Syntactic Analysis

Introduction to Bottom-Up Parsing

Parsing Techniques

Top-down parsers *(LL(1), recursive descent)*

- Start at the Root of the Parse Tree and grow toward Leaves
- Pick a Production & Try to Match the Input
- Bad "pick" ⇒ May need to Backtrack
- Some Grammars are backtrack-free *(predictive parsing)*

Bottom-up parsers *(LR(1), operator precedence)*

- Start at the Leaves and grow toward Root
- As input is consumed, encode possibilities in an internal state
- Start in a state valid for legal first tokens
- Bottom-up parsers handle a large class of Grammars

Bottom-up Parsing (definitions)

The point of parsing is to construct a derivation

A derivation consists of a series of rewrite steps

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

- Each γ_i is a sentential form
  - If γ contains only terminal symbols, γ is a sentence in $L(G)$
  - If γ contains ≥ 1 non-terminals, γ is a sentential form
- To get γ_i from γ_{i-1}, expand some NT $A \in \gamma_{i-1}$ by using $A \rightarrow \beta$
  - Replace the occurrence of $A \in \gamma_{i-1}$ with β to get γ_i
  - In a leftmost derivation, it would be the first NT $A \in \gamma_{i-1}$

A *left-sentential form* occurs in a *leftmost* derivation

A *right-sentential form* occurs in a *rightmost* derivation

Bottom-up Parsing

A bottom-up parser builds a derivation by working from the input sentence back toward the start symbol S

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

bottom-up

To reduce γ_i to γ_{i-1} match some *rhs* β against γ_i then replace β with its corresponding *lhs*, A. *(assuming the production $A \rightarrow \beta$)*

In terms of the parse tree, this is working from leaves to root

- Nodes with no parent in a partial tree form its *upper fringe*
- Since each replacement of β with A shrinks the upper fringe, we call it a *reduction*.

The parse tree need not be built, it can be simulated

$$|\textit{parse tree nodes}| = |\textit{words}| + |\textit{reductions}|$$

Finding Reductions

Consider the simple grammar

1	*Goal*	→	a *A B* e
2	*A*	→	*A* b c
3		|	b
4	*B*	→	d

Sentential Form	*Next Reduction Prod'n*	*Pos'n*
abbcde	3	2
a *A* bcde	2	4
a *A* de	4	3
a *A B* e	1	4
Goal	—	—

And the input string abbcde

The trick is scanning the input and finding the next reduction

The mechanism for doing this must be efficient

Finding Reductions (Handles)

The parser must find a substring β of the tree's frontier that

matches some production $A \rightarrow \beta$ that occurs as one step in the rightmost derivation *($\Rightarrow \beta \rightarrow A$ is in RRD)*

Informally, we call this substring β a *handle*

Formally,

A *handle* of a right-sentential form γ is a pair $\langle A \rightarrow \beta, \boldsymbol{k} \rangle$ where $A \rightarrow \beta \in P$ and $\boldsymbol{k}$ is the position in γ of β's rightmost symbol.

If $\langle A \rightarrow \beta, \boldsymbol{k} \rangle$ is a handle, then replacing β at $\boldsymbol{k}$ with A produces the right sentential form from which γ is derived in the rightmost derivation.

Because γ is a right-sentential form, the substring to the right of a handle contains only terminal symbols

$\Rightarrow$ the parser doesn't need to scan past the handle *(very far)*

so it needs to recognize a ***viable prefix*** of a right sentential form

Finding Reductions (Handles)

Critical Insight *(Theorem?)*

If G is unambiguous, then every right-sentential form has a unique handle.

If we can find those handles, we can build a derivation !

Sketch of Proof:

1 G is unambiguous $\Rightarrow$ rightmost derivation is unique

2 $\Rightarrow$ a unique production $A \rightarrow \beta$ applied to derive γ_i from γ_{i-1}

3 $\Rightarrow$ a unique position $\boldsymbol{k}$ at which $A \rightarrow \beta$ is applied

4 $\Rightarrow$ a unique handle $\langle A \rightarrow \beta, \boldsymbol{k} \rangle$

This all follows from the definitions

Example (a very busy slide)

1	*Goal*	→	*Expr*
2	*Expr*	→	*Expr* + *Term*
3		|	*Expr* - *Term*
4		|	*Term*
5	*Term*	→	*Term* * *Factor*
6		|	*Term* / *Factor*
7		|	*Factor*
8	*Factor*	→	number
9		|	id
10		|	(*Expr*)

The expression grammar

Prod'n.	*Sentential Form*	*Handle*
—	*Goal*	—
1	*Expr*	1,1
3	*Expr* - *Term*	3,3
5	*Expr* - *Term* * *Factor*	5,5
9	*Expr* - *Term* * <id,y>	9,5
7	*Expr* - *Factor* * <id,y>	7,3
8	*Expr* - <num,2> * <id,y>	8,3
4	*Term* - <num,2> * <id,y>	4,1
7	*Factor* - <num,2> * <id,y>	7,1
9	<id,x> - <num,2> * <id,y>	9,1

Handles for rightmost derivation of x - 2 * y

Handle-pruning, Bottom-up Parsers

The process of discovering a handle & reducing it to the appropriate left-hand side is called *handle pruning*

Handle pruning forms the basis for a bottom-up parsing method

To construct a rightmost derivation

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow w$$

Apply the following simple algorithm

for $i \leftarrow n$ *to 1 by* -1

Find the handle $<A_i \rightarrow \beta_i, k_i>$ in γ_i

Replace β_i *with* A_i *to generate* γ_{i-1}

This takes *2n* steps

Handle-pruning, Bottom-up Parsers

One implementation technique is the *shift-reduce parser*

```
push INVALID
token ← next_token( )
repeat until (top of stack = Goal and token = EOF)
    if the top of the stack is a handle A→β
        then     // reduce β to A
            pop |β| symbols off the stack
            push A onto the stack
        else if (token ≠ EOF)
            then // shift
                push token
                token ← next_token( )
        else    // need to shift, but out of input
            report an error
```

How do errors show up?

