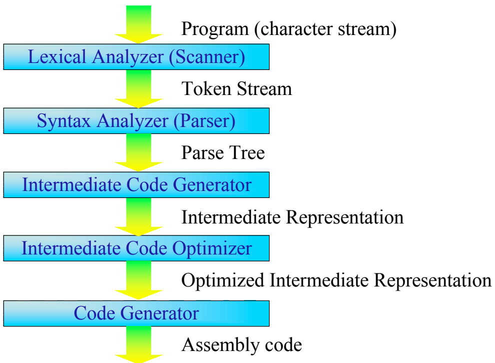
Control Flow Analysis

Outline

- Overview of Optimizations
- Control-Flow Analysis
- Dominators
- Graph Traversal
- Reducible Graphs
- Interval Analysis
- Few Definitions

Anatomy of a Compiler



Example

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, y;
    x = 0;
    y = 0;
    for(i = 0; i <= N; i++) {
      x = x + (4*a/b)*i + (i+1)*(i+1);
      x = x + b*y;
    }
    return x;
}
```

Example in Assembly

```
test:
   subu  $fp, 16
   sw    zero, 0($fp)            # x = 0
   sw    zero, 4($fp)            # y = 0
   sw    zero, 8($fp)            # i = 0
lab1:                            # for(i=0;i<N; i++)
   mul   $t0, $a0, 4             # a*4
   div   $t1, $t0, $a1           # a*4/b
   lw    $t2, 8($fp)             # i
   mul   $t3, $t1, $t2           # a*4/b*i
   lw    $t4, 8($fp)             # i
   addui $t4, $t4, 1             # i+1
   lw    $t5, 8($fp)             # i
   addui $t5, $t5, 1             # i+1
   mul   $t6, $t4, $t5           # (i+1)*(i+1)
   addu  $t7, $t3, $t6           # a*4/b*i + (i+1)*(i+1)
   lw    $t8, 0($fp)             # x
   add   $t8, $t7, $t8           # x = x + a*4/b*i + (i+1)*(i+1)
   sw    $t8, 0($fp)
   ...
```

Example in Assembly

```
...
lw    $t0, 4($fp)          # y
mul   $t1, $t0, a1         # b*y
lw    $t2, 0($fp)          # x
add   $t2, $t2, $t1        # x = x + b*y
sw    $t2, 0($fp)

lw    $t0, 8($fp)          # i
addui $t0, $t0, 1          # i+1
sw    $t0, 8($fp)
ble   $t0, $a3, lab1

lw    $v0, 0($fp)
addu  $fp, 16
b     $ra
```

Let's Optimize...

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, y;
    x = 0;
    y = 0;
    for(i = 0; i <= N; i++) {
      x = x + (4*a/b)*i + (i+1)*(i+1);
      x = x + b*y;
    }
    return x;
}
```

Constant Propagation

```
int sumcalc(int a, int b, int N)
{
    int i;
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```

Algebraic Simplification

```
int sumcalc(int a, int b, int N)
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      x = x;
    }
    return x;
}
```

Copy Propagation

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, y;
    x = 0;
    y = 0;
    for(i = 0; i <= N; i++) {
      x = x + (4*a/b)*i + (i+1)*(i+1);
      x = x;
    }
    return x;
}
```

Copy Propagation

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Common Sub-expression Elimination (CSE)

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int sumcalc(int a, int b, int N)
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    }
    return x;
}
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Common Sub-expression Elimination (CSE)

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, y, t;
    x = 0;
    y = 0;
    for(i = 0; i <= N; i++) {
      t = i+1;
      x = x + (4*a/b)*i + t * t;

    }
    return x;
}
```

Dead Code Elimination

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, y, t;
    x = 0;
    y = 0;
    for(i = 0; i <= N; i++) {
      t = i+1;
      x = x + (4*a/b)*i + t * t;

    }
    return x;
}
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      t = i+1;
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    }
    return x;
}
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    int i;
    int x, t;
    x = 0;

    for(i = 0; i <= N; i++) {
      t = i+1;
      x = x + (4*a/b)*i + t * t;

    }
    return x;
}
```

Loop Invariant Removal

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, t;
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    for(i = 0; i <= N; i++) {
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Loop Invariant Removal

