The C++0x "Concepts" Effort

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Abstract

C++0x is the working title for the revision of the ISO standard of the C++ programming language that was originally planned for release in 2009 but that was delayed to 2011. The largest language extension in C++0x was "concepts", that is, a collection of features for constraining template parameters. In September of 2008, the C++ standards committee voted the concepts extension into C++0x, but then in July of 2009, the committee voted the concepts extension back out of C++0x.

This article is my account of the technical challenges and debates within the "concepts" effort in the years 2003 to 2009. To provide some background, the article also describes the design space for constrained parametric polymorphism, or what is colloquially know as constrained generics. While this article is meant to be generally accessible, the writing is aimed toward readers with background in functional programming and programming language theory. This article grew out of a lecture at the Spring School on Generic and Indexed Programming at the University of Oxford, March 2010.

1 Introduction

The inclusion of the Standard Template Library (STL) [87] into C++ in 1994 transformed the development of software libraries in C++. Before 1994, C++ library developers primarily applied object-oriented techniques to facilitate software reuse. Beginning in 1994, library developers began to use the methodology of *generic programming*, which was created by Alexander Stepanov, David Musser, and colleagues in the 1980s [45, 48, 63, 64]. Generic programming is a methodology for decoupling algorithms from the data structures on which they operate, thereby enabling the use of a single algorithm implementation with many different data structures. Section 1.2 of this article gives a brief overview of generic programming and Section 2 goes into depth regarding how generic programming is ac-

complished in C++. Many software methodologies claim to be "silver bullets". Generic programming is not a silver bullet per se; developing generic libraries is a difficult task and requires considerable training and mathematical expertise. However, generic programming can provide significant cost savings in the long run.

Generic programming, as realized in C++, relies heavily on the template language feature to provide type parameterization of functions and classes. As programmers applied generic programming to larger and more complex libraries in the late 1990s and early 2000s, both the developers and users of these libraries began to suffer from several shortcomings of C++ templates.

- Minor errors in the use of template libraries produced extraordinarily long and complex error messages.
- Library developers could not easily check that their library code adhered to its specification.
- The run-time behavior of libraries could differ in unexpected ways based on usage context because of non-modular rules for name resolution.
- Applications using C++ template libraries suffered long compilation times.

In 2003, two groups, one at Indiana University and the other at Texas A&M University, set out to solve these problems through the addition of language features for constraining template parameters. The work by these two groups ultimately led to a language extension that was voted into the working draft of the C++ standard on September 2008. However, in July of 2009, the standards committee voted to remove the language extension.

The aim of this article is to describe the technical challenges and debates within the "concepts" effort from 2003 to 2009 and to provide the background for understanding the design trade offs. The remainder of this introduction starts with an introduction to generic programming methodology and then gives an overview of the history of language support for generic programming.

1.1 Higher-Order Polymorphic Programming

Generic programming builds on the style of higher-order, polymorphic programming that was developed in LISP [57] in the 1960s and that was crystallized in type systems of the 1970s such as System F of Girard [30] and Reynolds [72] and the Hindley-Milner [61] type system of Standard ML [62]. A canonical example of higher-order, polymorphic programming is the fold! function, shown below and

written in ML. The first argument to foldl is a binary function, the second argument can be thought of as a running total, and the third argument is a list. The foldl function applies the binary function to each element of the list and the running total. The two equations below that define foldl match on the third argument, with nil matching the empty list and x::xs matching a non-empty list, binding x to the element at the front and binding xs to the rest of the list.

```
fun fold f y nil = y

| fold f y (x::xs) = fold f (f (x,y)) xs
```

This fold function is quite flexible; it can compute sums, products, and many other things. In the first line below, we use addition as the binary function, 0.0 as the initial value for the running total, and [1.0,2.0,3.0,4.0] as the list. The result of fold in this case is 10.0, which is the sum of the integers in the list.

```
> foldl op + 0.0 [1.0,2.0,3.0,4.0];
val it = 10.0 : real
> foldl op * 1 [1,2,3,4];
val it = 24 : int
> foldl op @ [] [[1,2],[3],[4,5,6]];
val it = [4,5,6,3,1,2] : int list
```

The foldl function is *polymorphic* with respect to the element type of the list. In the first example, the elements are real numbers. In the second, they are integers, and in the third, they are lists of integers. The foldl function is *higher-order* because it takes a function as a parameter, which controls whether foldl computes the sum, product, or concatenation in the above examples.

To be specific, the kind of polymorphism provided in Standard ML and in System F is called *parametric polymorphism*. This kind of polymorphism is particularly important because it enables both modular type checking and separate compilation. A language provides *modular type checking* when 1) a call to a function, or similarly, an instantiation of a generic, can be type checked using only its type and not its implementation and 2) the definition of function or generic can be type checked in isolation, without using any information about call sites or points of instantiation. Modular type checking is critical for the development of libraries and large software systems. Modular type checking shields the client of a library from the internals of the library. Dually, modular type checking provides library developers with an automated means to rule out large classes of bugs before distributing the library, especially bugs regarding inconsistencies between the

library's specification and implementation.

A language enables *separate compilation* if it is possible to produce an executable machine program in time proportional to the size of the source code of the main program only, even though the program uses many software libraries. This definition is somewhat indirect because the more straightforward and traditional definition is meaningless in the presence of just-in-time compilation. Separate compilation is critical to the development of large software systems, as it provides a means to reduce compilation times during the software development cycle.

System F is a simple model for understanding how to ensure modular type checking and separate compilation in a language with parameterized types. The definition of System F is concise enough that we can discuss the entire language in a few paragraphs. The syntax of System F is minimal; it only supports two language features: functions and generics, both of just one parameter.

```
term variables x,y,z type variables \alpha,\beta integers n types \tau ::= int |\alpha| \tau \to \tau | \forall \alpha. \tau expressions e ::= n | x | \lambda x : \tau. e | e e | \Lambda \alpha. e | e[\tau]
```

The types of System F includes type variables, function types, and universal types, which give types to generics. The terms of System F include term variables, anonymous functions (the λ form), function application (which is the juxtaposition of two expressions, the first should evaluate to a function and the second, should evaluate to its argument), anonymous generics (the Λ form), and the *explicit instantiation* of generics ($e[\tau]$).

The modularity of System F's type system can be seen by inspecting its definition, shown in Figure 1. The horizontal bars should be read as if-then statements. The statements above the bar are premises and the statement below the bar is the conclusion. We use Γ to denote an environment, which is a sequence of the in-scope term variables (with their type bindings) and type variables. The type system disallows duplicate type variables in environments. We write $x:\tau\in\Gamma$ to mean that the first binding for x in Γ is τ . More formally, $x:\tau\in\Gamma$ if and only if $\Gamma_i=x:\tau$ for some i such that there is no i0 where i1 where i2 in the scope defined by i3. The notation i4 is well typed and has type i6 in the scope defined by i6. The notation i6 is for the capture-avoiding substitution of i7 for i8 in i9.

To see the modularity of the type system, first look at the rule for function application, with the conclusion $\Gamma \vdash e_1 \ e_2 : \tau'$. This rule requires that the parameter

$$\begin{array}{c|c} \underline{x:\tau\in\Gamma} \\ \hline \Gamma\vdash n: \mathsf{int} \end{array} \begin{array}{c} \underline{x:\tau\in\Gamma} \\ \hline \Gamma\vdash x:\tau \end{array} \\ \\ \underline{\Gamma,x:\tau\vdash e:\tau'} \\ \hline \Gamma\vdash \lambda x:\tau.e:\tau\to\tau' \end{array} \begin{array}{c} \Gamma\vdash e_1:\tau\to\tau' \quad \Gamma\vdash e_2:\tau \\ \hline \Gamma\vdash e_1e_2:\tau' \end{array} \\ \\ \underline{\Gamma\vdash e_1e_2:\tau'} \\ \hline \Gamma\vdash \Lambda\alpha.e:\forall\alpha.\tau \end{array} \begin{array}{c} \underline{\Gamma\vdash e:\forall\alpha.\tau} \\ \hline \Gamma\vdash e[\tau']:[\alpha:=\tau']\tau \end{array}$$

Figure 1: The typing rules for System F.

type τ of the function e_1 is the same type as the type of e_2 . Note that this rule does not require any knowledge of the body of the function being applied, just its type $\tau \to \tau'$. Similarly, look at the rule for instantiating generics, with the conclusion $\Gamma \vdash e[\tau'] : [\alpha := \tau']\tau$. Again, the rule does not require any knowledge of the body of the generic. Instead, it just requires knowledge of the generic's type.

On the flip side, consider the typing rules for creating functions and generics. A function knows that its input x has type τ , but it has no information about what value will be bound to x. Analogously, a generic knows that its parameter α is a type, but it does not know which type will be bound to α . The type system ensures that a well-typed generic does not depend on which type is bound to α , that is, it ensures that the generic will work with any choice for α . The way the type system ensures this is that it considers α different from every other type. (So α is only equal to itself.) This property of the type system comes from its use of syntactic identity to check whether two types are equal. Consider the rule for function application: the parameter type τ has to be syntactically identical to the type of the argument e_2 . For example, the following term is not well typed (after adding support for integers to System F).

$$\Lambda \alpha. \lambda x: \alpha \rightarrow \alpha. (x 1)$$

In the application $(x \ 1)$, the parameter type is α but the argument type is int. From the preceding discussion, we see that System F provides both polymorphism and modular type checking: an instantiation of a generic can be type checked without referring to the generic's body and the body of a generic can be type checked without referring to any instantiations.

