substitution into Pythagoras ✓ answer (3)

4.5	$F(8;6)$ G(20 ; 0) $(8-20)^2+(6-0)^2=r^2$ $r^2=180$ $(x-20)^2+y^2=180$ $x^2+y^2-40x+220=0$	✓ substitution of F and G ✓ value of r^2 ✓ equation of circle ✓ answer (4)
4.6	Smaller circle $r=2\sqrt{5}$ Larger circle $r=6\sqrt{5}$ G(20 ; 0) $k=20-(6\sqrt{5}-2\sqrt{5})$ or $k=20+(6\sqrt{5}-2\sqrt{5})$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ = 11,06 units = 28,94 units **OR** Smaller circle $r=2\sqrt{5}$ $k=2\left(2\sqrt{5}\right)+20-8\sqrt{5}$ or $k=2\left(6\sqrt{5}\right)+20-8\sqrt{5}$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ = 11,06 units = 28,94 units **OR** $x^2+y^2-40x+220=0$ $y=0$ $\therefore x^2-40x+220=0$ $\therefore x=20+6\sqrt{5}$ or $x=20-6\sqrt{5}$ $\therefore k=20+6\sqrt{5}-\sqrt{20}$ or $k=20-6\sqrt{5}+\sqrt{20}$ $\therefore k=20+4\sqrt{5}$ $\therefore k=20-4\sqrt{5}$ = 11,06 units = 28,94 units	✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ x-intercepts ✓ method ✓ answer ✓ answer (4)
		[20]

QUESTION/VRAAG 5

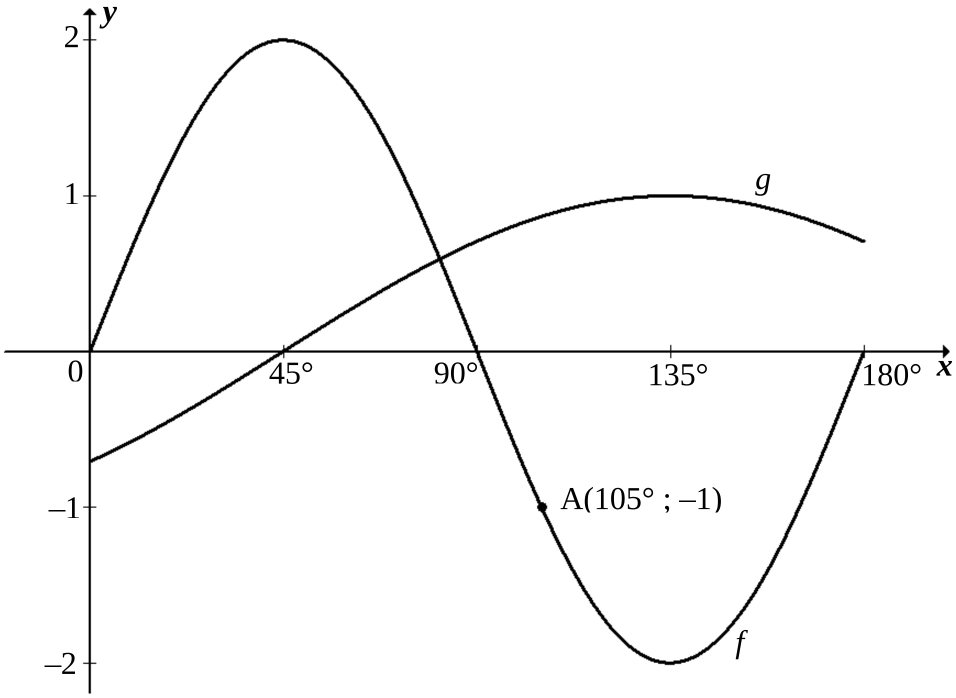
5.1.1	$\sin\beta = \frac{1}{3}$ $\quad \beta \in (90°;270°)$  $x = -\sqrt{8} = -2\sqrt{2}$ $\cos\beta$ $= \frac{-2\sqrt{2}}{3}$ **OR** $\sin\beta = \frac{1}{3}$ $\quad \beta \in (90°;270°)$ $\cos^2\beta = 1 - \sin^2\beta$ $\cos^2\beta = 1 - \left(\frac{1}{3}\right)^2$ $\cos^2\beta = \frac{8}{9}$ $\cos\beta = \frac{-\sqrt{8}}{3}$ $= \frac{-2\sqrt{2}}{3}$	✓ $x^2 + y^2 = r^2$ ✓ $x = -2\sqrt{2}$ ✓ answer (3) ✓ square identity ✓ $\cos^2\beta$ ✓ answer (3)
5.1.2	$\sin 2\beta$ $= 2\sin\beta\cos\beta$ $= 2\left(\frac{1}{3}\right)\left(\frac{-\sqrt{8}}{3}\right)$ $= \frac{-2\sqrt{8}}{9}$ **OR** $2\left(\frac{-2\sqrt{2}}{9}\right)$ $= \frac{-4\sqrt{2}}{9}$	✓ double angle ✓ substitution ✓ answer (3)
5.1.3	$\cos(450° - \beta)$ $= \cos(90° - \beta)$ $= \sin\beta$ $= \frac{1}{3}$ **OR**	✓ $\cos(90° - \beta)$ ✓ co-ratio ✓ answer (3)