- **failure to find a handle**
- **hitting EOF & needing to shift (final else clause)**

Either generates an error

Back to x - 2 * y

Stack	Input	Handle	Action
$	id – num * id	*none*	shift
$ id	– num * id		

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

Back to x - 2 * y

Stack	Input	Handle	Action
$	id – num * id	*none*	shift
$ id	– num * id	9,1	red. 9
$ *Factor*	– num * id	7,1	red. 7
$ *Term*	– num * id	4,1	red. 4
$ *Expr*	– num * id		

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

Back to x - 2 * y

Stack	Input	Handle	Action
$	id - num * id	*none*	shift
$ id	- num * id	9,1	red. 9
$ *Factor*	- num * id	7,1	red. 7
$ *Term*	- num * id	4,1	red. 4
$ *Expr*	- num * id	*none*	shift
$ *Expr* -	num * id	*none*	shift
$ *Expr* - num	* id		

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

Back to x - 2 * y

Stack	Input	Handle	Action
$	id - num * id	*none*	shift
$ id	- num * id	9,1	red. 9
$ *Factor*	- num * id	7,1	red. 7
$ *Term*	- num * id	4,1	red. 4
$ *Expr*	- num * id	*none*	shift
$ *Expr* -	num * id	*none*	shift
$ *Expr* - num	* id	8,3	red. 8
$ *Expr* - *Factor*	* id	7,3	red. 7
$ *Expr* - *Term*	* id		

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

Back to x - 2 * y

Stack	Input	Handle	Action
$	id - num * id	*none*	shift
$ id	- num * id	9,1	red. 9
$ *Factor*	- num * id	7,1	red. 7
$ *Term*	- num * id	4,1	red. 4
$ *Expr*	- num * id	*none*	shift
$ *Expr* -	num * id	*none*	shift
$ *Expr* - num	* id	8,3	red. 8
$ *Expr* - *Factor*	* id	7,3	red. 7
$ *Expr* - *Term*	* id	*none*	shift
$ *Expr* - *Term* *	id	*none*	shift
$ *Expr* - *Term* * id			

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

Back to x - 2 * y

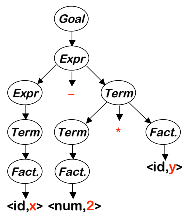
Stack	Input	Handle	Action
$	id - num * id	*none*	shift
$ id	- num * id	9,1	red. 9
$ *Factor*	- num * id	7,1	red. 7
$ *Term*	- num * id	4,1	red. 4
$ *Expr*	- num * id	*none*	shift
$ *Expr* -	num * id	*none*	shift
$ *Expr* - num	* id	8,3	red. 8
$ *Expr* - *Factor*	* id	7,3	red. 7
$ *Expr* - *Term*	* id	*none*	shift
$ *Expr* - *Term* *	id	*none*	shift
$ *Expr* - *Term* * id		9,5	red. 9
$ *Expr* - *Term* * *Factor*		5,5	red. 5
$ *Expr* - *Term*		3,3	red. 3
$ *Expr*		1,1	red. 1
$ *Goal*		*none*	accept

5 shifts +
9 reduces +
1 accept

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

Example

Stack	Input	Action
$	id – num * id	shift
$ id	– num * id	red. 9
$ *Factor*	– num * id	red. 7
$ *Term*	– num * id	red. 4
$ *Expr*	– num * id	shift
$ *Expr* –	num * id	shift
$ *Expr* – num	* id	red. 8
$ *Expr* – *Factor*	* id	red. 7
$ *Expr* – *Term*	* id	shift
$ *Expr* – *Term* *	id	shift
$ *Expr* – *Term* * id		red. 9
$ *Expr* – *Term* * *Factor*		red. 5
$ *Expr* – *Term*		red. 3
$ *Expr*		red. 1
$ *Goal*		accept



Shift-Reduce Parsing

Shift reduce parsers are easily built and easily understood

A shift-reduce parser has just four actions

- *Shift* — next word is shifted onto the stack
- *Reduce* — right end of handle is at top of stack
  - Locate left end of handle within the stack
  - Pop handle off stack & push appropriate *lhs*
- *Accept* — stop parsing & report success
- *Error* — call an error reporting/recovery routine

Accept & Error are simple

Shift is just a push and a call to the scanner

Reduce takes |*rhs*| pops & 1 push

If handle-finding requires state, put it in the stack ⇒ 2x work

Handle finding is key
- handle is on stack
- finite set of handles

⇒ use a DFA !

An Important Lesson about Handles

To be a handle, a substring of a sentential form γ must have two properties:

- It must match the right hand side β of some rule $A \rightarrow \beta$
- There must be some rightmost derivation from the goal symbol that produces the sentential form γ with $A \rightarrow \beta$ as the last production applied

- Simply looking for right hand sides that match strings is not good enough
- Critical Question: How can we know when we have found a handle without generating lots of different derivations?
  - Answer: we use look ahead in the grammar along with tables produced as the result of analyzing the grammar.
  - *LR(1)* parsers build a DFA that runs over the stack & finds them