Question 1:

A) 1. $10011011_2 = 1\cdot2^7 + 0\cdot2^6 + 0\cdot2^5 + 1\cdot2^4 + 1\cdot2^3 + 0\cdot2^2 + 1\cdot2^1 + 1\cdot2$

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$ $= 128 + 16 + 8 + 2 + 1 \Rightarrow \boxed{155}$

2. $456_7 = 4\cdot7^2 + 5\cdot7^1 + 6\cdot7^0$
$= \boxed{237}$

3. $38A_{16} = 3(16^2) + 8\cdot(16^1) + 10(16^0)$
$= \boxed{906}$

4. $2214_5 = 2(5^3) + 2(5^2) + 1(5^1) + 4(5^0)$
$= \boxed{309}$

B) 1. $69_{10} \Rightarrow 69 \div 2 = 34\ R1 \Rightarrow \boxed{1000101_2}$

$34 \div 2 = 17\ R0$
$17 \div 2 = 8\ R1$
$8 \div 2 = 4\ R0$
$4 \div 2 = 2\ R0$
$2 \div 2 = 1\ R0$
$1 \div 2 = 0\ R1$

2. 485_{10} :

$$\begin{array}{cccccc} 485 & 229 & 101 & 37 & 5 & 1 \\ -256 & -128 & -64 & -32 & -4 & -1 \\ \hline 229 & 101 & 37 & 5 & 1 & 0 \\ 2^8 & 2^7 & 2^6 & 2^5 & 2^2 & 2^0 \end{array}$$

$= \boxed{111100101_2}$

3. $6D1A_{16} \Rightarrow \boxed{0110\ 1101\ 0001\ 1010_2}$

$A = 10 \Rightarrow 1\cdot2^3 + 0\cdot2^2 + 1\cdot2^1 + 0\cdot2^0 = 1010$
$1 = 0001$
$D = 13 \Rightarrow 1\cdot2^3 + 1\cdot2^2 + 0\cdot2^1 + 1\cdot2^0 = 1101$
$6 = 2^2 + 2^1 + 0\cdot2^0 = 0110$

(C) 1. $\underbrace{110}\underbrace{1011}_2 = \boxed{6B_{16}}$

$$1011 = 1\cdot2^3 + 0\cdot2^2 + 1\cdot2^1 + 1\cdot2^0 = 8+2+1 = 11 = B$$

$$110 = 1\cdot2^2 + 1\cdot2^1 + 0\cdot2^0 = 6$$

2. $895_{10} = \boxed{37F_{16}}$

$$895 \div 16 = 55 R 15 \Rightarrow F$$
$$55 \div 16 = 3 R 7 \Rightarrow 7$$
$$3 \div 16 = 0 R 3 \Rightarrow 3$$

Question 2:

1)
$$\begin{array}{r} \scriptstyle 1\,1\,1\,1\;\; \\ 7566_8 \\ +\,4515_8 \\ \hline 14303_8 \end{array}$$

2)
$$\begin{array}{r} \scriptstyle 1\,1\,1\,1\,1\,1\;\;\; \\ 10110011_2 \\ +\quad 1101_2 \\ \hline 11000000_2 \end{array}$$

3)
$$\begin{array}{r} \scriptstyle 1\,1\;\;\;\;\; \\ 7A66_{16} \\ +\,45C5_{16} \\ \hline C02B_{16} \end{array}$$

4)
$$\begin{array}{r} \scriptstyle 2\,9\,1\;\; \\ \cancel{3}\cancel{0}\cancel{2}2_5 \\ -\,2433_5 \\ \hline 0034_5 \end{array}$$

Question 3: Convert to 8-bits two's complement

(A) 1) $124_{10} = \boxed{01111100}$ 8 bit 2's comp

$$\begin{array}{r} 124 \\ -64 \\ \hline 60 \end{array} \quad \begin{array}{r} 60 \\ -32 \\ \hline 28 \end{array} \quad \begin{array}{r} 28 \\ -16 \\ \hline 12 \end{array} \quad \begin{array}{r} 12 \\ -8 \\ \hline 4 \end{array} \quad \begin{array}{r} 4 \\ -4 \\ \hline 0 \end{array}$$

$2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2$

01111100

2) $-124_{10} = \boxed{10000100}$ 8bit 2's comp

$$\begin{array}{r} \scriptstyle 1\,1\,1\,1\,1\;\;\;\;\;\; \\ 01111100 \\ +\,10000100 \\ \hline 100000000 \end{array}$$

