Please enter a positive integer:

5

1. 1 $(=2^0)$
2. 2 $(=2^1)$
3. 4 $(=2^2)$
4. 8 $(=2^3)$
5. 16 $(=2^4)$

line #	power of 2
1	2^0
2	2^1
3	2^2
4	2^3
⋮	⋮
count	$2^{count-1}$
⋮	⋮
n	2^{n-1}

n = 5

count = ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ 6

currPower = ~~1~~ ~~2~~ ~~4~~ ~~8~~ 16

Please enter a positive integer

4

```
* * * *
* * * *
* * * *
* * * *
```

line number	# of stars
1	n
2	n
⋮	⋮
n	n

```
*
* *
* * *
* * * *
```

line number	# of stars
1	1
2	2
3	3
⋮	⋮
k	k
⋮	⋮
n	n

n

n-1

```
      *
    * *
  * * *
* * * *
```

n

line number	# of spaces	# of stars
1	n-1	1
2	n-2	2
3	n-3	3
⋮	⋮	⋮
k	n-k	k
⋮	⋮	⋮
n	0	n

Please enter a sequence of positive integers.
End by typing -1:

5
17
29
4
6
-1

Sum is 61

Average is ______

Number of evens is ______

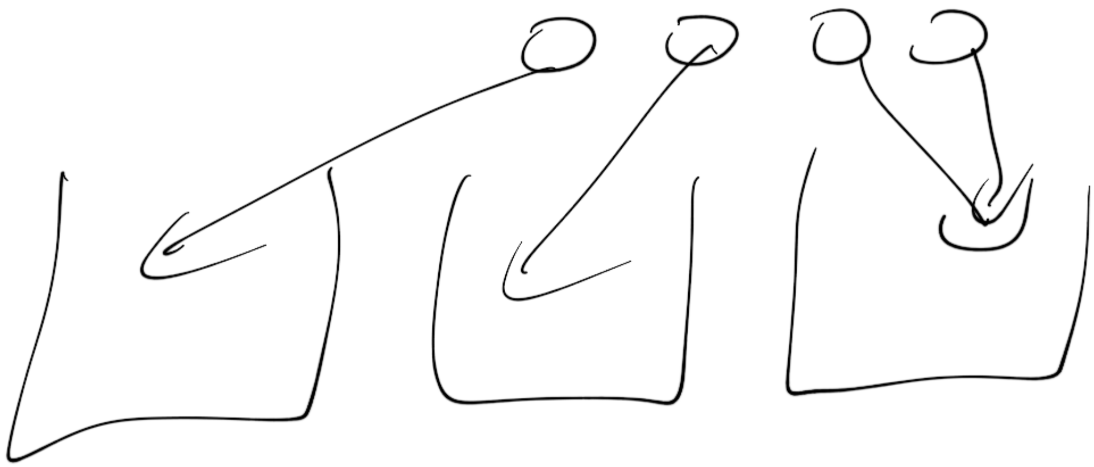
currElem = ~~5~~ ~~17~~ ~~29~~ ~~4~~ ~~6~~ -1

sum = ~~0~~ ~~5~~ ~~22~~ ~~51~~ ~~55~~ 61

seenEndOfSeq = ~~false~~ true

The pigeonhole principle:

If you split $(k+1)$ objects into k boxes, then there will be a box with > 1 object in it

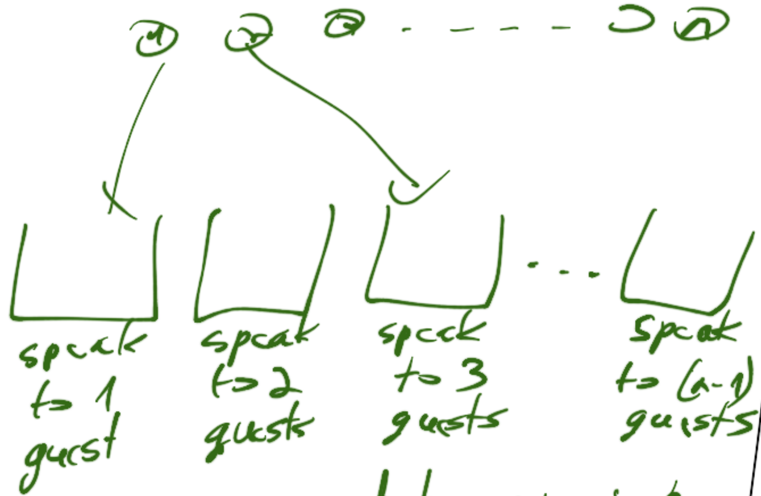


$k=3$

Ex 1: Number the people $1, 2, 3, \ldots n$

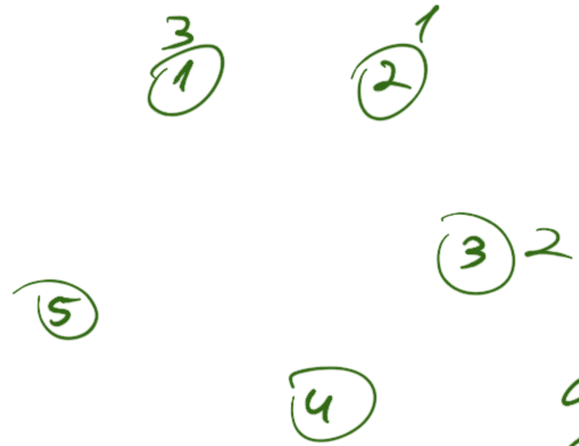
Let a_i = # of guests person i speaks to

Note that a_i is in the range $1-(n-1)$



by the pigeonhole principle we get that there would be two guests who speak to the same number of people

$n=5$



$a_1 = 3$

$a_2 = 1$

$a_3 = 2$

$\vdots$

Ex 2:

Take: $a_1 = 5$

$a_2 = 55$

$a_3 = 555$

$a_4 = 5555$

$a_5 = 55555$

$a_6 = 555555$

$a_7 = 5555555$

$a_8 = 55555555$

Consider the remainders each one leaves, when divided by 7.

The possible remainders are: 0-6 (7 possible remainders)

By the pigeonhole principle we get that there are two numbers with the same remainder.

Assume $a_i > a_j$, and both have the same rem.

$a_i = k_1 \cdot 7 + r$

$a_j = k_2 \cdot 7 + r$

Look at $a_i - a_j$:

$a_i - a_j = (k_1 - k_2) \cdot 7$

1) $a_i - a_j$ is a multiple of 7

2) $\begin{array}{r} 555\ldots5 \\ -\quad 5\ldots5 \\ \hline 55\,0\ldots0 \end{array} \Rightarrow$ $a_i - a_j$ has only 5s and 0s

$58 \div 7 = 8(2)$

$58 = 8 \cdot 7 + 2$

$$\begin{array}{r} 5555 \\ -\quad 55 \\ \hline 5500 \end{array}$$

Ex3: Look at the following 90 numbers:

1) 45 numbers: $a_1, a_2, \ldots, a_{45}$

2) 45 numbers: $a_1+9, a_2+9, \ldots, a_{45}+9$

Note that:

1) $a_1, a_2, \ldots, a_{45}$ are in the range 1-80

2) $a_1+9, a_2+9, \ldots, a_{45}+9$ are in the range 10-89

$\Downarrow$

All 90 numbers are in the range 1-89

By the pigeonhole principle we get that two numbers are equal

Since all a_i's are different $\Rightarrow$ all a_i+9 are different

$\Downarrow$

There exists i, j $\quad a_i = a_j + 9 \Rightarrow a_i - a_j = 9$