Advanced Signal Processing

Image Processing

Gustavo Andrade Miranda PhD

gxavier.andradem@ug.edu.ec

Facultad de Ingeniería en Sistemas, Electrónica e Industrial
Universidad Técnica de Ambato

PROGRAMA DE MAESTRÍA EN TELECOMUNICACIONES

Outline

Introduction

Sampling-Quantization

Simple image operations

Image manipulations

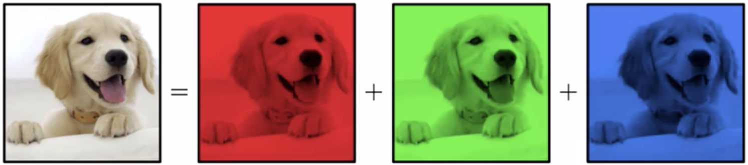
Overview

- Images as multidimensional digital signals
- 2D signal representations

- two dimensional signals $x[n_1, n_2]$ $n_1, n_2 \ \in \mathbb{Z}$
- indices indicate a position on a grid (pixels)
- the $x[n_1, n_2]$ values refer to pixel's apperance

Grayscale vs Color

- grayscale images: scalar pixel values
- color images: multidimensional pixel values in a color space (RGB, HSV, YUV, etc)
- we can consider the single components separately:



From one to two dimensions

- something still work
- something breaks down
- something is new

What works:

- linearity, convolution
- Fourier transform
- interpolation sampling

What breaks down (less well):

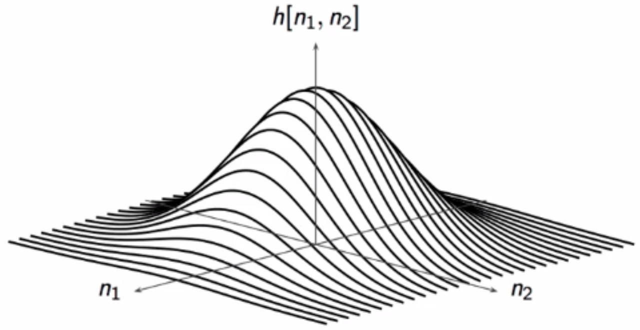
- Fourier analysis less relevant
- filter design hard, IIRs rare
- linear operators only mildly useful
- Images are very diverse signals, they have different texture and objects

What is new:

- new manipulations: affine transforms
  - rotation
  - scaling
  - skewing
- images are finite-support signals
- images are very specialized signals, designed for a very specific processing system, i.e. The human brain!!. Lots of semantic is extremely hard to deal with

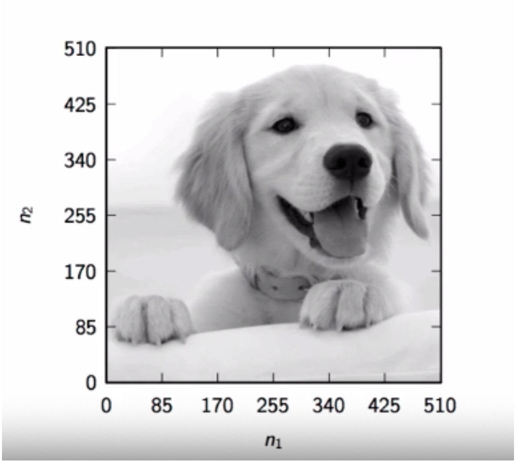
What is new

- a two dimensional discrete-space signal:
  $x[n_1, n_2], \quad n_1, n_2 \in \mathbb{Z}$