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, t, u;
    x = 0;
    u = (4*a/b);
    for(i = 0; i <= N; i++) {
      t = i+1;
      x = x + u *i + t * t;

    }
    return x;
}
```

Strength Reduction

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, t, u;
    x = 0;
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    for(i = 0; i <= N; i++) {
      t = i+1;
      x = x + u*i + t * t;

    }
    return x;
}
```

```
u*0,  v=0,
u*1,  v=v+u,
u*2,  v=v+u,
u*3,  v=v+u,
u*4,  v=v+u,
...   ...
```

Strength Reduction

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, t, u, v;
    x = 0;
    u = (4*a/b);
    v = 0;
    for(i = 0; i <= N; i++) {
      t = i+1;
      x = x + u*i + t*t;
      v = v + u;
    }
    return x;
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    x = 0;
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    for(i = 0; i <= N; i++) {
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      x = x + v + t*t;
      v = v + u;
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    return x;
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Strength Reduction

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      t = i+1;
      x = x + v + t*t;
      v = v + u;
    }
    return x;
}
```

Strength Reduction

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, t, u, v;
    x = 0;
    u = ((a<<2)/b);
    v = 0;
    for(i = 0; i <= N; i++) {
      t = i+1;
      x = x + v + t*t;
      v = v + u;
    }
    return x;
}
```

Register Allocation

Local variable **x** ← fp
Local variable **y**
Local variable **i**

Register Allocation

~~Local variable **x**~~ ← fp
~~Local variable **y**~~
~~Local variable **i**~~

```
$t9 = x
$t8 = t
$t7 = u
$t6 = v
$t5 = i
```

Optimized Example

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, t, u, v;
    x = 0;
    u = ((a<<2)/b);
    v = 0;
    for(i = 0; i <= N; i++) {
      t = i+1;
      x = x + v + t*t;
      v = v + u;
    }
    return x;
}
```

Optimized Example in Assembly

```
test:
   subu  $fp, 16
   add   $t9, zero, zero      # x = 0
   sll   $t0, $a0, 2   # a<<2
   div   $t7, $t0, $a1 # u = (a<<2)/b
   add   $t6, zero, zero      # v = 0
   add   $t5, zero, zero      # i = 0

lab1:                  # for(i=0;i<N; i++)
   addui$t8, $t5, 1    # t = i+1
   mul   $t0, $t8, $t8 # t*t
   addu  $t1, $t0, $t6 # v + t*t
   addu $t9, t9, $t1   # x = x + v + t*t

   addu $6, $6, $7     # v = v + u

   addui$t5, $t5, 1    # i = i+1
   ble   $t5, $a3, lab1

   addu  $v0, $t9, zero
   addu  $fp, 16
   b     $ra
```

Optimized Example in Assembly

Unoptimized Code

```
test:
      subu      $fp, 16
      sw        zero, 0($fp)
      sw        zero, 4($fp)
      sw        zero, 8($fp)
lab1:
      mul       $t0, $a0, 4
      div       $t1, $t0, $a1
      lw        $t2, 8($fp)
      mul       $t3, $t1, $t2
      lw        $t4, 8($fp)
      addui     $t4, $t4, 1
      lw        $t5, 8($fp)
      addui     $t5, $t5, 1
      mul       $t6, $t4, $t5
      addu      $t7, $t3, $t6
      lw        $t8, 0($fp)
      add       $t8, $t7, $t8
      sw        $t8, 0($fp)
      lw        $t0, 4($fp)
      mul       $t1, $t0, a1
      lw        $t2, 0($fp)
      add       $t2, $t2, $t1
      sw        $t2, 0($fp)
      lw        $t0, 8($fp)
      addui     $t0, $t0, 1
      sw        $t0, 8($fp)
      ble       $t0, $a3, lab1

      lw        $v0, 0($fp)
      addu      $fp, 16
      b         $ra
```

4*ld/st + 2*add/sub + br +
N*(9*ld/st + 6*add/sub + 4* mul + div + br)
= 7 + N*21

Execution time = 43 usec

Optimized Code

