Compilers

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Syntactic Analysis

Sample Exercises and Solutions

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Problem 1: Give the definition of Context Free Grammar (CFG).

Answer: The tuple G = {NT, T, S ∈ NT, P: NT → (NT ∪ T)*}, i.e., a set of non-terminal variable symbols, a set of terminal symbols (or tokens), a start symbol from the set of non-terminals and a set of productions that map a given non-terminal to a finite sequence of non-terminal and terminal symbols (possibly empty).

Problem 2: Argue that the language of all strings of the form {{...{}...}} (equal number of '{' and '}') is not regular. Give CFG that accepts precisely that language.

Answer: If it were regular there would be a DFA that would recognize it. Let's suppose that there is such a machine M. Given that the length of the input string can be infinite and that the states of M are in finite number, then, there must be a subsequence of the input string that leads to a cycle of states in M. Without loss of generality we can assume that that substring that induces a cycle in the states of M has only '{' (we can make this string as long as you want). If in fact there were such a machine that could accept this long string then it could also accept the same string plus one occurrence of the sub-string (idea of the pumping lemma). But since this sub-string does not have equal number of '{' and '}' then the accepted string would not be in the language contradicting the initial hypothesis. No such M machine can therefore exist. In fact this language can only be parsed by a CFG. Such CFG is for example, S → { S } | e where e is the epsilon or empty string.

Problem 3: Consider the following CFG grammar,

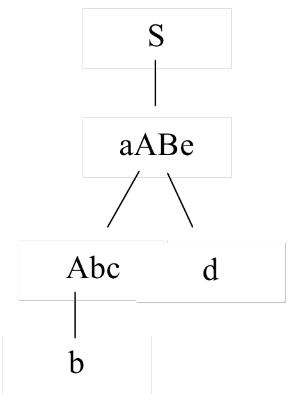
S → aABe
A → Abc | b
B → d

where 'a', 'b', 'c' and 'd' are terminals, and 'S' (start symbol), 'A' and 'B' are non-terminals.

a) Parse the sentence "abbcde" using left-most derivations.
b) Parse the sentence "abbcde" using right-most derivations.
c) Draw the parse tree.

Answer:

a) S → aABe → aAde → aAbcde → abbcde
b) S → aABe → aAbcBe → abbcBe → abbcde
c) Shown below:



Problem 4: Consider the following (subset of a) CFG grammar

stmt → NIL | stmt ';' stmt | ifstmt | whilestmt
ifstmt → IF bexpr THEN stmt END
whilestmt → WHILE bexpr DO stmt END

where NIL, ';', IF, THEN, END, WHILE and DO are terminals, and "stmt", "ifstmt", "whilestmt" and "bexpr" are non-terminals.

For this grammar answer the following questions:

a) Is it ambiguous? Is that a problem?
b) Design a non-ambiguous equivalent (subset of a) grammar.

Answers:

a) Is it ambiguous? Why? Is that a problem?

Yes, this language is ambiguous as there are two distinct parse trees for a specific input string. For example the input

b) Design a non-ambiguous equivalent (subset of a) grammar.

Problem 5: Consider the following Context-Free Grammar G = ({S,A,B},S,{a,b},P) where P is

$S \rightarrow AaAb$
$S \rightarrow Bb$
$A \rightarrow \varepsilon$
$B \rightarrow \varepsilon$

(a) Compute the FIRST sets for A, B, and S.
(b) Compute the FOLLOW sets for A, B and S.
(c) Is the CFG G LL(1)? Justify

Answers:

a. The FIRST of a sentential form is the set of terminal symbols that lead any sentential from derived from the very first sentential form. In this particular case A and B only derive the empty string and as a result the empty string is the FIRST set of both non-terminal symbols A and B. The FIRST of S, however, includes “a” as in the first production once can derive a sentential forms that starts with an “a” given that A can be replaced by the empty string. A similar reasoning allows you to include “b” in the FIRST(S). In summary: FIRST(A) = {ε}, FIRST(B) = {ε}, FIRST(S) = {a,b}

b. The FOLLOW set of a non-terminal symbo is the set of terminals that can appear after that non-terminal symbol in any sentential form derived from the grammar’s start symbol. By definition the follow of the start symbol will automatically include the $ sign which represents the end of the input string. In this particular case by looking at the productions of S one can determine right away that the follow of A includes the terminal “a” abd “b” and that the FOLLOW of B includes the terminal "b". Given that the non-terminal S does not appear in any productions, not even in its own productions, the FOLLOW of S is only \$. In summary: FOLLOW(S) = {\$}, FOLLOW(A) = {a,b}, FOLLOW(B) = {b}.

