Question 5:

HW2

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one-to-one? $f: \mathbb{Z} \to \mathbb{Z}$

I a) $f(n) = n - 1$

$$\forall a \forall b (f(a) = f(b) \to a = b)$$

$$f(n) = f(m) \quad n, m \in \mathbb{Z}$$
$$n - 1 = m - 1$$
$$n = m$$

$$\forall a \forall b (a \neq b \to f(a) \neq f(b))$$
$$n - 1 \neq m - 1$$
$$n \neq m$$

Therefore f is a one-to-one function

b) $f(n) = n^2 + 1$

To demonstrate: $f(1) = (1)^2 + 1 = 2$
$f(-1) = (-1)^2 + 1 = 2$

Because two different values 1 and -1 of the domain $\mathbb{Z}$ have the same assignment value of 2

Also, suppose $f(n) = f(m) \quad n, m \in \mathbb{Z}$
$$n^2 + 1 = m^2 + 1$$
$$n = \pm m$$

Thus f is not a one-to-one function

c) $f(n) = n^3$

To demonstrate: $f(1) = (1)^3 = 1$ $\quad f(-1) = -1$

$f(2) = (2)^3 = 8$ $\quad f(-2) = -8$

$f(3) = (3)^3 = 27$ $\quad f(-3) = -27$

$f(n) = f(m) \quad n, m \in \mathbb{Z}$

$n^3 = m^3$

$n = m$

$f(n) = n^3$ is a one-to-one function because no two values in domain $\mathbb{Z}$ are assigned to the same function value

d) $f(n) = \lceil \frac{n}{2} \rceil$

To demonstrate: $f(1) = \lceil \frac{1}{2} \rceil = 1$

$f(2) = \lceil \frac{2}{2} \rceil = 1$

The two values of 1 and 2 of domain $\mathbb{Z}$ are assigned to the same value 1

Therefore f is not a one-to-one function

ONTO? $f: \mathbb{Z} \to \mathbb{Z}$

II) a) $f(n) = n-1 \quad n \in \mathbb{Z}$

To demonstrate : $f(n) = m \Rightarrow n - 1 = m$

$n = m+1$

Thus $f(n) = n-1$ is an onto function because for every integer m there is an integer n where $f(n) = m$ or $\forall m \exists n (f(n) = m)$ where n is the domain and m is the codomain of the function

b) $f(n) = n^2+1 \quad n \in \mathbb{Z}$

To demonstrate : $f(1) = (1)^2+1 = 2$
$f(2) = (2)^2+1 = 5$

$f(n) = n^2+1$ is not an onto function because no integers of n in n^2+1 can equal to 3

c) $f(n) = n^3 \quad n \in \mathbb{Z}$

To demonstrate : $f(1) = (1)^3 = 1$
$f(2) = (2)^3 = 8$

$f(n) = n^3$ is not an onto function because no integers of n in n^3 can equal to 3 integer value.

d) $f(n) = \lceil \frac{n}{2} \rceil \quad n \in \mathbb{Z}$

$f(1) = \lceil \frac{1}{2} \rceil = 1$

$f(3) = \lceil \frac{3}{2} \rceil = 2$

$f(n) = m$ if and only if $\lceil \frac{n}{2} \rceil = m$

Thus, $f(n) = \lceil \frac{n}{2} \rceil$ is an onto function

Question 6: Bijection $f: R \to R$

a) $f(x) = -3x+4$

Let's suppose that $f(x) = f(y)$

$$-3x+4 = -3y+4$$
$$-3x = -3y$$
$$x = y$$ thus the function is a one-to-one

To show $f(x) = y \Rightarrow y = -3x+4$

$$x = \frac{y-4}{-3}$$
$$x = \frac{4-y}{3}$$ thus for every element y of the R co-domain $\exists$ an element x of the R domain

$x = \frac{4-y}{3}$, $f\left(\frac{4-y}{3}\right) = y$

Thus if the function is both onto and one-to-one we can say that the function is a bijection

b) $f(x) = -3x^2+7$

Suppose $f(x) = f(y)$

$$-3x^2+7 = -3y^2+7$$
$$-3x^2 = -3y^2$$
$$\sqrt{x^2} = \sqrt{y^2}$$
$$x = \pm y$$

