

Programming With Two-Level Type Theory

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What is this about

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- High-level: FP abstractions, generic programming, strong types.
- High-performance: control over code generation, memory layout, allocation.

Non-goal: “systems” programming.

- We'll have substantial RTS with GC & full memory safety.

Past implementations: smaller demo [Kov22], Agda & Typed TH embedding [Kov24]

Currently in early stage of development: <https://github.com/AndrasKovacs/2ltt-impl>

Motivation

I'm interested in high-performance implementations of dependent type systems.

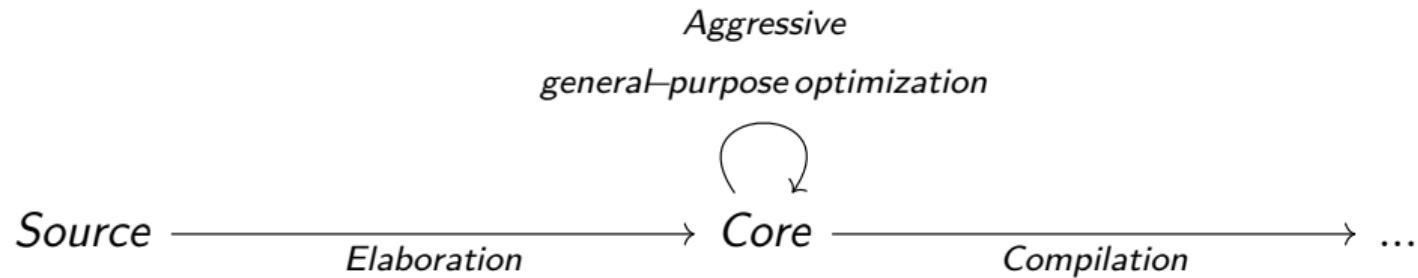
GHC Haskell has been my choice for performance:

- GC focused on throughput, OK machine code output, unboxed types, compact regions, efficient laziness (for the few cases when it's needed).

There are some performance issues or significant inconveniences in every other language I know.

Still: *performance issues with GHC too* - motivating this research.

The GHC pipeline



The optimizer is

- Complex.
- Unstable across GHC versions.
- Supports limited user control.

A lot of idiomatic Haskell relies on it for acceptable performance.

GHC example 1

Source:

```
f :: Reader Bool Int
f = do
  b <- ask
  if b then return 10
        else return 20
```

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-00 Core output:

```
dict1 :: Monad (Reader Int)
dict1 = MkMonad ...
dict2 :: MonadReader (Reader Int)
dict2 = MkMonadReader ...
```

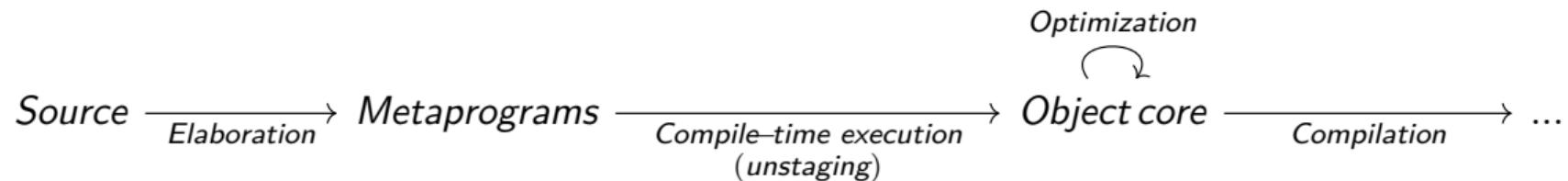
```
f :: Reader Bool Int
f = (>>=) dict1 (ask dict2) (\b ->
  case b of True  -> return dict1 10
            False -> return dict1 20)
```

GHC example 2

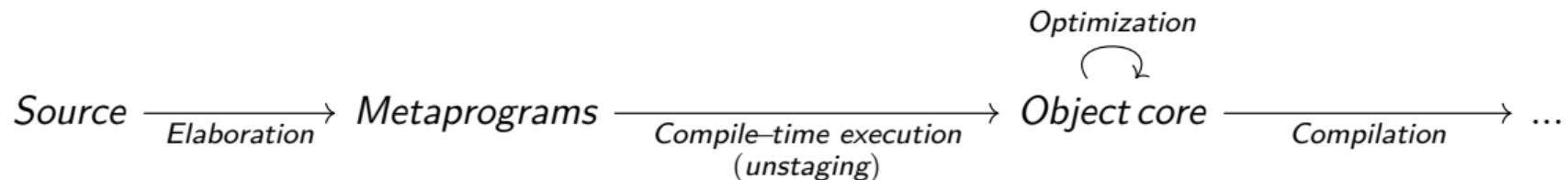
`mapM` is third-order & rank-2 polymorphic, but almost all use cases should compile to first-order monomorphic code.

```
mapM :: Monad m => (a -> m b) -> m [a] -> m [b]
```