c. YES, because the intersection of the FIRST for every non-terminal symbol in empty. This leads to the parsing table for this LL method as indicated below. As there are no conflict in this entry then grammar is clearly LL(1).

	a	b	\$
S	S→AaAb	S → Bb	
A	A→ε	A→ε	
B		B→ε	

Problem 6: Construct a table-based LL(1) predictive parser for the following grammar G = {bexpr, {bexpr, bterm, bfactor},{not, or, and, (,), true, false}, P } with P given below.

bexpr → bexpr or bterm | bterm
bterm → bterm and bfactor | bfactor
bfactor → not bfactor | (bexpr) | true | false

For this grammar answer the following questions:

(a) Remove left recursion from G.
(b) Left factor the resulting grammar in (a).
(c) Computer the FIRST and FOLLOW sets for the non-terminals.
(d) Construct the LL parsing table.
(e) Verify your construction by showing the parse tree for the input string "true or not (true and false)"

Answers:

(a) Removing left recursion:

bexpr → bterm E'

E' → or bterm E'
→ ε

T' → and bfactor T'
→ ε

bterm → bfactor T'

bfactor → not bfactor
| (bexpr)
| true
| false

(b) Left factoring: The grammar is already left factored.

(c) First and Follow for the non-terminals:

First(bexpr) = First(bterm) = First (bfactor) = {not, (, true, false}
First(E') = {or, ε}
First(T') = {and, ε}
Follow(bexpr) = {$,)}
Follow(E') = Follow(bexpr) = {$,)}
Follow(bterm) = First(E') ∪ Follow(E') = {or,), $}
Follow(T') = Follow(bterm) = {or,), $}
Follow(bfactor) = First(T') ∪ Follow(T') = {and, or,), $}

(d) Construct the parse table:

	or	and	not	(	)	True/false	$
bexpr			bexpr → bterm E'	bexpr → bterm E'		bexpr → bterm E'	
E'	E' → or bterm E'				E'→ ε		E' → ε
bterm			bterm → bfactor T'	bterm → bfactor T'		bterm → bfactor T'	
T'	T' → ε	T' → and bfactor T'			T'→ ε		T'→ ε
bfactor			bfactor → not bfactor	bfactor → (bexpr)		bfactor → true/false	

(e) To verify the construction, at each point we should find the right production and insert it into the stack. These productions at the end create the parse tree. Starting from the initial state and using the information in the parse table:

Stack	Input	Production
$	true or not (true and false)	bexpr-> bterm E'
$E' bterm	true or not (true and false)	bterm-> bfactor T'
$E' T' bfactor	true or not (true and false)	bfactor-> true
$E' T' true	true or not (true and false)	
$E' T'	or not (true and false)	T'->epsilon
$E'	or not (true and false)	E'->or bterm E'
$E' bterm or	or not (true and false)	
$E' bterm	not (true and false)	bterm-> bfactor T'
$E' T' bfactor	not (true and false)	bfactor-> not bfactor
$E' T' bfactor not	not (true and false)	
$E' T' bfactor	(true and false)	bfactor-> (bexpr)
$E' T') bexpr (	(true and false)	
$E' T') bexpr	true and false)	bexpr-> bterm E'
$E' T') E' bterm	true and false)	bterm-> bfactor T'
$E' T') E' T' bfactor	true and false)	bfactor-> true
$E' T') E' T' true	true and false)	
$E' T') E' T'	and false)	T'->and bfactor T'
$E' T') E' T' bfactor and	and false)	
$E' T') E' T' bfactor	false)	bfactor-> false
$E' T') E' T' false	false)	
$E' T') E' T'	)	T'->epsilon
$E' T') E'	)	E'->epsilon
$E' T')	)	
$E' T'	$	T'->epsilon
$E'	$	E'->epsilon
$	$	

bexpr-> bterm E' -> bfactor T' E' -> true T' E' -> true E' -> ture or bterm E' -> true or bfactor T' E' -> true or not bfactor T' E' -> true or not (bexpr) T' E'-> true or not (bterm E') T' E' -> true or not (bfactor T' E') T' E' -> true or not (true T' E') T' E' -> true or not (true and bfactor T' E') T' E' -> or not (true and false T' E') T' E' -> or not (true and false E') T' E'-> or not (true and false) T' E'-> or not (true and false) E' -> or not (true and false)

So there is a leftmost derivation for the input string.