Furthermore, the design of System F enables separate compilation but does not require it. The run-time behavior of the body of a generic may not depend on the instantiating type. Thus, it is possible to emit machine code that works for all instantiations of the same generic, modulo some restrictions on the calling conventions. In particular, the calling convention for data associated with a type parameter must be uniform, that is, it cannot depend on the instantiating type. The most common uniform representation is "boxing", that is, storing data on the heap and referring to it through a pointer. The compiler for a programming language with parametric polymorphism may choose to support separate compilation and use boxing, such as Java [7] and Objective Caml [54], or it may choose not to for the sake of efficiency, such as the MLton [15] compiler for Standard ML [62]. Better yet, a compiler may support separate compilation in general but sometimes optimize the instantiation of a generic when the body of the generic is available [47].

1.2 Generic Programming

Generic programming is a methodology that builds upon higher-order, polymorphic programming, scaling it up to deal with large families of algorithms and data structures. For example, the STL includes a function named accumulate that generalizes foldl to work on any representation of a sequence, not just linked lists. Going further, the STL includes a comprehensive set of sorting and searching functions. Beyond the STL, programmers applied generic programming in the development of libraries in the areas of computational geometry [6], quantum mechanics [2], graph algorithms (the author's Boost Graph Library [84]), and many more [21, 51, 71, 78].

The main programming language capability that is needed for higher-order, polymorphic programming to scale to large software libraries is the capability to organize and group together parameters, such as the parameters f and y of foldl. The number of such parameters grows large for interesting algorithms, upwards of 20 parameters! The key to organizing these parameters is that there are relationships between subgroups of parameters, and that these relationships form coherent abstractions. For example, in the use of foldl with multiplication, the choice of 1 as the argument to parameter y was not by chance. The integer 1 is the *identity element* for multiplication, that is, for any integer x, 1 * x = x. (Imagine using 0 for y instead. The result of foldl would be uninteresting.)

Mathematicians have been discovering such abstractions for quite some time, defining *algebraic structures* to describe these abstractions and proving reusable theorems about these abstractions [100]. An algebraic structure consists of: one or more sorts, signatures for operations over the sorts, and axioms that describe

the semantics of the operations. For example, the algebraic structure that matches the needs of foldl is Monoid. A Monoid consists of a sort S together with a binary operation on S that is associative and that has an identity element in S. An *instance* of an algebraic structure is a set for each sort, and an operation for each signature, such that the axioms are true. The set of integers with addition and 0 is an instance of Monoid.

In the late 1970s and early 1980s, Burstall and Goguen [10, 11] and Kapur, Musser, and Stepanov [45] noticed that it is helpful to think of computer algorithms as operating on algebraic structures instead of concrete data structures. To quote Stepanov:

That is the fundamental point: algorithms are defined on algebraic structures [73].

In this setting, algebraic structures are analogous to interfaces (as in Java) and instances are analogous classes that implement the interfaces.

However, the analogy between algebraic structures and interfaces is superficial because they differ both with regards to both purpose and semantics. Interfaces are primarily created to categorize classes whereas algebraic structures are created to express the requirements of theorems (in mathematics) and algorithms (in computer science). With respect to semantics, a class may inherit from an interface if it provides methods that satisfy the variance rules required by *subtyping* (contravariant parameter types and covariant return types). On the other hand, a class implements an algebraic data structure if *substituting* the class for the structure's sort in the structure's operation signatures yields a set of signatures that are implemented by the class. This subtle difference is significant: interfaces suffer from the binary method problem whereas algebraic structures do not [8].

The late 1970s and 1980s saw considerable developments both in the practice of generic programming and in the design of language features that support generic programming. Kershenbaum, Musser, and Stepanov [48] developed generic sorting and searching algorithms in Scheme and then Musser and Stepanov [64] developed similar algorithms in Ada. Recall that Scheme is a dynamically typed language, so it is straightforward to express polymorphic algorithms in Scheme. Ada, on the other hand, was one of the early statically typed languages to support parametric polymorphism. However, neither Scheme or Ada provided support for organizing parameters of generic algorithms. (This support was added to Ada later, in 1995, with the addition of generic package parameters.)

```
 \begin{aligned} &\textbf{create} \ \mathsf{semigroup}(S: \mathsf{set}, +: \mathsf{S} \! \times \! \mathsf{S} \to \mathsf{S}) \\ &\textbf{with} \ \mathsf{x} + (\mathsf{y} + \mathsf{z}) = (\mathsf{x} + \mathsf{y}) + \mathsf{z}; \end{aligned} \\ &\textbf{create} \ \mathsf{monoid}(S: \mathsf{semigroup}, \, 0 \colon (\mathsf{I}) \to \mathsf{S}) \\ &\textbf{with} \ \mathsf{0} + \mathsf{x} = \mathsf{x} + \mathsf{0} = \mathsf{x}; \end{aligned} \\ &\textbf{create} \ \mathsf{sequence}(S: \mathsf{set}, \, \mathsf{E}: \mathsf{set}, \, \mathsf{isnull}: \, \mathsf{S} \to \mathbf{bool}, \, \mathsf{head}: \, \mathsf{S} \to \mathsf{E}, \, \mathsf{tail}: \, \mathsf{S} \to \mathsf{S}); \end{aligned} \\ &\textbf{provide} \ \mathsf{sequence} \ \textbf{of} \ \mathsf{monoid} \ \textbf{with} \\ &\text{reduction:} \\ &x \to \mathbf{if} \ \mathsf{isnull}(\mathsf{x}) \ \textbf{then} \ \mathsf{0} \ \mathbf{else} \ \mathsf{head}(\mathsf{x}) + \mathsf{reduction}(\mathsf{tail}(\mathsf{x})) \\ &\mathsf{integers}(\mathsf{I}: \mathsf{set}, +: \mathsf{I} \times \mathsf{I} \to \mathsf{I}, \, *: \mathsf{I} \times \mathsf{I} \to \mathsf{I}, \, \mathsf{0}: (\mathsf{I} \to \mathsf{I}, \, \mathsf{1}: (\mathsf{I} \to \mathsf{I})) \\ &\mathsf{instantiate} \ \mathsf{monoid} \ \textbf{of} \ \mathsf{integers} \ (\mathsf{S=I}, + = +, \, \mathsf{0} = \mathsf{0}) \\ &\mathsf{instantiate} \ \mathsf{monoid} \ \textbf{of} \ \mathsf{integers} \ (\mathsf{S=I}, + = *, \, \mathsf{0} = \mathsf{1}) \end{aligned}
```

Figure 2: A generic reduction function written in Tecton.

1.3 Programming Language Support for Generic Programming

There were several lines of research in the 1980s to support the specification and use of algebraic structures in programming. Burstall and Goguen [10, 11] and Kapur et al. [46] developed the specification languages CLEAR and Tecton, respectively. An example of defining some algebraic structures and using them in a generic reduction function (a generalization of foldl), is shown in Figure 2, written in Tecton. Burstall collaborated with MacQueen on the HOPE language [12], which in turn inspired MacQueen's work on the *signatures* and *functors* of the Standard ML module system [56]. In the field of computer algebra, Jenks and Trager [40] created the Scratchpad language. Liskov et al. [55], while not explicitly targeting algebraic structures, did include a feature called *type set* in CLU that could be used to express algebraic structures. CLU was meant to support *abstract data types*, but it turns out that abstract data types and algebraic structures are closely related.

By the 1980s, object-oriented languages were gaining momentum, and work was underway to integrate parametric polymorphism into object-oriented languages. Cardelli and Wegner [14] developed *bounded polymorphism*, that is, using subtyping to express constraints on type parameters. Canning et al. [13]

generalized bounded polymorphism to enable recursive bounds, which laid the foundation for generics in Java [7] and C# [47, 104].

In 1988, Stroustrup [90] added support for generic programming to C++ with the addition of templates. Stroustrup considered using bounded polymorphism, but it was not a natural fit for many of the use cases he had in mind [92]. Stepanov suggested using a design similar to Ada's generic packages, with explicit instantiation, but Stroustrup disagreed with such a design because he thought explicit instantiation would be an unreasonable burden on clients of a generic libraries [88, 91, 93]. Stroustrup also looked at the design choices in ML [89], but he did not model C++ templates after ML's functors for the same reason: functors required explicit instantiation. However, some confluence in the two designs can be seen in C++'s template argument deduction for function templates. The algorithm is based on the mathematical notion of *matching*, which is the single-sided version of the *unification* algorithm at the heart of Hindley-Milner type inference.

Templates were designed to be safer than C preprocessor macros but just as efficient [90]. Macros had been used for years as "a poor man's generics". The design for type checking templates followed a similar model to that of macros: type checking occurs after instantiation. (For templates, some checking happens before instantiation, but not much.) This design choice had some significant advantages and disadvantages. The primary advantages were extreme flexibility and expressiveness: it opened the door to template metaprogramming [1, 3, 101]. The primary disadvantage was the lack of modular type checking. We discuss the semantics and type system for templates in more detail in Section 2. The flexibility of templates, combined with function overloading, provided a means to write type-parameterized functions and to implicitly parameterize over operations, thereby making it relatively convenient to write generic algorithms in C++. In concert with Stroustrup's work on templates, Stepanov began developing a C++ component library [86] along the lines of his prior work in Scheme and Ada. This C++ library was the direct precursor to the Standard Template Library.

The 1980s ended with an important result: Kaes [44] and Wadler and Blott [102] discovered parametric overloading, then Wadler and Blott [102] expanded the idea to create the *type class* feature for Haskell. Type classes were not explicitly designed to support algebraic structures. Instead they were a generalization of ML's equality types. Nevertheless, type classes provide excellent support for generic programming, combining a modular type system with the convenience of implicit instantiation of generics (in contrast to the explicit functor application of Standard ML). If only the divide between the imperative and functional programming communities had not been so large!

