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MATH 341

Homework 5

1. Take $x \in F$
   If $FE = \emptyset, x \in E^c \; which \; x \in FE^c$
   If $FE \neq \emptyset, x \in E \; or \; \in E^c \; such \; that \; x \in FE \; or \; x \in FE^c$
   So, $F \subseteq FE \cup FE^c$

   Take $x \in FE \cup FE^c$
   $x \in FE \; or \; x \in FE^c, so \; x \in F \; and \; x \in E \; or \; x \in F \; and \; x \in E^c$
   $x \in F$
   $So, FE \cup FE^c \subseteq F$
   $F = FE \cup FE^c$

2. A) (total - In Language)/total = $\frac{100-50}{100} = .5 = 50\%$

   B) $\frac{14+10+8}{100} = \frac{32}{100} = .32 = 32\%$

   C) $(\binom{50}{2} + \binom{50}{1}\binom{50}{1})/\binom{100}{2}$ = 0.7525 = 75.225%

3. $P[(A \cup B) \cap C)] = P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$
   $P[(A \cup B) \cap C)] \to P[(A \cap C) \cup (B \cap C)] \; Through \; the \; distributive \; property$
   $= P[E \cup F] \; where \; E = A \cap C \;\&\; F = B \cap C$
   $= P(E) + P(F) - P[EF] \; Through \; Proposition \; 4.3$
   $= P(A \cap C) + P(B \cap C) - P[(A \cap C) \cap (B \cap C)] \; redistributing \; variables$
   $= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$ ✓

4. $P(E_1) = \binom{100}{15}/\binom{110}{15} = 0.22$
   $P(E_2) = \binom{85}{15}/\binom{110}{15} = 0.015$
   $P(E_3) = \binom{80}{15}/\binom{110}{15} = 0.0056$
   $P(E_4) = \binom{65}{15}/\binom{110}{15} = 0.00018$
   $P(E_1E_2) = \binom{75}{15}/\binom{110}{15} = 0.001939$
   $P(E_1E_3) = \binom{70}{15}/\binom{110}{15} = 0.000614$
   $P(E_1E_4) = \binom{55}{15}/\binom{110}{15} = 0.00001012$
   $P(E_2E_3) = \binom{55}{15}/\binom{110}{15} = 0.00001012$
   $P(E_2E_4) = \binom{40}{15}/\binom{110}{15} = 0.00000003$

   Freshman - 10
   Sophomore - 25
   Junior - 30
   Senior - 45

$P(E_3E_4) = \binom{35}{15}/\binom{110}{15} = 2.762 * 10^{-9}$
$P(E_1E_2E_3) = \binom{45}{15}/\binom{110}{15} = 0.00000029$
$P(E_1E_2E_4) = \binom{30}{15}/\binom{110}{15} = 1.3193 * 10^{-10}$
$P(E_1E_3E_4) = \binom{25}{15}/\binom{110}{15} = 2.78 * 10^{-12}$
$P(E_2E_3E_4) = \binom{10}{15}/\binom{110}{15} = 0$
$P(E_1E_2E_3E_4) = \binom{0}{15}/\binom{110}{15} = 0$

$P(E_1) + P(E_2) + P(E_3) + P(E_4) - P(E_1E_2) - P(E_1E_3) - P(E_1E_4) - P(E_2E_3) - P(E_2E_4) - P(E_3E_4) + P(E_1E_2E_3) + P(E_1E_2E_4) + P(E_1E_3E_4) + P(E_2E_3E_4) - P(E_1E_2E_3E_4)$

= 0.2382 = 23.82%

5. If $P(\bigcup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} P(E_i)$, one case must be that all events are mutually exclusive (noted with a '*'). Meaning $\bigcup_{i=1}^{\infty} E_i^* = \sum_{i=1}^{\infty} P(E_i^*)\ as\ \bigcap_{i=1}^{\infty} E_i^* = \emptyset$. If one or more of the cases are not mutually exclusive, then $\bigcap_{i=1}^{\infty} E_i \neq \emptyset\ so\ \bigcup_{i=1}^{\infty} E_i < \sum_{i=1}^{\infty} E_i$ as to not over-count the intersection between two or more events. So, $P(\bigcup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} P(E_i)$ as it accounts for all cases of possible intersection between the events.

