Lexical Analysis

DFA Minimization

&

Equivalence to Regular Expressions

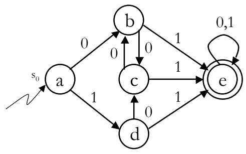
DFA State Minimization

- How to Reduce the Number of States of a DFA?
  - Find unique minimum-state DFA (up to state names)
  - Need to recognize the same language
- Normalization
  - Assume every state has a transition on every symbol
  - If not, just add missing transitions to a dead state
- Key Idea
  - Find string *w* that distinguishes states *s* and *t*
- Algorithm
  - Start with accepting *vs.* non-accepting states partition of states
  - Refine state groups on all input sequences, i.e. by tracing all transitions
  - Until no refinement is possible

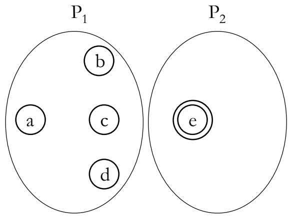
DFA State Minimization

- Algorithm
  - Start with accepting *vs.* non-accepting states partition of states
  - Refine state groups on all input sequences, i.e. by tracing all transitions
  - Until no refinement is possible
- Does this Terminate?
  - Refinement will end; in the limit 1 partition is 1 state
- What to do When Refinement Terminates?
  - Elect representative state for each partition
  - Merge edges
  - Remove unneeded states in each partition

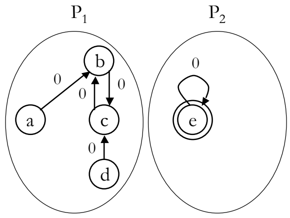
Minimization Example



Minimization Example

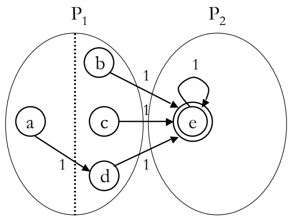


Minimization Example



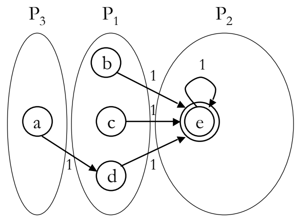
- Label 0 does not split any partition!

Minimization Example



- Label 1 splits P_1 and P_2 partitions!

Minimization Example



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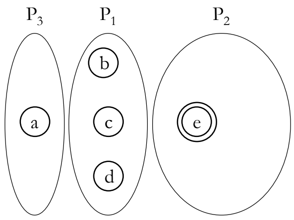

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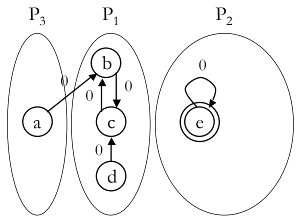

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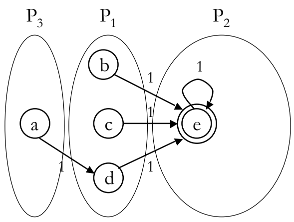

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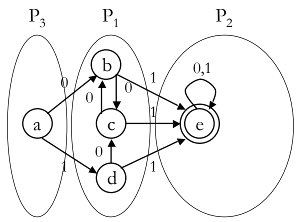
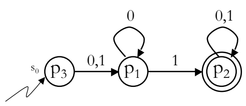


- Elect Representative and Merge Edges

Minimization Example



- Elect Representative and Merge Edges

DFA State Minimization: Algorithm

DFA = $\{D, \Sigma, d, s_0, D_F\}$

$P \leftarrow \{D_F, \{D - D_F\}\}$
while (P is still changing)
  $T \leftarrow \varnothing$
  for each set $p \in P$
    $T \leftarrow T \cup \text{Split}(p)$
$P \leftarrow T$

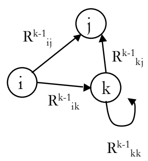
Split(S)
  for each $c \in \Sigma$
    if c splits S into s_1 and s_2
      then return $\{s_1, s_2\}$
  return S

DFA to RE: Kleene Construction

- Path Problem over the DFA
  - Starting from state s_1 (numbering of states is 1 … N - **important**)
  - Label all edges through all states to an accepting state
  - What to do with cycles in the DFA, as they are infinite paths?
- Kleene Construction
  - Iterate and merge path expressions for every pair of nodes i and j not going through any node with label higher then k
  - Increase k up to N
  - In the end do the union of all path expressions that start at s_1 and end in a final state.

DFA to RE: Kleene Construction

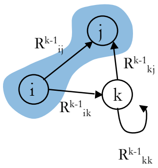
for i = 1 to N
  for j = 1 to N
    $R^0_{ij} = \{a \mid \delta(s_i,a) = s_j\}$
    if (i = j) then
      $R^0_{ij} = R^0_{ij} \mid \{\varepsilon\}$
for k = 1 to N
  for i = 1 to N
    for j = 1 to N
      $R^k_{ij} = R^{k-1}_{ik} (R^{k-1}_{kk})^* R^{k-1}_{kj} \mid R^{k-1}_{ij}$
$L = \mid s_j \in S_F\ R^N_{1j}$



