Entrega 3

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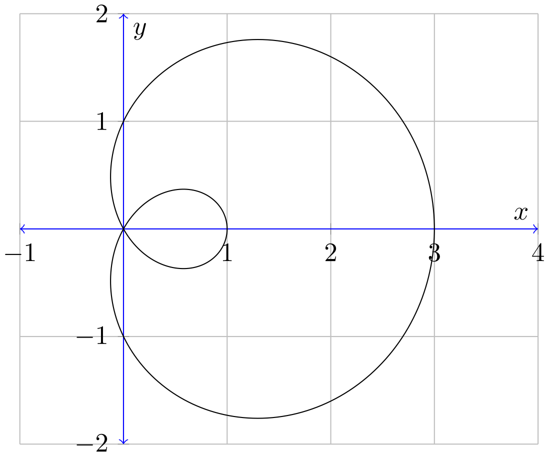
Problema:

Sea $\alpha : [0, 2\pi] \longrightarrow \mathbb{R}^2$ dada por $\alpha(t) = (2\cos t - 1)(\cos t, \operatorname{sen} t)$.

i) Representar $\alpha([0, 2\pi])$.



ii) ¿Es α simple?

No es simple por no ser inyectiva. $\alpha(\frac{\pi}{3}) = \alpha(\frac{5\pi}{3}) = (0, 0)$.

iii) ¿Es α convexa?

No es convexa, evidentemente la recta tangente en $\alpha(0)$ que es $x = 1$ corta la curva en otros dos puntos. Además veremos que solo tiene 2 vértices, por el teorema de los 4 vértices no puede ser convexa.

iv) Calcular los vértices de α.

Calculamos la curvatura y su derivada.

$$\alpha(t) = ((2\cos t - 1)\cos t, (2\cos t - 1)\operatorname{sen} t)$$

$$\alpha'(t) = (-2\operatorname{sen} t\cos t - (2\cos t - 1)\operatorname{sen} t, -2\operatorname{sen} t\operatorname{sen} t + (2\cos t - 1)\cos t) =$$

$$= (\operatorname{sen} t - 4\cos t\operatorname{sen} t, 2(\cos^2 t - \operatorname{sen}^2 t) - \cos t)$$

$$\alpha''(t) = (\cos t - 4(-\operatorname{sen} t\operatorname{sen} t + \cos t\cos t), 2(-2\cos t\operatorname{sen} t - 2\operatorname{sen} t\cos t) + \operatorname{sen} t) =$$

$$= (\cos t + 4(\operatorname{sen}^2 t - \cos^2 t), \operatorname{sen} t(1 - 8\cos t))$$

$$k_2(t) = \frac{\alpha''(t)\cdot \mathcal{J}\alpha'(t)}{\|\alpha'(t)\|^3} =$$

$$= \frac{(\cos t + 4(\operatorname{sen}^2 t - \cos^2 t), \operatorname{sen} t(1 - 8\cos t))\cdot \mathcal{J}(\operatorname{sen} t - 4\cos t\operatorname{sen} t, 2(\cos^2 t - \operatorname{sen}^2 t) - \cos t)}{\|(\operatorname{sen} t - 4\cos t\operatorname{sen} t, 2(\cos^2 t - \operatorname{sen}^2 t) - \cos t)\|^3} =$$

$$= \frac{-(\cos t + 4(\operatorname{sen}^2 t - \cos^2 t))(2(\cos^2 t - \operatorname{sen}^2 t) - \cos t) + (\operatorname{sen} t - 8\operatorname{sen} t\cos t)(\operatorname{sen} t - 4\cos t\operatorname{sen} t)}{\left(\sqrt{(\operatorname{sen} t - 4\cos t\operatorname{sen} t)^2 + (2(\cos^2 t - \operatorname{sen}^2 t) - \cos t)^2}\right)^3} =$$

Abreviamos $s = \operatorname{sen} t$ y $c = \cos t$

$$= \frac{-(c + 4(s^2 - c^2))(2(c^2 - s^2) - c) + (s - 8sc)(s - 4cs)}{(\sqrt{(s - 4cs)^2 + (2(c^2 - s^2) - c)^2})^3} =$$

