Logic for Computer Science - Week 1
Introduction to Informal Logic

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1 Propositions

A *proposition* is a statement that can be true or false. Propositions are sometimes called *sentences*. Here are examples of propositions:

1. "I wear a blue shirt."
2. "You own a laptop computer."
3. "You own a laptop computer and a tablet computer, but no smartphone."
4. "I own: a laptop, a tablet, a smartphone, a server.".
5. "I will buy a laptop or a tablet".
6. "I can install the software on my smartphone or on my tablet."
7. "It is raining outside, but I have an umbrella."
8. "If I get a passing grade in Logic, I will buy everyone beer."
9. "I play games often and I study very well."
10. "Snow is white."
11. "It is raining."
12. "It is not raining."
13. "I will pass Logic only if I study hard."
14. "Either white wins or black wins in a game of chess."
15. "$2 + 2 = 4$." ("Two plus two is four.")
16. "$1 + 1 = 1$." ("One plus one is one.")
17. "$1 + 1 \neq 1$." ("One plus one is not one.")

18. "If $1 + 1 = 1$, then I'm a banana."
19. "All natural numbers are integers."
20. "All rational numbers are integers."

Here are examples of things that are not propositions:

1. "Red and Black." (not a statement)
2. "π." (not a statement)
3. "Is it raining?" (question, not a statement)
4. "Go fish!" (imperative)
5. "x is greater than 7." (cannot tell unless I know who x is)
6. "This sentence is false."

Sometimes it is debatable whether something is truly a proposition. For example, we generally agree that "Snow is white" is true, but someone might argue that they have seen black snow, so the status of "Snow is white" is put in question. Arguing about whether something is a proposition or not is more a matter of philosophical logic than computer science logic and we will therefore not be too concerned about these sort of issues.

2 Arguments

An *argument* is a sequence of propositions. The last proposition is called the *conclusion* of the argument, while the other propositions (all except the last) are its *premisses*. Typically the conclusion of an argument is preceded by a word such as "therefore" or "so".

Here is an example of an argument:

A1:

1. "At university I will study books or I will play games."
2. "I will not study books at university."
3. Therefore, "I will play games."

Another example:

A2:

1. "All men are mortal."
2. "Socrates is a man."
3. So, "Socrates is mortal."

The arguments above seem pretty convincing, and we will see in a minute what makes an argument convincing. But note that according to our definition, this is also an argument:

A3:

1. "All students are smart."
2. "John is a man."
3. So, "The Earth is round."

Albeit not a very convincing one. What makes an argument convincing? There are arguably two things that makes an argument compelling:

1. The conclusion follows without failure from the premisses.
2. The premisses are true.

Any argument satisfying the first item is called *deductively valid*. An argument satisfying both items is call *sound*.

An argument is deductively valid if, whenever all premisses are true, the conclusion is absolutely guaranteed to hold as well. Otherwise put, an argument is deductively valid if it is logically impossible for the premisses to be true but the conclusion to be false. A valid argument with true premisses is a sound argument. In a sound argument, the conclusion is necessarily true.

A2 is an example of a sound argument. A1 is valid, but it is not sound, since the second premiss is false (I know it is false because I will study books at university). A3 is not valid, because we can image a fictional world where all students are smart, where John is a man, but where Earth is not round. In other words, in A3, the conclusion is not absolutely guaranteed to be true when the premisses are true.

A4:

1. "It rains."
2. Therefore, "I will get wet."

Argument A4 is pretty convincing, but it is not deductively valid, because getting wet is not absolutely guaranteed when it rains. For example, you could have an umbrella or be in a building, or it could rain around you and so on. Arguments where the conclusion is very probable given the premisses are called inductively valid. A4 is an example of an inductively valid argument. Here is another inductively valid argument:

1. "No glass of wine that was bottled killed anyone."
2. "You are drinking a glass of bottled wine."
3. So, "The present glass of wine will not kill you."

The argument is pretty compelling, but the conclusion is not absolutey foolproof, even provided the premisses hold. For example, it could be that someone poured poison into the bottle of wine, poison that will kill you now just to make the conclusion false.

