Closure-Free Functional Programming in a Two-Level Type Theory

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3rd Sept 2024, ICFP 2024, Milan

Source:

```haskell
f :: Reader Bool Int
f = do
  b <- ask
  if b then return 10
       else return 20
```

Compiling monads in GHC Haskell

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```

-O0 Core output:

```haskell
dict1 :: Monad (Reader Int)
dict1 = MkMonad ...

dict2 :: MonadReader (Reader Int)
dict2 = MkMonadReader ...

f :: Reader Bool Int
f = (>>=) dict1 (ask dict2) (\b ->
  case b of
    True  -> return dict1 10
    False -> return dict1 20)
```
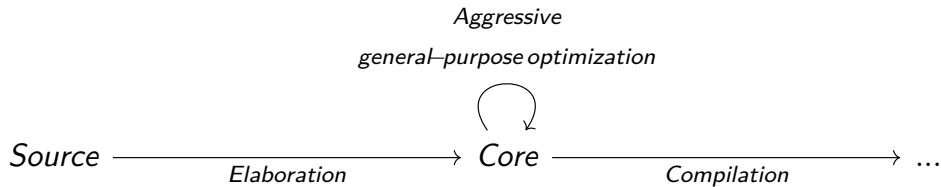
-O1 output:

```haskell
f :: Bool -> Int
f b = case b of
  True  -> 10
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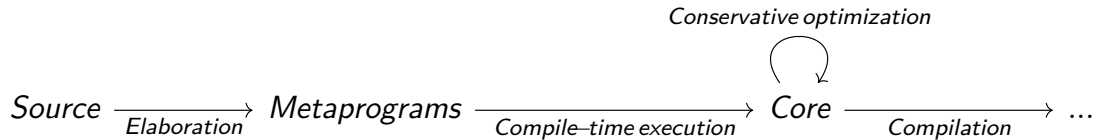
Optimization is hard!

Example: `mapM` is third-order & rank-2 polymorphic, but almost all use cases should compile to first-order monomorphic code.

```haskell
mapM :: Monad m => (a -> m b) -> [a] -> m [b]
```

Aggressive

general–purpose optimization

Source $\xrightarrow{\quad\quad Elaboration \quad\quad}$ *Core* $\xrightarrow{\quad\quad Compilation \quad\quad}$ ...

Source $\xrightarrow[\text{Elaboration}]{}$ Metaprograms $\xrightarrow[\text{Compile-time execution}]{}$ Core $\xrightarrow[\text{Compilation}]{}$ ...

Conservative optimization

Proposal

Source in WIP language:

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f : Reader Bool Int
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```

- Looks similar to Haskell.
- Desugaring & elaboration does slightly more work.
- Compilation to efficient code is formally guaranteed.

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Setup

- *Two-level type theory (2LTT)*:
    - Metalanguage (compile time): dependently typed.
    - Object language (runtime): simply typed.
    - The two are smoothly integrated.
- Most optimizations are implemented in libraries instead of compiler internals.

Closure-freedom

Many abstractions are usually implemented with closures:

- Higher-order functions
- Type classes
- ML functors

A big part of GHC's **-01** work is to get rid of closures.

Closure-freedom is a good indicator of *low-cost* abstraction.

Essential usage of closures is surprisingly rare!

- `MetaTy`: universe of meta-level types. Supports Π, Σ, inductive families.
- `Ty`: universe of object-level types. Polarization: `ValTy` and `CompTy` are sub-universes.

A meta-level program:

```
id : {A : MetaTy} -> A -> A
id x = x
```

An object-level program:

```
data List (A : ValTy) := Nil | Cons A (List A)

myMap : List Int -> List Int
myMap ns := case xs of
  Nil       -> Nil
  Cons n ns -> Cons (n + 10) (myMap ns)
```

- **Lifting**: for `A : Ty`, we have `⇑A : MetaTy`, as the type of metaprograms that produce `A`-typed object programs.
- **Quoting**: for `t : A` and `A : Ty`, we have `<t>` as the metaprogram which immediately returns `t`.
- **Splicing**: for `t : ⇑A`, we have `~t : A` which runs the metaprogram `t` and inserts its output in some object-level code.
- Definitional equalities: `~<t> ≡ t` and `<~t> ≡ t`.

Staged example