1.4 Road Map

In the next section we briefly survey the style of generic programming that is used to develop modern C++ template libraries, as *concepts* are meant to support this style of programming. After that, we discuss the preliminary language design work at Texas and Indiana (Section 3). We then take a look at the two proposals to the C++ standards committee, first the Indiana proposal (Section 4) and then the Texas rebuttal (Section 5). The two teams formed a compromise at the Adobe meeting (Section 6) which lead to the final design for concepts that was voted into the C++ standard working draft (Section 7). Unfortunately, the compromise unraveled and concepts were voted back out of C++ (Section 8), leaving much in doubt regarding the future of concepts in C++ (Section 9).

2 Generic Programming and C++ Templates

In this section we take a closer look at the modern practice of generic programming in C++. The examples are from the Standard Template Library [87], though they could have easily come from many other generic libraries. The presentation here is in the style of the SGI version of the STL [4].

In the parlance of modern C++, *concept* means algebraic structure and *model* means an instance of an algebraic structure [4, 88]. The shift away from using the term "algebraic structure" makes sense because many of the abstractions have little to do with algebra. Also, the term "structure" is too close to **struct**, which has a very different meaning in C++. Unfortunately, the term "concept" is rather generic.

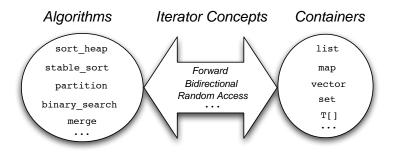


Figure 3: Organization of the Standard Template Library.

Figure 3 depicts the organization of the Standard Template Library. The STL contains 112 generic algorithms and 12 container classes. More importantly, the STL provides a handful of iterator concepts that decouple the algorithms from the containers. Because of the iterator abstractions, there only needs to be 112 algorithm implementations and 12 iterator implementations to enable the algorithms to work with all of the containers. (At least, for all the combinations that makes sense.) Using traditional procedural or object-oriented techniques, there would be 112×12 algorithm implementations, as each algorithm is implemented by different code for each container class.

Figure 4 depicts the hierarchy of iterator concepts within the STL. The arrows indicate the *refinement* relation between concepts (analogous to inheritance between interfaces). A concept refines another concept if it includes all of the capabilities of the other concept. Or put another way, concept C_1 refines C_2 if all the models of C_1 are also models of C_2 . The Random Access Iterator concept requires the most capabilities, in particular it requires the capability of jumping forward or backwards an arbitrary number of positions in constant time. The Input and Output Iterator concepts require the least capabilities, just requiring the ability to make a single pass through a sequence, respectively reading or writing to the elements.



Figure 4: The Iterator Concepts of the Standard Template Library.

The iterator hierarchy provides a fine-grained categorization of iterator capabilities to enable each algorithm to precisely specify its requirements. The precision is necessary to maximize reusability: each algorithm should use the minimal requirements necessary to efficiently carry out its task, thereby maximizing the number of situations in which it can be used. An important point to stress is that, in generic programming, the raison d'être for concepts is to provide concise means for algorithms to specify their requirements. This viewpoint contrasts with object-oriented methodologies in which interfaces are created to categorize classes independently of their use in algorithms.

Figure 5 shows the description of the Input Iterator concept from the SGI STL [4]. In the description, the type variable X is used as a place holder for the

Description

An input iterator provides the capability of traversing through a sequence once and reading the values of the elements.

Associated Types

- iterator_traits<X>::value_type, this is the element type of the sequence.
- iterator_traits<X>::difference_type, this is for measuring distances between positions in the sequence.

Valid Expressions

(X is the iterator type, T is the value type)

expression	return type	semantics		
*i	Convertible to T	Returns the value at position i		
++i	X&	Moves the iterator to the next posi-		
		tion.		
i == j	bool	Returns true if i and j are at the same		
		position. i == j implies *i == *j		
i != j	bool	Equivalent to $!(i == j)$.		

Complexity guarantees

All operations are amortized constant time.

Figure 5: The Input Iterator Concept

modeling type. The valid expressions specify the operations that must be provided by the modeling type. The complexity guarantees enable generic algorithms to in turn state guarantees regarding their execution time. Associated types are types that have an auxiliary role in the concept and whose identity depends on (is a function of) the modeling type. (Associated types are related to virtual types [9, 25, 67, 99], but avoid many of the difficulties by being accessed through types instead of objects. Associated types are most closely related to abstract types of a signature in ML.) In C++, associated types are usually accessed through a template trick known as traits [65].

Figure 6 lists the merge function template from the STL, which serves here as a typical example of a generic algorithm. The documentation for merge states that the type parameters Inlter1 and Inlter2 must model Input Iterator. The need for this requirement can be seen in that the algorithm increments, dereferences, and uses the not-equal operator on the the iterators first1, last1, first2, and last2. The Outlter type is required to model Output Iterator, which enables the dereference

template <typename InIter1, typename InIter2, typename OutIter> OutIter merge(InIter1 first1, InIter1 last1, InIter2 first2, InIter2 last2, OutIter result) { while (first1 != last1 && first2 != last2) { if (*first2 < *first1) { *result = *first2; ++first2; } else { *result = *first1; ++first1; } ++result; } return copy(first2, last2, copy(first1, last1, result)); }</pre>

Figure 6: The Merge Function Template of the STL.

and assignment to result as well as the incrementing of result. Also, to enable the assignment, the value type of Outlter must be the same as the value types of Inlter1 and Inlter2. And finally, the if-statement compares the elements of the two input iterators, so the associated value type of Inlter1 must model the Less Than Comparable concept. The merge function calls the copy function template, which is also part of the STL. The copy template in turn places requirements on its type parameters, and those requirements are a subset of the type requirements for merge.

2.1 The Semantics of C++ Templates

The main idea behind the semantics of templates is that different versions of a template are stamped out for different uses. The min function template and its use in main in Figure 7, serves to demonstrate the key ideas. At the point where min is defined, a C++ type checker looks at any expressions in the body of min whose type does not depend on the type parameters. In this case, there are none, so no type checking occurs at the point of definition of min.

Moving on to the main function, the call to std::min is an example of template argument deduction. A C++ compiler deduces that **int** should be chosen for parameter T by pattern matching the argument types against the parameter types. So

```
namespace std {
  template <class T>
  T min(T a, T b) {
    if (b < a) return b; else return a;
  }
}
int main() {
  return std::min(3, 4);
}</pre>
```

Figure 7: A simple function template: the min operation.

the compiler transforms the call to include an explicit instantiation:

```
int main() {
  return std::min<int>(3, 4);
}
```

A C++ compiler then generates a version of std::min that is specialized for **int**. The following is a source code representation of the generated code (which usually only exists in the internal representation of the compiler).

```
namespace std {
  template<>
  int min<int>(int a, int b) {
    if (b < a) return b; else return a;
  }
}</pre>
```

A C++ compiler type checks the specialized code, which in this case is well typed. If we change the example to apply min to a type that does not provide a less-than comparison operator, as follows,

```
struct A {};
int main() {
   A a;
   return std::min(a, a);
}
```

we get an error message that points inside the min function template.

```
error1.cpp: In function 'T std::min(T, T) [with T = A]': error1.cpp:8: instantiated from here
```

```
error1.cpp:3: error: no match for 'operator<' in 'b < a'
```

For such a small template, this error is not difficult to debug, but for most templates in the STL, such errors are infamously difficult to comprehend. For example, the following program that mistakenly applies stable_sort to a linked list results in the error message in Figure 8. Can you see the problem?

```
int main() {
    list<int> I;
    stable_sort(I.begin(), I.end());
}
```

The stable_sort template requires the iterators to provide random access, but the iterators of a double linked list are merely bidirectional.

These examples demonstrate that C++ does not have a modular type system. When checking the use of a template, the C++ type checker looks at the body of the template. To be modular, C++ would need the notion of the type of a template, like the universal type of System F, and it would have to check uses of templates against such types, but it does not.

As more programmers began to use the STL in the late 1990s and early 2000s, such error messages became a constant aggravation and deterred many programmers from using the STL. Meyers [58], an author of many popular C++ books, writes

Perhaps most daunting, even the smallest STL usage error often led to a blizzard of compiler diagnostics, each thousands of characters long, most referring to classes, functions, or templates not mentioned in the offending source code, almost all incomprehensible.

Building on an idea of Stepanov's, I developed a C++ library to check requirements on type parameters and thereby improve the error messages [79]. However, using the library required writing tedious and error prone code, and the quality of the error messages was dependent on the compiler.

2.2 Templates and Separate Compilation

The specialization-based semantics of C++ templates rules out the possibility of separate compilation. Other languages, such as ML and Java provide generics but retain separate compilation, so why can't C++? It turns out that not only is specialization the main implementation approach for compiling templates, but the semantics of the language forces it to be the only implementation approach.

```
stl_algo.h: In function 'void std::__inplace_stable_sort(_RandomAccessIterator, _RandomAccessIterator)
[with RandomAccessIterator = std:: List_iterator<int>]':
  tl_algo.h:3633: instantiated from 'void std::stable_sort(_RandomAccessIterator,_RandomAccessIterator)
[with_RandomAccessIterator = std::_List_iterator<int>]'
error1.cpp:8: instantiated from here
stl algo.h:2921: error: no match for 'operator' in ' last
stl_algo.h:3635: instantiated from 'void std::stable_sort(_RandomAccessIterator, _RandomAccessIterator)
[with _RandomAccessIterator = std:: _List_iterator<int>]'
error1.cpp:8: instantiated from here
stl_algo.h:3550: error: no match for 'operator' in '__last stl_algo.h:3551: error: no match for 'operator+' in '__first
stl_algo.h:3551: error: no match for 'operator+' in '__first + __len' stl_algo.h:3552: instantiated from 'void std::stable_sort(_RandomAccessIterator,_RandomAccessIterator)
  [with _RandomAccessIterator = std::_List_iterator<int>]
error1.cpp:8: instantiated from here
stl_algo.h:3564: error: no match for 'operator' in '__middle _
stl_algo.h:3564: error: no match for 'operator' in '_last __middle' stl_algo.h: In function 'void std::__insertion_sort(_RandomAccessIterator, _RandomAccessIterator)
[with_RandomAccessIterator = std::_List_iterator<int>]':
stl_algo.h:2923: instantiated from 'void std::__inplace_stable_sort(_RandomAccessIterator, _RandomAccessIterator)
[with_RandomAccessIterator = std::_List_iterator<int>]'
stl_algo.h:3633: instantiated from 'void std::stable_sort(_RandomAccessIterator, _RandomAccessIterator)
  [with RandomAccessIterator = std:: List iterator<int>]
error1.cpp:8: instantiated from here
```

Figure 8: The prefix of a typical error message that arises from misusing the STL.

