$A_{m\times n}$ $\quad car(A) = n \quad m > n$

$$PA = T = \left[\frac{R}{O}\right]_{m\times n} \qquad R_{n\times n} \text{ t.s invertível} \qquad P^T = P^{-1}$$

$$P^T = [\underbrace{X}_{n} \mid Y]_{m\times m}$$

$CS(X) = CS(A)$ $\quad$ As colunas de X formam uma base ortonormada de $CS(A)$

$\rightarrow \quad CS(X) \subseteq CS(A)$

$$PA = T \underset{\text{LHS } P^T}{\Rightarrow} \underbrace{P^T P}_{=I} A = P^T T \Rightarrow A = P^T T$$

$$\Rightarrow A = [X \mid Y]\left[\frac{R}{O}\right] = X\underbrace{R}_{\text{invertível}}$$

$$v \in CS(X) \Rightarrow \exists_w : v = Xw = \underbrace{XR}_{A}R^{-1}w = A(R^{-1}w)$$

$$\Rightarrow \quad v \in CS(A)$$

$$\dim CS(A) = car(A) = car(X) = \dim CS(X)$$

Logo $\quad CS(X) = CS(A)$

$$\underset{m\times n}{X} = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_n \\ | & | & & | \end{bmatrix} \qquad X^T = \begin{bmatrix} — x_1^T — \\ — x_2^T — \\ \vdots \\ — x_n^T — \end{bmatrix}$$

$$X^T X = \begin{bmatrix} - & x_1^T & - \\ & \vdots & \\ - & x_n^T & - \end{bmatrix} \begin{bmatrix} | & & | \\ x_1 & \cdots & x_n \\ | & & | \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} = I_n$$