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Week 1-2: Logic and Truth Tables

<Section 2.1: Logical Form & Logical Equivalences>

Warm Up Consider the following

Speific → General

1, 2, 3, 4, ..., n
1st, 2nd, 3rd, 4th, nth

"inductive logic"

General → Specific

"argument form"

All men are mortal (premise 1)
Socrates is a man (premise 2)
∴ socrates is mortal (conclusion)

"deductive Reasoning"

Example 1

If p then q

↳ If all integers are rational, then the number 1 is rational.
↳ All integers are rational

→ p
∴ q

Therefore, the number 1 is rational

Def^n (statement)

A statement is a sentence that is true, or false, but not both

Example 2 "Steve is a student." → just a sentence
↳ could be both true and false
↳ not a statement

● We can combine statements with "NOT", "AND", "OR"

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$p :=$ today is cold $\qquad q :=$ today is windy

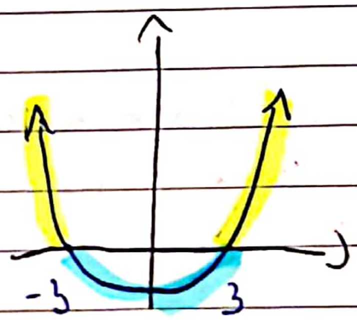
Meaning	symbol	Name	Example
NOT	$\sim$	negation	$\sim p$: Today is <u>not</u> cold
AND	$\wedge$	Conjunction	$p \wedge q$: Today is cold <u>and</u> today is windy
OR	$\vee$	disjunction	$p \vee q$: Today is cold OR today is windy.

Example 3 Write this sentence symbolically

Today is neither cold nor windy

Today is not cold and not windy

$$\boxed{\sim p \wedge \sim q}$$



$x \geq 3$ means $[x > 3$ or $x = 3]$

$-3 \leq x \leq 3$ means $[x \geq -3$ and $x \leq 3]$

<u>inequalities</u>
- $x \leq a$ means $x < a$ or $x = a$
- $a \leq x \leq b$ means $x \geq a$ and $x \leq b$

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Example 4: Let say we assign as follows:

$$\begin{cases} p := 0 < x \\ q := x < 3 \\ r := x = 3 \end{cases}$$

Write the following symbolically:

1. $x \leq 3 \Rightarrow q \vee r$
2. $0 < x < 3 \Rightarrow p \wedge q$
3. $0 < x \leq 3 \Rightarrow p \wedge (q \vee r)$

Truth Values & Truth Table

To find the truth values of statements, we use "Truth table"

(Note: Each statement is either true or false.)

p	$\sim p$
T	F
F	T

Truth table for two statements: p, q

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

$(p \vee q) \wedge \sim(p \wedge q)$

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Example 5: Build the truth table for $(p \vee q) \wedge \sim(p \wedge q)$
(Tip! start with $p, q, p \vee q, p \wedge q$, and so on)

p	q	$p \vee q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

- This is the truth table for "exclusive or" $p \oplus q$
- "one or the other, but not both!"

Take-Home Example: Build a truth table for the statements
$(p \wedge q) \vee \sim r$

p	q	r	$p \wedge q$	$\sim r$	$(p \vee q) \vee \sim r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

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Week 1-3: Logical Equivalences & Conditional Statements

<sec 2.1: Logical Equivalences>

Basic idea If two statement forms have identical truth values, then they are logically equivalent.

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

same truth values

$\therefore p$ is equivalent to $\sim(\sim p)$

$$p \equiv \sim(\sim p)$$

"Useful Negation Laws: De Morgan's Laws"

De Morgan's Laws

$$\boxed{\begin{aligned} \sim(p \wedge q) &\equiv \sim p \vee \sim q \\ \sim(p \vee q) &\equiv \sim p \wedge \sim q \end{aligned}}$$

warning!
$\sim(p \wedge q) \equiv \sim p \wedge \sim q$
wrong!

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F	F
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F
F	F	T	T	F	T	T	T

→ clearly, $\sim(p \wedge q) \not\equiv \sim p \wedge \sim q$

This shows that
$\sim(p \wedge q) \equiv \sim p \vee \sim q$

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$\boxed{\text{Example 6:}}$ Use De Morgan's Laws to write negation of $-1 < x \leq 4$.

Let $p := -1 < x$

$q := x < 4$

$r := x = 4$

$-1 < x \leq 4$ means $p \wedge (q \vee r)$

so $\sim[p \wedge (q \vee r)] \equiv \sim p \vee \sim(q \vee r)$ By De Morgan's Laws

$\equiv \sim p \vee \sim q \wedge \sim r$

$x \leq -1$ or ($x \geq 4$ and $x \neq 4$)

Ans: $\boxed{x \leq -1 \text{ or } x > 4}$

Tautologies and Contradictions

- Tautology: is a statement form that is always true
- Contradiction: is a statement form that is always false

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

tautology: $p \vee \sim p \equiv t$

contradiction: $p \wedge \sim p \equiv c$

$x \cdot 1 = x$

- $p \wedge t \equiv p$
- $p \wedge c \equiv c$

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Theorem 2.1.1.
Logical equivalences

statement variables: p, q, r tautology t, contradiction c

1. Commutative laws $p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
2. Associative laws $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
   $(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
   $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws $p \wedge t \equiv p$
   $p \vee c \equiv p$
5. Negation laws $p \vee \sim p \equiv t$
   $p \wedge \sim p \equiv c$
6. Double negative law: $\sim(\sim p) \equiv p$
7. Idempotent laws: $p \wedge p \equiv p$ $p \vee p \equiv p$
8. Universal bound laws: $p \vee t \equiv t$ $p \wedge c \equiv c$
9. De Morgan's laws: $\sim(p \wedge q) \equiv \sim p \vee \sim q$
   $\sim(p \vee q) \equiv \sim p \wedge \sim q$