	$\cos(450° - \beta)$ $= \cos 450° \cos\beta + \sin 450° \sin\beta$ $= \cos 90° \cos\beta + \sin 90° \sin\beta$ $= \sin\beta$ $= \frac{1}{3}$	✓ expansion ✓ reduction ✓ answer (3)
5.2.1	LHS = $\frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \frac{\cos^2 x\left(\cos^2 x + \sin^2 x\right)}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ = RHS	✓ factors ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $\cos^2 x = 1-\sin^2 x$ ✓ factors (4)
	OR LHS = $\frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \frac{\cos^4 x + \left(1-\cos^2 x\right)\cos^2 x}{1+\sin x}$ $= \frac{\cos^4 x + \cos^2 x - \cos^4 x}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ = RHS	✓ $\sin^2 x = 1-\cos^2 x$ ✓ expansion ✓ $\cos^2 x = 1-\sin^2 x$ ✓ factors (4)
	OR RHS $= 1-\sin x$ $= (1-\sin x) \times \frac{1+\sin x}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{\cos^2 x}{1+\sin x}$ $= \frac{\cos^2 x\left(\sin^2 x + \cos^2 x\right)}{1+\sin x}$ $= \frac{\cos^4 x + \cos^2 x.\sin^2 x}{1+\sin x}$ = LHS	✓ $\times \frac{1+\sin x}{1+\sin x}$ ✓ product ✓ $1-\sin^2 x = \cos^2 x$ ✓ $1 = \cos^2 x + \sin^2 x$ (4)

5.2.2	$\sin x + 1 = 0$ $\sin x = -1$ ref. $\angle = 90°$ $x = 270°$	✓ $\sin x + 1 = 0$ ✓ $x = 270°$ (2)
5.2.3	$y = \dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $y = 1 - \sin x$ $\therefore$ Minimum = 0	✓✓ Minimum = 0 (2)
5.3.1	sin (A – B) $= \cos\left[90° - (A - B)\right]$ $= \cos\left[(90° - A) - (-B)\right]$ $= \cos(90° - A)\cos(-B) + \sin(90° - A)\sin(-B)$ $= \sin A \cos B + \cos A(-\sin B)$ $= \sin A \cos B - \cos A \sin B$ **OR** sin (A – B) $= \cos\left[90° - (A - B)\right]$ $= \cos\left[(90° + B) - A\right]$ $= \cos(90° + B)\cos A + \sin(90° + B)\sin A$ $= -\sin B \cos A + \cos B \sin A$ $= \sin A \cos B - \cos A \sin B$	✓ co-ratio ✓ compound angle ✓ reduction (3) ✓ co-ratio ✓ compound angle ✓ reduction (3)
5.3.2	$\sin 48° \cos x - \cos 48° \sin x = \cos 2x$ $\sin(48° - x) = \cos 2x$ $\sin(48° - x) = \sin(90° - 2x)$ $48° - x = 90° - 2x + k.360°$ or $48° - x = 180° - (90° - 2x) + k.360°$ $x = 42° + k.360°$ $-3x = 42° + k.360°$ $x = -14° - k.120°$; $k \in Z$ **OR** $\sin 48° \cos x - \cos 48° \sin x = \cos 2x$ $\sin(48° - x) = \cos 2x$ $\cos(90° - 48° + x) = \cos 2x$ $\cos(42° + x) = \cos 2x$ $42° + x = 2x + k.360°$ or $42° + x = 360° - 2x + k.360°$ $-x = -42° + k.360°$ $3x = 318° + k.360°$ $x = 42° - k.360°$ $x = 106° + k.120°$; $k \in Z$	✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k \in Z$ (5) ✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k \in Z$ (5)