In this course, we will be concerned only with deductively valid arguments.

Just to make sure that you understand what a deductively valid argument is, I will give one more example:

1. "I have fallen of the roof of a skyscraper."
2. Therefore, "I am dead."

The argument is not valid (deductively), because it is conceivable that I am not dead. For example, I could have had a parachute or maybe there was a net to catch me.

A5:

1. "All students are smart."
2. "John is a student."
3. So, "John is smart."

This argument is valid. Note that the arguments A2 and A5 are instances of the following argument pattern:

1. "All P are Q."
2. "x is P."
3. Therefore, "x is Q."

If you replace P and Q by any property (such as the property of being a student, of being smart, of being mortal, of being a man, etc.) and x by any name (e.g., John, Socrates, etc.) you will obtain a valid argument.

Consider the following argument A6:

1. "I will learn chess or I will learn Go."
2. "I will not learn chess."
3. So, "I will learn Go."

It shares the following argument pattern with argument A2:

1. "P or Q."
2. "It is not the case that P."
3. So, "Q."

Any instance of this argument pattern will also make a valid argument. The individual propositions that occur in the argument patterns above are called *logical forms* or, in more modern terminology, *logical formulae* or simply *formulae*. Formulae such as "P or Q" and "not P" are studied in Propositional Logic, while formulae such as "All P are Q" are studied as part of First-Order Logic. There are several logics today, but we will concentrate first on one of the simplest, namely Propositional Logic.

3 Examples of Arguments

1. You are attending this course. Therefore, you are a student. (Not (deductively) valid)
2. At university I will either study books or I will play games. I will not study books. Therefore, I will play games. (Valid, but hopefully not sound)
3. (forall X) Oranges are either fruits or musical instruments. Oranges are not fruits. Therefore, oranges are musical instruments. (Valid, but not sound)
4. (forall X) London is in England. Beijing is in China. Therefore, Paris is in France. (Not valid, even if the premisses and the conclusion are all true)
5. No Englishman is a great poet. Eminescu is an Englishman. Therefore, Eminescu is not a great poet. (Valid despite the fact that the premisses and the conclusion are all false)
6. All computer scientists are weird. John is a computer scientist. So John is weird. (Valid, not sound)
7. Either John is in the library or in the bar. John is not in the library. Therefore, John is in the bar. (Valid)
8. (Peter Smith. An Introduction to Formal Logic) Cups of coffee that looked and tasted just fine haven't killed you before. The present cup of coffee looks and tastes fine. Therefore, the present cup of coffee will not kill you. (Inductively valid, but not (deductively) valid – perhaps someone poured poison in your coffee)
9. All computer science students are very smart. John is very smart. Therefore, John is a computer science student. (Not valid)
10. Most students are smart people. Most smart people read newspapers. So, at least some students read newspapers. (Invalid)
11. (Peter Smith. An Introduction to Formal Logic) Some computer science students admire all logicians. No computer science student admires any bad teacher. So, no logician is a bad teacher. (Valid, but it takes a while to check this; one part)

4 Informal Propositional Logic

Propositional Logic is the logic of propositions linked together with *logical connectives* such as *or*, *and* and *not*. In this section, we will go over the basics of propositional logic.

4.1 Atomic Propositions

Some propositions are atomic, in that they cannot be decomposed further into smaller propositions:

1. "I wear a blue shirt."

2. "You own a laptop computer."

3. "$2 + 2 = 4$." ("Two plus two is four.")

4.2 Conjunctions

Others however seem to be composed of smaller parts. For example, the proposition "I play games often and I study very well" is composed of two smaller propositions "I play games often" and "I study very well", joined together by "and". When two propositions "φ" and "ψ" are joined by an "and", the resulting proposition "φ and ψ" is called a *conjunction* or *the conjunction of φ and ψ*. The propositions "φ" and "ψ" are called the *conjuncts* of the proposition "φ and ψ".

Intuitively, a conjunction is true if both of its conjuncts are true. For example, the proposition "I play games often and I study very well." is true if both "I play games often." and "I study very well." are true. In particular, as I do not play games often, this proposition is false (when I say it).

Note that a conjunction need not use explicitly the word "and". For example, the proposition "It is raining outside, but I have an umbrella." is also a conjunction, and it conjuncts are "It is raining outside" and "I have an umbrella.". This particular conjunction uses the adversative conjunction "but".

Exercise 4.1. *Find the conjuncts of "I play at home and I study at school.".*

Exercise 4.2. *Give an example of a conjunction that is false.*

4.3 Disjunctions

Disjunctions are propositions linked together by "or". For example, "I can install the software on my smartphone or on my tablet." is a disjunction between "I can install the software on my smartphone" and "I can install the software on my tablet.". The two parts of the disjunction are called the *disjuncts*.

In the example above, note that the English grammar allows us to omit "I can install the software ...", as it is implicit in our understanding of the language. However, when we find the disjuncts, is helps to state them explicitly.

Exercise 4.3. *Find the disjuncts of "I will buy a laptop or a tablet.".*

A disjunction is true if at least one of the disjuncts is true. For example, "I am Darth Vader or I teach." is true because "I teach" is true (I do not have to worry about being Darth Vader). "I teach or I program." is also true (it happens that both disjuncts are true).

This meaning of disjunctions is called the *inclusive or*. It is standard in mathematics. Sometimes people use "or" in natural language to mean *exclusive or*. For example "Either white wins or black wins in a game of chess." is an example where the "or" is exclusive. The meaning of the sentence is that "white wins" or "black wins", but not both. Here is an example of a false proposition that uses "exclusive or": "Either I program or I teach.". When you see "either" in a sentence, it is a sign that you are dealing with an "exclusive or". In this case, do not say that it is a disjunction.

Exercise 4.4. *Give an example of a false disjunction.*

Exercise 4.5. *When is a disjunction "φ or ψ" false?*

4.4 Implications

Implications are propositions of the form "if φ then ψ". The proposition φ is called the *antecedent* and the proposition ψ is called the *conclusion* of the implication.

An example of an implication is "If I get a passing grade in Logic, I will buy everyone beer.". The antecedent is "I get a passing grade in Logic." and the conclusion is "I will buy everyone beer.". When is an implication true? Actually, it is easier to say when it is false. An implication is false if and only if the antecedent is true but the conclusion is false. Assume that I got a passing grade in Logic. Therefore, the proposition "I get a passing grade in Logic" is true. However, I will not buy beer for everyone (just a few select friends). Therefore the proposition "I will buy everyone beer" is false. Therefore the implication "If I get a passing grade in Logic, I will buy everyone beer." as a whole is false (antecedent is true, but conclusion is false).

The meaning of implications is worth a more detailed discussion as it is somewhat controversial. This is mostly because implication as we understand it in mathematics can sometimes be subtly different from implication as we understand it in everyday life. In everyday life, when we say "If I pass Logic, I buy beer.", we understand that there is a cause-and-effect relation between passing Logic and buying beer. This subtle cause-and-effect relation is evident in a number of "if-then" statements that we use in real life: "If I have money, I will buy a car.", "If you help me, I will help you", etc. We would never connect two unrelated sentences with an implication: the proposition "If the Earth is round, then 2+2=4." would not be very helpful, even though it is true (both the antecedent and the conclusion are true).

This implication is called *material implication* or *truth functional implication*, because the truth value of the implication as a whole depends only on the truth values of the antecedent and the conclusion, not on the antecedent and the conclusion itself. This meaning of implication sometimes does not correspond to the meaning of natural language implications, but it turns out that it is the only sensible interpretation of implications in mathematics (and computer science).

In particular, we will take both the propositions "If the Earth is flat, then $2 + 2 = 5$." and "If the Earth is flat, then $2 + 2 = 4$." to be true, because the antecedent is false. Implications that are true because the antecedent is false are called *vacuously true*.

