Essential Laws of Propositional Logic

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\begin{aligned}
&\text{Double Negation} && \{\neg(\neg p) \models\!\!\dashv p \\
&\text{Excluded Middle} && \{p \vee \neg p \models\!\!\dashv 1 \\
&\text{Contradiction} && \{p \wedge \neg p \models\!\!\dashv 0 \\
&\text{Idempotence} && \begin{cases} p \wedge p \models\!\!\dashv p \\ p \vee p \models\!\!\dashv p \end{cases} \\
&\text{Identity} && \begin{cases} p \wedge 1 \models\!\!\dashv p \\ p \vee 0 \models\!\!\dashv p \end{cases} \\
&\text{Domination} && \begin{cases} p \wedge 0 \models\!\!\dashv 0 \\ p \vee 1 \models\!\!\dashv 1 \end{cases} \\
&\text{Commutativity} && \begin{cases} p \wedge q \models\!\!\dashv q \wedge p \\ p \vee q \models\!\!\dashv q \vee p \\ p \leftrightarrow q \models\!\!\dashv q \leftrightarrow p \end{cases} \\
&\text{Associativity} && \begin{cases} p \wedge (q \wedge r) \models\!\!\dashv (p \wedge q) \wedge r \\ p \vee (q \vee r) \models\!\!\dashv (p \vee q) \vee r \end{cases} \\
&\text{Distributivity} && \begin{cases} p \wedge (q \vee r) \models\!\!\dashv (p \wedge q) \vee (q \wedge r) \\ p \vee (q \wedge r) \models\!\!\dashv (p \vee q) \wedge (q \vee r) \end{cases} \\
&\text{Implication} && \{p \to q \models\!\!\dashv \neg p \vee q \\
&\text{Contrapositive} && \{p \to q \models\!\!\dashv \neg q \to \neg p \\
&\text{Equivalence} && \{p \leftrightarrow q \models\!\!\dashv (p \to q) \wedge (q \to p) \\
&\text{De Morgan} && \begin{cases} \neg(p \wedge q) \models\!\!\dashv \neg p \vee \neg q \\ \neg(p \vee q) \models\!\!\dashv \neg p \wedge \neg q \end{cases} \\
&\text{Absorption I} && \begin{cases} p \wedge (p \vee q) \models\!\!\dashv p \\ p \vee (p \wedge q) \models\!\!\dashv p \end{cases} \\
&\text{Absorption II} && \begin{cases} (p \vee q) \wedge (\neg p \vee q) \models\!\!\dashv q \\ (p \wedge q) \vee (\neg p \wedge q) \models\!\!\dashv q \end{cases}
\end{aligned}
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Rules of Formal Deduction

(Abbr.)	From	Conclude	Rule
(Ref)	$\emptyset$	$A \vdash A$	Reflexivity
$(+)$	$\Sigma \vdash A$	$\Sigma, \Sigma' \vdash A$	Addition of premises
$(\neg\ -)$	$\Sigma, \neg A \vdash B$ $\Sigma, \neg A \vdash \neg B$	$\Sigma \vdash A$	$\neg$ elimination
$(\to -)$	$\Sigma \vdash A \to B$ $\Sigma \vdash A$	$\Sigma \vdash B$	$\to$ elimination (modus ponens)
$(\to +)$	$\Sigma, A \vdash B$	$\Sigma \vdash A \to B$	$\to$ introduction
$(\wedge -)$	$\Sigma \vdash A \wedge B$	$\Sigma \vdash A$ $\Sigma \vdash B$	$\wedge$ elimination
$(\wedge +)$	$\Sigma \vdash A$ $\Sigma \vdash B$	$\Sigma \vdash A \wedge B$	$\wedge$ introduction
$(\vee -)$	$\Sigma, A \vdash C$ $\Sigma, B \vdash C$	$\Sigma, A \vee B \vdash C$	$\vee$ elimination
$(\vee +)$	$\Sigma \vdash A$	$\Sigma \vdash A \vee B$ $\Sigma \vdash B \vee A$	$\vee$ introduction
$(\leftrightarrow -)$	$\Sigma \vdash A \leftrightarrow B$ $\Sigma \vdash A$	$\Sigma \vdash B$	$\leftrightarrow$ elimination
	$\Sigma \vdash A \leftrightarrow B$ $\Sigma \vdash B$	$\Sigma \vdash A$	