Problem 7: Consider the following grammar for variable and class declarations in Java:

```
<Decl>       → <VarDecl>
              | <ClassDecl>
<VarDecl>    → <Modifiers> <Type> <VarDec> SEM
<ClassDecl>  → <Modifiers> CLASS ID LBRACE <DeclList> RBRACE
<DeclList>   → <Decl>
              | <DeclList> <Decl>
<VarDec>     → ID
              | ID ASSIGN <Exp>
              | <VarDec> COMMA ID
              | <VarDec> COMMA ID ASSIGN <Exp>
```

For this grammar answer the following questions:

a. Indicate any problems in this grammar that prevent it from being parsed by a recursive-descent parser with one token look-ahead. You can simply indicate the offending parts of the grammar above.

b. Transform the rules for <VarDec> and <DeclList> so they can be parsed by a recursive-descent parser with one token look-ahead i.e., remove any left-recursion and left-factor the grammar. Make as few changes to the grammar as possible. The non-terminals <VarDec> and <DeclList> of the modified grammar should describe the same language as the original non-terminals.

Answers:

a. This grammar is left recursive which is a fundamental problem with recursive descendent parsing either implemented as a set of mutually recursive functions of using a tabel-driven algorithm implementation. The core of the issue hás to deal with the fact that when this parsing algorithm tries to expand a production with another production that starts (either by direct derivation or indirect derivation) with the same non-teminal that was the leading (or left-most non-terminal) in the original sentential form, it will have not onsumed any inputs. This means that it can reapply the same derivation sequence without consuming any inputs and continue to expand the sentential form. Given that the size of the sentential fom will have grown and no input tokens will have been consumed the process never ends and the parsing eventually fails due to lack of resources.

b. We can apply the immediate left-recursion elimination technique for <VarDec> and <DecList> by swapping the facotr in the left-recusive production and including an empty production. This results in the revised grammar segments below:

```
<DeclList>   → <Decl> <DecList>
              |   ε

<VarDec>     → ID <VarDec>
              | ID ASSIGN <Exp>
              | ID  COMMA <VarDec>
              | ε
```

Problem 8: Consider the CFG G = {NT = {E,T,F}, T = {a,b,+,*}, P, E } with the set of productions as follows:

(1) E → E + T
(2) E → T
(3) T → T F
(4) T → F
(5) F → F *
(6) F → a
(7) F → b

For the above grammar answer the following questions:

(a) Compute the FIRST and FOLLOW for all non-terminals in G.
(b) Consider the augmented grammar G' = { NT, T, { (0) E' -> E$ } + P, E' }. Compute the set of LR(0) items for G'.
(c) Compute the LR(0) parsing table for G'. If there are shift-reduce conflicts use the SLR parse table construction algorithm.
(d) Show the movements of the parser for the input w = "a+ab*$".
(e) Can this grammar be parsed by an LL (top-down) parsing algorithm? Justify.

Answers:

(a) We compute the FIRST and FOLLOW for the augmented grammar (0) E' → E$

FIRST(E)	= FIRST(T) = FIRST(F) = {a,b}
FOLLOW(E)	= {+,$}
FOLLOW(T)	= FIRST(F) + FOLLOW(E) = {a,b,+,$}
FOLLOW(F)	= {*,a,b,+,$}

(b) Consider the augmented grammars E' → E$ we compute the LR(0) set of items.