DFA to RE: Kleene Construction

Direct Path

for i = 1 to N
  for j = 1 to N
    $R^0_{ij} = \{a \mid \delta(s_i,a) = s_j\}$
    if (i = j) then
      $R^0_{ij} = R^0_{ij} \mid \{ \varepsilon \}$
for k = 1 to N
  for i = 1 to N
    for j = 1 to N
      $R^k_{ij} = R^{k-1}_{ik} (R^{k-1}_{kk})^* R^{k-1}_{kj} \mid R^{k-1}_{ij}$
$L = |_{s_j \in S_F} R^N_{1j}$



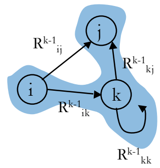
DFA to RE: Kleene Construction

Direct Path

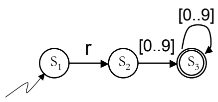
for i = 1 to N
  for j = 1 to N
    $R^0_{ij} = \{a \mid \delta(s_i,a) = s_j\}$
    if (i = j) then
      $R^0_{ij} = R^0_{ij} \mid \{ \varepsilon \}$

Indirect Path

for k = 1 to N
  for i = 1 to N
    for j = 1 to N
      $R^k_{ij} = R^{k-1}_{ik} (R^{k-1}_{kk})^* R^{k-1}_{kj} \mid R^{k-1}_{ij}$
$L = |_{s_j \in S_F} R^N_{1j}$



DFA to RE: Example

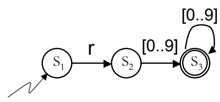


$R^0_{12} = r$

$R^0_{23} = [0..9]$

$R^0_{33} = [0..9] \mid \varepsilon$

DFA to RE: Example



$R^1_{13} = R^0_{11} (R^0_{11})^* R^0_{13} \mid R^0_{13} = nil$

$R^1_{23} = R^0_{21} (R^0_{11})^* R^0_{13} \mid R^0_{23} = [0..9]$

$R^1_{33} = R^0_{31} (R^0_{11})^* R^0_{13} \mid R^0_{13} = [0..9] \mid \varepsilon$

$R^0_{12} = r$

$R^0_{23} = [0..9]$

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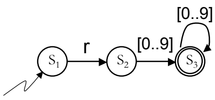
$R^0_{kk} =$ nil otherwise

DFA to RE: Example



$R^0_{12} = r$

$R^0_{23} = [0..9]$

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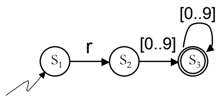
$R^1_{33} = R^0_{31} (R^0_{11})^* R^0_{13} \mid R^0_{13} = [0..9] \mid \varepsilon$

$R^2_{13} = R^1_{12} (R^1_{22})^* R^1_{23} \mid R^1_{13} = r \,.\, \varepsilon^* [0..9]$

$R^2_{33} = R^1_{32} (R^1_{22})^* R^1_{23} \mid R^1_{33} = [0..9] \mid \varepsilon$

$R^2_{33} = R^1_{32} (R^1_{22})^* R^1_{23} \mid R^1_{33} = [0..9] \mid \varepsilon$

DFA to RE: Example



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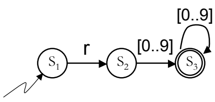
$R^3_{13} = R^2_{13} (R^2_{33})^* R^2_{33} \mid R^2_{13}$

DFA to RE: Example



$R^0_{12} = r$

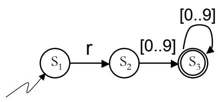
$R^0_{23} = [0..9]$

$R^0_{33} = [0..9] \mid \varepsilon$

$R^0_{kk} =$ nil otherwise

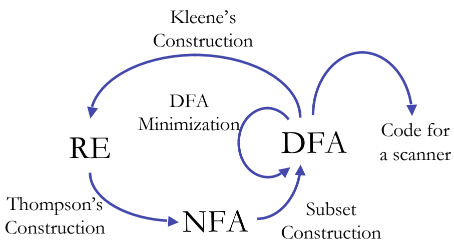
$$R^3_{13} = R^2_{13} (R^2_{33})^* R^2_{33} \mid R^2_{13}$$
$$= (r \,.\, \varepsilon^* [0..9])([0..9]^*)([0..9]) \mid r \,.\, \varepsilon^* [0..9]$$
$$= (r \,.\, [0..9]+) \mid r \,.\, [0..9]$$
$$= (r \,.\, [0..9]+)$$

DFA to RE: Example

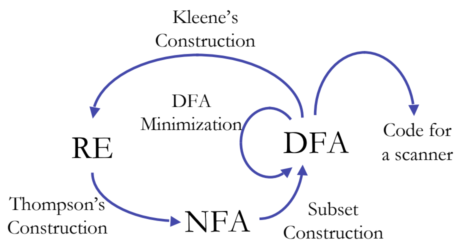


$L(M) = R^3_{13} = r \,.\, [0..9]+$

DFA, NFA and REs



DFA, NFA and REs



Regular Expressions and FA are Equivalent

Summary

- DFA Minimization
  - Find sequence *w* that discriminates states
  - Iterate until no possible refinement
- DFA to RE
  - Kleene construction
  - Combine Path Expression for an increasingly large set of states
- DFA and RE are Equivalent
  - Given one you can an equivalent representation in the other