Seminario 4

Andoni Latorre Galarraga

Sustituimos $x_n = \epsilon_n + \alpha$ en $x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$.

$$\epsilon_{n+1} = \epsilon_n - \frac{\epsilon_n - \epsilon_{n+1}}{f(\epsilon_{n+1} + \alpha) -} f(\epsilon_n + \alpha)$$

Sustituimos el desrrollo de Taylor $f(x) \approx f'(\alpha)(x-\alpha) + \frac{1}{2} f''(\alpha)(x-\alpha)^2$.

$$f(\epsilon_n + \alpha) \approx \epsilon_n f'(\alpha)(1 + \frac{f''(\alpha)}{2f'(\alpha)}$$

$$\epsilon_{n+1} \approx \epsilon_n - \frac{\epsilon_n(1 + \frac{f''(\alpha)}{2f'(\alpha)}\epsilon_n)}{1 + \frac{f''(\alpha)}{2f'(\alpha)}(e_n + \epsilon_{n+1})} = \frac{\frac{f''(\alpha)}{2f'(\alpha)}\epsilon_n \epsilon_{n+1}}{1 + \frac{f''(\alpha)}{2f'(\alpha)}(e_n + \epsilon_{n+1})}$$

$$\approx \frac{f''(\alpha)}{2f'(\alpha)}\epsilon_n \epsilon_{n+1}$$

Ahora:

$$|\epsilon_{n+1}| = \lambda |\epsilon_n|^p \iff |\frac{f''(\alpha)}{2f'(\alpha)}||\epsilon_n||\epsilon_{n+1}| \approx \lambda |\epsilon_n|^p$$

$$|\epsilon_n| \approx \left(\frac{|\frac{f''(\alpha)}{2f'(\alpha)}|}{\lambda} \right)^{\frac{1}{p-1}} |\epsilon_{n+1}|^{\frac{1}{p-1}} \Rightarrow \lambda = \left(\frac{|\frac{f''(\alpha)}{2f'(\alpha)}|}{\lambda} \right)^{\frac{1}{p-1}}$$

Entonces, $p = \frac{1}{p-1}$. Por lo que $p = \frac{1+\sqrt{5}}{2}$, que es la razón aurea.