I0 = closure({[E' → •E$]})
= E' → •E$
E → •E + T
E → •T
T → •T F
T → •F
F → •F *
F → •a
F → •b

I1 = goto (I0,E)
= closure({[E' → E•$],[E → E• + T]})
= E' → E• $
E → E• + T

I2 = goto (I0,T)
= closure({[E → T•],[T → T• F]})
= E → E•
T → T•F
F → •F *
F → •a
F → •b

I 3 = goto (I0, F)
= closure({[T → F•],[F →F• *]})
= T → F•
F →F• *

I 4 = goto (I2,a)
= closure({[F → a•]})
= F → a•

I 5 = goto (I2,b)
= closure({[F → b•]})
= F → b•

I 6 = goto (I1, +)
= closure({[E → E+•T]})
= E → E+•T
T → •T F
T → •F
T → •F *
F → •a
F → •b

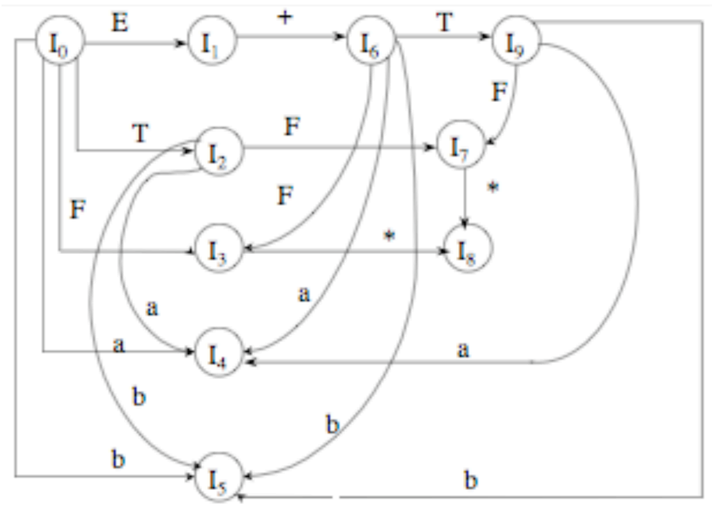
I 3 = goto(I0 ,F)
= closure({[T → F•], [F →F•*]})

I 7 = goto (I2,F)
= closure({[T → TF•],[F → F•*]})
= T → T F•
F → F•*

I 8 = goto (I3,*)
= closure({[F → F*•]})
= F → F*•

I 9= goto (I6,T)
= closure({[E → E+T•], [E → T•F]})
= E → E + T•
E → T•F
F → •F *
F → •a
F → •b

goto (I 9,a) = I4
goto (I 9,b) = I5
goto (I ,F) = I7



(c) We cannot construct an LR(0) parsing table because states I1, I2, I3, I7 and I9 have shift-reduce conflicts. We use the FOLLOW sets to eliminate the conflicts and build the SLR parsing table below.

State	Action					Goto		
	a	b	+	*	$	E	T	F
0	s4	s5				g1	g2	g3
1			s6		acc			
2	s4	s5	r2		r2			g3
3	r4	r4	r4	s8	r4			
4	r6	r6	r6	r6	r6			
5	r7	r7	r7	r7	r7			
6	s4	s5					g9	g3
7	r3	r3	r3	s8	r3			
8	r5	r5	r5	r5	r5			
9	s4	s5	r1		r1			g7

(d) For example if input = a+ab*$ the parsing is:

$0	s4	
$0a4	r6	F → a
$0F3	r4	T → F
$0T2	r2	E → T
$0E1	s6	
$0E1+6	s4	
$0E1+6a4	r6	F → a
$0E1+6F3	r4	T → F
$0E1+6T9	s5	
$0E1+6T9b5	r7	F → b
$0E1+6T9F7	s8	
$0E1+6T9F7*8	r5	F → F*
$0E1+6T9F7	r3	T → TF
$0E1+6T9	r4	E →E+T
$0E1		accept

(e) No because the grammar is left-recursive.

Problem 9: Consider the following Context-Free Grammar G = ({S,A,B},S,{a,b},P) where P is

(1) S → Aa
(2) S → bAc
(3) S → dc
(4) S → bda
(5) A → d

Show that this grammar is LALR(1) but not SLR(1). To show this you need to construct the set of LR(0) items and see that there is at least one multiply defined entry in the SLR table. Then compute the set of LR(1) items and show that the grammar is indeed LALR(1). Do not forget to use the augmented grammar with the additional production { S '→ S $ }.

Answer:

To begin with, we compute the FIRST and FOLLOW sets for S and A, as FIRST(S) = {b,d} and FIRST(A) ={d} and FOLLOW(A) = {a,c}, FOLLOW(S) = {$} used in computing the SLR table.