Revised pipeline



Revised pipeline

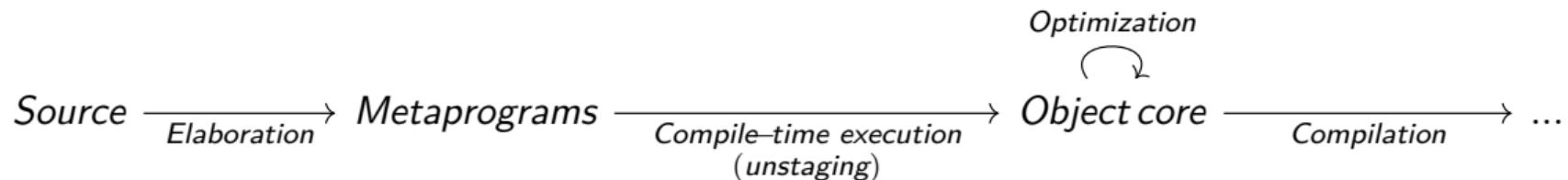


The *metalinguage* and the *object language* should be different.

- Simple object language supports better compilation & performance.
- Dependent type theory as metalinguage.

Main design question: explicit control in object language vs. optimizations in the compiler.

Revised pipeline



The *metalinguage* and the *object language* should be different.

- Simple object language supports better compilation & performance.
- Dependent type theory as metalinguage.

Main design question: explicit control in object language vs. optimizations in the compiler.

- The object language **can** be tedious as long as we can address tedium with metaprogramming!

Setup

Universes for stages:

- **Set** : **Set** contains **dependent** meta-level types.
- **Ty** : **Set** contains **simple** object-level types.
- **ValTy** : **Set** and **CompTy** : **Set** are subtypes of **Ty** (polarization!).

Setup

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Interaction between stages:

- **Lifting**: for $A : \text{Ty}$, we have $\uparrow A : \text{Set}$, as the type of metaprograms that produce A -typed object programs.
- **Quoting**: for $t : A$ and $A : \text{Ty}$, we have $\langle t \rangle : \uparrow A$ as the metaprogram which immediately returns t .
- **Splicing**: for $t : \uparrow A$, we have $\sim t : A$ which runs the metaprogram t and inserts its output in some object-level code.
- Definitional equalities: $\sim \langle t \rangle \equiv t$ and $\sim \sim t \equiv t$.

The object level

An object-level program:

```
data List (A : ValTy) := nil | cons@Hp A (List A)
```

```
f : List Int → List Int
```

```
f xs := case xs of
```

```
  nil       → nil
```

```
  cons x xs → cons (x + 10) (f xs)
```

Polarization:

- Functions have value arguments and are computations.
- ADTs have value fields and are values.

The object level

Explicit type former for closures:

```
Close : CompTy → ValTy
close : A → Close A
open  : Close A → A
```

Mapping with closures:

```
map : Close (Int → Int) → List Int → List Int
map f xs = case xs of
  nil      → nil
  cons x xs → cons (open f x) (map f xs)
```

Closures are surprisingly rarely needed in practical programming!

Staging

```
inlineInt : ↑Int  
inlineInt = <100>
```

```
myInt : Int  
myInt := 200
```

```
id : {A : Ty} → ↑A → ↑A  
id x = x
```

```
f : Int → Int  
f x := x + ~inlineInt
```

```
g : Int → Int  
g x := ~(id <x>)
```

unstage

==>

```
myInt : Int  
myInt := 200
```

```
f : Int → Int  
f x := x + 100
```

```
g : Int → Int  
g x := x
```

Staging

Fully explicit `map`:

```
map : {A B : ValTy} → ( $\uparrow$ A →  $\uparrow$ B) ->  $\uparrow$ (List A) →  $\uparrow$ (List B)
map {A}{B} f as = <
  let go : List ~A → List ~B
    go as := case as of
      nil       → nil {~B}
      cons a as → cons {~B} ~( $f$  <a>) (go as)
  go ~as>
```

```
monoMap : List Int -> List Int
monoMap xs := ~(map ( $\lambda$  x. < $\sim$ x + 10>) <xs>)
```

Staging

Unstaged output:

```
monoMap : List Int → List Int
monoMap xs :=
  let go : List Int → List Int
    go as := case as of
      nil      → nil {Int}
      cons a as → cons {Int} (a + 10) (go as)
  go xs
```

Inference & elaboration

Quotes and splices are almost always inferable!

```
map : {A B : ValTy} → (A → B) → List A → List B
map f as =
  let go as := case as of
    nil      → nil
    cons a as → cons (f a) (go as)
  go as
```

```
monoMap : List Int → List Int
monoMap := map (λ x. x + 10)
```

How to compile: monads

Not easy! We want

- guaranteed closure-freedom for everything except CPS monads
- guaranteed fusion for straight-line code (e.g. no intermediate constructor allocations in **Maybe**)
- proper handling of join points and tail calls
- modest extra noise compared to Haskell

How to compile: monads

Not easy! We want

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- proper handling of join points and tail calls
- modest extra noise compared to Haskell

Ingredients of the solution:

- The bulk of the logic is in a plain library.
- We use type classes & *implicit coercions*.
- Extra desugaring logic in **do**-blocks.
- Tail calls are handled by the downstream optimizer.