One might think that a C++ compiler could support both separate compilation and specialization by postponing template specialization to run time. However, the reader may recall that the definition of separate compilation (Section 1.1) requires that the amount of time used for compilation (whether it be at compile time or run time) be proportional to the size of the application, not including the size of the (template) libraries that it uses. Thus, even an implementation using run-time specialization would not provide separate compilation.

There are several language features in C++ that force specialization; the one we describe here is user-defined template specialization. The example in Figure 9 demonstrates that many phases of a C++ compiler, from front-end type checking to back-end code generation, require specialization. The example shows a function template f that refers to class template C, using its nested type num to declare an array A. The call f(z) causes C to be instantiated with the type argument **char**, which results in the user-defined specialization of C and the typedef of num to **int**. So **sizeof**(U) is 4 bytes (assuming a 32 bit architecture) and therefore **sizeof**(U)/4 – 1 is zero, causing x to be assigned to the first element of the array. The call f(42) causes C to be instantiated with **int**, which results in the instantiation of the primary template and the typedef of num to **double**. So **sizeof**(U) is 8 bytes and therefore **sizeof**(U)/4 – 1 is one, causing x to be assigned to the second element of

```
template<typename T>
struct C {
 typedef double num;
};
template<>
struct C<char> {
 typedef int num;
};
template<typename T>
void f(T x) {
 typedef typename C<T>::num U;
  U A[2];
 A[\mathbf{sizeof}(U)/4 - 1] = x;
}
int main() {
 f('z');
 f(42);
 f(pair(1,2));
}
```

Figure 9: User-defined specialization inhibits separate compilation.

the array. The call f(pair(1,2)) results in a type error because pair is not convertible to **double**.

This example shows that C++ templates are not a form of parametric polymorphism (as technically defined) because the behavior of a template may depend on its type arguments. But more importantly, it shows that compilation time is a function of the number of template instantiations. For example, the type checking of f's body must be repeated for each unique instantiation because there can be a different outcome for each one: f(42) was well typed but not f(pair(1,2)). Furthermore, the generated assembly code for the instantiations of f would need to differ in many ways. For example, most architectures provide different instructions for storing **int** versus **double**, so the assignment of x into A would require different instructions. Thus, in general, a C++ compiler needs to generate different code sequences for different instantiations.

So the compilation time for a C++ application must be a function of the size of all the templates libraries it uses, transitively. For modern C++ applications, the compilation time can stretch from minutes into hours.

2.3 Templates and Name Lookup

While non-modular error messages are the most obvious problem with the semantics of C++ templates, there are also subtle problems regarding name lookup. Consider the min template again, but this time we apply it to a user-defined type that provides a less-than operator.

```
namespace L {
    class interval { };
    bool operator<(interval x, interval y) { ... }
}
int main() {
    L::interval i, j, k;
    k = std::min(i,j);
}</pre>
```

The above example is well typed, but how? How does the use of the less-than operator inside std::min resolve to the less-than operator in namespace L? The answer is *argument dependent name lookup* (ADL). When a C++ compiler performs name lookup for the function name in a function call, it not only considers the names that are in lexical scope but it also analyzes the argument types and then includes all the names in the namespaces in which those types were defined. In

```
namespace lib {
   template <class T> void load(T, string) { printf("Proceeding as normal!\n"); }
   template <class T> void initialize(T x) { load(x, "file"); }
}
namespace N {
   struct b { int n; };
   template <class T> void load(T, const char*) { printf("Hijacked!\n"); }
   template <class T> void shoot(T x) { load(x, "ammunition"); }
}
int main() {
   N::b a;
   lib::initialize(a);
}
// Output: Hijacked!
```

Figure 10: Argument dependent lookup can lead to unintended run-time behavior.

this case, the less-than operator is applied to arguments of type interval, and interval was defined in namespace L. So the less-than operator in namespace L is among the entities returned by name lookup in this case.

While argument dependent name lookup may seem like a good thing, it is not modular and causes bugs when building large software systems. The example in Figure 10 is distilled from a real bug that showed up in an application using the Boost C++ libraries. The initialize function template in namespace lib intends to call another function template in lib named load. Unbeknownst to the author of lib, there is another function named load in namespace N that is meant to be a helper function for shoot. Because of argument dependent lookup, the call to load inside of initialize does not resolve to lib::load, but instead resolves to N::load. Thus, the output of the above program is "Hijacked!". What is particularly troubling about this example is that the run-time behavior of lib::initialize depends on the context in which it is used, analogous to the infamous dynamic scoping of Lisp [43].

2.4 Tag Dispatching

On the positive side, the semantics of C++ templates enables some powerful programming styles. For example, there is a natural tension between performance and generality in algorithm design. Often times there exist multiple algorithms to solve the same problem, where one algorithm is more efficient than another, but

```
template <class InIter, class Dist>
void advance_aux(InIter& i, Dist n, input_iterator_tag)
    { while (n) ++i; }

template <class RandIter, class Dist>
void advance_aux(RandIter& i, Dist n, random_access_iterator_tag)
    { i += n; }

template <class InIter, class Dist>
void advance(InIter& i, Dist n) {
    typename iterator_traits<InIter>::iterator_category cat;
    advance_aux(i, n, cat);
}
```

Figure 11: An example of the tag dispatching idiom.

requires more capabilities from the types it operates on. The advance function of the STL is a particularly simple example of this. The advance function moves an iterator forward n positions within the sequence. If the iterator models Random Access Iterator, then the iterator can directly jump forward n positions in constant time. On the other hand, if an iterator only models Input Iterator, then it takes linear time to move forward n positions.

An obvious solution to this problem is to have two differently named functions for advancing iterators. However, what if the need to advance iterators appears in the context of a generic algorithm? For example, the lower_bound function template of the STL calls advance, but is itself parameterized on the iterator types. In that context we do not know which iterators are being used. The solution to this problem in C++ is the tag dispatching idiom, shown in Figure 11¹. We create a single entry point named advance and then use traits to inquire about the capability of the iterator. The resulting tag is used to influence the function overload resolution for the call to the helper functions named advance_aux. The tag dispatching trick works because the overload resolution of the call to advance_aux inside of advance does not occur until after advance has been instantiated on particular concrete iterator types.

¹Kiselyov and Peyton-Jones [50] describe a solution to this problem in Haskell.

2.5 Evaluation of C++ Templates

To briefly evaluate the design of C++ templates, on the positive side C++ effectively enables generic programming by providing type parameterization and the convenient use of function templates through template argument deduction and argument dependent lookup. Furthermore, the run-time performance of C++ templates can match that of hand-coded procedures because the specialization approach leaves behind little or no run-time overhead. Last but not least, the lack of point-of-definition type checking for templates provided the flexibility needed for template metaprogramming.

On the negative side, the type checking of templates is not modular which leads to confusing error messages for users of generic libraries and opens the door to discrepancies between a library's documentation and implementation. Further, because argument dependent lookup is non-modular, the run-time behavior of template libraries can silently change in unexpected ways in different usage scenarios. Finally, the specialization semantics of C++ templates prevents separate compilation, so developers of large C++ applications endure long compilation times.

3 Preliminary Research on Concepts

In 2003, two teams set out to design an extension to C++ to improve C++ templates by providing modular type checking. To type check the body of a template independently of any instantiation, the type checker needs to know what assumptions are being made about the type parameters. As discussed in the previous section, these assumptions were commonly stated in the documentation for the template; what was needed was a way to make these assumptions explicit in the code. The two teams set out to add support for type constraints, using concepts to express the constraints. At Texas A&M University, Bjarne Stroustrup (having just moved from AT&T Labs—Research) and then post-doc Gabriel Dos Reis began the Pivot project and produced several technical reports for the C++ standards committee. We discuss these reports in Section 3.3.

At Indiana University, Professor Andrew Lumsdaine, post-doc Jaakko Järvi, and the graduate students Ronald Garcia, Jeremiah Willcock, and I also began working on concepts and constrained generics. We did not have prior experience in language design, but we had considerable experience in the design and implementation of generic libraries [27, 37, 38, 53, 77, 78, 84]. Our first ma-

	C++	SML	Haskell	Eiffel	Java	C#
Multi-type concepts	-	•	•*	0	0	0
Multiple constraints	-	\odot	•	\circ	•	
Associated type access	•	•	\bigcirc	\bigcirc	\bigcirc	\bigcirc
Retroactive modeling	-	•	•	\circ	\circ	\circ
Type aliases	•	•	•	\circ	\circ	\circ
Separate compilation	\circ	•	•	•	•	•
Template arg. deduction	•	\circ	•	0	•	•
Concise syntax	•	•	•	0	•	\circ

^{*}Using the multi-parameter type class extension to Haskell.

