Recurrencias

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1

$$t(n) = \log_2(n) + t(\frac{n}{2}) = \log_2(n) + \log_2(\frac{n}{2}) + t(\frac{n}{4})$$
$$= 2\log_2(n) - 1 + t(\frac{n}{4}) = 2\log_2(n) - 1 + \log_2(\frac{n}{4}) + t(\frac{n}{8})$$
$$= 3\log_2(n) - 1 - 2 + t(\frac{n}{8}) = 4\log_2(n) - 1 - 2 - 3 + t(\frac{n}{16})$$
$$= k\log_2 n - \frac{k(k-1)}{2} + t(\frac{n}{2^k})$$
$$1 = \frac{n}{2^k} \quad \Leftrightarrow \quad k = \log_2(n)$$
$$t(n) = \log_2(n)\log_2 n - \frac{\log_2(n)(\log_2(n)-1)}{2} + t(1) = \log_2^2(n) - \frac{\log_2^2(n) - \log_2(n)}{2} + 1$$
$$t(n) = \log_2^2(n) - \frac{\log_2(n)}{2} + 1$$

2

$$t(n) = n^2 + t(n-1) = n^2 + (n-1)^2 + t(n-2) = \cdots = n^2 + (n+1)^2 + \cdots + 0^2 + t(0)$$
$$t(n) = \frac{n(n+1)(2n+1)}{6} + 1$$

3

Recursion.py
Es evidente que el metodo de expansión no lleva a ninguna parte. Intentemos encontrar una cota superior $t_{\text{sup}}(n)$. Y una cota inferior $t_{\text{inf}}(n) = 1$

$$t_{\text{sup}}(n) = 1 + 2 \cdot t(n-1) = 1 + 2 + 4 \cdot t(n-2)$$
$$\cdots = \frac{k^2+1}{2} + 2^k \cdot t(n-k)$$
$$t_{\text{sup}}(n) = \frac{n^2+1}{2} + 2^n \cdot t(0) = \frac{n^2+1}{2} + 2^n$$
$$t(n, K) \in O(2^n) \wedge t(n, k) \in \Omega(1)$$