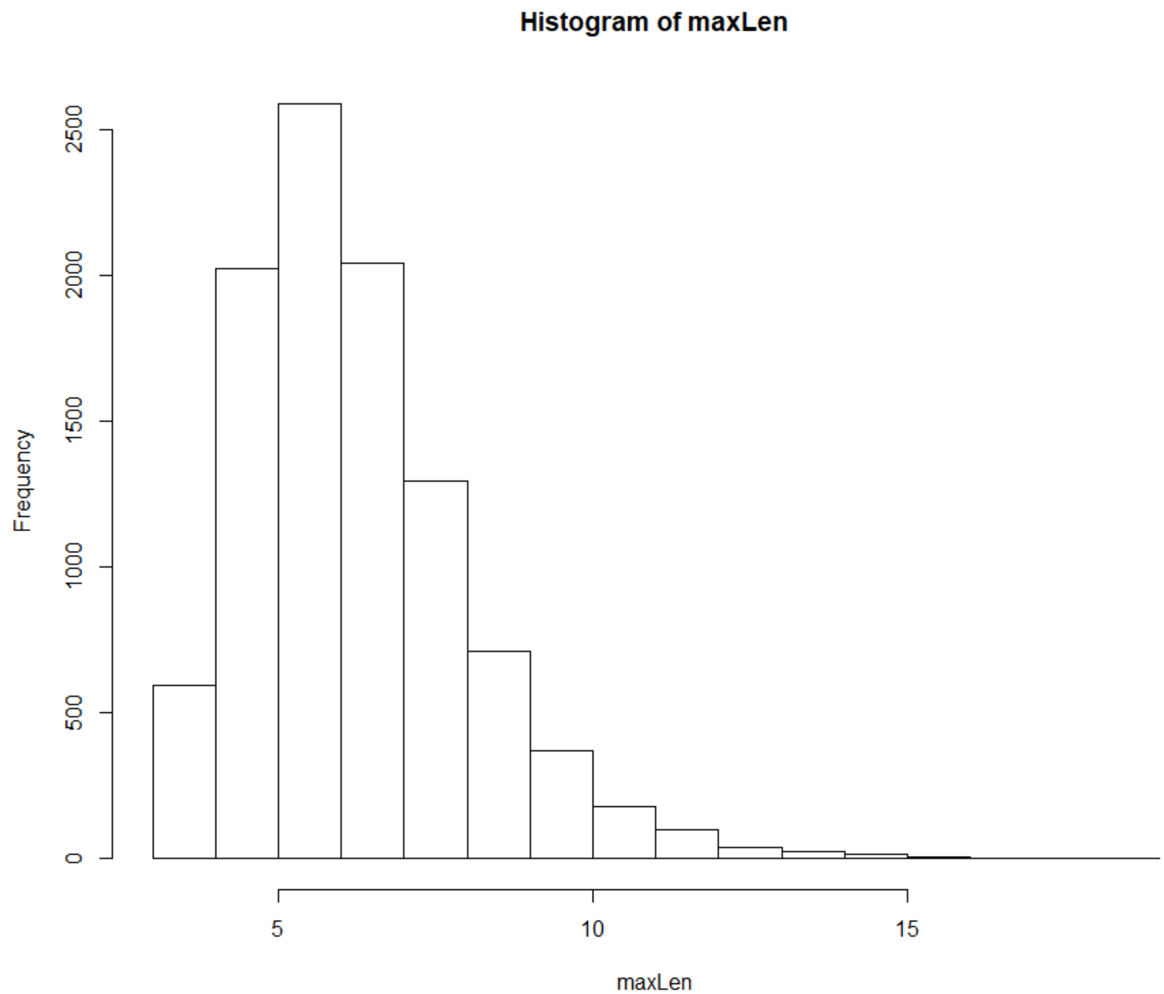
6.

(Histogram for part A below)

A) The likelihood of Baltimore winning 10 games in a row in one season is relatively low, sitting at approximately 0.025 or 2.5% (from the data). While the likelihood of them winning 15 games in a row is even lower (low enough that I cannot visually estimate its value). So while winning ten games in a row is unlikely, it is far more unlikely that they will win 1 games in a row.

B) When I simulated 10000 seasons, there were 31 seasons where Cleveland Indians got a 22 or higher game win streak. 31/10000 = 0.0031 = 0.31%, which is a very unlikely possibility that they will win 22 or more games in a row.

C) When I simulated 10000 seasons, there were 12 seasons where the Los Angeles Dodgers lost 11 or more games in a row. 12/10000 = 0.0012 = 0.12%, which is a very unlikely possibility that they will lose 11 or more games in a row.



7. A) $P(A|B) = 1 - \frac{P(A \cap B)}{P(A)} = 1 - \frac{.27}{.79} = 1 - 0.34 = 0.66 = 66\%$

   B) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.27}{.38} = 0.7105 = 71.05\%$