Table 1: Results from the study of language support for generic programming. A black circle indicates full support, a white circle indicates poor support, and a half-filled circle indicates partial support. The rating of "-" in the C++ column indicates that C++ does not explicitly support the feature, but one can still program as if the feature were supported due to the permissiveness of C++ templates.

jor undertaking was to study support for generic programming in state of the art programming languages, discussed in the following section.

3.1 Indiana: A Comparative Study

In our study [28] we implemented several generic algorithms from the Boost Graph Library [84] in six languages: C++, Standard ML, Haskell, Eiffel, Java, and C#. (In a follow-on study, we added OCaml and Cecil [29].) We sought to use best practices for each language. In the course of the study, we took note of when the presence or absence of a language feature made the implementation more or less difficult. Table 1 summarizes the results of the study.

The following explains the language features that form the rows of the table. "Multi-type concepts" indicates whether multiple types can be simultaneously constrained. "Multiple constraints" indicates whether more than one constraint can be placed on a single type parameter. "Associated type access" rates the ease in which types can be mapped to other types within the context of a generic function. "Retroactive modeling" indicates the ability to add new modeling relationships after a type has been defined. "Type aliases" indicates whether a mechanism for creating shorter names for types is provided. "Separate compilation" indicates whether generic functions are type-checked and compiled independently from their use. "Template arg. deduction" indicates that type arguments can be deduced without requiring explicit syntax for instantiation. "Concise syntax" in-

dicates whether the syntax required to compose layers of generic components is independent of the scale of composition.

Haskell faired particularly well in this study, with Standard ML not too far behind, while Eiffel, Java, and C# did not do as well. The underlying pattern was that the three object-oriented languages relied on F-bounded polymorphism [13], whereas Standard ML and Haskell did not. Standard ML supports generic programming through Functors and signatures and Haskell supports generic programming via type classes. The take-away point for us was that a design for concepts in C++ should be based on the best features of Haskell and Standard ML, and not F-bounded polymorphism. Our case study influenced other programming language researchers. For example, Chakravarty et al. [16, 17] added associated types to Haskell, filling in the only half-circle for Haskell in Table 1.

3.2 Texas: Proposal at Kona

At the October, 2003 C++ standards committee meeting in Kona, Stroustrup and Dos Reis presented three technical reports, N1510 [92], N1522 [95], and N1536 [96] that sketched a design for concepts.

There were five important aspects to the Texas design. First, they proposed *usage patterns* as the mechanism for specifying which operations are required of the modeling type, that is, the type implementing the concept. Usage patterns mimic the notation that had become common for documenting generic libraries. The following example shows a concept that requires the modeling type to provide a copy constructor, an addition operator, and an assignment operator.

```
concept Add {
  constraints(Add x) { Add y = x; x = x+y; }
};
```

Second, the design proposed *composing concepts* using the logical operators && (and), || (or), and ! (not). In the following, the concepts C1 and C2 are combined in different ways to constrain the T type parameter of three classes.

```
template <(C1 && C2) T> class X { ... };
template <(C1 || C2) T> class Y { ... };
template <(C1 && !C2) T> class Z { ... };
```

Third, the proposal did not include support for associated types, but instead proposed *parameterized concepts*. In the following Forward_iterator concept, the value type of the iterator is a type parameter.

```
template <Value_type V> concept Forward_iterator {
  constraints(Forward_iterator p) {
    Forward_iterator q = p; V v = *p; p++; ++p;
  }
}
```

Fourth, the proposal included *concept-based overloading*. That is, extending the rules for function overload resolution to take into account the constraints on type parameters. This feature was meant to replace the tag dispatching idiom. For example, below are the two advance algorithms, the first requiring only an Input Iterator and the second requiring a Random Access Iterator.

```
template <InputIterator InIter>
void advance(InIter& i, InIter::difference_type n)
    { while (n) ++i; }

template <RandomAccessIterator RandIter>
void advance(RandIter& i, RandIter::difference_type n)
    { i += n; }
```

Fifth and finally, the proposal used *implicit modeling*. That is, a C++ compiler would automatically deduce whether a class models a concept. In the following example, we instantiate the f function template with class A. Template f requires A to model the Add concept, so the C++ compiler would check that A implements all of the operations required by the Add concept.

```
class A { };
A operator+(A x, A y) { ... }

template<Add T> void f(T x) { ... }

int main() {
    A a;
    f(a); // compiler deduces that A models Add
}
```

At the Kona meeting and in the following months, several problems with the proposal were identified. With respect to usage patterns, there was an open question regarding implementation. To type check the body of a template, the usage patterns would need to be converted to type signatures for use in the typing environment (that is, the symbol table). Supposing that hurdle could be overcome,

there was also an issue regarding usability. For example, it seems that the following template h, which seems fine, would in fact be ill typed.

```
template<Add T> void g(T x, T y) { ... }

template<Add T> void h(T x, T y) {
    g(x, x + y); // error: no matching function for call to 'g(T, T1)'
}
```

To explain, the usage patterns for the Add concept, defined above, would imply that, given a type T that models Add, addition would have the following signature:

```
T1 operator+(T,T); where T1 is some type convertible to T
```

Thus, in the call to g, the types of the two arguments would be T and T1, respectively. But g requires that the two arguments be of the same type.

The third issue regarding usage patterns was that any C++ expression was allowed as a usage pattern. Together with implicit modeling and concept-based overloading, the C++ compiler would need the ability backtrack out of arbitrarily deep chains of template instantiations, which would require significant reengineering of the industry's C++ compilers.

With respect to composing concepts, the && operator was straightforward and expected, but the || and ! operators were both unexpected and problematic. They were unexpected because their was no use of them in the documentation of C++ template libraries. The || operator was problematic because it seemed that adding this operator would cause type checking to become exponential. The ! operator was problematic because its semantics was unclear: how does the type checker make use of such a constraint inside the body of a template? And on the instantiation side, the ! operator might require the close-world assumption and therefore be incompatible with separate type checking.

With respect to parameterized concepts, the Texas design was similar to Haskell's type classes, which supported multi-parameter type classes but not associated types. However, our comparative study showed that associated types provide a nice mechanism for reducing the number of type parameters needed in generic algorithms [28, 29].

Finally, the combination of implicit modeling and concept-based overloading produces some dangerous situations [32]. Consider the vector class from the STL shown in Figure 12. It has two constructors that accept an iterator range. The first only requires the iterators to model Input Iterator, so the algorithm can only make a single pass through the iterator range and must continually resize the vector's

```
template<typename T>
class vector {
   template<InputIterator InIter> vector(InIter first, InIter last);
   template<ForwardIterator FwdIter> vector(FwdIter first, FwdIter last);
   ...
};
int main() {
   istream_iterator<int> i(cin), j;
   vector<int> v(i, j); // Silently dispatches to the wrong constructor!
}
```

Figure 12: Implicit modeling and concept-based overloading don't mix.

memory to fit the incoming elements. The second constructor requires the iterators to model Forward Iterator, so the algorithm makes a first pass through the iterator range to determine its length, then resizes the vector's memory to the appropriate size, and finally makes a second pass through the iterator range, filling in the elements of the vector.

Consider the initialization of a vector from an istream_iterator<int> (an iterator over integers from the standard input). To determine which constructor to call, the C++ compiler would need to deduce whether istream_iterator implements Forward Iterator and Input Iterator. Based on the syntactic constraints captured in the concept definitions, istream_iterator appears to model both concepts, so overload resolution would choose the constructor for the more refined concept, Forward Iterator (Figure 4). However, istream_iterator does not, in fact, model Forward Iterator because it lacks the ability to make multiple passes through its range. Thus, the run-time behavior would be rather surprising: instead of filling the vector with the first n integers from standard input, it would use the n+1 through 2n integers to fill in the vector (supposing there is that much input available).

The ability to make multiple passes through a range is an example of a *semantic constraint*. Another example of a semantic constraint, from the Equality Comparable concept, is that equality should be reflexive, symmetric, and transitive, that is, an equivalence relation. The Equality Comparable concept, from the compiler's view, only requires that a type implement **operator**==, but does not check that the **operator**== is actually an equivalence relation. To fully support semantic constraints requires a highly expressive logic and support for theorem proving.

```
template <InputIterator Iter1, OutputIterator Iter2>
Iter2 copy(Iter1 first, Iter1 last, Iter2 result);

template <RandomAccessIterator Iter1, OutputIterator Iter2>
Iter2 copy(Iter1 first, Iter1 last, Iter2 result);

template <InputIterator InIter1, InputIterator InIter2, OutputIterator OutIter>
OutIter merge(InIter1 first1, InIter1 last1, InIter2 first2, InIter2 last2, OutIter result) {
    ...
    return copy(first2, last2, copy(first1, last1, result));
}
```

Figure 13: An example that demonstrates late versus early-bound overloading.

For example, the Isabelle proof assistant supports semantic constraints with its axiomatic type classes [103]. However, there are major research and educational challenges that need to be resolved before we can transplant those ideas into an mainstream, imperative language such as C++. Thus, the concepts design did not include semantic constraints, only syntactic ones. The recommended practice is to document semantic constraints in English, as is done for the Standard Template Library.

3.3 Indiana: Two Designs

Meanwhile Jaakko Järvi, Jeremiah Willcock, and I began designing a prototype language with concepts and constrained templates. Before we proceeded very far, we realized that some of our design goals were in conflict. It was clear that concept-based overloading was an important language feature, but it seemed to conflict with being able to separately compile templates. Even worse, there was tension between concept-based overloading and modular type checking.

Consider the excerpt from the STL merge function together with two overloads of the copy function template in Figure 13. To separately compile merge, the C++ compiler needs to decide which overload of copy should be called without knowing the actual iterator types. If the C++ compiler performed overload resolution only based on the type constraints for merge, it would resolve to the overload of copy for Input Iterator. However, consider what happens when using merge with the iterators from a vector, which model Random Access Iterator. With the

resolution already decided, merge would call the slow version of copy.