We now compute the set of LR(0) items

I0 = closure({S '→•S $}) =
S '→ • S $
S → • Aa
S → • bAc
S → • bda
S → • dc
A → • d

I1 = goto(I0,S) = closure({S '→ S •$}) =
S '→ S •$

I2 = goto(I0,A) = closure({S → A•a}) =
S → A•a

I3 = goto(I0,b) = closure({S → b•Ac, S → b•da }) =
S → b•Ac
S → b•da
A → •d

I4 = goto(I0,d) = closure({S → d•c, A → d• }) =
S → d•c
A → d•

I5 = goto(I2,a) = closure({S → Aa•}) =
S → Aa•

I6 = goto(I3,a) = closure({S → bA•c}) =
S → bA•c

I7 = goto(I3,d) = closure({S → bd•a, A→ d•}) =
S → bd•a
A→ d•

I8 = goto(I4,c) = closure({S → dc•}) =
S → dc•

I9 = goto(I6,c) = closure({S → bAc•}) =
S → bAc•

I10 = goto(I7,a) = closure({S → bda•}) =
S → bda•

The parsing table for this grammar would have a section corresponding to states I4 and I7 with conflicts. In states I4 on the terminal c the item S → d•c would prompt a shift on c but since FOLLOW(A) = {a,c} the item A → d• would create a reduce action on that same entry, thus leading to a shift/reduce conflicts. A similar situation arises for state I7 but this time for the terminal a. As such this grammar is not SLR(1).

To show that this grammar is LALR(1) we construct its LALR(1) parsing table. We need to compute first the LR(1) sets of items.

I0 = closure({S '→•S \$, \$}) =
S '→ • S \$, \$
S → • Aa, \$
S → • bAc, \$
S → • bda, \$
S → • dc, \$
A → • d, \$/a

I1 = goto(I0,S) = closure({S '→ S •\$, \$}) =
S '→ S •\$, \$

I2 = goto(I0,A) = closure({S → A•a, \$}) =
S → A•a, \$

I3 = goto(I0,b) = closure({[S → b•Ac, \$], [S → b•da,\$]})
=
S → b•Ac, \$
S → b•da, \$
A → •d, \$/c

I4 = goto(I0,d) = closure({[S → d•c ,\$],[A → d•, \$/a]}) =
S → d•c,\$
A → d•, \$/a

I5 = goto(I2,a) = closure({S → Aa•,\$}) =
S → Aa•,\$

I6 = goto(I3,a) = closure({S → bA•c,\$}) =
S → bA•c,\$

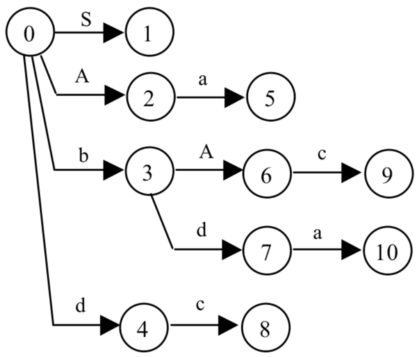
I7 = goto(I3,d) = closure({S → bd•a,\$, A→ d•,\$c}) =
S → bd•a,\$
A→ d•,\$/c

I8 = goto(I4,c) = closure({S → dc•,\$}) =
S → dc•,\$

I9 = goto(I6,c) = closure({S → bAc•,\$}) =
S → bAc•,\$

I10 = goto(I7,a) = closure({S → bda•,\$}) =
S → bda•,\$

In this case, and since we do not have two sets with identical core items, the LALR(1) and LR(1) parsing tables are identical. The DFA build from the set of items and the table is shown below.



State	Action					Goto	
	a	b	c	D	\$	S	A
0		shift 3		shift 4		1	2
1					accept		
2	shift 5						
3				shift 7			6
4	reduce (5)		shift 8		reduce (5)		
5					reduce (1)		
6			shift 9				
7	shift 10		reduce (5)		reduce (5)		
8					reduce (3)		
9					reduce (2)		
10					reduce (4)		

This is an LALR(1) parsing table without any conflicts, thus the grammar is LALR(1).

