Canonicity for Indexed Inductive-Recursive Types

ANONYMOUS AUTHOR(S)

We prove canonicity for a Martin-Löf type theory that supports a countable universe hierarchy where each universe supports indexed inductive-recursive (IIR) types. We proceed in two steps. First, we construct IIR types from inductive-recursive (IR) types and intensional identity types, in order to simplify the subsequent canonicity proof. The constructed IIR types support the same definitional computation rules that are available in Agda's native IIR implementation. Second, we give a canonicity proof for IR types, building on the well-known method of Artin gluing. The main idea is to represent the canonicity predicate for each IR type using a metatheoretic IIR type. In short, we use metatheoretic IIR types to prove canonicity for IR types.

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1 Introduction

Induction-recursion (IR) was first used by Martin-Löf in an informal way [**?**], then made formal by Dybjer and Setzer [**?**], who also developed set-theoretic and categorical semantics [**?**]. A common application of IR is to define custom universe hierarchies inside a type theory. In the proof assistant Agda, we can use IR to define a universe closed under a choice of type formers.

TODO

This can be used in building metatheory for type theories with universe hierarchies. For example, IR has been used to prove normalization [**?**] and to build semantics for first-class universe levels [**?**] and to prove canonicity for them [**?**]. Another application is in generic programming, where IR is used to define custom universes of datatype descriptions [**?**].

References

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