2D Signals: image representation

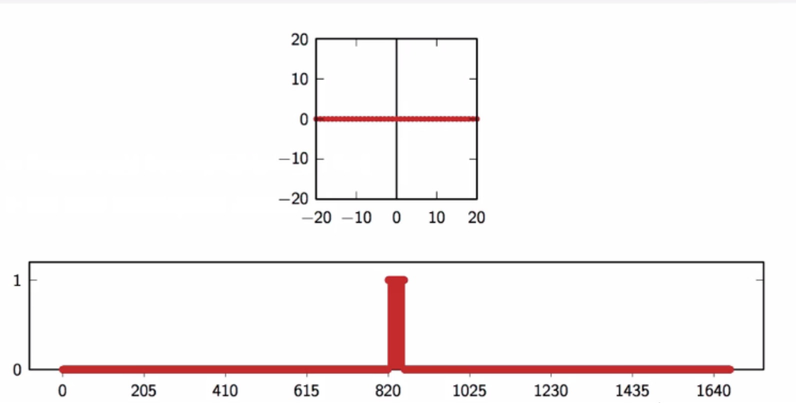
- medium has a certain dynamic range (paper, screen)
- images values are quantized (usually to 8 bits, or 256 levels)
- the eyes does the interpolation in space provided the pixel density is high enough



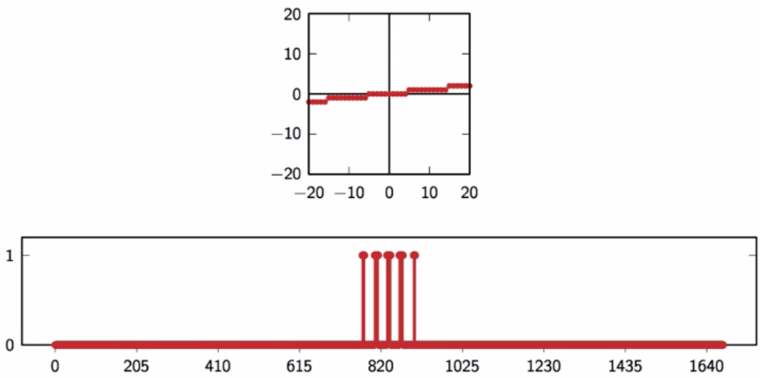
Why 2D??

- images could be unrolled (printers, fax)
  but what about spatial correlation??

