

```
In [1]: n = 15
```

```
In [2]: Zn = IntegerModRing(n)
```

```
In [3]: Zn.list()
```

```
Out[3]: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]
```

```
In [5]: Zn(-19)
```

```
Out[5]: 11
```

```
In [6]: Zn(10)+Zn(8)
```

```
Out[6]: 3
```

```
In [7]: Zn(10)* Zn(8)
```

```
Out[7]: 5
```

```
In [8]: Zn(2)*Zn(8)
```

```
Out[8]: 1
```

```
In [9]: 1/Zn(2)
```

```
Out[9]: 8
```

```
In [10]: euler_phi(15)
```

```
Out[10]: 8
```

Sejam p e q primos distintos, e $n = pq$. Definimos $m = \varphi(n) = (p-1)(q-1)$. Seja $e \in \mathbb{Z}_m^*$; ou seja, $e \in \mathbb{Z}_m$ tal que $(e, m) = 1$.

A Chave Pública é (n, e) .

```
In [11]: p = random_prime(2^32, 2^30)
         q = random_prime(2^32, 2^30)
         p ,q
```

```
Out[11]: (3345955037, 1178504321)
```

```
In [12]: n = p*q
```

```
In [15]: #m = euler_phi(n)  !!! NÃO FAZER ISTO !!!
```

```
In [16]: m = (p-1)*(q-1)
```

```
In [18]: Zm = IntegerModRing(m)
         e = Zm.random_element()
         gcd(e, m)
```

Out[18]: 1

```
In [19]: PubKey = (n, e)
```

Bob publica $PubKey = (n, e)$.

Alice quer enviar $mens = 1234$ a Bob.

Alice calcula $cifr = mens^e \mod n$ e envia $cifr$ a Bob.

```
In [20]: mens = 1234
         Zn = IntegerModRing(n)
         cifr = Zn(mens)^e
         cifr
```

Out[20]: 1283134510482137927

```
In [21]: d = 1/e
         d
```

Out[21]: 2765008457644335157

d é a chave privada de Bob.

Bob calcula $cifr^d \mod n$.

```
In [23]: cifr^d
```

Out[23]: 1234

```
In [27]: mod(55, 16)
```

Out[27]: 7

```
In [28]: crt(13, 11, 16, 15)
```

Out[28]: 221

```
In [29]: crt(5, 0, 16, 15)
```

Out[29]: 165

```
In [30]: crt(8, 0, 16, 15)
```

Out[30]: 120

```
In [31]: 16*15*13
```

Out[31]: 3120

In [32]: `mod(1421, 16), mod(1421, 15), mod(1421, 13)`

Out[32]: (13, 11, 4)

In [33]: `mod(1100, 16), mod(1100, 15), mod(1100, 13)`

Out[33]: (12, 5, 8)

In [34]: `crt(crt(9, 1, 16, 15), 12, 16*15, 13)`

Out[34]: 2521

In [35]: `1421+1100`

Out[35]: 2521

In []: