

MATH 135 Fall 2020: Extra Practice 1**Warm-Up Exercises**

WE01. Determine if the following quantified statements are true or false. No justification is needed.

- (a) $\forall x \in \mathbb{R}, \sin^2 x + \cos^2 x = 1$
True, by the Pythagorean identity.
- (b) $\exists y \in \mathbb{Z}, 6y - 3 = 28$
False, since $6y - 3 = 28 \implies 6y = 31 \implies y = \frac{31}{6}$, which is undefined in \mathbb{Z} .
- (c) $\forall p \in \mathbb{Q}, \exists q \in \mathbb{Z}, |p - q| \leq 1$
True, select $q = \lfloor p \rfloor$.

Recommended Problems

RP01. Which of the following are statements? If it is a statement, determine if it is true or false. No justification is needed.

- (a) $3 \leq \pi$
Statement, true.
- (b) $2x - 3 \geq -1$
Not a statement, depends on x .
- (c) $x^2 - y^3 = 1$
Not a statement, depends on x and y .
- (d) N is a perfect square.
Not a statement, depends on N .
- (e) $x^2 + 5x - 2$
Not a statement or an open sentence.
- (f) $x \leq x + 1$
Not a statement, depends on x .
- (g) There is a largest real number.
Statement, false.
- (h) There is a smallest positive number.
Statement, true.
- (i) Every real number is either positive or negative.
Statement, false.
- (j) Some triangles are right triangles.
Statement, true.

RP02. For each of the following statements, identify the four parts of the quantified statement (quantifier, variables, domain, and open sentence). Next, express the statement in symbolic form using as few words as possible and then write down the negation of

the statement (when possible, without using any negative words such as “not” or the \neg symbol, but negative math symbols like \neq are okay). Finally, determine if the original statement is true or false. No justification is needed.

- (a) The equation $x^2 + 2x - 3 = 0$ has a real solution.

$$\exists x \in \mathbb{R}, x^2 + 2x - 3 = 0$$

Quantifier: existential; variable: x ; domain: \mathbb{R} ; open sentence: $x^2 + 2x - 3 = 0$.

Negation:

$$\forall x \in \mathbb{R}, x^2 + 2x - 3 \neq 0$$

The statement is *true*.

- (b) No matter which real value x we choose, $-1 + \cos x$ will always be positive

$$\forall x \in \mathbb{R}, -1 + \cos x > 0$$

Quantifier: universal; variable: x ; domain: \mathbb{R} ; open sentence: $-1 + \cos x > 0$.

Negation:

$$\exists x \in \mathbb{R}, -1 + \cos x \leq 0$$

The statement is *false*.

- (c) Every natural number can be expressed as the product of two integers.

$$\forall n \in \mathbb{N}, \exists a, b \in \mathbb{Z}, n = ab$$

Quantifier: universal/existential; variables: n, a, b ; domain: \mathbb{N}, \mathbb{Z} ; open sentence: $n = ab$. Negation:

$$\exists n \in \mathbb{N}, \forall a, b \in \mathbb{Z}, n \neq ab$$

The statement is *true*.

- (d) There is a perfect square which is also a perfect cube.

$$\exists n \in \mathbb{Z}, \exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, n = x^2 = y^3$$

Quantifier: existential; variables: n, x, y ; domain: \mathbb{Z} ; open sentence: $n = x^2 = y^3$.

Negation:

$$\forall n \in \mathbb{Z}, \forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, n \neq x^2 \wedge n \neq y^3$$

The statement is *true*.

RP03. Negate the following statements without using words or the \neg symbol. For each statement determine whether it or its negation is true.

- (a) $\exists a \in \mathbb{Z}, \forall b \in \mathbb{Z}, 3a = b$: false

$\forall a \in \mathbb{Z}, \exists b \in \mathbb{Z}, 3a \neq b$: true

- (b) $\forall a \in \mathbb{Z}, \exists b \in \mathbb{Z}, 3a = b$: true

$\exists a \in \mathbb{Z}, \forall b \in \mathbb{Z}, 3a \neq b$: false

- (c) $\forall a \in \mathbb{R}, \forall b \in \mathbb{R}, \exists c \in \mathbb{R}, \frac{a}{c} = b$: true

$\exists a \in \mathbb{R}, \exists b \in \mathbb{R}, \forall c \in \mathbb{R}, \frac{a}{c} \neq b$: false

RP04. Express the following statement symbolically without using any words: *Every integer is a perfect square.*

$$\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}, n = m^2$$