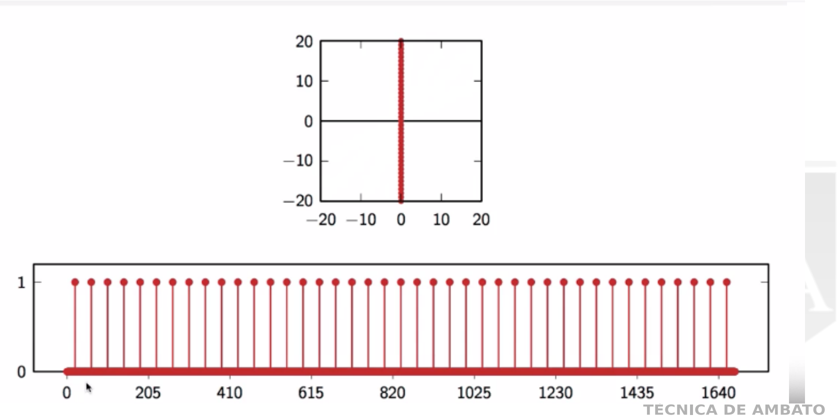
2D vs raster scan



2D vs raster scan



2D vs raster scan



Outline

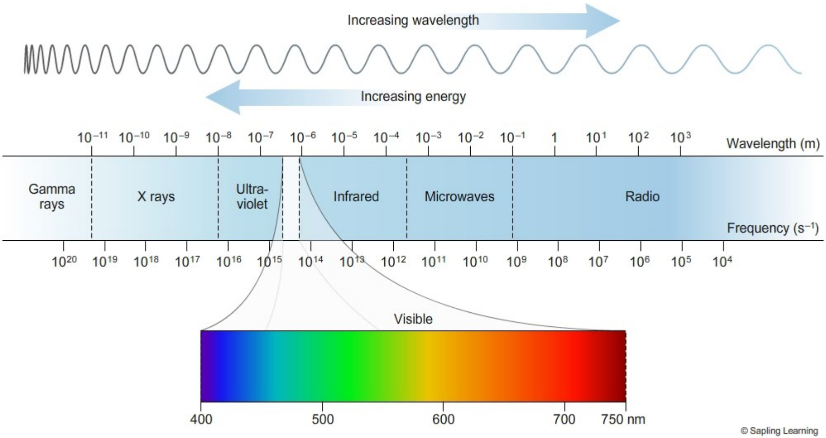
Introduction

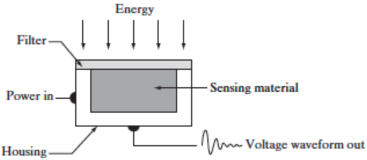
Sampling-Quantization

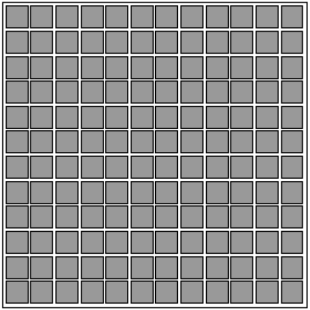
Simple image operations

Image manipulations

- to create a digital image, we need to convert the continuos sensed data into digital form. This involves two processes:
  - Sampling - digitizing the coordinate values
  - Quantization - Digitizing the amplitudes values


Increasing wavelength
Increasing energy
10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10^{1} 10^{2} 10^{3}
Wavelength (m)
Gamma rays
X rays
Ultra-violet
Infrared
Microwaves
Radio
Frequency (s^{-1})
10^{20} 10^{19} 10^{18} 10^{17} 10^{16} 10^{15} 10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^{9} 10^{8} 10^{7} 10^{6} 10^{5} 10^{4}
Visible
400
500
600
700
750 nm
© Sapling Learning


Energy
Filter
Power in
Sensing material
Housing
Voltage waveform out



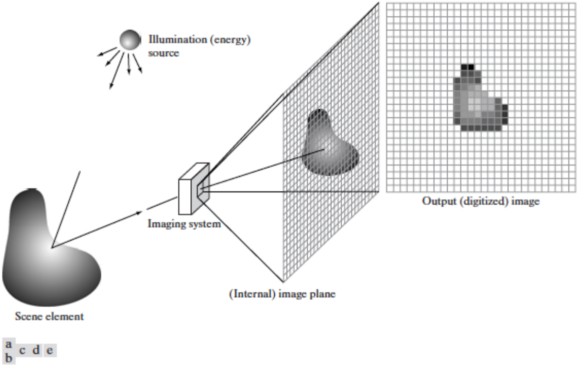
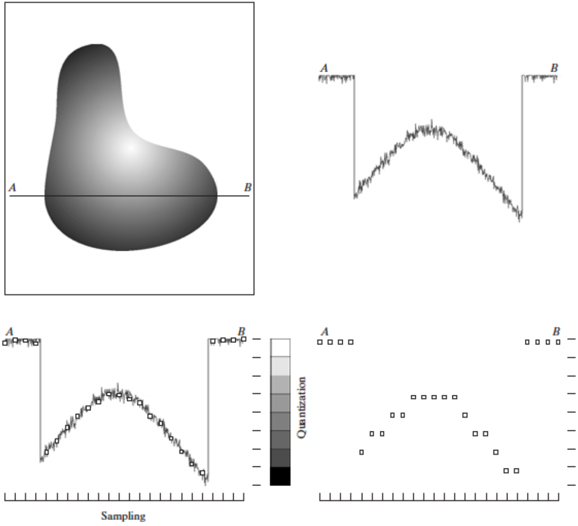
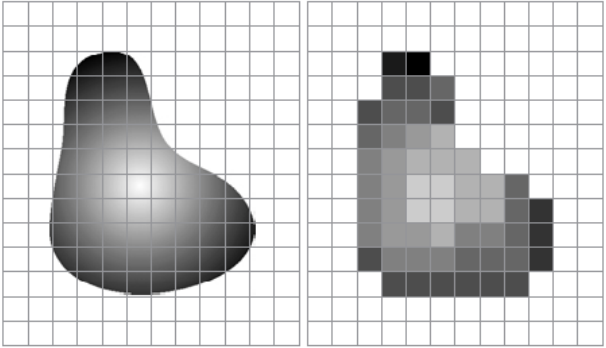


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.