Sustituimos $s^2 = 1 - c^2$

$$= \frac{-(c + 4(1 - c^2 - c^2))(2(c^2 - (1 - c^2)) - c) + (s - 8sc)(s - 4cs)}{(\sqrt{(s - 4cs)^2 + (2(c^2 - (1 - c^2)) - c)^2})^3} =$$

$$= \frac{-(c + 4(1 - c^2 - c^2))(2(c^2 - 1 + c^2) - c) + (s - 8sc)(s - 4cs)}{(\sqrt{(s - 4cs)^2 + (2(c^2 - 1 + c^2) - c)^2})^3} =$$

$$= \frac{-(c+4(1-2c^2))(2(2c^2-1)-c)+(s-8sc)(s-4cs)}{(\sqrt{(s-4cs)^2+(2(2c^2-1)-c)^2})^3} =$$

$$= \frac{-(c+4-8c^2)(4c^2-2-c)+(s-8sc)(s-4cs)}{(\sqrt{(s-4cs)^2+(2(2c^2-1)-c)^2})^3} =$$

$$= \frac{-(c+4-8c^2)(4c^2-2-c)+s^2(1-8c)(1-4c)}{(\sqrt{(s-4cs)^2+(2(2c^2-1)-c)^2})^3} =$$

$$= \frac{-(c+4-8c^2)(4c^2-2-c)+(1-c^2)(1-8c)(1-4c)}{(\sqrt{(s-4cs)^2+(2(2c^2-1)-c)^2})^3} =$$

$$= \frac{-(c+4-8c^2)(4c^2-2-c)-32c^4+12c^3+31c^2-12c+1}{(\sqrt{(s-4cs)^2+(2(2c^2-1)-c)^2})^3} =$$

$$= \frac{-(-32c^4+12c^3+31c^2-6c-8)+-32c^4+12c^3+31c^2-12c+1}{(\sqrt{(s-4cs)^2+(2(2c^2-1)-c)^2})^3} =$$

$$= \frac{9-6c}{(\sqrt{(s-4cs)^2+(2(2c^2-1)-c)^2})^3} =$$

$$= \frac{9-6c}{(\sqrt{s^2(1-4c)^2+(2(2c^2-1)-c)^2})^3} =$$

$$= \frac{9-6c}{(\sqrt{(1-c^2)(1-4c)^2+(2(2c^2-1)-c)^2})^3} =$$

$$= \frac{9-6c}{(\sqrt{(1-c^2)(1-8c+16c^2)+(4c^2-2-c)^2})^3} =$$

$$= \frac{9-6c}{(\sqrt{-16c^4+8c^3+15c^2-8c+1+(4c^2-2-c)^2})^3} =$$

$$= \frac{9-6c}{(\sqrt{-16c^4+8c^3+15c^2-8c+1+16c^4-8c^3-15c^2+4c+4})^3} =$$

$$= \frac{9-6c}{(\sqrt{5-4c})^3} =$$

$$= \frac{9-6\cos t}{(5-4\cos t)^{3/2}}$$

Derivamos

$$k_2'(t) = -\frac{(9-6\cos t)((5-4\cos t)^{3/2})'-(9-6\cos t)'((5-4\cos t)^{3/2})}{(5-4\cos t)^3}$$

$$= -\frac{(9-6\cos t)\frac{3}{2}(5-4\cos t)^{1/2}(4\operatorname{sen} t)-6\operatorname{sen} t(5-4\cos t)^{3/2}}{(5-4\cos t)^3} =$$

$$= -\frac{(5-4\cos t)^{1/2}((9-6\cos t)\frac{3}{2}(4\operatorname{sen} t)-6\operatorname{sen} t(5-4\cos t))}{(5-4\cos t)^3} =$$

$$= -\frac{(9-6\cos t)(6\operatorname{sen} t)-6\operatorname{sen} t(5-4\cos t)}{(5-4\cos t)^{5/2}} =$$

$$= -\frac{6\operatorname{sen} t(9-6\cos t-5+4\cos t)}{(5-4\cos t)^{5/2}} =$$

$$= -\frac{6\operatorname{sen} t(4-2\cos t)}{(5-4\cos t)^{5/2}} =$$

$$= \frac{12\operatorname{sen} t(\cos t-2)}{(5-4\cos t)^{5/2}}$$

Tenemos ceros, y por lo tanto vértices, en $0, \pi$ y 2π, es decir, en $(3,0)$ y $(1,0)$.