Universidade de Aveiro
2017-2018

Quantum Noise

Diamantino Silva
(diamantinosilva@ua.pt)

Department of Electronics, Telecommunications and Informatics,
University of Aveiro, Aveiro, Portugal
Instituto de Telecomunicações, Aveiro, Portugal

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Quantum Noise

Quantum noise is an intrinsic property of light, it is a manifestation of fluctuations of the quantum field. In fact, there is a minimum energy level ??? (ver mark fox) of uncertanty given by the Heisenberg uncertanty

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{1}$$

The objective of this work is to model quantum noise in a double homodyne detection system and validate the numerical model with experimental data.

Quantum Noise

The studied communication system is based on the transmission of coherent states.
We start by defining a number state, $|n\rangle$, which has exactly n photons. The action of the creation $\hat{a}^\dagger$ and annihilation $\hat{a}$ operators are

$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle \quad (2)\ , \qquad \hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle \quad (3)\ , \qquad \hat{n}\,|n\rangle = n\,|n\rangle \quad (4)$$

in which $\hat{n}$ is the number operator.

Quantum Noise

Mathematically, a coherent state is represented in the number states basis as

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{5}$$

in which the complex number α is the sole parameter that characterizes it. The measurement of quadratures is based in the following quantum operators:

$$\hat{X} = \frac{1}{2}\left(\hat{a}^{\dagger} + \hat{a}\right) \;\; (6)\;, \qquad \hat{Y} = \frac{i}{2}\left(\hat{a}^{\dagger} - \hat{a}\right) \;\; (7)$$

In fact, the expected value of this two operators are:

$$\langle\alpha|\hat{X}|\alpha\rangle = \mathrm{Re}(\alpha), \qquad \langle\alpha|\hat{Y}|\alpha\rangle = \mathrm{Im}(\alpha)$$

Quantum Noise - Mathematics

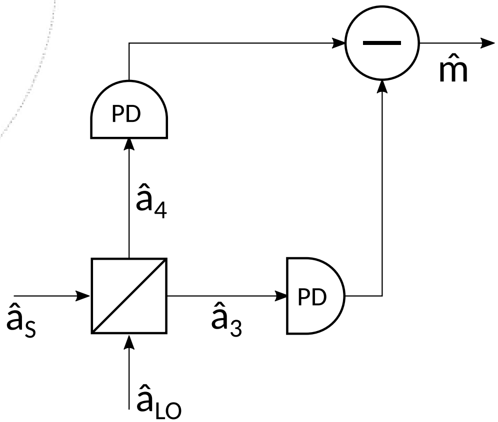
The variance of these two operators is given by:

$$\mathrm{Var}(\hat{X}) = \mathrm{Var}(\hat{Y}) = \frac{1}{4} \tag{8}$$

This result show us that for both quadratures, the variance of measurement is the same and independent of the value of α.

Homodyne Detection

The measurement of quadratures is made by the homodyne technique. The quantum description of the detection is described by the following illustration:



This technique basically measures the phase difference between a input signal S and a local oscillator LO. Figure xx shows the quantum mechanical description of the technique, in which $\hat{m}$ is the operator of the output difference between the photocurrents.

Homodyne Detection

Given the input signal given by the state $|\alpha\rangle$ ($\alpha = |\alpha|e^{i\theta_\alpha}$) and the local oscillator given by state $|\beta\rangle$ ($\beta = |\beta|e^{i\theta_\beta}$), the expected value of $\hat{m}$ and it's variance will be

$$\langle m\rangle = 2|\alpha||\beta|\cos(\theta_\alpha - \theta_\beta) \quad (9)\ , \qquad \mathrm{Var}(m) = |\alpha|^2 + |\beta|^2 \quad (10)$$

If we normalize the units of m by $2|\beta|$ and knowing that $\alpha \ll \beta$, then it can be simplified to

$$\langle m\rangle = |\alpha|\cos(\theta_\alpha - \theta_\beta) \quad (11)\ , \qquad \mathrm{Var}(m) \approx \frac{1}{4} \quad (12)$$

Homodyne Detection

In fact, we can measure the two quadratures X and Y at the same time, using the double homodyne detection. This tecnique consists in simply divide the signal in two beams with half the power of the original one and one of them is measure in-phase with the local oscillator, and the other is measured with a phase difference of $\pi/2$ relative to the first one???????

$$\langle m_X \rangle = \left| \frac{\alpha}{\sqrt{2}} \right| \cos(\theta_\alpha - \theta_\beta) \quad (13), \qquad \mathrm{Var}(m_X) \approx \frac{1}{4} \quad (14)$$

$$\langle m_Y \rangle = \left| \frac{\alpha}{\sqrt{2}} \right| \sin(\theta_\alpha - \theta_\beta) \quad (15), \qquad \mathrm{Var}(m_Y) \approx \frac{1}{4} \quad (16)$$

E-mail: diamantinosilva@ua.pt