A
B
A
B
A
B
Quantization
A
B
Sampling



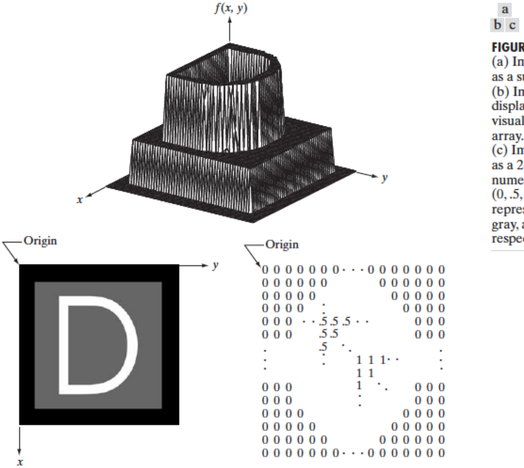
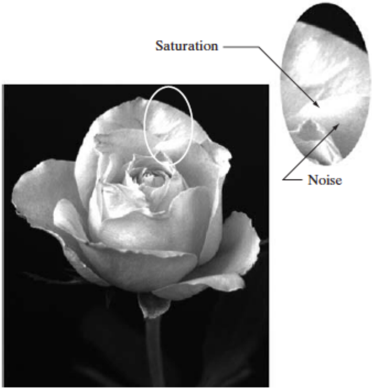
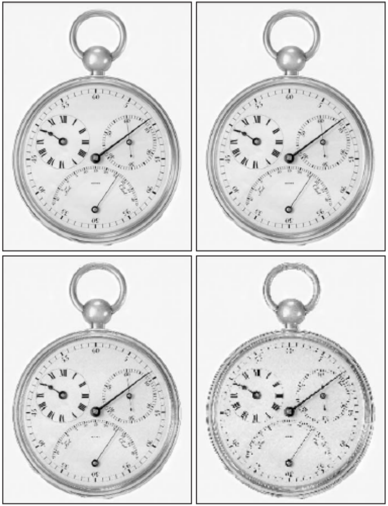


FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

FIGURE 2.19 An image exhibiting saturation and noise. Saturation is the highest value beyond which all intensity levels are clipped (note how the entire saturated area has a high, *constant* intensity level). Noise in this case appears as a grainy texture pattern. Noise, especially in the darker regions of an image (e.g., the stem of the rose) masks the lowest detectable true intensity level.



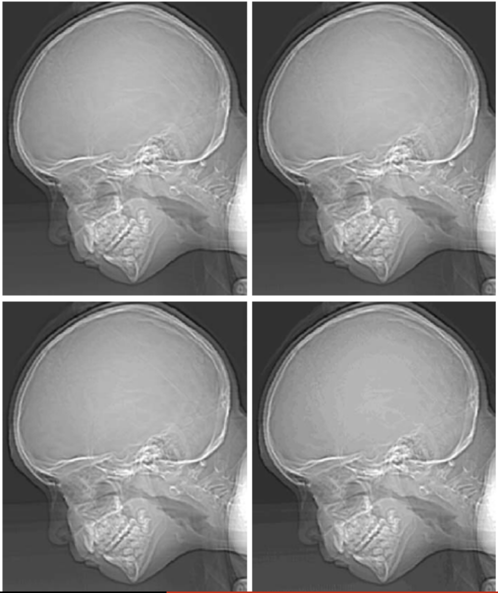


a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

a b
c d

FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 intensity levels, while keeping the image size constant.



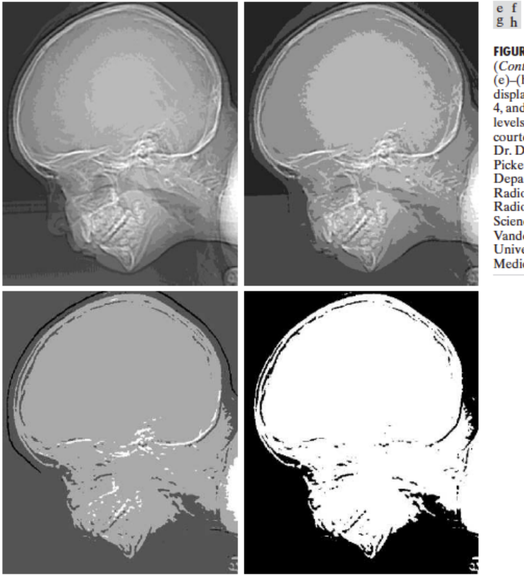


FIGURE 2.21 (*Continued*) (e)–(h) Image displayed in 16, 8, 4, and 2 intensity levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

