

```
In [1]: p = 13
```

```
In [2]: a = 5
```

```
In [3]: legendre_symbol(a, p)
```

```
Out[3]: -1
```

```
In [4]: [power_mod(k, 2, p) for k in range(0, p+1)]
```

```
Out[4]: [0, 1, 4, 9, 3, 12, 10, 10, 12, 3, 9, 4, 1, 0]
```

```
In [5]: legendre_symbol(10, p)
```

```
Out[5]: 1
```

```
In [122... b = 20  
p = 29
```

```
In [123... Zp = IntegerModRing(29)  
Zp
```

```
Out[123]: Ring of integers modulo 29
```

```
In [124... legendre_symbol(b, 29)
```

```
Out[124]: 1
```

```
In [125... Pol = PolynomialRing(Zp, 'x')  
Pol
```

```
Out[125]: Univariate Polynomial Ring in x over Ring of integers modulo 29
```

```
In [126... f = Pol(x^2-b)  
f
```

```
Out[126]: x^2 + 9
```

```
In [127... R = PolynomialQuotientRing(Pol, f, 'a')  
R
```

```
Out[127]: Univariate Quotient Polynomial Ring in a over Ring of integers modulo 29 with modulus x^2 + 9
```

```
In [128... a = R(x)
```

```
In [129... f1 = R(1+a)  
f2 = R(2+3*a)
```

```
In [130... f1, f2
```

```
Out[130]: (a + 1, 3*a + 2)
```

```
In [131... f1*f2
```

Out[131]:  $5a + 4$

```
In [132... z = Zp.random_element()  
z
```

Out[132]: 7

```
In [133... elemento = R(1+z*a)^((p-1)//2)  
elemento
```

Out[133]: 28

```
In [134... elemento[1]
```

Out[134]: 0

```
In [135... while elemento[1] == 0:  
    z = Zp.random_element()  
    elemento = R(1+z*a)^((p-1)//2)
```

```
In [136... u, v = elemento  
elemento
```

Out[136]:  $4a$

```
In [137... u, v
```

Out[137]: (0, 4)

```
In [138... type(v)
```

Out[138]: <class 'sage.rings.finite\_rings.integer\_mod.IntegerMod\_int'>

```
In [139... sol1, sol2, sol3 = -u/v, (1-u)/v, (-1-u)/v
```

```
In [140... sol1^2 == b, sol2^2 == b, sol3^2 == b
```

Out[140]: (False, True, True)

```
In [141... sol1^2, sol2^2, sol3^2
```

Out[141]: (0, 20, 20)

```
In [142... sol2
```

Out[142]: 22

```
In [67]: sol3
```

Out[67]: 6

```
In [68]: b
```

Out[68]: 10