Problem 10: Given the following CFG grammar G = ({SL}, S, {a, "(",")",","), P) with P:

S → (L) I a
L → L, S I S

answer the following questions:

a) Is this grammar suitable to be parsed using the recursive descendent parsing method? Justify and modify the grammar if needed.
b) Compute the FIRST and FOLLOW set of non-terminal sysmbols of the grammar resulting from your answer in a)
c) Construct the corresponding parsing table using the predictive parsing LL method.
d) Show the stack contents, the input and the rules used during parsing for the input string w = (a,a)

Answers:

a) No because it is left-recursive. You can expand L using a production with L as the left-most symbol without consuming any of the input terminal symbols. To eliminate this left recursion we add another non-terminal symbol, L' and productions as follows:

S → (L) I a
L → S L'
L' → , S L' I ε

b) FIRST(S) = { (, a }, FIRST(L) = { (, a } and FIRST(L') = { , , ε}
FOLLOW(L) = {) }, FOLLOW(S) = { , ,) , $ }, FOLLOW(L') = {) }

c) The parsing table is as shown below:

	(	)	a	,	$
S	S→(L)		S → a		
L	L→S L'		L→ S L'		
L'		L'→ε		L'→ , S L'	

d) The stack and input are as shown below using the predictive, table-driven parsing algorithm:

STACK	INPUT	RULE/OUTPUT
$S	(a,a)$	
$) L (	(a,a)$	S → (L)
$) L	a,a)$	
$) L' S	a,a)$	L → S L'
$) L' a	a,a)$	S → a
$) L'	,a)$	
$) L' S ,	,a)$	L' → , S L'
$) L' S	a)$	
$) L' a	a)$	S → a
$) L'	)$	
$)	)$	S → ε
$	$	

Problem 11: In class we saw an algorithm used to eliminate left-recursion from a grammar G. In this exercise you are going to develop a similar algorithm to eliminate ε-productions, i.e., productions of the form A → ε. Note that if ε is in L(G) you need to retain a least one ε-production in your grammar as otherwise you would be changing L(G). Try your algorithm using the grammar G = {S, {S},{a,b},{S→aSbS, S→bSa, S→ ε}}.

Answer:
Rename all the non-terminal grammar symbols, A1, A2, …, Ak, such that an ordering is created (assign A1 = S, the start symbol).

(1) First identify all non-terminal symbols, Ai, that directly or indirectly produce the empty-string (i.e. epsilon production)

Use the following 'painting' algorithm:

1. For all non-terminals that have an epsilon production, paint them blue.
2. For each non-blue non-terminal symbol Aj, if Aj→W1W2…Wn is a production where Wi is a non-terminal symbol, and Wi is blue for i=1,…,n, then paint Aj blue.
3. Repeat step 2, until no new non-terminal symbol is painted blue.

(2) Now for each production of the form A → X1 X2 … Xn, add A→W1 W2 … Wn such that:

(i) if Xi is not painted blue, Wi = Xi
(ii) if Xi is painted blue, Wi is either Xi or empty
(iii) not all of Wi are empty.

Finally remove all A→ epsilon productions from the grammar.

If S → ε, augment the grammar by adding a new start symbol S' and the productions:

S' → S
S' → ε

Applying this algorithm to the grammar in the question yields the equivalent grammar below:

S'→ S | ε
S → aSb | bSa | abS | ab | ba

Problem 12: Given the grammar G = {S,{S,A,B},{a,b,c,d},P} with set of productions P below compute;

a. LR(1) sets of items
b. The corresponding parsing table for the corresponding shift-reduce parse engine
c. Show the actions of the parsing engine as well as the contents of the symbol and state stacks for the input string w ="bda$".
d. Is this grammar LALR(1)? Justify.

(1) S → Aa
(2) | bAc
(3) | Bc
(4) | bBa
(5) A → d
(6) B → d

Do not forget to augment the grammar with the production S' → S$

Answers:

(a) LR(1) set of items

I0 = closure{[S' → .S, $]}
S'→ .S, $
S → .Aa, $
S → .bAc, $
S → .Bc, $
S →.bBa, $
A →.d, a
B →.d, c