```
test:
      subu      $fp, 16
      add       $t9, zero, zero

      sll       $t0, $a0, 2
      div       $t7, $t0, $a1
      add       $t6, zero, zero

      add       $t5, zero, zero

lab1:
      addui     $t8, $t5, 1

      mul       $t0, $t8, $t8
      addu      $t1, $t0, $t6
      addu      $t9, t9, $t1

      addu      $6, $6, $7

      addui     $t5, $t5, 1

      ble       $t5, $a3, lab1

      addu      $v0, $t9, zero
      addu      $fp, 16
      b         $ra
```

6*add/sub + shift + div + br +
N*(5*add/sub + mul + br)
= 9 + N*7

Execution time = 17 usec

Question: Can you optimize…

```
int foobar(int N)
{
   int i, j, k, x, y;
   x = 0;
   y = 0;
   k = 256;
   for(i = 0; i <= N; i++) {
      for(j = i+1; j <= N; j++) {
         x = x + 4*(2*i+j)*(i+2*k);
         if(i>j)
           y = y + 8*(i-j);
         else
           y = y + 8*(j-i);
      }
   }
   return x;
}
```

Question: Can you optimize…

```
int foobar(int N)
{
    int i, j, k, x, y;
    x = 0;
    y = 0;
    k = 256;
    for(i = 0; i <= N; i++) {
       for(j = i+1; j <= N; j++) {
           x = x+8*i*i+4096*i+j*(4*i+2048);
        }
     }
     return x;
}
```

Question: Can you optimize…

```
int foobar(int N)
{
    int i, j, x, t0, t1;
    x = 0;
    t1 = 2048;
    for(i = 0; i <= N-1; i++) {
       t0 = (i*i)<<3 + i<<12;
        x = x + (N-i)*t0;
       for(j = i+1; j <= N; j++) {
          x = x + t1*j;
        }
       t1 = t1 + 4;
     }
     return x;
}
```

Question: Can you optimize…

```
int foobar(int N)
{
    int i, j, x, t0, t1;
    x = 0;
    t1 = 1024;
    for(i = 0; i <= N-1; i++) {
       t0 = (i*i)<<3 + i<<12;
        x = x + (N-i)*t0 + t1*(N*(N+1)-i*(i+1));
       t1 = t1 + 2;
     }
     return x;
}
```

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Constant Propagation

```
int sumcalc(int a, int b, int N)
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    y = 0;
    for(i = 0; i <= N; i++) {
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        x = x + b*y;
    }
    return x;
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```

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      x = x + b*0;
    }
    return x;
}
```

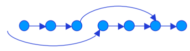
Implementing Constant Propagation

- Find an RHS expression that is a Constant
- Replace the use of the LHS variable with the RHS Constant given that:
  - All paths to the use of LHS passes the assignment of the LHS with the constant
  - There are no intervening definition of the RHS variable
- Need to know the "control-flow" of the program

Representing the Control Flow of a Program

- Most instructions
  - execute the next instruction
  - straight line control-flow

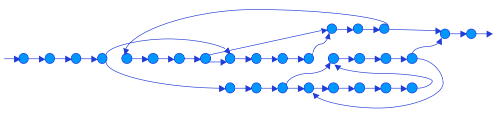
- Jump instructions
  - execute form different location
  - jump in control-flow



- Branch instructions
  - execute either the next instruction or from a different location
  - fork in the control-flow

Representing Control Flow

- Forms a Graph



- A Very Large Graph
- Observations
  - lots of straight-line connections
  - simplify the graph by grouping some instructions

Representing Control Flow

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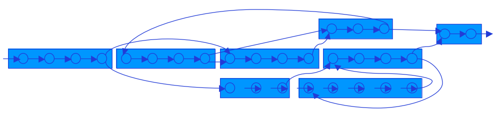

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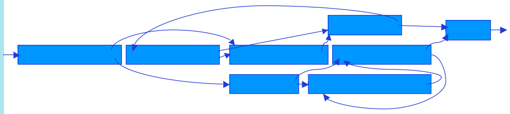

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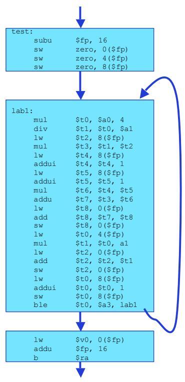
Basic Blocks

- A Basic Block is a maximal sequence of instructions such that
  - Only the first instruction can be reached from outside the basic block
  - All the instructions are executed consecutively if the first instruction is executed
    - No branch or jump instructions in the basic block
    - Except the last instruction
    - No labels within the basic block
    - Except before the first instruction

Basic Blocks: Algorithm

- Input: Sequence of Three-Address Instructions
- Output: A list of Basic Blocks
- Algorithm:
  - Determine the set of *leaders*, the head of a basic block using the following:
    - The first statement of the program is a *leader*
    - Any statement that is the target of a goto (either conditional or not) is a *leader*
    - Any statement that immediately follows a goto or unconditional goto statement is a *leader*
  - For each *leader*, its basic block consists of the *leader* and all the statements up to but not including the next *leader* or the end of the program.

Basic Blocks: Example



Control Flow Graph (CFG)

- Control-Flow Graph G = <N, E>
- Nodes(N): Basic Blocks
- Edges(E): (x,y) ∈ E iff first instruction in the basic block y follows the last instruction in the basic block x
  - First instruction in y is the target of branch or jump instruction (last instruction) in the basic block x
  - first instruction of y is next after the last instruction of x in memory and the last instruction of x is not a jump instruction

Control Flow Graph (CFG)

- Block with the first instruction of the procedure is the entry node (block with the procedure label)
- The blocks with the return instruction (jsr) are the exit nodes.
  - Can make a single exit node by adding a special node

Why Control-flow Analysis ?

- Loops are important to optimize
  - Programs spend a lot of times in loops and recursive cycles
  - Many special optimizations can be done on loops
- Programmers organize code using structured control-flow (if-then-else, for-loops *etc*)
  - optimizer can exploit this
  - but need to discover them first

Challenges in Control-Flow Analysis

- Unstructured Control Flow
  - Use of goto's by the programmer
  - Only way to build certain control structures

