

Closure-Free Functional Programming in a Two-Level Type Theory

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Compiling monads in GHC Haskell

Input:

```
f :: Reader Bool Int
f = do
  b <- ask
  if b then return 10
      else return 20
```

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-OO output:

```
dict1 :: Monad (Reader Int)
dict1 = MkMonad ...

dict2 :: MonadReader (Reader Int)
dict2 = MkMonadReader ...

f :: Reader Bool Int
f = (>=) dict1 (ask dict2) (\b ->
  case b of
    True  -> return dict1 10
    False -> return dict1 20)
```

Compiling monads in GHC Haskell

-01 output:

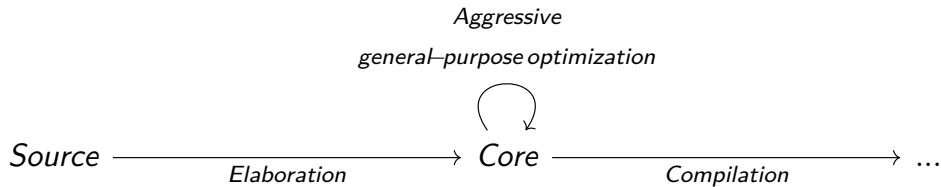
```
f :: Bool -> Int
f b = case b of
  True  -> 10
  False -> 20
```

Optimization is hard!

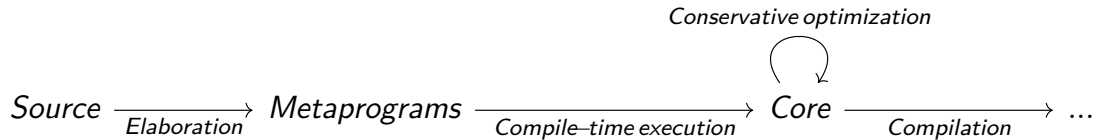
Example: **mapM** is third-order & rank-2 polymorphic, but almost all use cases should compile to first-order monomorphic code.

```
mapM :: Monad m => (a -> m b) -> [a] -> m [b]
```

Compiling monads in GHC Haskell



Proposal



Proposal

Input in WIP language:

```
f : Reader Bool Int
f := do
  b <- ask
  if b then return 10
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```

- Looks similar to Haskell.
- Desugaring & elaboration does slightly more work.
- Compiles to efficient code *with a formal guarantee, without general-purpose optimization.*

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Setup

- We use a *two-level type theory (2LTT)*:
 - Metalanguage (compile time): dependently typed, fancy features.
 - Object language (runtime): simpler & lower-level.
 - The two are smoothly integrated.
- Monadic programs are *metaprograms* which generate efficient runtime code.
- Most optimizations are implemented in libraries instead of compiler internals.

How do we know that an abstraction feature is “low-cost”?

It's a good indicator if the generated code is **free of closures**.

- Functional abstractions are usually implemented by closures: higher-order functions, classes, ML functors.
- A big part of GHC's **-01** work is to get rid of closures.

The 2LTT

- **MetaTy**: universe of meta-level types. Supports Π , Σ , inductive families.
- **Ty**: universe of object-level types. Only simple types. Polarized to *computation & value* types.

A meta-level program:

```
id : {A : MetaTy} → A → A
id x = x
```

An object-level program:

```
data List (A : ValTy) := Nil | Cons A List

myMap : List Int → List Int
myMap ns := case xs of
  Nil      → Nil
  Cons n ns → Cons (n + 10) (myMap ns)
```

The 2LTT - interaction between stages

- **Lifting**: for $A : \mathbf{Ty}$, we have $\uparrow A : \mathbf{MetaTy}$, as the type of metaprograms that produce A -typed object programs.
- **Quoting**: for $t : A$ and $A : \mathbf{Ty}$, we have $\langle t \rangle$ as the metaprogram which immediately returns t .
- **Splicing**: for $t : \uparrow A$, we have $\sim t : A$ which runs the metaprogram t and inserts its output in some object-level code.
- Definitional equalities: $\sim \langle t \rangle \equiv t$ and $\langle \sim t \rangle \equiv t$.

Staged example