Suppose one forsakes separate compilation in favor of a design that allows late binding (after instantiation) for concept-based overloading. The next question is whether such a design can support modular type checking. The answer to this question is not straightforward, and turns out to be analogous to the problem of modular type checking in languages with multi-methods [19, 20, 59, 60]. It is also analogous to the problem of dealing with overlapping instance declarations in Haskell [69]. It is possible to achieve modular type checking, but the resulting type systems are complicated.

Facing this fork in the design space, we split in two directions: I investigated a design with separate compilation and early-bound overloads and Järvi and Willcock explored a design with late-bound overloads but not separate compilation. My investigation led to the design of a calculus for generics, named $F^{\mathcal{G}}$ [80, 81] and a prototype language named \mathcal{G} [75, 82, 83]. The exploration by Järvi and Willcock did not immediately lead to a concrete design, but their work later became the foundation for the Indiana proposal for concepts in C++, which we discuss in Section 4.

3.4 A Calculus for Generic Programming

Here we present a streamlined version of $F^{\mathcal{G}}$ to give the reader a feel for the design work that went on at Indiana. The purpose of a calculus like $F^{\mathcal{G}}$ is not to define a language that is for programmers to use, but it is instead a mathematical tool to help a language designer better understand the impact of different design decisions. In particular, $F^{\mathcal{G}}$ was meant to check whether my language design provided modular type checking. In addition to informing the design for concepts, $F^{\mathcal{G}}$ inspired the *implicits* feature of the Scala language [68].

The inspiration for $F^{\mathcal{G}}$ came primarily from Haskell and Standard ML. The $F^{\mathcal{G}}$ language borrows type classes and instance declarations from Haskell, with some modification, to form concepts and models. The main differences are that 1) $F^{\mathcal{G}}$ does not infer constraints but instead allows an operation name to appear in more than one concept, and 2) model declarations are lexically scoped in $F^{\mathcal{G}}$ instead of being essentially global. With respect to Standard ML, $F^{\mathcal{G}}$ borrows abstract types and type sharing from ML signatures to provide support for associated types.

Figure 14 defines the syntax of $F^{\mathcal{G}}$. As the name implies, $F^{\mathcal{G}}$ is an extension to System F [30, 72], which we reviewed in Section 1. $F^{\mathcal{G}}$ is also inspired by the qualified types of Jones [41]. For $F^{\mathcal{G}}$, we retain the explicit instantiation syntax of System F $(e[\tau])$ to avoid complexity in well-understood areas of the design. The

```
concept names
                                  c
integers
                                  n
                                           ::= \  \, \operatorname{int} \mid \tau \to \tau \mid \forall \alpha. \ \tau \mid C \Rightarrow \tau \mid \Pi
types
model identifier m
                                         ::= c < \overline{\tau} >
type identifier
                                  П
                                          := \alpha \mid m.\Pi
constraints
                                          := m \mid \tau = \tau
                                           ::= n \mid e \mid \lambda y : \tau \cdot e \mid \Lambda \alpha \cdot e \mid e[\tau]
expressions
                                                    C \Rightarrow e \mid \pi
                                                    concept c < \overline{\alpha} > \{ \overline{\beta}; \ \overline{C}; \ \overline{x : \tau} \} in e
                                                    model c < \overline{\tau} > \{ \overline{\beta = \tau}; \ \overline{x = e} \} in e
                                                     type \alpha = \tau in e
term identifier
                                          ::= x \mid m.\pi
                                  \pi
environments
                                  Γ
                                           ::= \Gamma, x : \tau \mid \Gamma, \alpha \mid \Gamma, C
                                             \Gamma, concept c < \overline{\alpha} > \{\overline{\beta}; \overline{C}; \overline{x} : \overline{\tau}\}
```

Figure 14: Syntax of $F^{\mathcal{G}}$.

language \mathcal{G} (presented in Section 3.5), however, does provides implicit instantiation (template argument deduction). The overbar notation \overline{a} indicates a list of items in the syntactic category a. For example, $\overline{\tau}$ is a list of types: $\tau_1, \tau_2, \ldots, \tau_n$.

 $F^{\mathcal{G}}$ adds several syntactic forms to System F: $C \Rightarrow e$ is a constrained expression, with the syntactic category C for expressing constraints. A constraint is either a concept constraint of the form $c < \overline{\tau} >$ or a same-type constraint of the form $\tau = \tau$. The expression π is a term identifier, which is a variable possibly prefixed by a sequence of model identifiers. The purpose of such expressions is to refer to members of a model. The expression (concept $c < \overline{\alpha} > \{\beta; C; x : T\}$ in e) defines a concept named c for use in e. The β 's are requirements for associated types, \overline{C} are nested constraints, and $\overline{x}:T$ are the required members of the concept. The expression (model $c < \overline{T} > \{ \overline{\beta = T}; \ \overline{x = e} \}$ in e) establishes that the types \overline{T} together model the concept c. (The common case is for there to be only one modeling type.) The expression (type $\alpha = \tau$ in e) aliases the type τ to the variable α . (In some sense, type aliases are not necessary, because the programmer could manually substitute τ for α in e. But type alias are important from a software engineering point of view because, in generic programming, type expressions can grow quite large, so typing τ many times inside e, and maintaining the resulting code, becomes rather burdensome [28, 29].)

The type system for $F^{\mathcal{G}}$ is defined in Figure 15. As can be seen in the rule for function application, type equality is not simply syntactic equality in $F^{\mathcal{G}}$; the addition of same-type constraints means that the type system needs to perform equational reasoning over types. The judgment $\Gamma \vdash \tau = \tau$ gives the declarative rules for type equality. These rules can be implemented efficiently using a congruence closure algorithm [24, 66]. Otherwise, the typing rules for function and generics is the same as System F.

The introduction (creation) of an expression of type $C\Rightarrow \tau$ is explicit whereas the elimination (use) of such an expression is implicit. This design is intended to make it convenient to use generic libraries without adding too much complexity to the type system. (We also want to avoid overburdening implementors of the language). To type check a constrained expression $C\Rightarrow e$, we type check e with C added as an assumption in the environment. If C is a concept constraint, then we also add all constraints that are nested inside the concept into the environment, using the notation $\flat(C)_\Gamma$ defined as follows:

$$\begin{split} \flat(\tau_1 = \tau_2)_{\Gamma} &= (\tau_1 = \tau_2) \\ \flat(c < \overline{\tau} >)_{\Gamma} &= c < \overline{\tau} >, [\overline{\alpha} := \overline{\tau}] \overline{\flat(C)_{\Gamma}} \quad \text{ if concept } c < \overline{\alpha} > \{\overline{\beta}; \ \overline{C}; \ \overline{x : T}\} \in \Gamma \end{split}$$

To type check the elimination (use) of an expression of type $C\Rightarrow \tau$, we check that that the constraint C can be satisfied in the current environment, for which we use the judgment $\Gamma \vdash C$. A concept constraint $c < \overline{\tau} >$ is satisfied if there is a model for that model identifier in the environment. A same-type constraint $\tau_1 = \tau_2$ is satisfied if the types are equal in the current environment.

Note how the typing rules for introducing and eliminating constrained expression are nearly mirror images of each other. The introduction rule makes some assumptions and the elimination rule discharges an assumption. This kind of mirror imaging is required to create a type system that ensures type safety. With that in mind, it is worth noting a discrepancy here: the elimination rule only discharges one assumption whereas the introduction rule adds many assumptions to the environment. The extra assumptions come from nested constraints, and if you look at the typing rule for models you see the check for the nested constraints there.

Type checking a concept definition expression is straightforward, we add the concept to the environment and proceed to check the rest of the program. Type checking a model definition expression, of the following form, deserves some explanation.

model
$$c < \overline{\tau_1} > \{ \overline{\beta = \tau_3}; \ \overline{x = e} \}$$
 in e

First, there must be a concept named c in the environment. Second, the nested

constraints C within c must be satisfied in the current environment. Third, the expressions \overline{e} must be of the right types, as specified by the concept. Finally, we type check the rest of the program e in an environment where we have added the model $c < \overline{\tau} >$ as an assumption, as well as the type equalities for its choices regarding the associated types.

In the definition of the *lookup* function, the notation $\Gamma|_c$ means the environment obtained by removing everything but the concept and same-type constraints from Γ .

Although $F^{\mathcal{G}}$ is a small calculus, with the addition of a few standard features such as lists, integers, and fix [70], it can express simple generic algorithms. Figure 16 shows the definition of a generic foldl. The example begins with the definition of three concepts that are used to specify constraints on the type parameters of foldl. The Semigroup concept is used indirectly, as a nested requirement inside Monoid. The concept Seq has one associated type E for the element type of the sequence. The fold function has one parameter type S for the sequence, and constrains S to be something that models Seq. We alias the element type of the sequence to the name E, and constrain E to be a model of Monoid. The fix operator is to make fold a recursive function. We access the binary operator and identity element of the monoid and use them to implement the core logic of the foldl. Much of the syntactic noise in this part of the function is because $F^{\mathcal{G}}$ does not support function overloading and therefore cannot implicitly introduce concept members into the scope of a constrained expression without running into problems with name conflicts. The language \mathcal{G} , on the other hand, adds support for function overloading.

