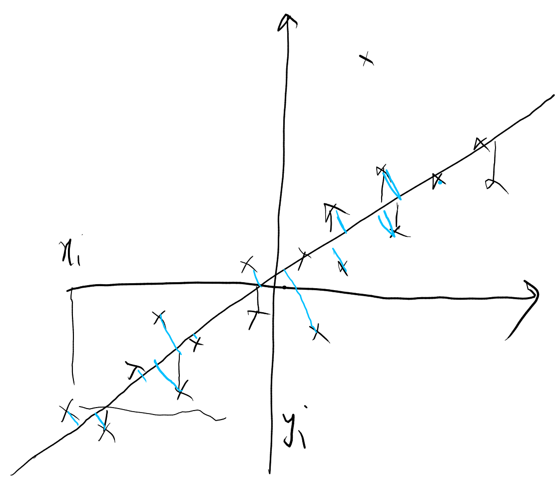
PCA



$$\begin{bmatrix} a_i \\ b_i \end{bmatrix} \quad ; \quad \begin{bmatrix} \bar{a}_i \\ \bar{b}_i \end{bmatrix}$$

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} a_i \\ b_i \end{bmatrix} - \begin{bmatrix} \bar{a}_i \\ \bar{b}_i \end{bmatrix}$$

$$D = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \end{bmatrix}$$

$A_{n\times n}$; polinómio característico de A

$\Delta_A(\lambda) = |\lambda I - A|$; $|M|$ denota o determinante de M

$\deg \Delta_A(\lambda) = n$

As raízes de $\Delta_A(\lambda)$ chamam-se valores próprios de A

$\sigma(A)$ = espectro de A ; $\sigma(A) = \{\lambda \in \mathbb{C} : \Delta_A(\lambda) = 0\}$

$\lambda \in \sigma(A)$; i.e., $\Delta_A(\lambda) = 0$ i.e., $|\lambda I - A| = 0$

$(\lambda I - A)x = 0$ é possível indeterminado

i.e., $\exists\, v \neq 0 : (\lambda I - A)v = 0 \iff \boxed{Av = \lambda v}$

v chama-se vector próprio assoc. ao v.p. $\lambda \in \sigma(A)$

Ex. $A \in \mathbb{R}^{n \times n}$ c/ $\sigma(A) \not\subseteq \mathbb{R}$

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ 1 & \lambda \end{bmatrix}$$

$$|\lambda I - A| = \lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i$$

$$\sigma(A) = \{\pm i\}$$

Def. $H_{n \times n}$ hermítica se $H^* = H$

↳ transconjugada de H

Prop. H hermítica

a) $\sigma(H) \subseteq \mathbb{R}$

b) vectores próprios assoc. a valores próprios $\neq$'s são $\perp$

c) $H = U D U^{-1}$ c/ U unitária, D diagonal

As colunas de U são os vect. p. de H assoc. v.p.

$\lambda_1, \lambda_2, \dots, \lambda_n$ v.p. H , $\lambda_i \in \mathbb{R}$

$v_1, v_2, \dots, v_n$ vect. prop assoc. $\lambda_1, \dots, \lambda_n$

$\|v_i\| = 1$, $v_i \perp v_j$, $i \neq j$

SPG $\lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \dots \leq \lambda_n$

$$D = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} \qquad U = \begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{bmatrix}$$

TODAS AS MATRIZES SÃO REAIS

S simétrica se $S^T = S$

U é ortogonal se $U^{-1} = U^T$

$U \in \mathbb{R}^{n \times n}$; U é ortogonal sse as colunas de U formarem uma base ortonormada de $R(U)$

$A_{m \times n}$ real. ; AA^T , A^TA são simétricas

Defn. S simétrica. Semi-definida positiva (SDP) se $\langle v, Sv\rangle \geq 0$, $\forall v$

DP (definida positiva) se $\langle v, Sv\rangle > 0$, $\forall v \neq 0$

Teor. S SDP $\Rightarrow \sigma(S) \subseteq \mathbb{R}_0^+$

S DP $\Rightarrow \sigma(S) \subset \mathbb{R}^+$

Teor. AA^T simétrica SDP

dem. seja v qq.

$$\langle v, AA^T v\rangle = v^T A A^T v = (A^T v)^T (A^T v)$$

$$= \langle A^T v, A^T v\rangle = \|A^T v\|^2 \geq 0 \quad \square$$

Teor. $\lambda \in \sigma(AA^T)\setminus\{0\} \iff \lambda \in \sigma(A^T A)\setminus\{0\}$

$\sigma(AA^T) \subseteq \mathbb{R}_0^+$; $\sigma(A^T A) \subseteq \mathbb{R}_0^+$

m atributos , n leituras

$$x_i = \begin{bmatrix} x_i^1 \\ x_i^2 \\ \vdots \\ x_i^m \end{bmatrix} \qquad x_1, \dots, x_n$$

$$B = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_n \\ | & | & & | \end{bmatrix}$$

$\frac{1}{n-1} BB^T$ =: matriz covariância

$\alpha \neq 0$, M SDP

αM $\qquad \sigma(\alpha M) = \{\alpha \lambda : \lambda \in \sigma(M)\}$