HW 8

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Question 5 :

a) $P(n)$: 3 divide n^3+2n , n = positive integer

Basis step $P(1)$, $n=1$

$(1)^3+2(1)=3$; 3 is divisible by 3 so

$P(1)$ base step is true

Inductive Step $P(n)$

$n^3+2n=3p$ $\quad p$ = pos integer

If $P(n)$ is true, $P(n+1)$ is also true

$$(n+1)^3+2(n+1) = n^3+3n^2+3n+1+2n+2$$
$$=(n^3+2n)+3(n^2+n+1)$$
$$=3p+3(n^2+n+1)$$
$$=3(p+n^2+n+1)$$

$\Rightarrow (n+1)^3+2(n+1)=3(p+n^2+n+1)$ is divisible by 3

and $p+n^2+n+1$ is an integer

therefore $P(n+1)$ is also true and

by induction $P(n)$ is true for all n positive

integers

b) Strong induction

every positive integer $n (n \geq 2)$ is a product of primes

Basis Step: $n \geq 2$ is a prime number if the positive integers that divide n are 1 and n.

Thus, 2 is a prime number

Inductive Step:

$2 \leq j \leq K$

Case1: $P(K+1)$ if $K+1$ = prime, $k+1$ is the product of a prime, itself, thus $P(K+1)$ is true

Case2: $K+1$ is composite

$k+1$ as a product of two positive integers a and b

$2 \leq a \leq b \leq K+1$

a and b are positive integers greater or equal to 2 and not exceeding K so by induction a and b are product of primes. If $K+1$ is composite, it can be written as the product of primes namely the primes in the factorization of a and in b.

By strong induction $\forall n P(n)$ is true.