The dynamic semantics of $F^{\mathcal{G}}$ is defined through a type-directed translation to System F, which we refer to here as $\mathcal{C}(\cdot)$. Without loss of generality, the semantics is defined for programs (closed expressions) of type int. Let \longrightarrow be the standard single-step reduction relation for call-by-value System F and \longrightarrow^* be its reflexive, transitive closure [70]. Then the dynamic semantics of $F^{\mathcal{G}}$ is defined by the following partial function named eval.

$$eval(e) = \begin{cases} n & \text{if } C(e) \longrightarrow^* n \\ \uparrow & \text{if } C(e) \text{ diverges} \end{cases}$$

The translation uses the dictionary-passing approach that is commonly used for Haskell type classes [36, 102]. Associated types are translated into extra type parameters. We do not give the translation here, but refer the interested reader to the paper *Essential Language Support for Generic Programming* [80, 81].

Figure 15: The type system of $F^{\mathcal{G}}$.

```
concept Semigroup<\alpha> { ; ; binary op : \alpha \to \alpha \to \alpha } in
concept Monoid<\alpha> { ; Semigroup<\alpha>; identity elt : \alpha } in
concept Seq<S> { E ; ; isnull : S \to \mathbf{bool}, head : S \to E, tail : S \to S } in
let foldl = (\Lambda S. \text{Seq} < S > \Rightarrow
             type E = \text{Seq} < S > .E \text{ in }
             Monoid < E > \Rightarrow
             fix (\lambda r : S \to E. \lambda ls : S.
                    let binary op = Monoid<E>.Semigroup<E>.binary op in
                    let identity_elt = Monoid<E>.identity_elt in
                    if Seq < S >.isnull Is then identity elt
                    else binary_op(Seq<S>.head ls, r(Seq<S>.tail ls))))
in
model Semigroup<int> { ; binary op = \lambda x:int. \lambda y:int. x + y } in
model Monoid<int> { ; identity_elt = 0 } in
model Seq<int list> {E=int; isnull=\lambda ls. null? ls, head=\lambda ls. car ls, tail=\lambda ls. cdr ls} in
foldl[int list] [2,3,4]
```

Figure 16: Example of a generic fold in $F^{\mathcal{G}}$.

Getting back to the main point of $F^{\mathcal{G}}$, does it provide modular type checking? The answer is yes and that can be seen by inspecting the type rules in Figure 15. The rules for functions and generics are the same as those of System F, which we inspected for modularity in Section 1.1. The important addition is the constrained expression $C \Rightarrow e$. The introduction rule for constrained expressions only refers to the constraint C and body e. The elimination rule for constrained expressions only depends on the type of expression e, that is, the type $C \Rightarrow \tau$, and the current environment Γ .

However, an experienced language designer knows that it is easy to create a modular type system by cheating, that is, by creating a type system that does not guarantee type safety. Thus, the type safety result for $F^{\mathcal{G}}$ plays an important role in verifying that $F^{\mathcal{G}}$ has a modular type system. The type safety theorem for $F^{\mathcal{G}}$ is stated as follows.

```
Theorem 1 (Type Safety). If \vdash e: int, then either eval(e) = n for some integer n or eval(e) = \uparrow.
```

System F is type safe, so proving type safety for $F^{\mathcal{G}}$ amounts to proving that the translation \mathcal{C} is type preserving. Indeed, I proved the following lemma [80, 81].

```
Lemma 1 (\mathcal{C} is type preserving). If \vdash e : \tau, then \vdash \mathcal{C}(e) : \mathcal{C}(\tau).
```

The proof of type safety then proceeds as follows.

Proof of Type Safety. We are given that $\vdash e$: int. By Lemma 1, we have $\vdash C(e)$: int (we have C(int) = int). Then by the type safety of System F, C(e) either reduces to a value of type int, in which case the value is an integer, or C(e) diverges. In the first case, we have eval(e) = n and in the second case, $eval(e) = \uparrow$. Either way, our proof is complete.

In addition to having a modular type system, $F^{\mathcal{G}}$ supports separate compilation, as $F^{\mathcal{G}}$ translates to System F, and it is straightforward to separately compile System F as we discussed in Section 1.1.

3.5 A Prototype Language for Generic Programming

To gain some practical experience in using this language design, I developed a prototype language named \mathcal{G} [75, 82, 83]. Then, as a case study, I ported a large portion of the STL to \mathcal{G} . In addition to the features in $F^{\mathcal{G}}$, \mathcal{G} included template argument deduction, function overloading, concept refinement, parameterized models, a basic module system, and simple classes. The overall experience of porting the STL to \mathcal{G} was pleasant: short error messages and short compile times.

As mentioned earlier, in deciding to support separation compilation, the design for \mathcal{G} gave up late-bound concept-based overloading. However, \mathcal{G} does provide early-bound concept-based overloading, and in the process of porting the STL to \mathcal{G} , I discovered workarounds for mimicking late-bound overloads using early-bound overloads.

Recall the merge function from Figure 13. When ported to \mathcal{G} , the calls to copy always resolve to the slower version for Input Iterator, even when merge is used on a sequence that models Random Access Iterator. Figure 17 shows the workaround for mimicking late-bound overloading. The main idea is to create a new concept for the overloaded function, in this case for copy. Here we name the concept CopyRange. Instead of calling copy directly, merge adds CopyRange to its constraints (in \mathcal{G} , constraints go in the where clause), and then calls the copy_range operation instead of copy.

With this change, the choice of which model of CopyRange is used, and therefore which copy, is made at the point where merge is instantiated, where the concrete iterator is known. However, it would place a burden on users of merge if they had to create extra model definitions for CopyRange. To solve this problem, we create two parameterized model definitions for CopyRange, one for models

```
concept CopyRange<I1,I2> {
  fun copy range(11,11,12) -> 12;
};
model < lter1, lter2> where { Input lterator < lter1>, ... }
CopyRange<Iter1,Iter2> {
  fun copy_range(Iter1 first, Iter1 last, Iter2 result) -> Iter2
    { return copy(first, last, result); }
};
model < lter1, lter2> where { RandomAccessIterator< lter1>, ... }
CopyRange<Iter1,Iter2> {
  fun copy range(Iter1 first, Iter1 last, Iter2 result) -> Iter2
    { return copy(first, last, result); }
};
fun merge<lter1,lter2,lter3>
where { ..., CopyRange<Iter2,Iter3>, CopyRange<Iter1,Iter3> }
(Iter1 first1, Iter1 last1, Iter2 first2, Iter2 last2, Iter3 result) -> Iter3 {
  ... return copy_range(first2, last2, copy_range(first1, last1, result));
}
```

Figure 17: Workaround for mimicking late-bound overloading.

of Input Iterator and one for models of Random Access Iterator. With these two parameterized models, the user of merge does not need to do any extra work.

The attentive reader may wonder how merge, using the above workaround, can be separately compiled. The reason is that \mathcal{G} uses the same dictionary-passing implementation model as $F^{\mathcal{G}}$. Thus, \mathcal{G} generates just one version of merge that takes a dictionary for CopyRange, which includes a pointer to the appropriate copy_range function. The cost of separate compilation, and this dictionary-passing model, is an extra level of indirection and therefore some run-time overhead. However, the \mathcal{G} compiler is free to optimize calls to merge, through function specialization and constant-propagation through the dictionaries, to remove the run-time overhead in situations where performance outweighs separate compilation. Jones [42] describes this optimization in the context of Haskell type classes.

4 The Indiana Proposal

After the Kona meeting in 2003, two C++ standards meetings (one year) elapsed without an updated proposal from the Texas A&M team. In the Fall of 2004, the Indiana team began to worry about the progress on concepts with respect to getting an extension into C++0x. Around the same time, Douglas Gregor arrived at Indiana University to do a post-doc with Andrew Lumsdaine, and Douglas was excited to join in the design effort and to implement concepts in the GNU C++ compiler. The prototype would be named ConceptGCC. The Indiana team began in earnest to develop a proposal for C++0x in the Fall of 2004. The design differed from \mathcal{G} in several respects, which we discuss in the next few paragraphs.

Achieving separate compilation for C++ was a non-starter because we had to maintain backwards compatibility, and existing features of C++ such as user-defined template specializations interfere with separate compilation (as discussed in Section 2.2). With that in mind, it made sense to use the late-bound approach to concept-based overload resolution [39].

In general, performance is a high priority for C++ programmers, so we needed an implementation that would yield zero run-time overhead. The dictionary-passing implementation used in \mathcal{G} comes with roughly the same overhead as virtual method dispatch. To improve upon this, we devised an approach that instead resolves all uses of concept operations at compile time [33].

The third difference between the Indiana proposal for C++ and \mathcal{G} was that models would not be lexically scoped but instead they would reside in the same namespace as their concept. This design choice was made primarily to simplify the implementation but it meant that model definitions would not be modular.

Compared to the Texas proposal, there were also several differences. Instead of usage patterns to specify concept operations, the Indiana proposal went with pseudo-signatures, a design that the Texas team had considered but discarded [92]. The idea with pseudo-signatures is to relax the rules for matching up functions to concept operations. Consider the example in Figure 18 in which class A models concept C (written in the syntax of the Indiana proposal). The concept requires a function f taking two parameters of type A and B and has return type A. (In this case, A is substituted for T.) However, the only function named f in the scope of the model definition has both parameters at type A and a return type of B. With pseudo-signatures, this function f satisfies the requirement because B is implicitly convertible to A.

The strength of pseudo-signatures is that it provides flexibility for the client of a template while at the same time providing precision for the implementor of

```
class A {};
class B : public A {};

template<typeid T>
concept C {
    A f(T, B);
};

B f(A, A) { return B(); }

model C<A> { };
// This model is OK because B is convertible to A.
```

Figure 18: Example of a pseudo-signature.

a template. The pseudo-signature approach was straightforward to implement. On the client side, finding implementations that match a pseudo-signature can be accomplished with the normal algorithm for function overload resolution. On the implementation side of a template, a pseudo-signature is treated just like a normal signature for the purposes of type checking. That is, what you see is what you get.