Spatial and Intensity Resolution

REMARKS

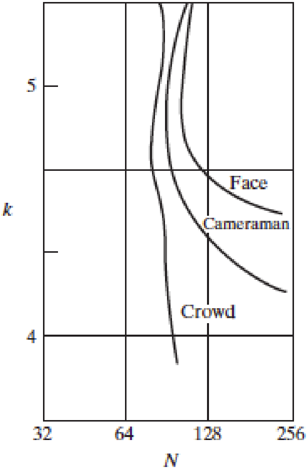
- The digitization process requires that decisions be made regarding the values for M, N, and for the number, L, of discrete intensity levels
- the number of intensity levels typically is an integer power of 2: $L = 2^k$ in the interval $[0, L-1]$
- the number of bits required to store a digitized images is: $b = M \times N \times k$

Isopreference curves

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

Isopreference curves



Outline

Introduction

Sampling-Quantization

Simple image operations

Image manipulations

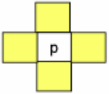
Neighborhood

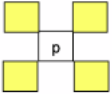
- 4-neighbors of p $N_4(p)$

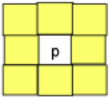
  $(x+1, y), (x-1, y), (x, y+1), (x, y-1)$
- diagonal-neighbors of p $N_D(p)$

  $(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$
- 8-neighbors of p $N_8(p)$

  $(x+1, y), (x-1, y), (x, y+1), (x, y-1), (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$







Adjacency

- let V be the set of intensity values used to define adjacency, in a binary image $V = 1$
- in the adjacency of pixels with a range of possible intensity values 0 to 255, set V could be any subset of these 256 values

we consider 3 types of adjacency:

- 4-adjacency.Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$
- 8-adjacency.Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$
- m-adjacency (mixed adjacency).Two pixels p and q with values from V are m-adjacent if:
  - q is in $N_4(p)$, or
  - q is in $N_D(p)$ and the set $N_4(p)N_D(p)$ has no pixels whose values are from V

Digital path or curve

- A (digital) path (or curve) from pixel p with coordinates (x, y) to pixel q with coordiantes (s, t) is a sequence of distinct pixels with coordinates:

  $(x_0, y_0), (x_1, y_1), ..., (x_n, y_n)$ where $(x_0, y_0) = (x, y)$ and $(x_n, y_n) = (s, t)$
- pixels (x_i, y_i) and (x_{i-}, y_{i-1}) are adjacent for $1 \leq i \leq n$
- We can define 4-, 8-, or m-paths depending on the type of adjacency specified.

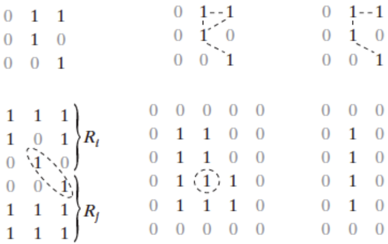
Connectivity

- Let S represent a subset of pixels in an image.Two pixels p and q are said to be connected in S if there exists a path between them consisting entirely of pixels in S
- For any pixel p in S, the set of pixels that are connected to it in S is called a connected component of S

Region

- Let R be a subset of pixels in an image.We call R a region of the image if R is a connected set.
- Two regions R_i and R_j are said to be adjacent if their union forms a connected set.
- Regions that are not adjacent are said to be disjoint.We consider 4- and 8-adjacency when referring to regions.

Region



a	b	c
d	e	f

FIGURE 2.25 (a) An arrangement of pixels. (b) Pixels that are 8-adjacent (adjacency is shown by dashed lines; note the ambiguity). (c) m-adjacency. (d) Two regions (of 1s) that are adjacent if 8-adjecency is used. (e) The circled point is part of the boundary of the 1-valued pixels only if 8-adjacency between the region and background is used. (f) The inner boundary of the 1-valued region does not form a closed path, but its outer boundary does.