3) 109_{10} = $\boxed{01101101_{\text{8 bit 2's comp}}}$

$109 \div 2 = 54 R1$ $\quad 13 \div 2 = 6 R1$ $\quad 1 \div 2 = 0 R1$

$54 \div 2 = 27 R0$ $\quad 6 \div 2 = 3 R0$

$27 \div 2 = 13 R1$ $\quad 3 \div 2 = 1 R1$

4) -79_{10} = $\boxed{10110011_{\text{8 bit 2's comp}}}$

$79 \div 2 = 38 R1$ $\quad 9 \div 2 = 4 R1$ $\quad 1 \div 2 = 0 R1$

$38 \div 2 = 19 R0$ $\quad 4 \div 2 = 2 R0$

$19 \div 2 = 9 R1$ $\quad 2 \div 2 = 1 R0$

```
 11111111
  01001101
+ 10110011
----------
 100000000
```

(B) 1) $00011110_{\text{8 bit 2's comp}}$ = $\boxed{30_{10}}$

$1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1$

$= 30$

2) $11100110_{\text{8 bit 2's comp}}$ = $\boxed{-26_{10}}$

$1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^5 + 1 \cdot 2^6 + 1 \cdot 2^7$

$= 230$

```
 1111 111
  00011010
+ 11100110
----------
 100000000
```

00011010

$1 \cdot 2^4 + 2^3 + 2^1$

$= 26$

3) $00101101_{\text{8 bit 2's comp}}$ = $\boxed{45_{10}}$

$1 \cdot 2^0 + 2^2 + 2^3 + 2^5 = 45$

4) $10011110_{\text{8 bit 2's comp}}$ = $\boxed{-98_{10}}$

```
 1111111
  01100010
+ 10011110
----------
 100000000
```

01100010

$2^1 + 2^5 + 2^6 = 98$

Question 4:

(A) a) $\{x \in R \mid x$ is an integer greater than 1
True

b) $\{2, \{2\}\}$
True

c) $\{x \in R \mid x$ is the square of an integer$\}$
False

d) $(\{2\}, \{\{2\}$
False

e) $\{\{2\}, \{2, \{2\}\}\}$
False

f) $\{\{\{2\}\}\}$
False

(B) a) $x \in \{x\}$
True

b) $\{x\} \subseteq \{x\}$
True

c) $\{x\} \in \{x\}$
False

d) $\{x\} \in \{\{x\}\}$
True

e) $\emptyset \subseteq \{x\}$
True

f) $\emptyset \in \{x\}$
False

(C) $A \in B$ and $A \subseteq B$

$A = \{2, 3\}$

$B = \{2, 3, 4\}$

(D) i. 1st is a subset of 2nd, not vice versa

ii. Neither is a subset of the other

iii. 1st is a subset of 2nd, not vice versa

Question 5:

$A = \{a, b, c, d, e\}$ $B = \{a, b, c, d, e, f, g, h\}$

a) $A \cup B = \{a, b, c, d, e, f, g, h\}$

b) $A \cap B = \{a, b, c, d, e\}$

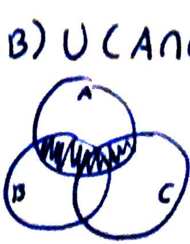
c) $A - B = \{\}$

d) $B - A = \{f, g, h\}$

Question 6:

a) $A \cap (B - C)$

b) $(A \cap B) \cup (A \cap C)$



c) $(A \cap \bar{B}) \cup (A \cap \bar{C})$

Question 7:

a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

A	B	C	$B \cap C$	$A \cup (B \cap C)$ *	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$ *
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

Columns for $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$ are the same, thus the identity is valid

b) $(B - A) \cup (C - A) = (B \cup C) - A$

A	B	C	$B - A$	$C - A$	$(B-A) \cup (C-A)$	$B \cup C$	$(B \cup C) - A$
1	1	1	0	0	0	1	0
1	1	0	0	0	0	1	0
1	0	1	0	0	0	1	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1
0	0	1	0	1	1	1	1
0	0	0	0	0	0	0	0

Columns for $(B-A) \cup (C-A)$ and $(B \cup C) - A$ are the same, thus the identity is valid

c) $\overline{(A \cap B \cap C)} = (\bar{A} \cup \bar{B} \cup \bar{C})$

A	B	C	$A \cap B \cap C$	$\overline{A \cap B \cap C}$	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{A} \cup \bar{B} \cup \bar{C}$
1	1	1	1	0	0	0	0	0
1	1	0	0	1	0	0	1	1
1	0	1	0	1	0	1	0	1
1	0	0	0	1	0	1	1	1
0	1	1	0	1	1	0	0	1
0	1	0	0	1	1	0	1	1
0	0	1	0	1	1	1	0	1
0	0	0	0	1	1	1	1	1

Columns for $\overline{(A \cap B \cap C)}$ and $(\bar{A} \cup \bar{B} \cup \bar{C})$ are not the same, thus the identity is invalid

d) $(A-C) \cap (C-B) = \varnothing$

A	B	C	$A-C$	$C-B$	$(A-C) \cap (C-B)$	$\varnothing$
1	1	1	0	0	0	0
1	1	0	1	0	0	0
1	0	1	0	1	0	0
1	0	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	1	0	0
0	0	0	0	0	0	0

Columns for $(A-C) \cap (C-B)$ and $\varnothing$ are the same, thus the identity is valid

e) $(A-B) - C \subseteq (A-C)$

A	B	C	$A-B$	$(A-B)-C$	$(A-C)$
1	1	1	0	0	0
1	1	0	0	0	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

All of the elements in $(A-B)-C$ are in $(A-C)$, thus the statement is true.