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10. Absorption Laws: $p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

11. Negations of t and c: $\sim t \equiv c$ $\quad \sim c \equiv t$

$\boxed{\text{Example 7:}}$ Use Theorem 2.1.1. to simplify $\sim(p \wedge q) \wedge (p \vee q)$

$$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv [\sim(\sim p) \vee \sim q] \wedge p \vee q$$

By De Morgan's Laws

$$\equiv (p \vee \sim q) \wedge p \vee q$$

By Double Negative Law

$$\equiv p \vee (\sim q \wedge q)$$

By Distributive Laws

$$\equiv p \vee c$$

By Negation Laws

$$\equiv p$$

by Identity Laws

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< Section 2.2 Conditional Statements >

statement form: if p then q
symbol: $p \to q$

This is conditional because the truth of statement q is conditioned on the truth of p.

Warm Up Example: If $\underbrace{\text{you study hard}}_{p}$ then $\underbrace{\text{you will earn an A}}_{q}$

Question: When is this statement false?

Answer: $\underbrace{\text{You studied hard}}_{T}$, $\underbrace{\text{but not getting an A}}_{F}$

"из нетачног следи све"

Truth Table for $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Warm Up Example: Determine true or false

if $\underbrace{1=2}_{F}$ then $\underbrace{3=0}_{F}$

Note: Hypothesis is False. This means $p \to q$ is true by default! (i.e. vaccuously true)

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Order of Operation

1. $\sim$
2. $\wedge$ $\vee$
3. $\rightarrow$ $\leftarrow$ $\leftrightarrow$

Example 1: Construct a truth table for $p \vee \sim q \rightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim p$	$p \vee \sim q \rightarrow \sim p$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

hypothesis conclusion Answer

Your turn! Find the truth tables for

a) $p \rightarrow q$ b) $\sim p \vee q$

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\boxed{p \rightarrow q \equiv \sim p \vee q}$ $\therefore$ They are logically equivalent!

$\sim(p \rightarrow q) \equiv \sim(\sim p \vee q)$ By De Morgan's Laws

$\equiv \sim(\sim p) \wedge \sim q$ By Double Negative Laws

$\equiv p \wedge \sim q$

$$\boxed{\sim(p \rightarrow q) \equiv p \wedge \sim q}$$

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$\boxed{\text{Example 2:}}$ Negate the statement

If $\underbrace{\text{my car is in the repair shop}}_{p}$, then $\underbrace{\text{I cannot get to class}}_{q}$

$$\boxed{\sim(p \to q) \equiv p \land \sim q}$$

Ans. $\boxed{\text{My car is in the repair shop AND I can get to class}}$

Contrapositive, Converse, and Inverse of $p \to q$

$\boxed{\text{Example 3:}}$ Consider the statement.

If $\underbrace{\text{today is Easter}}_{p}$, then $\underbrace{\text{tomorrow is Monday}}_{q}$.

Type	Symbol	Example
Contrapositive	$\sim q \to \sim p$	If tomorrow is not Monday then today is not Easter. (T)
Converse	$q \to p$	If tomorrow is Monday, then ~~today~~ is Easter (F)
Inverse	$\sim p \to \sim q$	If today is not Easter, then tomorrow is NOT Monday (F)

p is sufficient for q.

p is not necessary for q.

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+ Biconditional Statements

Hot tip! "only if" " → " } "if and only if"
"if" " ← " } ⟷

p only if q

- p can happen only if we have q
- if we don't have q, then p can't happen
- [contrapositive] if p, then q $\boxed{p \to q}$

versus:

p if q

- p can happen if we have q (sufficient)
- q is sufficient to have p.
- if q, then p.
- $q \to p$

combine them

$$p \leftrightarrow q$$

This is called "biconditional of p and q"

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Summary

- symbol: $\leftrightarrow$ (if and only if) $\to$ iff
- write: $p \leftrightarrow q$

$$\boxed{p \to q \text{ and } q \to p}$$

Example 4: Construct truth table for $p \leftrightarrow q$
(Tip: $p \leftrightarrow q$ is equivalent to $p \to q$ and $q \to p$)

p	q	$p \to q$	$q \to p$	$(p \to q) \land (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Sufficient and Necessary Conditions

Def^n if p and q are statements:

- p is a sufficient condition for q: $\boxed{\text{if } p \text{ then } q}$
- p is a necessary condition for q: if $\sim p$ then $\sim q$
  $\boxed{\text{if } q \text{ then } p}$

Example 5 Consider $p \to q$ where

$\boxed{p \leftrightarrow q}$ X $\qquad$ $\boxed{p \to q}$ ✓

p: = Today is Easter
q: = Tomorrow is Monday

- Today being Easter is sufficient for tomorrow to be Monday
- Check for necessary condition:
  if today is not Easter, then tomorrow is not Monday $\boxed{q \to p}$ X
  NOT True! Today being Easter is not necessary for tomorrow to be Not Monday.