```
map : {A B : ValTy} -> (⇑A -> ⇑B) -> ⇑(List A) -> ⇑(List B)
map f as = <letrec go as := case as of
                Nil       -> Nil
                Cons a as -> Cons ~(f <a>) (go as)
             in go ~as>

myMap : List Int -> List Int
myMap ns := ~(map (\x. <~x + 10>) <ns>)
```

```
map : {A B : ValTy} -> (A -> B) -> List A -> List B
map f = letrec go as := case as of
            Nil        -> Nil
            Cons a as -> Cons (f a) (go as)
        in go

myMap : List Int -> List Int
myMap := map (\x. x + 10)
```

A monad for code generation

Type classes (and monads) only exist in the metalanguage.

```
class Monad (M : MetaTy -> MetaTy) where
  return : A -> M A
  (>>=)  : M A -> (A -> M B) -> M B
```

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Gen is a Monad whose effect is **generating object code**:

```
newtype Gen A = Gen {unGen : {R : Ty} -> (A -> ⇑R) -> ⇑R}
instance Monad Gen where ...

runGen : Gen (⇑A) -> ⇑A
runGen (Gen f) = f id
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runGen (Gen f) = f id
```

Generating an object-level **let**-definition:

```
gen : {A : Ty} -> ⇑A -> Gen (⇑A)
gen {A} a = Gen $ \k. <let x : A := ~a in ~(k <x>)>
```

A monad for code generation

Metaprogram:

```
foo : Int
foo := ~(runGen $ do
  x <- gen <10 + 10>
  y <- gen <~x * ~x>
  return <~x * ~y>)
```

Code output:

```
foo : Int
foo := let x := 10 + 10 in
       let y := x * x in
       x * y
```

Generating monadic code

We want to define efficient code generation for a monad `M`.

> *`M` extended with `Gen` at the bottom yields the corresponding code generator monad.*

For example:
- `ReaderT (⇑R) Gen (⇑A)` actions are code generators for `R -> A`.
- `StateT (⇑S) Gen (⇑A)` actions are code generators for `S -> (A, S)`.

In each case, we can convert back-and forth, e.g.

```
up   : ⇑(R -> A) -> ReaderT (⇑R) Gen (⇑A)
down : ReaderT (⇑R) Gen (⇑A) -> ⇑(R -> A)
```

Metaprogram:

```
action : Int -> Int
action := ~(down $ do
  x <- ask
  x <- ask
  return <~x + ~x>)
```

Output:

```
action : Int -> Int
action x := x + x
```

Meta-monads "fuse" at compile time.

Pattern matching on object-level values

Generativity: metaprograms cannot inspect the structure of object expressions.

But we can generate object-level pattern matches in **Gen**:

```
split : ⇑Bool -> Gen MetaBool
split b = Gen $ \k. <case ~b of
  True  -> ~(k MetaTrue)
  False -> ~(k MetaFalse)>
```

split generalizes to all object ADT-s and all **Gen**-based monads.

Example

```
f : Reader Bool Int          f : Reader Bool Int
f := do                      f := ~(down $ do
  b <- ask            ==>       b <- ask
  if b then return 10          split b >>= \case
       else return 20             MetaTrue  -> return <10>
                                  MetaFalse -> return <20>)


                 f : Reader Bool Int
         ==>     f = Reader (\b. case b of
                    True  -> 10
                    False -> 20)
```

More things

More in the paper and artifact:
- Generic handling of join points in monads.
- Powerful & general stream fusion.
- More metatheory.
- Adaptation as Agda and Typed Template Haskell libraries.

Work in progress:
- Standalone prototype targeting LLVM.
- Deploying the Template Haskell library in the Agda source code, to improve generics.

Thank you!