HW 5

Nam Nguyen
nn1338

Question 7:

a) $\underbrace{5n^3 + 2n^2 + 3n}_{f(n)} = \underbrace{\Theta(n^3)}_{g(n)}$

If we take $c1 = 7$

$c2 = 5$

$n0 = 5$

Then for all $n \geq n_0$ we have:

$$5n^3 \leq 5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^2 \leq 5n^3 + 2n^3 = 7n^3$$

$2n^2 + 3n \leq 0$

$2n^2 \leq -3n$

$n \leq -\frac{3}{2}$

$$5n^3 \leq 5n^3 + 2n^2 + 3n \leq 7n^3$$

Therefore $5n^3 + 2n^2 + 3n = \Theta(n^3)$

b) $\underbrace{\sqrt{7n^2 + 2n - 8}}_{f(n)} = \underbrace{\Theta(n)}_{g(n)}$

$c1 = 3$
$c2 = 7$
$n0 = \sqrt{7}$

$$\sqrt{7}n \leq \sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n^2} = \sqrt{9n^2}$$
$$= 3n$$

$2n - 8 \leq 0$

$n \leq 4$

$$\sqrt{7}n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$$

Therefore $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

Question 8: fair die never comes up an even # rolled 6x

Sample space = {1, 2, 3, 4, 5, 6}

Event = {1, 3, 5} for rolling odd #

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Each roll is independent

$\frac{1}{2^6} = \boxed{\frac{1}{64}}$ That a dice rolled 6x never comes up an even number

Question 9:

1001 0011 1100

Sample space = $2^3 = 8$

$P(E) = \boxed{\frac{3}{8}}$ That a 4-bit string with at least two consecutive zeros, given the first bit is a 1.

Question 10:

a) exactly three boys

C(5,3) since 5 children being born are independent from one another

chance of being a girl

$1 - 0.51 = 0.49$

$C(5,3) * (0.51)^3 (0.49)^2 = \boxed{0.318}$

b) at least one boy

if all 5 children are girls $(.49)^5$ were subtracted from 1, we would find probability of at least one boy ~~(1, 2, 3, 4 or all 5 boys)~~

$1 - (0.49)^5 = \boxed{0.97}$

c) at least one girl

$1 - (0.51)^5 = \boxed{0.965}$

d) all children of the same sexo

all 5 are boys $= (0.51)^5$

or

all 5 are girls $= (0.49)^5$

$(0.51)^5 + (0.49)^5 = \boxed{0.06275}$

e) First chilld is a boy or that the last two chilldren of the family are girls

$P(E) = 0.51$ => 1st chilld is a boy

$P(F) = 0.49 * 0.49$ = last two children are female

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$P(E \cap F) = (0.51)(0.49^2) = 0.122$

$P(E \cup F) = 0.51 + 0.49^2 + 0.122$

$= \boxed{0.6276}$

Question 11: 5 children, no boys

a) boy and girl equally likely

boy = girl = 1/2

$C(5,0)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$

$\frac{5!}{0!(5-0)!} = \frac{5!}{5!} = 1 \qquad 1 * \left(\frac{1}{32}\right) = \boxed{\frac{1}{32}}$

b) Probability of a boy is 0.51

since there's no boys

$C(5,0)(0.51)^0(0.49)^5 = \boxed{0.02824}$

c) i^{th} child is a boy is $0.51 - \frac{i}{100}$.

$1-(0.51 - i/100) \Rightarrow 49 + i/100$

$49 + i/100 = i^{th}$ child is a girl

For all 5 i^{th} child to be girls

$(0.49 + 1/100)(0.49 + 2/100)(0.49 + 3/100)$
$(0.49 + 4/100)(0.49 + 5/100)$

$= (0.5)(0.51)(0.52)(0.53)(0.54)$

$= \boxed{0.038}$

Question 12:

n = trials (Independant Bernoulli trials)

p = success (probability)

$q = 1 - p$ = failure

a) probability of no failures

also meaning all successes

$= \boxed{p^n}$

b) probability of at least one failure

take $1 - p^n$ or 1 - probability of all successes

$\boxed{1-p^n}$ => probability of at least 1 failure

c) probability of at most one failure

p No failures ~~and~~ p one failure

p^n $\quad$ $C(n,1)(1-p)(p^{n-1})$

$p^n + C(n,1)(1-p)(p^{n-1})$

$= \boxed{p^n + np^{n-1}(1-p)}$

d) probability of at least two failures

1 - probability of at most one failure

$\boxed{1 - p^n - np^{n-1}(1-p)}$