Monads: the bulk of the logic

Monads only exist at compile time.

```
class Monad (M : Set → Set) where
  pure  : {A : Set} → A → M A
  (≫=) : {A B : Set} → M A → (A → M B) → M B
```

Recipe for porting over a transformer stack from Haskell:

- ① We have an object-level type, same as in Haskell (but with polarities).
- ② We have a meta-level transformer stack, which has an extra monad at the bottom, having *code generation as an effect*.
- ③ We define back-and-forth conversion between the object-level type and the metamonad.

The Gen monad

```
record Gen (A : Set) : Set = gen {unGen : {R : Ty} → (A → ↑R) → ↑R}

instance Monad Gen where ...

runGen : Gen ↑A → ↑A
runGen (gen f) = f id

class Monad M => MonadGen M where
  liftGen : Gen A → M A

genLet : MonadGen M => ↑A → M ↑A
genLet a = liftGen λ k. <let x := ~a; ~(k <x>)>
```

The Gen monad

```
f : Int  
f := ~ (runGen do  
  x ← genLet <10 + 20>  
  y ← genLet <~x * 10>  
  pure <~x + ~y>)
```

unstage

==>

```
f : Int  
f :=  
  let x := 10 + 20  
  let y := x * 10  
  x + y
```

Case splitting in MonadGen

```
data BoolM : Set = trueM | falseM
data Bool : ValTy := true | false

down : BoolM → ↑Bool
down x = case x of trueM → <true>; falseM → <false>

up : ↑Bool → BoolM
up = ?                                -- impossible!
```

Case splitting in MonadGen

```
data BoolM : Set = trueM | falseM
data Bool : ValTy := true | false

down : BoolM → ↑Bool
down x = case x of trueM → <true>; falseM → <false>

up : ↑Bool → BoolM
up = ?                                -- impossible!
```

However:

```
up : MonadGen M => ↑Bool → M BoolM
up b = liftGen λ k. <case ~b of true → ~(k trueM); false → ~(k falseM)>
```

Case splitting in MonadGen

We add **extra desugaring** in **MonadGen** do-blocks for case splitting.

$f : \text{Bool} \rightarrow \text{Bool}$		$f : \text{Bool} \rightarrow \text{Bool}$
$f\ b := \text{runGen}\ \text{do}$	elaborate	$f\ b := \sim(\text{runGen}\ \text{do}$
$\text{case}\ b\ \text{of}$	\implies	$b \leftarrow \text{up}\ $
$\text{true} \rightarrow \text{pure}\ \text{false}$		$\text{case}\ b\ \text{of}$
$\text{false} \rightarrow \text{pure}\ \text{true}$		$\text{trueM} \rightarrow \text{pure}\ <\text{false}>$
		$\text{falseM} \rightarrow \text{pure}\ <\text{true}>)$

Every object-level case split can be handled analogously!

Monads in general

Example: conversion between object-level type and a meta-monad.

$$\begin{array}{c} \uparrow(\mathbf{StateT\ Int\ (ReaderT\ Bool\ Identity)\ A}) \\ \quad \uparrow \downarrow \\ \mathbf{StateTM\ (\uparrow Int)\ (ReaderTM\ (\uparrow Bool)\ Gen)\ (\uparrow A)} \end{array}$$

Generally: this conversion can be defined by recursion on the transformer stack (using e.g. typeclasses) [Kov24].

Monads in general

Code example:

M : Ty

M = StateT Int (ReaderT Bool Identity)

f : M ()

f := do

 b <- ask

 n <- get

 case b of

 true → put \$ n + 10

 false → put \$ n * 10

elab +

unstage

==>

f : M ()

f := stateT λ s. readerT λ r.

case r of

 true → let s' := s + 10

 identity ((), s')

 false → let s' := s * 10

 identity ((), s')

Monads in general

- A modest amount of extra noise compared to Haskell (but no native implementation yet!)
- All of `mtl` works. Closures are only needed in `ContT`.
- Note: `Reader` and `State` are computation types, so we need to wrap them in `Close` to store them in data structures.

