GtG Ljubljana

January 23-24, 2025

Abstracts

Arithmeticity and thinness of hypergeometric groups

24 Jan
11:30

Jitendra Bajpai
Kiel University

The monodromy groups of hypergeometric differential equations are often called hypergeometric groups. These are subgroups of the general linear groups. Recently, these groups have caught a lot of attention. In the talk, a gentle introduction and recent progress of the theory of hypergeometric groups will be presented.

Diameter of random Schreier graphs with many generators

23 Jan
11:30

Daniele Dona
Alfréd Rényi Institute of Mathematics

In the 1990s Roichman proved that, for a group G of size n and a random set $S \subseteq G$ of size $k \geq (\log n)^{1+\varepsilon}$ for some $\varepsilon > 0$, the diameter of the corresponding Cayley graph is $O(\log n / \log k)$ with high probability. The proof involves studying the mixing time of random walks, and uses defining properties of Cayley graphs, such as vertex-transitivity. In a joint work with Sabatini, we use more flexible combinatorial techniques and generalize the bound above to Schreier graphs of transitive group actions on sets of size n, bypassing the limitations of the previous approach. The bound $O(\log n / \log k)$ is sharp in general, and so is the condition $k \geq (\log n)^{1+\varepsilon}$.

Diameter bounds for finite classical groups generated by transvections

23 Jan
10:00

Sean Eberhard
University of Warwick

The diameter of a group G with respect to a symmetric generating set X is the smallest integer d such that every element of G is the product of at most d elements of X. A well-known conjecture of Babai predicts that every nonabelian finite simple group G has diameter $(\log |G|)^{O(1)}$ with respect to any generating set. This is known to be true for bounded-rank groups of Lie type (Helfgott; Pyber–Szabo; Breuillard–Green–Tao), but the conjecture is wide open for high-rank groups. There has been a good deal of progress recently for generating sets containing either special elements or random elements. In this talk I will outline the proof that the conjecture holds for the classical groups $\mathrm{SL}_n(q)$, $\mathrm{Sp}_{2n}(q)$, $\mathrm{SU}_n(q)$ and any generating set containing a transvection. The proof is based essentially on (a) the positive resolution of Babai's conjecture in bounded rank and (b) a result of Kantor classifying finite irreducible linear groups generated by transvections.

Boolean designs and Binary Hamming codes

24 Jan
16:30

Giovanni Falcone
University of Palermo

In this paper I report on a joint study with Marco Pavone. We consider a finite-dimensional vector space P over the Galois field $GF(2)$, and the family B_k (respectively, B_k^*) of all the k-sets of elements of P (respectively, of $P^* = P \setminus \{0\}$) summing up to zero. We compute the parameters of the 3-design (P, B_k) for any (necessarily even) k, and of the 2-design (P^*, B_k^*) for any k. Moreover, we find the automorphism groups of the above designs by characterizing the permutations of P, respectively of $P*$, that induce permutations of B_k, respectively of B_k^*. As an application, this allows one to relax the definitions of the permutation automorphism groups of the binary Hamming code.

On a family of binary Lie algebras and diassociative loops

23 Jan
16:30

Ágota Figula
University of Debrecen

The classical Baker-Campbell-Hausdorff series over binary Lie algebras (anti-commutative algebras in which any two elements generate a Lie subalgebra) defines local analytic diassociative loops (any two elements generate a local subgroup). If the adjoint map of a binary Lie algebra has no nonzero purely imaginary eigenvalues, then the exponential map is globally defined and determines a diassociative loop on the entire binary Lie algebra. This loop multiplication is glued together from partial Lie group multiplications defined on the Lie subalgebras of the binary Lie algebra. In general, the analytic property of this loop multiplication is not inherited from the partial Lie group multiplications. The goal of our paper is to find the closed form for the Baker-Campbell-Hausdorff series of this multiplication on a family of solvable binary Lie algebras that determines global analytic diassociative loops.

The binary Lie algebras studied are semidirect sums of the two-dimensional non-abelian Lie algebra and an abelian Lie algebra and at the same time extensions of the one-dimensional algebra by a nilpotent Lie algebra, the Lie subalgebras generated by 2-frames have dimension 2 or 3. Considering their isomorphic matrix Lie algebras, we express the corresponding Lie group multiplications on the Lie subalgebras of the binary Lie algebra. We show that they define the closed form of the Baker-Campbell-Hausdorff series of the loop multiplication on the entire binary Lie algebra that yields the desired global analytic diassociative loops.

On the Gowers trick for classical simple groups

24 Jan
10:00

Francesco Fumagalli
Università degli Studi di Firenze

Let A, B, C be three subsets of a finite group G. Let $\mathrm{Prob}(A, B; C)$ be the probability that if a and b are uniformly and randomly chosen elements from A and B respectively, then $ab \in C$. Let k be the minimal degree of a non-trivial complex irreducible character of G. W.T. Gowers proved the following beautiful theorem.

