Question 7:

How many strings of 8 uppercase English letters

a) Letters repeated?

26^8

b) No letters repeated

$P(26,8) = 62990928000$

c) Starting with X, letters repeated...

26^7

d) starting with X, no letters repeated...

$P(25,7) = 2422728000$

Question 8: One-to-one function one set w/ 5 elements

a) 4; no one-to-one function as elements in set A is greater than elements in set B

$a > b$ or $5 > 4$

b) 5; A set of 5 elements can be mapped to another set of 5 elements by using the product rule to figure out the possible number of one-to-one functions

$5 \times 4 \times 3 \times 2 \times 1 = 120$

c) 6; a_1 a_2 a_3 a_4 a_5 — b_1 b_2 b_3 b_4 b_5 b_6

6 choices from a set of 5 elements.

$6 \times 5 \times 4 \times 3 \times 2 = 720$

d) 7i $7 \times 6 \times 5 \times 4 \times 3 = \boxed{2520}$ possible one-to-one functions from 5 elements in the domain and 7 elements in the codomain

Question 9:

Set A has 100 elements

The number of subsets with more than one element or 'x' elements can be denoted as 2^x (2^x subsets)

the # of subsets of $A = 2^{100}$

However we will need to subtract the number of subsets of A with ≤ 1 element which is 0 and 100 subsets or 101 elements.

Thus, # of subset of A with > 1 element can be denoted as $\boxed{2^{100} - 101}$

Question 10:

a) Unique combinations : Bride must be in the picture. 10 people total
Permutation

$9P_5 = 9\times8\times7\times6\times5 = 15,120$ * ways of arranging the 5 remaining people

$15,120 \times 6 = \boxed{90,720}$ * 6 ways bride can be in the picture

b) $8P_4 = 8\times7\times6\times5 = 1,680$

$6\times5\times1680 = \boxed{50,400}$ * Bride in the pic : 6 ways
Then Groom in the pic : 5 ways

c) exactly one of the bride and groom in the picture

$8P_5 = 8\cdot7\cdot6\cdot5\cdot4 = 6,720$ * exactly one = 5
* 10-2 = 8

$6\cdot2\cdot6720 = \boxed{80,640}$ Two slot ways of arranging bride and groom

Question 11: bits of Strings of length 12

a) exactly three 1s?

$C = \frac{n!}{k!(n-k)!}$

$\frac{12!}{3!(9)!} = \frac{12\times11\times10}{3\times2\times1} = \boxed{220}$

b) at most three 1s?

$12C_3 + 12C_2 + 12C_1 + 12C_0 \Rightarrow 220 + \frac{12\times11}{2} + \frac{12}{1} + 1$

$= 220 + 66 + 12 + 1 = \boxed{299}$

c) at least three 1s?

$= 12C_3 + 12C_{k+1} \ldots + 12C_{12} \Rightarrow \boxed{4017}$

Question 12: Permutations ABCDEFGH

a) contain string DE ABC[DE]FGH => 7 letters

$= 7! = \boxed{5040}$

b) E and D are not next to one another

8! to arrange the string

ED = 7! ways of being together

$8! - 7! = \boxed{35,280 \text{ ways}}$

c) contain either CD or DE?

7! for CD $7! + 7! = 10,080$

7! for DE $10,080 - 6! = \boxed{9360}$

6! for CDE