Type-Theoretic Signatures for Algebraic Theories and Inductive Types

Theses of the Ph.D. Dissertation

Author:
András Kovács

Supervisor:
Ambrus Kaposi

Eötvös Loránd University
Doctoral School of Informatics
Head of School: Erzsébet Csuhaj-Varjú
Foundation and Methodology of Informatics Doctoral Program
Program Director: Zoltán Horváth

March 2022

[KK20b] András Kovács and Ambrus Kaposi. Large and infinitary quotient inductive-inductive types. In Holger Hermanns, Lijun Zhang, Naoki Kobayashi, and Dale Miller, editors, *LICS '20: 35th Annual ACM/IEEE Symposium on Logic in Computer Science, Saarbrücken, Germany, July 8-11, 2020*, pages 648–661. ACM, 2020. `doi:10.1145/3373718.3394770`.

[KKA19] Ambrus Kaposi, András Kovács, and Thorsten Altenkirch. Constructing quotient inductive-inductive types. *Proc. ACM Program. Lang.*, 3(POPL):2:1–2:24, 2019. `doi:10.1145/3290315`.

[KKL19] Ambrus Kaposi, András Kovács, and Ambroise Lafont. For finitary induction-induction, induction is enough. In Marc Bezem and Assia Mahboubi, editors, *25th International Conference on Types for Proofs and Programs, TYPES 2019, June 11-14, 2019, Oslo, Norway*, volume 175 of *LIPIcs*, pages 6:1–6:30. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2019. `doi:10.4230/LIPIcs.TYPES.2019.6`.

[Uni13] The Univalent Foundations Program. *Homotopy Type Theory: Univalent Foundations of Mathematics*. `https://homotopytypetheory.org/book`, Institute for Advanced Study, 2013.

Thesis 2

We present syntax and semantics for finitary quotient inductive-inductive (FQII) signatures in Chapter 4 of the thesis. These are close in expressive power to Cartmell's generalized algebraic theories [Car86], but differ in formalization and what kind of semantics results and constructions are built around them.

- FQII signatures can describe most type theories in the wild, thus providing a model theory for them through the semantics of signatures.
- The theory of FQII signatures is specified compactly as a type theory, and it is itself amenable to algebraic specification.
- For each signature a finitely complete category of algebras is given. This category is presented as a cwf (category with families, see [CCD19]) with certain type formers, which makes it possible to exactly compute notions of induction. We show that induction is equivalent to initiality in each category of algebras.
- We show that initial algebras can be constructed from the syntax of FQII signatures, by a term algebra construction. In turn, we show that certain fragments of the syntax of FQII signatures can be reduced to basic type formers, thereby reducing some of the initial algebras to basic type formers.
- We show that substitutions of signatures can be viewed as model constructions, being functors between categories of algebras in the semantics. Additionally, under the assumption that initial FQII-algebras exist, every such functor has a left adjoint.

Thesis 3

In Chapter 5, we modify FQII signatures to obtain infinitary quotient inductive-inductive signatures. This allows us to describe infinitely branching trees as initial algebras.

- Real numbers, surreal numbers, ordinals and the cumulative hierarchy of sets [Uni13] can be now specified using signatures.