Exercise 4.6. *What are the truth values of "If $2 + 2 = 4$, then the Earth is flat." and "If $2 + 2 = 5$, then the Earth is flat."?*

The truth value of an implication "if φ then ψ" depending on the truth values of its antecedent φ and its conclusion ψ is summarized in the truth-table below:

φ	ψ	if φ then ψ
false	*false*	*true*
false	*true*	*true*
true	*false*	*false*
true	*true*	*true*

The following example aims at convincing you that the truth table above is the only good one. You must agree that every natural number is also an integer. Otherwise put, the proposition "for any number x, if x is a natural, then x is an integer" is true. In particular, you will agree that the proposition above holds for $x = -10$, $x = 10$ and $x = 1.2$. In particular, the propositions "If -10 is a natural, then -10 is an integer.", "If 10 is a natural, then 10 is an integer." and "If 1.2 is a natural, then 1.2 is an integer." must all be true. This accounts for the first, second and fourth lines of the truth table above (typically, the second line is controversial). As for the third line, false is the only reasonable truth value for an implication "if φ then ψ" where φ is true but ψ is false. Otherwise, we would accept propositions such as "If $2+2 = 4$, then $2+2 = 5$." (antecedent $2 + 2 = 4$ true, conclusion $2 + 2 = 5$ false) as being true.

Implications are sometimes subtle to spot and identify correctly. For example, in the proposition "I will pass Logic only if I study hard.", the antecedent is "I will pass Logic" and the conclusion is "I study hard". In particular, the above proposition does not have the same meaning as "If I study hard, then I will pass Logic.".

Implications can sometimes not make use of "if". For example, take the proposition "I will pass Logic or I will drop school.". This proposition can be understood as "If I do not pass Logic, then I will drop school".

4.5 Negations

A proposition of the form "it is not the case that φ" is the *negation* of φ. For example, "It is not raining." is the negation of "It is raining.". The negation of a proposition takes the opposite truth value. For example, as I am writing this text, the proposition “It is raining.” is false, and therefore the proposition “It is not raining.” is true.

The words “and”, “or”, “if-then”, “not” are called *logical connectives*, as they can be used to connect smaller propositions in order to obtain larger propositions.

Exercise 4.7. *Give an example of a false proposition that uses both a negation and a conjunction.*

4.6 Equivalence

A proposition of the form “φ if and only if ψ” is called an *equivalence*. Such a proposition, as a whole, it true if φ and ψ have the same truth value (both false or both true).

For example, when I am writing this text, “It is raining if and only if it is snowing” is true. Why? Because both of the propositions “It is raining.” and “It is snowing.” are false.

Exercise 4.8. *What is the truth value of the proposition “The number* 7 *is odd if and only if* 7 *is a prime.”?*

5 Ambiguities in Natural Language

We have described informally the language of propositional logic: atomic propositions connected with and, or, not, etc. So far, our approach has used English. However, English (or any other natural language) is not suitable for our purposes because it exhibits imprecisions in the form of ambiguitites.

Consider the following argument:

1. “John and Mary are married.”
2. “Mary is a student.”
3. So, “John’s wife is a student.”

Is the argument valid? It depends on how you resolve the ambiguity of the first sentence. The first sentence could mean any of the following two things:

1. that John and Mary are married to each other;
2. or that John and Mary are married, but not necessarily to each other (they could be married to unnamed 3rd parties).

In the first interpretation, the argument is valid, while in the second interpretation, it is not. In the study of the laws of logic, such ambiguities can get in the way, just like a wrong computation could impact the resistance of buildings or bridges in civil engineering.

The state of the art in Logic for over 2.000 years, from Aristotle up to the development of Symbolic Logic in the 19th century, has been to use natural language. Symbolic logic (formal logic) has changed the game by introducing languages so precise that there is no risk of misunderstandings.

The first such language that we study will be the language of propositional logic.

6 Bibliography

Depending on the performance you want to achieve, you should study the following material:

$O(2^n)$ The present lecture notes;

$O(n^2)$ Chapter 1, What is Logic? from P. D. Magnus. forall X An Introduction to Formal Logic (open source);

$O(n)$ Chapter 1, Basic Concepts of Logic from G. Hardegree. Symbolic Logic: A First Course (available online for free);

$O(log(n))$ Chapters 1-6 from P. Smith. An Introduction to Formal Logic. Cambridge University Press 2003; (you can order it from Amazon for about $32 plus shipping);