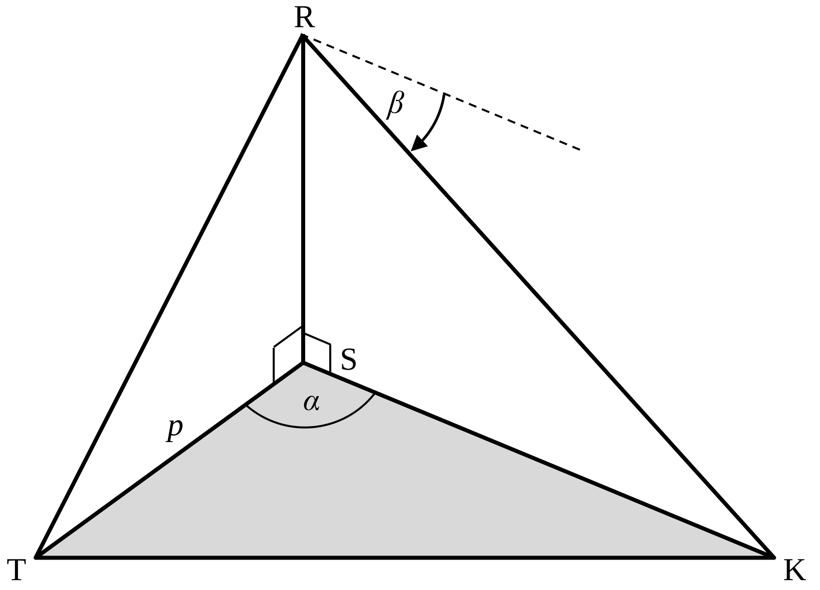
5.4	$\dfrac{\sin 3x+\sin x}{\cos 2x+1}$ $=\dfrac{\sin(2x+x)+\sin(2x-x)}{\cos 2x+1}$ $=\dfrac{\sin 2x\cos x+\cos 2x\sin x+\sin 2x\cos x-\cos 2x\sin x}{2\cos^2 x-1+1}$ $=\dfrac{2\sin 2x\cos x}{2\cos^2 x}$ $=\dfrac{2(2\sin x\cos x)\cos x}{2\cos^2 x}$ $=\dfrac{4\sin x\cos^2 x}{2\cos^2 x}$ $=2\sin x$	✓ $3x=(2x+x)$ ✓ expansion ✓ double angle of cos2*x* ✓ simplification ✓ $\sin 2x=2\sin x\cos x$ ✓ answer (6)
	OR	
	$\dfrac{\sin 3x+\sin x}{\cos 2x+1}$ $=\dfrac{\sin(2x+x)+\sin x}{2\cos^2 x-1+1}$ $=\dfrac{\sin 2x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$ $=\dfrac{2\sin x\cos x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$ $=\dfrac{\sin x\left(2\cos^2 x+\cos 2x+1\right)}{2\cos^2 x}$ $=\dfrac{\sin x\left(2\cos^2 x+2\cos^2 x-1+1\right)}{2\cos^2 x}$ $=2\sin x$	✓ $3x=(2x+x)$ ✓ double angle of cos2*x* ✓ expansion ✓ $\sin 2x=2\sin x\cos x$ ✓ common factor ✓ answer (6)
		[31]