Since $x = y$ is not shown, meaning in this case there are two values of x in the R domain being mapped on the y element in the R codomain, f is not a one-to-one function thus by definition $f(x) = -3x^2+7$ is not a bijection.

c) $f(x) = \frac{x+1}{x+2}$

Suppose $f(-2) = \frac{-2+1}{-2+2}$ = undefined

Since there is no assignment of a unique element of Y to each element of X,
$f(x) = \frac{x+1}{x+2}$ from R to R is not a function
and thus is not a bijection

d) $f(x) = x^5 + 1$

suppose $f(x) = f(y)$

$x^5+1 = y^5+1 \Rightarrow x^5 = y^5$

$x = y$ thus is a one-to-one function

suppose $f(x) = y$

$x^5+1 = y$

$x = \sqrt[5]{y-1}$

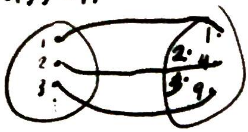
$\forall y \exists x (f(\sqrt[5]{y-1}) = y)$ thus the function f is an onto function.

By definition, since $f(x) = x^5+1$ is both onto and one-to-one function, f is a bijection function

Question 7:

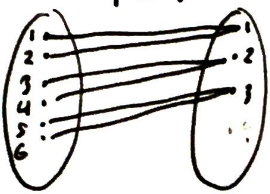
a) one-to-one, but not onto.

$f(n) = n^2$



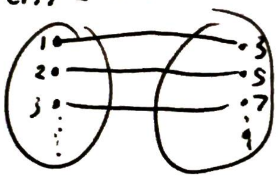
b) onto, but not one-to-one

$f(n) = \lceil \frac{n}{2} \rceil$



c) one-to-one and onto

$f(n) = 2n+1$



d) Neither one-to-one nor onto

$f(n) = n^2+1$

Question 8:

$f(x) = ax+b \quad , \quad g(x) = cx+d$

$f \circ g = g \circ f \; ?$

$f(g(x)) = g(f(x))$

$f(cx+d) = g(ax+b)$

$a(cx+d)+b = c(ax+b)+d$

$\cancel{acx} + ad + b = \cancel{cax} + cb + d$

$ad + b = cb + d$

Since a,b,c,d are constants,

$ad - d = cb - b$

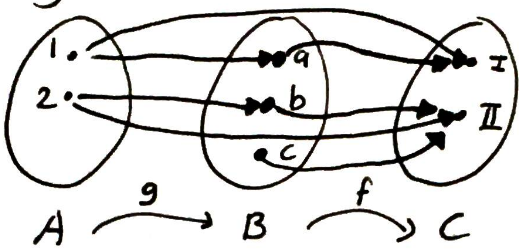
$d(a-1) = b(c-1)$

any combination of sets of integers can hold true for $f \circ g = g \circ f$

Question 9:

$g: A \to B$, $f: B \to C$

$f \circ g$ is an onto function

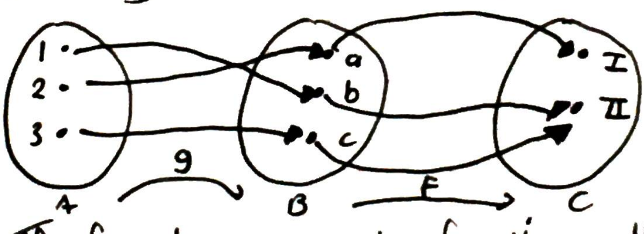


a) If $f \circ g$ is an onto function, then it does not follow that both f and g are onto functions. $g(1) = a$, $g(2) = b$

$f(a) = \mathrm{I}$, $f(b) = \mathrm{II}$, $f(c) = \mathrm{II}$

Thus, there does not exist for every element $b \in B$ there is an element $a \in A$

b) f and g are onto functions



If f and g are onto functions then it does follow that the composition of f and g or $f \circ g$ is an onto function as well.

$g(1) = b$, $g(2) = a$, $g(3) = c$

$f(a) = \mathrm{I}$, $f(b) = \mathrm{II}$, $f(c) = \mathrm{II}$

$\forall b \exists a (f(a) = b)$ and $\forall c \exists b (f(b) = c)$

Thus, $f \circ g$ is surjective