Adding Images

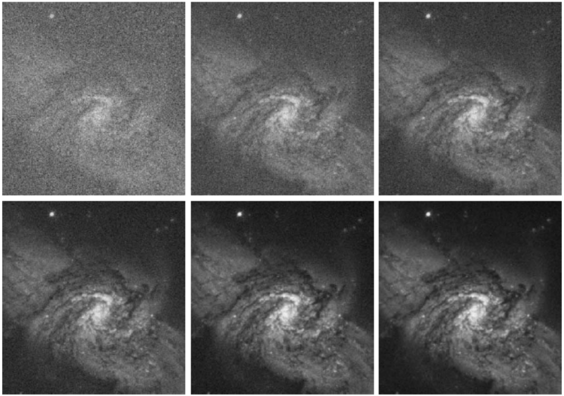
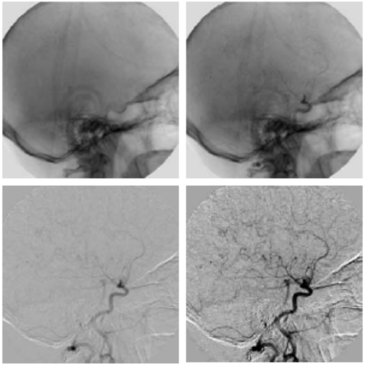


FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

Substracting Images

a b
c d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)



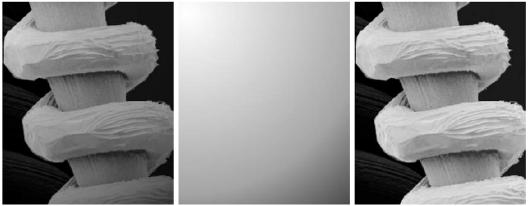
Substracting Images

- $g(x, y) = f(x, y) - h(x, y)$
- In the area of medical imaging, the mask mode radiography make use of image substraction
- The procedure consists of injecting an X-ray contrast medium into the patient's bloodstream, taking a series of images called live images [samples of which are denoted as $f(x, y)$] of the same anatomical region as $h(x, y)$, and subtracting the mask from the series of incoming live images after injection of the contrast medium.

Multiplication and division

Shading correction

- The shading might be caused by non-uniform illumination, non-uniform camera sensitivity, or even dirt and dust on glass (lens) surfaces.
- images can be modeled as the product of a “perfect image” times a shading function $g(x, y) = f(x, y)h(x, y)$



a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Multiplication and division

ROI detection

- consists simply of multiplying a given image by a mask image that has 1s in the ROI and 0s elsewhere.

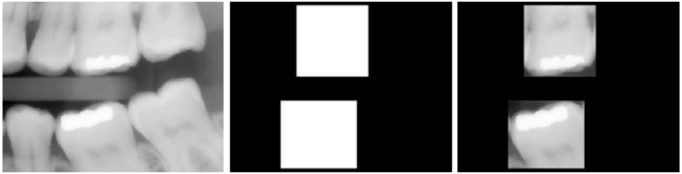


FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Logical operation

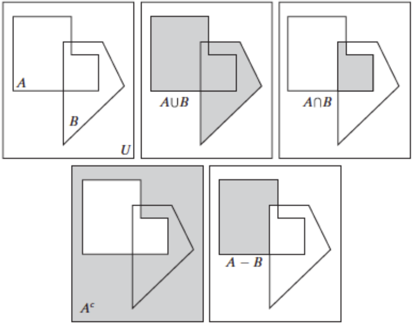
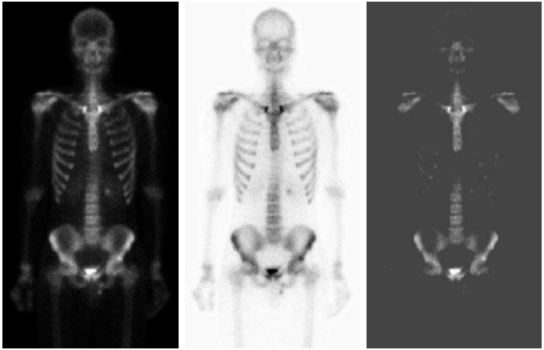


FIGURE 2.31 (a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the members of the set operation indicated.