QUESTION/*VRAAG* 6



6.1	Period = 180°	✓ 180° (1)
6.2	$y \in \left[-\frac{\sqrt{2}}{2}\ ;1\right]$ **OR** $y \in [-0,71;\ 1]$ **OR** $-\frac{\sqrt{2}}{2} \le y \le 1$	✓ $-\frac{\sqrt{2}}{2}$ ✓ $y \in \left[-\frac{\sqrt{2}}{2}\ ;1\right]$ (2)
6.3.1	$x \in (45°\ ;\ 90°)$ **OR** $45° < x < 90°$	✓✓ $x \in (45°\ ;\ 90°)$ (2)
6.3.2	$f(x)+1 \le 0$ $f(x) \le -1$ $x \in [105°\ ;\ 165°]$ **OR** $105° \le x \le 165°$	✓✓ $x \in [105°\ ;\ 165°]$ (2)
6.4	$p(x) = -2\sin 2x$ $-2\sin 2x = -1$ **OR** $2\sin 2x = 1$ k = 15° or k = 75°	✓ reading off $f(x) = 1$ or $-f(x) = -1$ ✓ 15° ✓ 75° (3)
6.5	$g(x) = -\cos(x+45°)$ $h(x) = -\cos(x+90°)$ $h(x) = \sin x$	✓ $-\cos(x+90°)$ ✓ answer (2)
		[12]

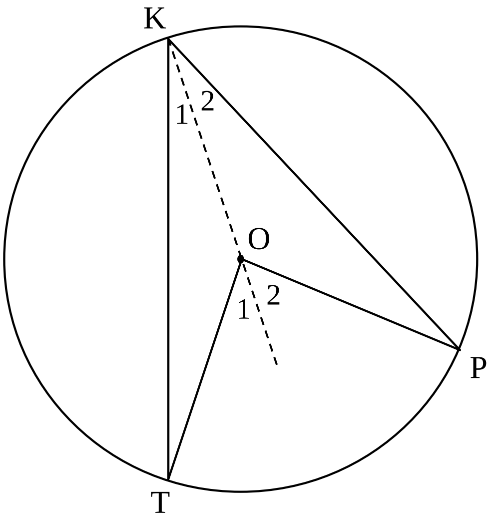
QUESTION/*VRAAG* 7



7.1	$\text{Area } \Delta\text{STK} = \frac{1}{2}p(\text{SK})\sin\alpha$ $q = \frac{1}{2}p(\text{SK})\sin\alpha$ $\text{SK} = \frac{q}{\frac{1}{2}p\sin\alpha}$ $= \frac{2q}{p\sin\alpha}$	✓ substitution into the correct formula ✓ answer (2)
7.2	$\text{R}\hat{\text{K}}\text{S} = \beta$ $\frac{\text{RS}}{\text{SK}} = \tan\beta$ $\text{RS} = \frac{2q\tan\beta}{p\sin\alpha}$ **OR** $\frac{\text{RS}}{\sin\beta} = \frac{\text{SK}}{\sin(90° - \beta)}$ $\text{RS}\cos\beta = \text{SK}\sin\beta$ $\text{RS} = \text{SK}\tan\beta$ $\text{RS} = \frac{2q\tan\beta}{p\sin\alpha}$	✓ $\text{R}\hat{\text{K}}\text{S} = \beta$ ✓ correct trig ratio (2) ✓ $\text{R}\hat{\text{K}}\text{S} = \beta$ ✓ $\tan\beta = \frac{\sin\beta}{\cos\beta}$ (2)
7.3	$70 = \frac{2(2500)\tan 42°}{80\sin\alpha}$ $\sin\alpha = \frac{25}{28}\tan 42°$ **OR** $\sin\alpha = 0,80...$ $\alpha = 53,51°$	✓ correct substitution of values into RS ✓ value of $\sin\alpha$ ✓ answer (3)
		[7]