The Indiana proposal also differed from the Texas proposal in that it did not support implicit modeling, but instead relied on model definitions to establish the modeling relationship between a class and a concept. We did not want to expose programmers to the kind of accidental run-time errors discussed in Section 3.3. With respect to operators for combining concepts, the Indiana proposal only included conjunction.

Figure 19 shows an example of a concept definition and constrained template from the Indiana proposal.

The Indiana proposal was completed and submitted as document number N1758 to the C++ standards committee in January of 2005 [85].

5 The Texas Rebuttal

Stroustrup and Dos Reis [97] submitted a revised proposal in April of 2005 and submitted a paper describing their design to POPL 2006 [22, 23]. The proposal moved closer to the Indiana proposal in several respects, but there were significant differences. The revised proposal included where clauses to enable the use of

```
template<typeid lter>
concept InputIterator : EqualityComparable<Iter>,
    CopyConstructible<Iter>, Assignable<Iter> {
  typename value type;
  typename reference;
  require Convertible<reference, value type>;
  lter& operator++(lter&);
  reference operator*(Iter);
};
template<typeid Inlter, typeid Outlter>
where { InputIterator<InIter>, OutputIterator<OutIter>,
         InIter::value type == OutIter::value type }
Outlter copy(InIter a, InIter b, Outlter out) {
  while (a != b) *out++ = *a++;
  return out;
}
```

Figure 19: Example of a concept and constrained template in the Indiana proposal.

multi-type concepts and added associated types. The revised proposal also added model declarations, under the name static_assert. However, unlike the Indiana proposal, model declarations were optional, so the Texas proposal retained implicit modeling. The Texas proposal kept usage patterns for specifying concept operations and an associated technical report defined an algorithm for converting usage patterns into type signatures [22]. The proposal also retained the || and ! operators for combining concepts.

To address the concern regarding accidental run-time errors, discussed in Section 3.3, Stroustrup and Dos Reis [97] proposed using negative model declarations, that is, declarations that a type does not implement a concept. For the example in Section 3.3, they suggested adding a declaration stating that istream_iterator does not model ForwardIterator.

At the Mont Tremblant C++ standards meeting in October 2005, there was considerable disagreement regarding the design of concepts. The Indiana team was not in favor of negative model declarations because the number of properties that a type does not satisfy is open ended and much larger than the number of properties that a type does satisfies. Thus, negative model declarations did not seem like an economical approach to solving this problem.

With respect to usage patterns and the || and ! operators for combining concepts, there was no publicly available compiler implementation, so doubt remained regarding the usability and efficiency of these design choices.

6 The Compromise at Adobe

Alexander Stepanov invited the Texas and Indiana teams to a meeting at Adobe Systems Inc. in San Jose. The goal of the meeting was to resolve the outstanding differences between the two proposals, and indeed, the teams were able to agree on a compromise design. The high points of the compromise were as follows. The joint design would include || and ! operators for combining concepts, but it would use pseudo-signatures instead of usage patterns. With respect to model declarations, the compromise was to have two kinds of concepts. The default kind of concept would require explicit model declarations whereas a concept that started with the keyword **auto** could be modeled implicitly. The Indiana team was receptive to this compromise because it balanced convenience and safety.

Over the next few months, the Texas and Indiana teams worked together to document the compromise design and published the results at OOPSLA 2006 [35]. One of the syntactic changes that happened during this time, thanks to research by Beman Dawes, was that the keyword model was replaced by concept_map to minimize the number of existing C++ programs that would break when upgraded to the new version of C++.

To avoid "too many cooks in the kitchen", Stroustrup and Gregor teamed up to write the compromise proposal to the C++ standards committee, which resulted in the document N2042 [34] in June of 2006 and the revision N2081 in September. The proposal was well received by the C++ standards committee.

7 Proposed Wording and Acceptance

In 2007, work began in earnest to draft the wording that would go into the C++ standard. The dialect of English used in the C++ standard is lovingly referred to as "standardese". Unfortunately, writing standardese is a slow process. Gregor and Stroustrup wrote the initial draft and several revisions, producing the documents N2193, N2307, and N2398. In the Fall of 2007, Widman and I began to pitch in with the writing, and helped produce N2421, N2501, and N2520.

During this time, the design evolved in a few respects. The where keyword

was changed to requires because requires would cause fewer existing C++ programs to break. Also, the || constraints were removed for lack of implementation experience.

The initial implementation approach for concepts required the generation of forwarding functions within each model [33]. While these function calls can be statically resolved (and usually inlined), there was still some run-time overhead with ConceptGCC. Also, some optimizations such as copy-elision could not be applied with the forwarding functions in the way. To address these issues, Gregor devised an alternative compilation approach that did not rely on forwarding functions but instead inlines the body of the forwarding functions into the template [31]. Unfortunately, this approach opened up another hole in the modularity of the type system: there could be ambiguities after instantiation that cause type errors.

Meanwhile, I revisited the decision that model declaration would live in the same namespace as their concept, proposing to allow model declarations to appear in any namespace and to use the normal (lexical) name lookup rules for concept maps [76]. This proposal was referred to as "scoped concept maps". In 2007, scoped concept maps were approved and proposed wording for the standard was drafted by Widman and I [74].

In addition to the work on the concepts feature itself, there was considerable work to update the C++ standard library to use concepts. Gregor, Halpern, Lumsdaine, Marcus, Witt, and I with the help of many others, drafted the changes to the C++ standard, producing N2500, N2502, N2677, and their revisions.

At the September 2008 meeting of the C++ standards committee in San Francisco, the concepts proposal was voted into the working draft for C++0x! In the next few months, the editor of the working draft, Pete Becker, began merging the proposed wording for concepts and the updates to the standard library into the C++ standard. At this point, we believed it would be smooth sailing, with the remaining work on the order of fixing typos and resolving ambiguities in the wording. We hoped that the entire working draft would be finalized in a year or so and then approved as the new ANSI and ISO standard for C++.

8 The Removal of Concepts

In the months following the San Francisco meeting, heated discussions occurred on the C++ standards committee mailing list. There were two threads of discussion. The first thread was kicked off by Howard Hinnant, with the email titled

"Are concepts required of Joe Coder?". The question was whether a programmer using the standard library would need to be aware of concepts. My simple answer is yes. Concepts inform the programmer regarding which types can be used with which templates. In fact, before the addition of the concepts feature to C++, programmers needed to be aware of concepts (in the form of documentation) for the same reason.

However, there was a concern that the addition of concepts would create a learning curve that would be too great for the average C++ programmer. In particular, there was concern that it would be too burdensome for programmers to write lots of concept maps. So an important question is: in which situations does a programmer need to write concepts maps?

The most common place for concept maps is immediately after a class definition. The author of the class knows that the class models various concepts and therefore documents these facts for users of the class, including the C++ compiler. This use of concept maps is analogous to a class inheriting from an abstract base class (that is, an interface). Millions of programmers have learned object-oriented languages and inheritance, so learning to use concept maps cannot be too great of a hurdle. Furthermore, many concepts would be **auto** concepts, for which concept maps are not required.

A less common place for concept maps is when a programmer wants to use one library with another library, but the library authors did not intended the libraries to be used together. In such cases a programmer can use concept maps to retroactively specify that a class implements a concept. Retroactive inheritance has long been desired in object-oriented languages. The *external polymorphism* design pattern provides a workaround [26] for languages without retroactive inheritance and there have been many language extensions that provide retroactive inheritance, such as signatures [5, 52] and aspects [49]. So this use of concept maps is a nice advance compared to traditional forms of inheritance.

In a reaction to the thread "Are concepts required of Joe Coder?" and to move closer to his original design, Stroustrup proposed to remove explicit concepts (concepts that require concept maps) and replace them with *explicit refinement* [94]. However, the semantics of explicit refinement was not clear, so it was very difficult for committee members to evaluate the proposal.

The second thread of discussion concerned the state of implementations of the concepts feature [98]. Although Gregor had implemented a prototype, Concept-GCC, the concept specification had moved beyond the prototype with changes and additions. Also, there were problems with the prototype: bugs and slow compile times, that made it difficult to use ConceptGCC with large generic libraries.

(Gregor is not to blame in this regard, as implementing concepts inside the Gnu C compiler was a heroic task.) The slow compile times also worried many committee members, even though it was an engineering issue and not a theoretical limit that was causing the slow down. An algorithm for fast type checking had been demonstrated in the \mathcal{G} prototype [75, 81].

At the Frankfurt meeting in July 2009, the C++ standard committee voted with the following options.

- 1. Continue with the current specification of concepts.
- 2. Remove explicit concepts and add explicit refinement.
- 3. Remove concepts from the working draft of the C++ standard.

Most of the committee members felt it was too late for major changes, and without Stroustrup supporting the status quo, the overwhelming majority voted to remove concepts. Needless to say, everyone who had been involved in the development of concepts was deeply disappointed.

9 Conclusion

So where do concepts go from here? The next round of C++ standardization will most likely be in five years. However, the more important question is whether C++ programmers, and the C++ committee in particular, will be able to gain experience using concepts so that they can better evaluate the tradeoffs regarding different designs. So there is great need for an implementation of concepts. However, for the next few years, most commercial C++ compiler vendors will be focused on implementing the features that made it into C++0x. Also, implementing concepts is an expensive endeavor, and without consensus in the C++ committee on the design, such an expense is risky from a business point of view.

There is some hope that Douglas Gregor, heading up the C++ compiler team at Apple, Inc., together with Andrew Lumsdaine's research group at Indiana University, will be able to implement concepts within the Clang LLVM-based compiler. On another front, I am leading a group at the University of Colorado to add concepts to the Chapel programming language [18] with funding from the U.S. Department of Defense. While Chapel is somewhat different from C++, this may give some programmers more exposure to concepts and generate feedback regarding the design.

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