Theorem 1 (Gowers) *If $\eta > 0$ is such that $|A||B||C| > |G|^3/\eta^2 k$, then*

$$(1-\eta)|C||G| < \mathrm{Prob}(A, B; C) < (1+\eta)|C||G|.$$

In this talk we will consider the special case when G is a classical simple group and when A, B, C are special kinds of sets. We will require at least two of these sets to be normal. (Recall that a subset of a group G is normal if it is invariant under conjugation by every element of G.) This is joint work with Attila Maróti.

Simple groups and derangements

23 Jan
15:00

Marco Fusari
University of Pavia and University of Milan-Bicocca

Let $G \leq \mathrm{Sym}(\Omega)$ be a finite transitive permutation group and recall that an element in G is a derangement if it has no fixed points on Ω. Let $\Delta(G)$ be the set of derangements in G and set $\delta(G) = \frac{|\Delta(G)|}{|G|}$ the proportion of derangements. By combining a theorem of Fulman and Guralnick with recent work by Larsen, Shalev and Tiep, it follows that $\delta(G) \geq 0.016$ and $G = (\Delta(G))^2$ for all sufficiently large simple transitive groups G.

In this presentation, I will discuss recent collaborative work with Tim Burness that aims to broaden both results. For example, it is possible to show that $\delta(G) \geq \frac{89}{325}$ and $G = (\Delta(G))^2$ for all finite simple primitive groups with soluble point stabilisers, and that the lower bound on $\delta(G)$ is best possible.

Some results about the diameter fo the commuting graph

23 Jan
12:30

Michele Gaeta
University of Salerno

The association of graphs to groups goes back to the 19th century when Cayley introduced a graph that encodes the abstract structure of a group. Later other graphs have been considered. More precisely, given a group property $\mathcal{P}$ and a group G one can consider the graph whose set of vertices is G and two vertices x and y are adjacent if and only if the subgroup generated by x and y has the property $\mathcal{P}$. In particular if the property $\mathcal{P}$ denotes commutativity, then the resulting graph is called the commuting graph of the group G. In discussing the connection and the diameter of this graph, it is customary to exclude the unit of the group and sometimes all universal vertices. This talk aims to provide some results about the connectivity and the diameter of the commuting graph, with particular attention to the case in which this graph has diameter 2.

On twisted conjugacy classes of finite groups

24 Jan
16:00

Chiara Nicotera
University of Salerno

We will investigate finite groups with some condition on twisted conjugacy classes.

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p-elements in profinite groups

24 Jan
14:30

Nowras Naufel Ali Mahamoud Otmen
Università degli Studi di Padova

Given a profinite group G, one can always consider the (normalized) Haar measure on it. In this talk, we'll consider a few types of measurable sets which concern the p-elements of G and see what can said about the structure of G whenever these sets have positive measure. First, we show that if p is odd and the probability of randomly choosing a p-element is positive, then G is virtually prosolvable. We'll then exhibit a counterexample to this when p is 2. Second, we prove that if G has the property that, for every p-element x, it is positive the probability that a randomly chosen element y of G generates with x a pro-p subgroup, then G is virtually pro-p. This is joint work with Andrea Lucchini.

Stabilizers in finite permutation groups

23 Jan
14:30

Luca Sabatini
University of Warwick

Let G be a permutation group on the finite set Ω. One of the most studied topics in permutation group theory is finding a partition of Ω whose stabilizer is trivial. For example, the base size and the distinguishing number are important notions in this investigation. In this seminar, we consider a small variation: we fix only the bare minimum (so one subset, or a pair of points) and we do not aim to find a trivial stabilizer, since this would be impossible in general, but to find a stabilizer that has at least some good properties. We will see that this can be done in a few interesting situations.

Profinite properties of ℓ^2-invariants

24 Jan
12:30

Giada Serafini
Heinrich Heine University Düsseldorf

Although higher ℓ^2-Betti numbers are in general not profinite invariants, some interesting information on the ℓ^2-cohomology of S-arithmetic groups still seems to be preserved under profinite commensurability. This leads us to prove a stability result for another ℓ^2-invariant: the sign of the Euler characteristic. The key steps needed to prove such a result can be seen in terms of local-global principles in Galois cohomology.

Fusion Systems and their applications in block theoretic conjectures

23 Jan
16:00

Patrick Serwene

TU Dresden

Fusion systems arise from both finite groups and finite group blocks. A conjecture suggests the equivalence of these construction approaches. We explore the conjecture's implications in Block Theory and discuss the broader applications of fusion systems, particularly in addressing key conjectures within this field.

Probability measure on finite groups induced by words are determined by their images

24 Jan
15:00

Shrinit Singh

Shiv Nadar Institution of Eminence

In this talk, we give a class of words in which words induce the probability measure on a finite group, which can be determined by the images of the corresponding word maps, in the sense that any two words that have the same image on every finite group must also induce the same probability measure.