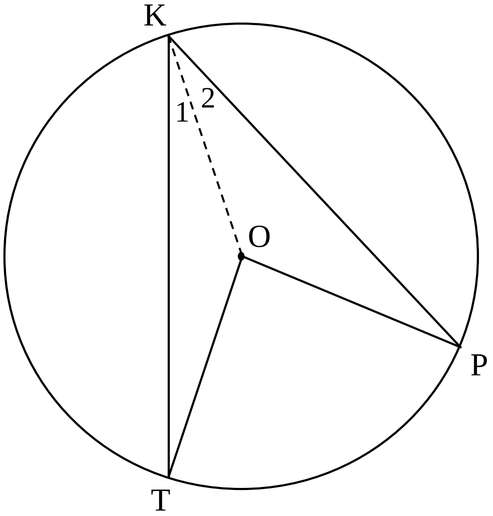
QUESTION/*VRAAG* 8

8.1



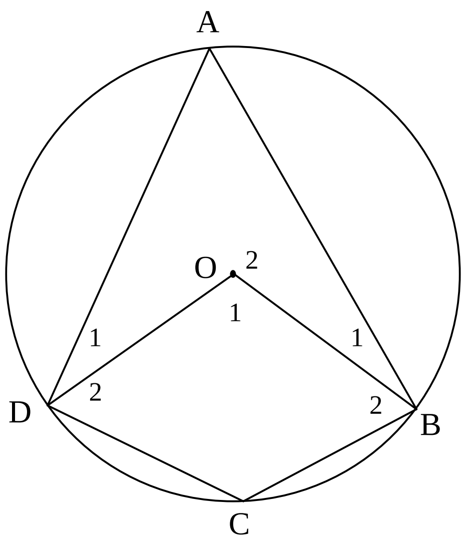
8.1	Construction: Draw KO produced	✓ construction
	$\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext ∠ of Δ]	
	But $\hat{K}_1 = \hat{T}$ [∠s opp equal sides]	✓ S / R
	$\therefore \hat{O}_1 = 2\hat{K}_1$	✓ S
	$\hat{O}_2 = \hat{K}_2 + P$ [ext ∠ of Δ]	
	But $\hat{K}_2 = P$ [∠s opp equal sides]	
	$\therefore \hat{O}_2 = 2\hat{K}_2$	✓ S
	$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$	✓ S
	$= 2\left(\hat{K}_1 + \hat{K}_2\right)$	
	$\therefore T\hat{O}P = 2T\hat{K}P$	(5)
	OR	

8.1



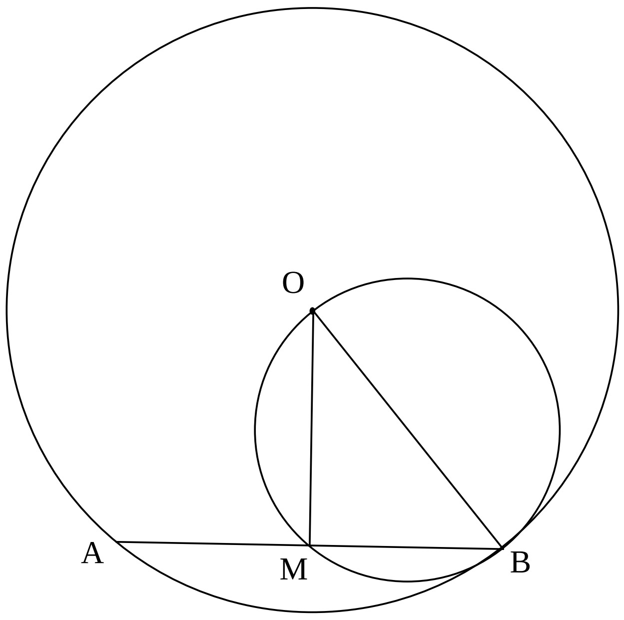
8.1	Construction: Draw KO $\hat{T} = \hat{K}_1$ [$\angle$s opp. equal sides] $\therefore K\hat{O}T = 180° - 2\hat{K}_1$ [sum of $\angle$s of ΔKOT] $\hat{P} = \hat{K}_2$ [$\angle$s opp. equal sides] $\therefore K\hat{O}P = 180° - 2\hat{K}_2$ [sum of $\angle$s of ΔKOP] $T\hat{O}P = 360° - \left(K\hat{O}T + K\hat{O}P\right)$ [$\angle$s around a point] $= 360° - \left(180° - 2\hat{K}_1 + 180° - 2\hat{K}_2\right)$ $= 2\hat{K}_1 + 2\hat{K}_2$ $= 2\left(\hat{K}_1 + \hat{K}_2\right)$ $\therefore T\hat{O}P = 2T\hat{K}P$	✓ construction ✓ S / R ✓ S ✓ S ✓ S (5)