I1 = goto(I0, S)
S'->S., $

I2 = goto(I0, A)
S → A.a, $

I3 = goto(I0, B)
S → B.c, $

I4 = goto(I0, b)
S → b.Ac, $
S → b.Ba, $
A →. d, c
B → .d, a

I5 = goto(I0, d)
A-> d., a
B-> d., c

I6 = goto(I2, a)
S->Aa., $

I7 = goto(I3, c)
S->Bc., $

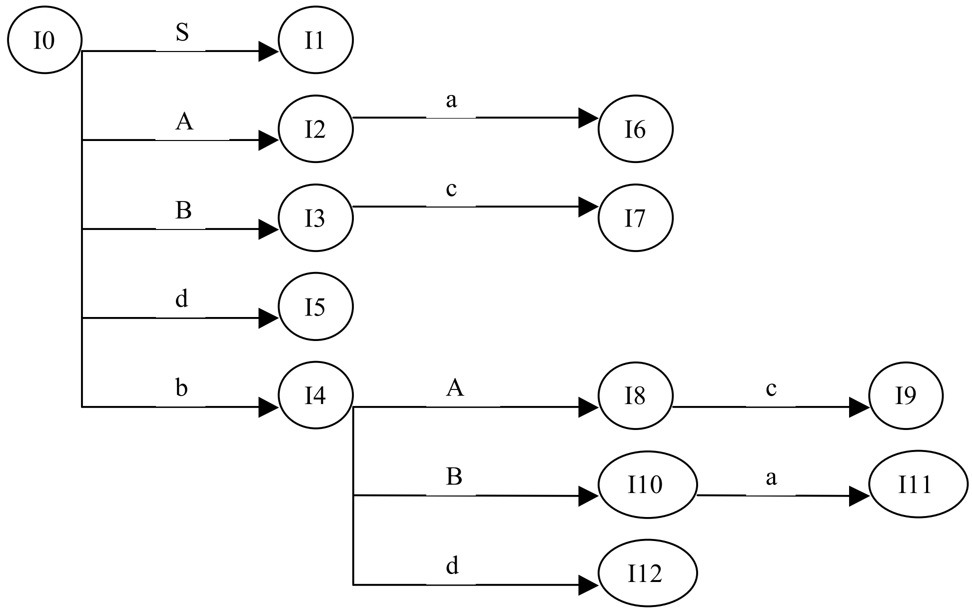
I8 = goto(I4, A)
S->bA.c, $

I9 = goto(I8, c)
S->bAc., $

I10 = goto(I4, B)
S->bB.a, $

I11 = goto(I10, a)
S->bBa., $

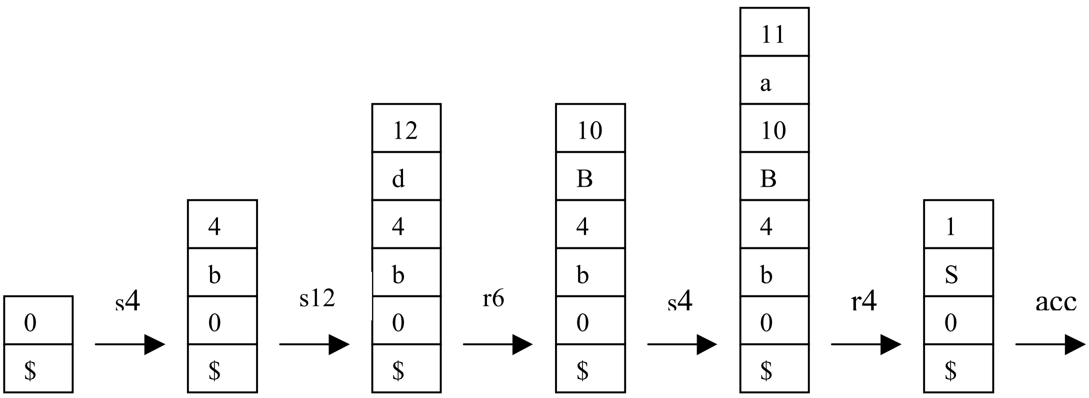
I12 = goto(I4, d)
A-> d., c
B-> d., a



(b) Parsing Table:

Start	Action					Goto		
	a	b	c	d	$	S	A	B
0		S4		S5		1	2	3
1					Acc			
2	S6							
3			S7					
4				S12			8	10
5	R5		R6					
6					R1			
7					R3			
8			S9					
9					R2			
10	S11							
11					R4			
12	R6		R5					

(c) For input string W=bda$, we follow the information in the table, starting from the bottom of the stack and state 0.



(d)