Question 8: Use set identities

a) $A - B = \overline{B} - \overline{A}$

$\overline{B} - \overline{A} =$

$=$

$\overline{B} \cap A =$

$A \cap \overline{B} \Rightarrow \overline{B} \cap A$ and $\overline{B} - \overline{A} = \overline{B} \cap A$

$X - Y = X \cap \overline{Y}$ Commutative Laws valid $A - B = \overline{B} - \overline{A}$

b) $(A \cap B) \cup (A \cap \overline{B}) = A$

$A \cap (B \cup \overline{B})$ Distributive Laws

$A \cap (\ U\ )$ Complement Laws

$A \cap U = A$ Dominance Laws

$A = A$

c) $A - (B - C) = (A - B) \cup (A - \overline{C})$

$A - (B \cap \overline{C}) \Rightarrow A \cap \overline{(B \cap \overline{C})} \Rightarrow (A \cap \overline{B}) \cup (A \cap \overline{\overline{C}}) \Rightarrow (A - B) \cup (A - \overline{C})$

$X - Y = X \cap \overline{Y}$ Distributive Law $X \cap \overline{Y} = X - Y$

Question 9: $A = B$

a) $A \cup C = B \cup C$?

if $A = \{1,2,3\}$ then $C = \{1,2,3,4,5\}$. $A \cup C = \{1,2,3,4,5\}$, $B \cup C = \{1,2,3,4,5\}$

$B = \{3,4,5\}$

Thus, $A \cup C = C$ and $B \cup C = C$ but $A \neq B$

b) $A \cap C = B \cap C$?

if $A = \{1,2,3\}$, $B = \{3,4,5\}$ then $C = \{3\}$. $A \cap C = \{3\} = C$, $B \cap C = \{3\} = C$

$A \cap C = C$, $B \cap C = C$ but $A \neq B$ Not all elements in A are in B, vice versa

c) $(A \cup C = B \cup C)$ and $(A \cap C = B \cap C)$?

Since each are false, this conclusion is also false $A = B$

$A \cup C = B \cup C$

A	B	C	$A \cup C$	$B \cup C$
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
1	0	0	1	0
0	1	1	1	1
0	1	0	0	1
0	0	1	1	1
0	0	0	0	0

$A \cap C = B \cap C$

A	B	C	$A \cap C$	$B \cap C$
1	1	1	1	1
1	1	0	0	0
1	0	1	1	0
1	0	0	0	0
0	1	1	0	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

Question 10:

$\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$

a) $A_i = \{-i, -i+1, \ldots, -1, 0, 1 \ldots i-1, i\}$

$A_1 = \{-1, 0, 1\}$

$A_2 = \{-2, -1, 0, 1, 2\}$

$A_3 = \{-3, -2, -1, 0, 1, 2, 3\}$

$$\boxed{\begin{aligned}&\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \ldots = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\} = \mathbb{Z} \\ &\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \ldots = \{-1, 0, 1\}\end{aligned}}$$

b) $A_i = \{-i, i\}$

$A_1 = \{-1, 1\}$

$A_2 = \{-2, 2\}$

$A_3 = \{-3, 3\}$

$$\boxed{\begin{aligned}&\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \ldots = \{-3, -2, -1, 1, 2, 3 \ldots\} = \mathbb{Z} \text{ without } \{0\} \\ &\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 = \emptyset\end{aligned}}$$

c) $A_i = [-i, i]$ $\quad -i \leq x \leq i$

$A_1 = [-1, 1]$

$A_2 = [-2, 2]$

$A_3 = [-3, 3]$

$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \ldots = [\ldots -3, -2, -1, 1, 2, 3 \ldots] = [\mathbb{R}^-, \mathbb{R}^+]$ not including $[0]$

$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \ldots = [-1, 1]$

d) $A_i = [-i, \infty) \quad x \geq -i$

$A_1 = [-1, \infty)$

$A_2 = [-2, \infty)$

$A_3 = [-3, \infty)$

$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \ldots = [\mathbb{R}^-, \infty)$

$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \ldots = [-1, \infty)$