

Question 7:

26 cards, diff alphabet letter on each

Vowel = 3 pts

Consonant = 1 pt

- a) Distribution of  $X$       b) Expected Value of  $X$       c) Variance of  $X$

$$P(\text{vowel}) = \frac{5}{26} * 3 = \frac{15}{26}$$

$$P(\text{consonant}) = \frac{21}{26}$$

$$1 - \frac{15}{26} = \frac{11}{26} \text{ new } P(\text{cons})$$

$$E(X) = \frac{15}{26}, \text{ expected value}$$

$$V(X) = \sum_{s \in S} (X(s) - E(X))^2 p(s)$$

$$= E(X^2) - E(X)^2$$

$$= 1.38$$

3 points \* 5 vowels = 15 points

1 point \* 21 consonants = 21 points

There are 36 total points in one deck of 26 cards

$$P(\text{drawing one vowel}) = \frac{1}{26} * \left(\frac{1}{3}\right) 3 = \frac{3}{26} = \frac{1}{12} \text{ Total points for randomly drawing 1 vowel}$$

$$P(\text{drawing one consonant}) = \frac{1}{26} \text{ Total points for drawing 1 consonant}$$

### Question 8:

Two decks of 26 cards each. Pick 1 card at random from each of two decks

$$\text{Vowels} = \{a, e, i, o, u\} = 5$$

$$\text{Cons} = 26 - 5 = 21$$

$$P(\text{consonant}) = \frac{{}^{21}C_1}{{}^{26}C_1} = \frac{21}{26}$$

$$P(\text{vowel}) = \frac{{}^5C_1}{{}^{26}C_1} = \frac{5}{26}$$

$P(x=2)$  Probability that two consonants are picked from both decks

$$= \left(\frac{21}{26}\right)\left(\frac{21}{26}\right) \text{ or } \left(\frac{21}{26}\right)^2$$

$P(x=4)$  Probability of picking one consonant and one vowel

$$= \left(\frac{21}{26}\right)\left(\frac{5}{26}\right) + \left(\frac{5}{26}\right)\left(\frac{21}{26}\right)$$

$P(x=6)$  Probability of picking two vowels  
 $\left(\frac{5}{26}\right)^2$

a) Distribution

x	2	4	6
P(x)	$\left(\frac{21}{26}\right)^2$	$2\left(\frac{21}{26}\right)\left(\frac{5}{26}\right)$	$\left(\frac{5}{26}\right)^2$

b) Expected Value

$$\begin{aligned} E(x) &= 2[P(x=2)] + 4[P(x=4)] + 6[P(x=6)] \\ &= \boxed{2.77} \end{aligned}$$

### Question 9:

a) Expected sum (two dice, 3 comes up 2x as often as other #s)

$$= \frac{1}{7}(1) + \frac{1}{7}(2) + \frac{1}{7}(3) + \frac{1}{7}(3) + \frac{1}{7}(4) + \frac{1}{7}(5) + \frac{1}{7}(6)$$

$$= \frac{24}{7} * 2 = \boxed{\frac{48}{7}}$$

b)  $S = \text{sum}$

$$E(S) = E(X_1 + X_2 + X_3)$$

$$= E(X_1) + E(X_2) + E(X_3)$$

Since 3 dices are fair

$$= 3 * E(X)$$

$$= 3 \left[ 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) \right]$$

$$= 3(3.5) = 10.5$$