```
L1: x = 0
    if (y > 0) goto
  L3
L2: if (y < 0) goto
  L1
L3: y = y + z
    goto L2
```

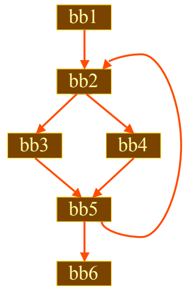
- Obscured Control Flow
  - Method Invocations
  - Procedure Variables
  - Higher-Order Functions
  - Jump Tables

```
Myobject->run()
```

Building CFGs

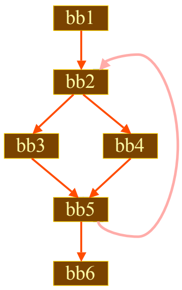
- Simple:
  - Programs are written in structured control flow
  - Has simple CFG patterns
- Not so!
  - Gotos can create different control-flow patterns than what is given by the structured control-flow
  - Need to perform analyses to identify true control-flow patterns

Identifying Recursive Structures Loops



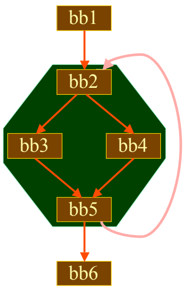
Identifying Recursive Structures Loops

- Identify back edges



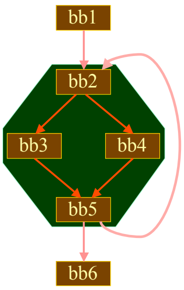
Identifying Recursive Structures Loops

- Identify back edges
- Find the nodes and edges in the loop given by the back edge



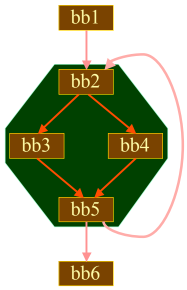
Identifying Recursive Structures Loops

- Identify back edges
- Find the nodes and edges in the loop given by the back edge
- Other than the back edge
  - Incoming edges only to the basic block with the back edge head
  - one outgoing edge from the basic block with the tail of the back edge



Identifying Recursive Structures Loops

- Identify back edges
- Find the nodes and edges in the loop given by the back edge
- Other than the back edge
  - Incoming edges only to the basic block with the back edge head
  - one outgoing edge from the basic block with the tail of the back edge
- How do I find the back edges?



Outline

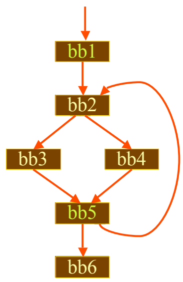
- Overview of Optimizations
- Control-Flow Analysis
- Dominators
- Graph Traversal
- Reducible Graphs
- Interval Analysis
- Few Definitions

Dominators

- Node x dominates node y (x dom y) if <u>every possible</u> execution path from entry to node y includes node x

Dominators

- Is bb1 dom bb5?