Logical operation

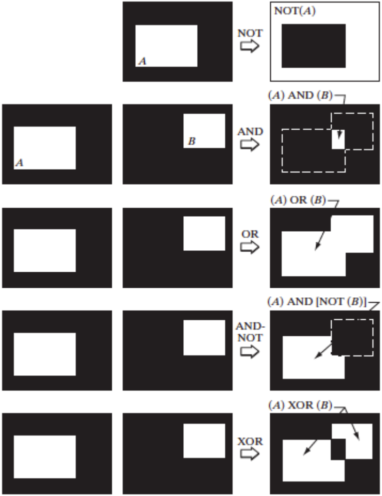


a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

Logical operation

FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.



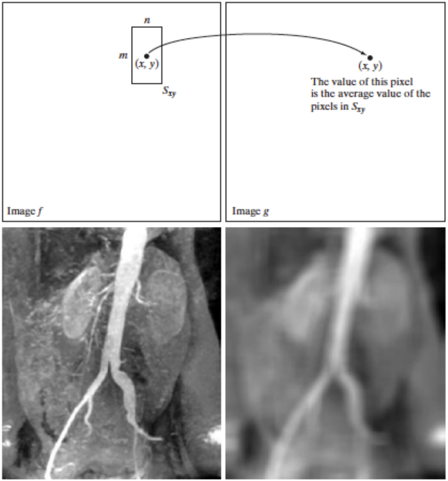
Single-pixel operations

- alter the values of its individual pixels based on their intensity
- $S = T(z)$ z is the intensity of a pixel in the original image and s is the (mapped) intensity of the corresponding pixel in the processed image

Neighbourhood Operation

a b
c d

FIGURE 2.35 Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.



Outline

Introduction

Sampling-Quantization

Simple image operations

Image manipulations

Affine Transforms

mapping $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ that reshapes the coordinate system:

$$\begin{bmatrix} t_1' \\ t_2' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} - \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

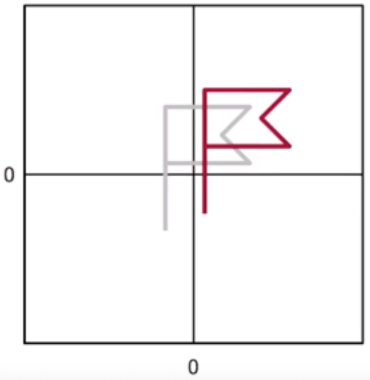
$$\begin{bmatrix} t_1' \\ t_2' \end{bmatrix} = \mathbf{A} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} - \mathbf{d}$$

Translations

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$\begin{bmatrix} t_1' \\ t_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} - \mathbf{d}$$

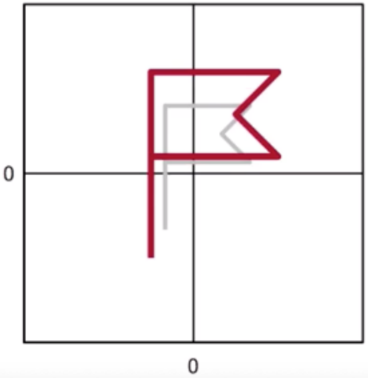


Scaling

$$\mathbf{A} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} = \mathbf{I}$$

$$\mathbf{d} = 0$$

- if $a_1 = a_2$ the aspect ratio is preserved

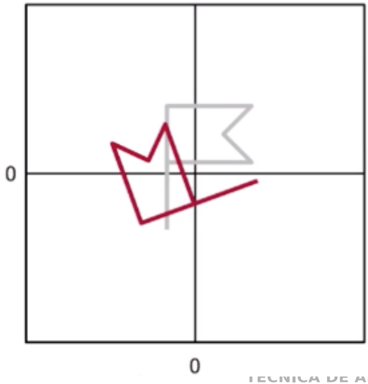


Rotation

$$\mathbf{A} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \mathbf{I}$$

$$\mathbf{d} = 0$$

- Example: $\theta > \frac{\pi}{2}$

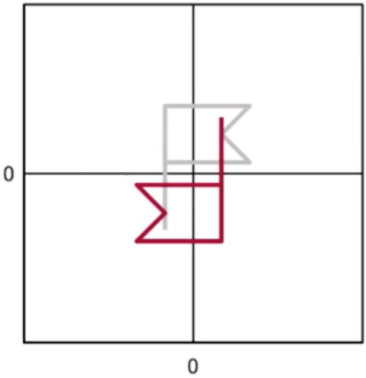


Rotation

$$\mathbf{A} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \mathbf{I}$$

$$\mathbf{d} = 0$$

- Example: $\theta = \pi$



Flips

Horizontal:

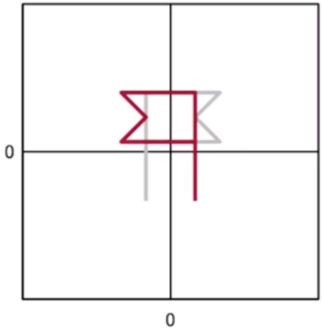
$$\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{d} = 0$$

Vertical:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathbf{d} = 0$$



Shear

Horizontal:

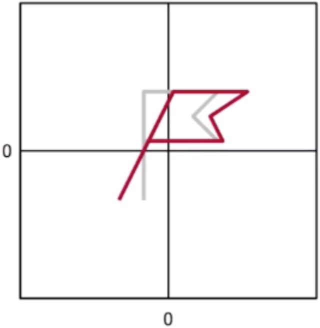
$$\mathbf{A} = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{d} = 0$$

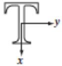
Vertical:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

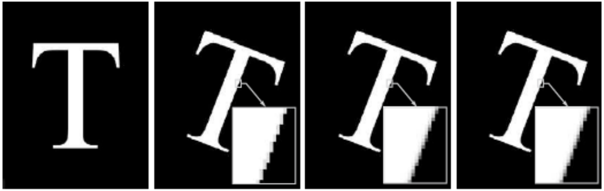
$$\mathbf{d} = 0$$



Affine Transformation -Summary

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

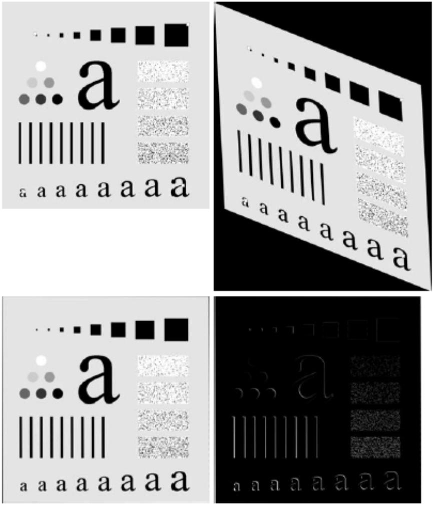
Affine Transformation -Examples



a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Affine Transformation -Examples



a b
c d

FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the border). (d) Difference between (a) and (c), showing more registration errors.

Affine transforms in discrete-space

- apply an affine transform to a digital image, we run into a fundamental problem.
- Remember, a digital image lives in discreet space.
- pixel coordinates are pairs of integers, they live in $\mathbb{Z}^2$

$$\begin{bmatrix} t_1' \\ t_2' \end{bmatrix} = \mathbf{A} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} - \mathbf{d} \in \mathbb{R}^2 \neq \mathbb{Z}^2$$

- The affine transform on the other hand, maps this original coordinates onto a point in $\mathbb{R}^2$, which means that the new pair of coordinates can lie anywhere on the plane and, in particular, it will most likely lie in between individual pixel coordinates.

Solution for images

- apply the inverse transform:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} m_1 + d_1 \\ m_2 + d_2 \end{bmatrix}$$

- interpolate from the original grid point to the "mid-point" $(t_1, t_2) = (\eta_1 + \tau_1, \eta_2 + \tau_2)$, $\eta_{1,2} \in \mathbb{Z}$, $0 \leq \tau_{1,2} < 1$

Interpolation

- basic tool used extensively in tasks such as zooming, shrinking, rotating, and geometric corrections
- interpolation is the process of using known data to estimate values at unknown locations

Bilinear Interpolation