Closure-Free Functional Programming in a Two-Level Type Theory

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Compiling monads in GHC Haskell

Source:

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       else return 20
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-O0 Core output:

```haskell
dict1 :: Monad (Reader Int)
dict1 = MkMonad ...

dict2 :: MonadReader (Reader Int)
dict2 = MkMonadReader ...

f :: Reader Bool Int
f = (>>=) dict1 (ask dict2) (\b ->
  case b of
    True  -> return dict1 10
    False -> return dict1 20)
```

Compiling monads in GHC Haskell

-O1 output:

```haskell
f :: Bool -> Int
f b = case b of
  True  -> 10
  False -> 20
```

Optimization is hard!

Example: `mapM` is third-order & rank-2 polymorphic, but almost all use cases should compile to first-order monomorphic code.

```haskell
mapM :: Monad m => (a -> m b) -> [a] -> m [b]
```

Compiling monads in GHC Haskell

$$\begin{tikzcd}[column sep=huge] \mathit{Source} \arrow[r, "\mathit{Elaboration}"'] & \mathit{Core} \arrow[r, "\mathit{Compilation}"'] \arrow[loop above, "\begin{array}{c}\mathit{Aggressive}\\ \mathit{general{-}purpose\ optimization}\end{array}"] & \ldots \end{tikzcd}$$

Proposal

$$
\begin{tikzcd}
\mathit{Source} \arrow[r, "\mathit{Elaboration}"'] & \mathit{Metaprograms} \arrow[r, "\mathit{Compile\text{–}time\ execution}"'] & \mathit{Core} \arrow[loop above, "\mathit{Conservative\ optimization}"] \arrow[r, "\mathit{Compilation}"'] & \ldots
\end{tikzcd}
$$

Proposal

Source in WIP language:

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f : Reader Bool Int
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- Looks similar to Haskell.
- Desugaring & elaboration does slightly more work.
- Compilation to efficient code is formally guaranteed.

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Setup

- *Two-level type theory (2LTT)*:
  - Metalanguage (compile time): dependently typed.
  - Object language (runtime): simply typed, *polarized*.
  - The two are smoothly integrated.
- Most optimizations are implemented in libraries instead of compiler internals.

The 2LTT

- **`MetaTy`**: universe of meta-level types.
- **`Ty`**: universe of object-level types. Polarization: **`ValTy`** and **`CompTy`** are sub-universes of **`Ty`**.

A meta-level program:

```
id : {A : MetaTy} -> A -> A
id x = x
```

An object-level program:

```
data List (A : ValTy) := Nil | Cons A (List A)

myMap : List Int -> List Int
myMap ns := case xs of
  Nil       -> Nil
  Cons n ns -> Cons (n + 10) (myMap ns)
```

Closure-freedom

In the object language:

- Closures are **values**.
- Statically known functions are **computations**.
- If we don't ask for closures, we don't get them!

Essential usage of closures is surprisingly rare!

Closure-freedom is a good indicator of low-cost abstraction.

The 2LTT - interaction between stages

- **Lifting**: for **A : Ty**, we have **⇑A : MetaTy**, as the type of metaprograms that produce **A**-typed object programs.
- **Quoting**: for **t : A** and **A : Ty**, we have **<t>** as the metaprogram which immediately returns **t**.
- **Splicing**: for **t : ⇑A**, we have **~t : A** which runs the metaprogram **t** and inserts its output in some object-level code.
- Definitional equalities: **~<t> ≡ t** and **<~t> ≡ t**.

Staged example

```
map : {A B : ValTy} -> (⇑A -> ⇑B) -> ⇑(List A) -> ⇑(List B)
map f as = <letrec go as := case as of
               Nil       -> Nil
               Cons a as -> Cons ~(f <a>) (go as)
             in go ~as>

myMap : List Int -> List Int
myMap ns := ~(map (\x. <~x + 10>) <ns>)
```

Staged example - with stage inference

```
map : {A B : ValTy} -> (A -> B) -> List A -> List B
map f = letrec go as := case as of
           Nil       -> Nil
           Cons a as -> Cons (f a) (go as)
        in go

myMap : List Int -> List Int
myMap := map (\x. x + 10)
```

A monad for code generation

Type classes (and monads) only exist in the metalanguage.

```
class Monad (M : MetaTy -> MetaTy) where
  return : A -> M A
  (>>=)  : M A -> (A -> M B) -> M B
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`Gen` is a Monad whose effect is **generating object code**:

```
newtype Gen A = Gen {unGen : {R : Ty} -> (A -> ⇑R) -> ⇑R}
instance Monad Gen where ...

runGen : Gen (⇑A) -> ⇑A
runGen (Gen f) = f id
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Generating an object-level **`let`**-definition:

```
gen : {A : Ty} -> ⇑A -> Gen (⇑A)
gen {A} a = Gen $ \k. <let x : A := ~a in ~(k <x>)>
```

A monad for code generation

Metaprogram:

```
foo : Int
foo := ~(runGen $ do
  x <- gen <10 + 10>
  y <- gen <~x * ~x>
  return <~x * ~y>)
```

Code output:

```
foo : Int
foo := let x := 10 + 10 in
       let y := x * x in
       x * y
```

Generating monadic code

We want to define efficient code generation for a monad **`M`**.

__`M`__ extended with __`Gen`__ at the bottom yields the corresponding code generator monad.

For example:

- **`ReaderT (⇑R) Gen (⇑A)`** actions are code generators for **`R -> A`**.
- **`StateT (⇑S) Gen (⇑A)`** actions are code generators for **`S -> (A, S)`**.

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For example:

- `ReaderT (⇑R) Gen (⇑A)` actions are code generators for `R -> A`.
- `StateT (⇑S) Gen (⇑A)` actions are code generators for `S -> (A, S)`.

In each case, we can convert back-and forth, e.g.

```
up   : ⇑(R -> A) -> ReaderT (⇑R) Gen (⇑A)
down : ReaderT (⇑R) Gen (⇑A) -> ⇑(R -> A)
```

Generating monadic code

Metaprogram:

```
action : Int -> Int
action := ~(down $ do
  x <- ask
  x <- ask
  return <~x + ~x>)
```

Output:

```
action : Int -> Int
action x := x + x
```

We get "fusion" for monadic code.

Pattern matching on object-level values

Generativity: metaprograms cannot inspect the structure of object expressions.

But we can generate object-level pattern matches in **`Gen`**:

```
split : ⇑Bool -> Gen MetaBool
split b = Gen $ \k. <case ~b of
  True  -> ~(k MetaTrue)
  False -> ~(k MetaFalse)>
```

`split` generalizes to all object ADT-s and all **`Gen`**-based monads.

Compiling monads - example

```
f : Reader Bool Int                  f : Reader Bool Int
f := do                              f := ~(down $ do
  b <- ask                  ==>        b <- ask
  if b then return 10                  split b >>= \case
       else return 20                     MetaTrue  -> return <10>
                                          MetaFalse -> return <20>)
```

```
        f : Reader Bool Int
==>     f = Reader (\b. case b of
          True  -> 10
          False -> 20)
```

More things

More in the paper and artifact:

- Handling join points in monads.
- Handling mutually recursive blocks.
- Powerful & general stream fusion.
- More metatheory.
- Adaptation as Agda and Typed Template Haskell libraries.

Work in progress:

- Standalone prototype targeting LLVM.
- Deploying the Template Haskell library in the Agda source code, in high-performance generics.

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Thank you!