8.2



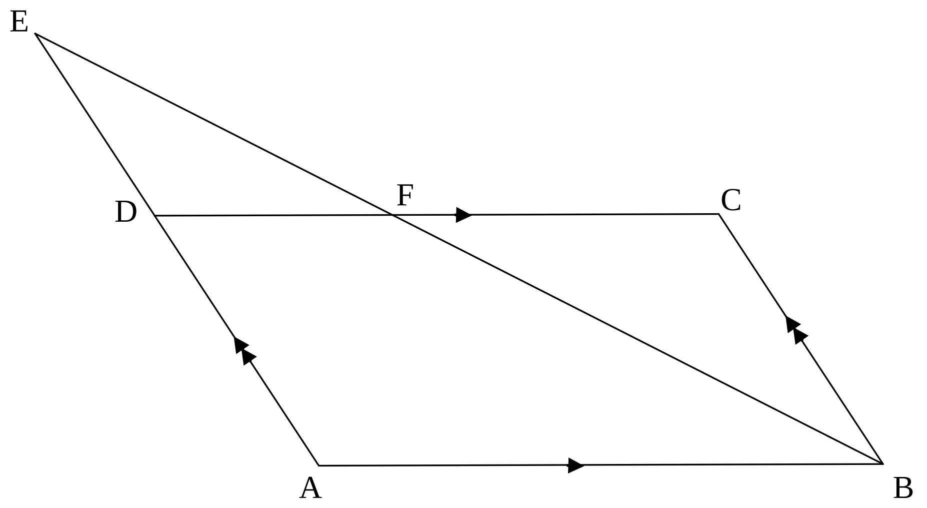
8.2	$\hat{O}_1 = 4x + 100°$ [given] $\therefore \hat{A} = 2x + 50°$ [$\angle$ at centre = 2 ×$\angle$ at circumference] $x + 34° + 2x + 50° = 180°$ [opp $\angle$s of cyclic quad] $3x = 96°$ $x = 32°$ **OR** $\hat{O}_2 = 2x + 68°$ [$\angle$ at centre = 2 ×$\angle$ at circumference] $4x + 100° + 2x + 68° = 360°$ [$\angle$s round a pt] $6x = 192°$ $x = 32°$ **OR** $\hat{O}_2 = -4x + 260°$ [$\angle$s round a pt] $2\hat{C} = -4x + 260°$ [$\angle$ at centre = 2 ×$\angle$ at circumference] $\hat{C} = -2x + 130°$ $x + 34° = -2x + 130°$ $3x = 96°$ $x = 32°$	✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5)

8.3



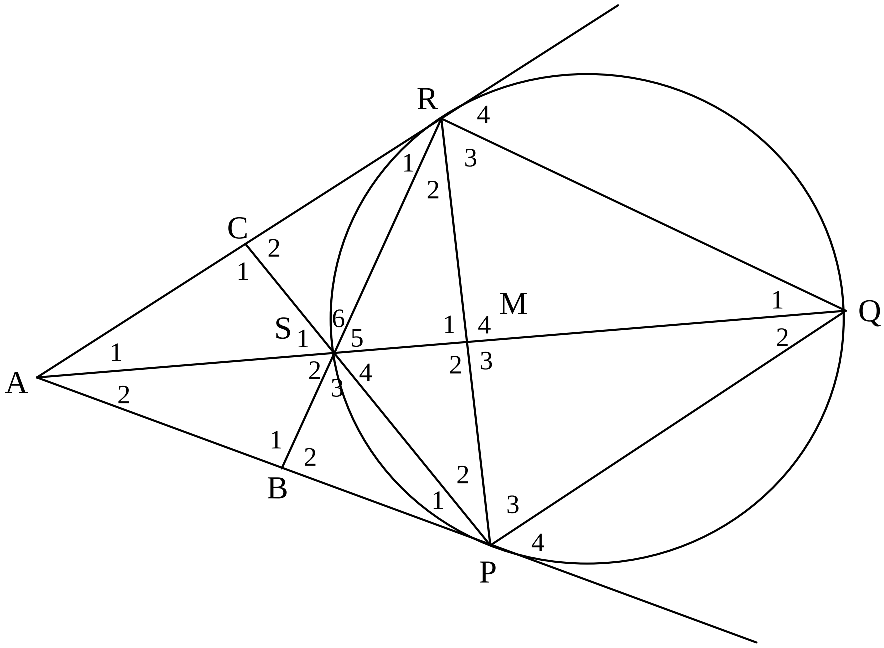
8.3.1	$O\hat{M}B = 90°$ [$\angle$ in semi circle]	✓ S ✓ R (2)
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$ $\therefore MB = 5\sqrt{3}$ [line from centre $\perp$ to chord] $OB^2 = OM^2 + MB^2$ [Pythagoras] $OB^2 = 5^2 + \left(5\sqrt{3}\right)^2$ OB = 10 units	✓ S ✓ R ✓ S ✓ answer (4)
		[16]