```
map : {A B : ValTy} → (↑A → ↑B) → ↑(List A) → ↑(List B)
```

```
map f as = <letrec go as := case as of
```

```
    Nil      → Nil
```

```
    Cons a as → Cons ~(f <a>) (go as)
```

```
in go ~as>
```

```
myMap : List Int → List Int
```

```
myMap ns := ~(map (λ x. <~x + 10>) <ns>)
```

Staged example - with stage inference

```
map : {A B : ValTy} → (A → B) → List A → List B
map f = letrec go as := case as of
      Nil      → Nil
      Cons a as → Cons (f a) (go as)
  in go
```

```
myMap : List Int → List Int
myMap := map (λ x. x + 10)
```

A monad for code generation

Type classes (and monads) only exist in the metalanguage.

```
class Monad (m : MetaTy → MetaTy) where  
  return : a → m a  
  (>>=)   : m a → (a → m b) → m b
```

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Gen is a Monad whose effect is **generating object code**:

```
newtype Gen A = Gen {unGen : {R : Ty} → (A → ↑R) → ↑R}  
instance Monad Gen where ...
```

```
runGen : Gen (↑A) → ↑A  
runGen (Gen f) = f id
```

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Generating an object-level **let**-definition:

```
gen : {A : Ty} → ↑A → Gen ↑A  
gen {A} a = Gen $ λ k. <let x : A := ~a in ~(k <x>)>
```


A monad for code generation

Staged input:

```
myAction : ↑Int → Gen ↑Int
myAction x = do
  y ← gen <~x + ~x>
  z ← gen <~y * ~y>
  return <~y * ~z>
```

```
foo : Int
foo := ~(runGen $ myAction <10>)
```

Output:

```
foo : Int
foo := let y := 10 + 10 in
       let z := y * y in
       y * z
```

Staging monads

We only program in meta-level monads, but also have back-and-forth translations between object-level types and metamonads.

```
down : ReaderT (↑R) Gen (↑A) → ↑(ReaderT. R Identity. A)
```

```
up    : ↑(ReaderT. R Identity. A) → ReaderT (↑R) Gen (↑A)
```

```
f : ReaderT. Bool Identity. Int
```

```
f := ~(down $ do
```

```
  b ← ask
```

```
  b' ← split b
```

```
  case b' of
```

```
    MetaTrue  → return <10>
```

```
    MetaFalse → return <20>)
```

In general: up/down is defined by recursion on a transformer stack. **Identity.** is related to **Gen.**

Case splitting on object values

```
split : MonadGen m => ↑Bool → m MetaBool
split b = liftGen $ Gen $ λ k. <case ~b of
  True  → ~(k MetaTrue)
  False → ~(k MetaFalse)>
```

```
f : ReaderT. Bool Identity. Int
f := ~(down $ do
  b ← ask
  b' ← split b
  case b' of
    MetaTrue  → return <10>
    MetaFalse → return <20>)
```

Polarization & Closure-Freedom

Computation and *value* types are tracked in the object language.

```
_→_      : ValTy → Ty → CompTy  
Closure : CompTy → ValTy  
List    : ValTy → ValTy  
...
```

Closures only appear at runtime if we use **Closure**!

We have to use **Closure** (**A** → **B**) to store functions in ADTs or pass them as function arguments.

(It's rare that closures are *really needed* in programming!)

Polarization & Closure-Freedom

How to compile this?

```
f : Bool → Int → Int  
f b = case b of True  → λ x. x + 10  
                  False → λ x. x * 10
```

And this?

```
f : Int → Int  
f x :=  
  let g y := x + y;  
  g x + 10
```

More things

- Conditionally accepted at ICFP 24: *Closure-Free Functional Programming in a Two-Level Type Theory*.
- More things in paper: join points, stream fusion, semantics, more about polarized types.
- Implementations:
 - In Agda and typed Template Haskell with some limitations.
 - Standalone implementation early WIP.

Thank you!