Memory layout control

All constructors are unboxed by default.

```
data Pair (A B : ValTy) := pair A B
data Sum (A B : ValTy) := left A | right B
```

Recursive constructors must be guarded by a *pointer to a region*. **Hp** is the general GC-d heap.

```
data List A := nil | cons@Hp A (List A)
```

Weird sum type with just one unboxed constructor:

```
data Sum A B := left A | right@Hp B
```

Tag-free GC & bit-stealing

GC is *almost tag-free*: only 1 bit metadata per heap object.

Bit-stealing: any data can be stored in unused bits in pointers or constructor fields.

Large space savings compared to pretty much any managed RTS language.

Tag-free GC & bit-stealing

Example: pure lambda terms with 32-bit variables.

```
data Tm := var UInt32 | app@Hp Tm Tm | lam@Hp Tm
```

Layout of `app (var 0) (var 1)`

```
| app | ptr |           -- 1 word  
    ↓  
| var | 0 | var | 1 |   -- 2 words
```

In GHC:

```
| app | ptr |           -- 1 word  
    ↓  
| app     | ptr       | ptr |       -- 3 words  
    ↓           ↓  
| var | 0 |     | var | 1 |   -- 4 words
```

Tag-free GC & bit-stealing

Implementation: explored back in the 90s [Tol94].

In a simple type theory, it's enough to know the types (memory layouts) of GC roots.

For each monotype, we generate code for GC scanning & copying.

Only *stack frames* need to store runtime type information about roots.

Regions

```
Location : Set
Hp       : Location
Region   : Set
```

There's implicit coercion from **Region** to **Location**. The object language supports dependent functions of the form **(R : Region) →**

Lists with cons cells in a specified location:

```
data List (L : Location) (A : ValTy) := nil | cons@L A (List L A)
```

Regions

Example: list in a local region.

```
sum : {R : Region} → List R Int → Int
sum xs := case xs of nil → 0; cons x xs → x + sum xs

countDown : {R : Region} → Int → List R Int
countDown x := case x of 0 → nil
                  n → cons x (countDown (x - 1))

f : Int → Int
f x :=
  let R : Region
  let xs : List R Int := countDown x
  sum xs
```

Type-directed GC scanning

Example 1: if a list contains no heap pointers, GC doesn't touch it!

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The region `R` itself is present at runtime, and it serves as a GC root for the whole region.

Type-directed GC scanning

Example 1: if a list contains no heap pointers, GC doesn't touch it!

Why? Assume we have `xs : List R Int` in scope where `R : Region`.

The region `R` itself is present at runtime, and it serves as a GC root for the whole region.

Since the whole region is kept alive by the `R` reference, there's no need to scan the list.

Type-directed GC scanning

Example 2: let's have `data HpPtr A := box@Hp A` and consider `List R (HpPtr Int)`.

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Example 3: `List R (Close (Int → Int))`. Closures can capture arbitrary data, including heap pointers, so we also need to scan them!

Example 4: `List Hp Int`. The cons cells are on the general heap, so we scan and possibly copy everything.

Existential regions

```
data InSomeRegion (L : Location) (F : Region → ValTy) :=  
  inSomeRegion@L (R : Region) (F R)
```

We can also *hash regions* and compare for them equality.

So we can store regions in data structures and manage them at runtime.

The more we know about lifetimes, the more we can speed up GC by moving data into regions.

If we don't care, we can ignore regions with *no burden* on programmers.

1. GHC compact regions [YCA⁺¹⁵]

- We can create compact regions.
- There's a primop to *deeply copy* GHC objects into the region.
- Hence: a compact region only contains internal pointers.
- GC doesn't scan regions. A region is alive as long as any object in it is alive.
- Very important in Agda!

Related

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2. Rust lifetimes

- Deterministic & precise tracking of where objects get destroyed.
- Sub-structural typing.

The problem with sub-structural typing

We don't know how to make it work nicely with staging.

Consider an application rule for linear functions:

$$\frac{\Gamma \vdash t : A \multimap B \quad \Delta \vdash u : A}{\Gamma, \Delta \vdash t \ u : B}$$

The semantic (operational) meaning of $\uparrow A \rightarrow \uparrow B$ in 2LTT:

- We map object term to object terms in *arbitrary* object contexts.
- The mapping *commutes with substitutions*.

We can't inhabit $\uparrow(A \multimap B) \rightarrow \uparrow A \rightarrow \uparrow B$. No idea if the free variables are disjoint.

Structuralizing things

1. Massage sub-structural features into structural shape

- Erasure control by Constantine Theocharis (unpublished).
- Closure capture control by AK (unpublished).

2. Other structural features

- ST monad, monadic regions.
- Disentanglement typing for thread-local GC [MBXW26].

Further topics

- Staged fusion.
- IR optimizations.
- Backend compilation, LLVM.

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- IR optimizations.
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Thank you!

András Kovács.

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