QUESTION/*VRAAG* 9



9.1	$\frac{FB}{EB}=\frac{DA}{EA}$ [prop theorem; DC ‖ AB] **OR** [line ‖ one side of Δ] $FB=\frac{4p\times 21}{7p}$ FB = 12 units	✓ S ✓ R ✓ answer (3)
9.2	In ΔEDF and ΔEAB: $\hat{E}$ is common $E\hat{D}F=\hat{A}$ [corresp ∠s; EA ‖ CB] $E\hat{F}D=E\hat{B}A$ [corresp ∠s; DC ‖ AB] ΔEDF ‖| ΔEAB [∠;∠;∠]	✓ S ✓ S/R ✓ S **OR** R (3)
9.3	$\frac{DF}{AB}=\frac{ED}{EA}$ [‖| Δs] $DF=\frac{3p\times 14}{7p}$ DF = 6 units FC = 8 units [DC = AB = 14 units; opp sides of $\|^{m}$] **OR** ΔEDF ‖| ΔBCF [∠;∠;∠] $\frac{ED}{BC}=\frac{DF}{CF}$ [‖| Δs] $\frac{3}{4}=\frac{14-FC}{FC}$ [BC = AD; opp sides of $\|^{m}$] $3FC=56-4FC$ $FC=8$	✓ S ✓ DF = 6 ✓ FC = 14 − DF (3) ✓ ΔEDF ‖| ΔBCF ✓ $\frac{3}{4}=\frac{14-FC}{FC}$ ✓ answer (3)
		[9]

QUESTION/*VRAAG* 10



10.1	$\hat{S}_3 = P\hat{Q}R$	[ext ∠ of cyclic quad]	✓ S ✓ R
	$\hat{R}_3 = P\hat{Q}R$	[∠s opp equal sides]	✓ S / R
	$\therefore \hat{S}_3 = \hat{R}_3$		
	But $\hat{S}_4 = \hat{R}_3$	[∠s in the same seg]	✓ S ✓ R
	$\therefore \hat{S}_3 = \hat{S}_4$		(5)
10.2	$\hat{R}_1 + \hat{R}_2 = P\hat{Q}R$	[tan chord theorem]	✓ S ✓ R
	$\hat{S}_4 = P\hat{Q}R$	[proved in 10.1]	
	$\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$		✓ S
	SMRC is a cyclic quad	[converse ext ∠ of cyclic quad]	✓ R
			(4)
10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$	[ext ∠ of Δ]	✓ S ✓ R
	$\hat{S}_4 = \hat{P}_1 + \hat{A}_2$	[ext ∠ of Δ]	✓ S
	$\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$		
	But $\hat{P}_1 = \hat{R}_2$	[tan chord theorem]	✓ S ✓ R
	$\therefore \hat{P}_2 = \hat{A}_2$		
	RP is a tangent to the circle	[converse tan chord theorem] **OR** [∠ between line and chord] **OR** [converse alt seg theorem]	✓ R
			(6)
	OR		

	In ΔMSP and ΔMPA		
	$\hat{M}_2$ is common		✓ S
	AR = AP	[tans from same point]	✓ S / R
	$\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$	[$\angle$s opp equal sides]	✓ S
	$\hat{S}_4 = \hat{R}_1 + \hat{R}_2$	[proved in 10.2]	
	$\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$		✓ S
	$\therefore \hat{P}_2 = \hat{A}_2$	[sum of $\angle$s in Δ]	✓ S
	RP is a tangent to the circle	[converse tan chord theorem]	✓ R
			(6)
			[15]

TOTAL/TOTAAL: 150