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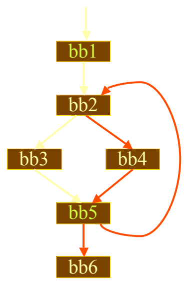

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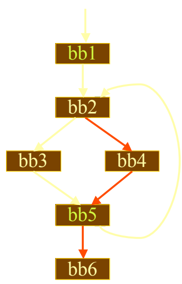

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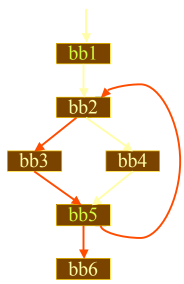

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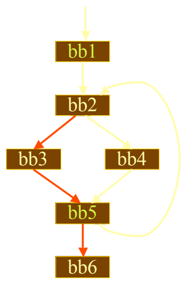

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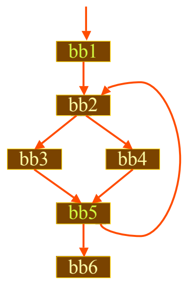

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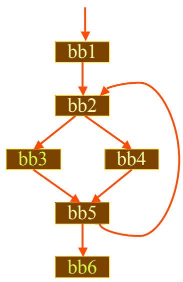
- Is bb1 dom bb5? Yes!
- Is bb3 dom bb6?



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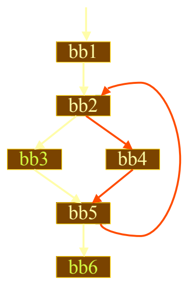

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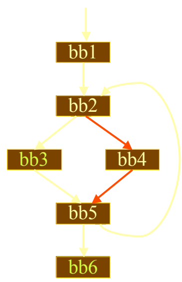

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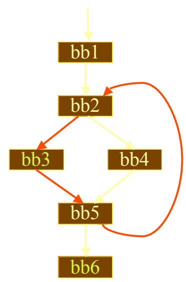

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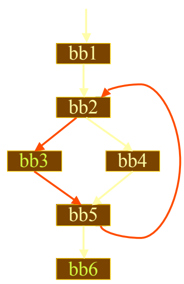

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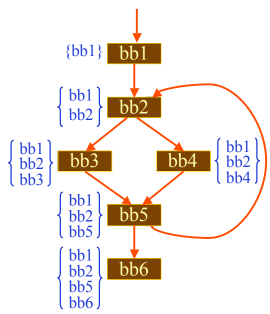

Computing Dominators

- A dom b iff
  - a = b or
  - a is the unique immediate predecessor of b or
  - a is a dominator of all immediate predecessor of b
- Algorithm
  - Make dominator set of the entry node has itself
  - Make dominator set of the rest have all the nodes
  - Visit the nodes in any order
  - Make dominator set of the current node intersection of the dominator sets of the predecessor nodes + the current node
  - Repeat until no change

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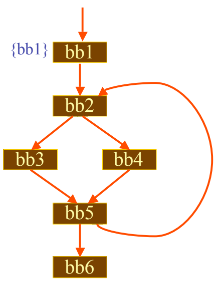

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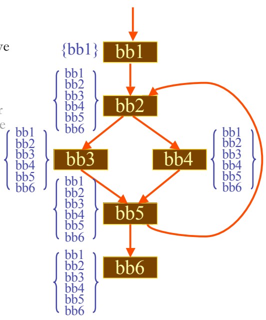

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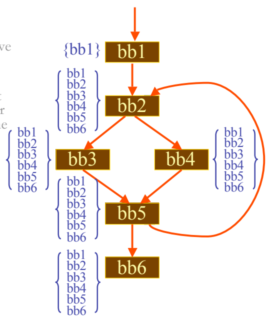

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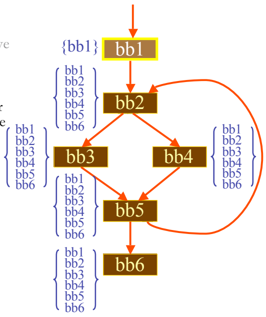

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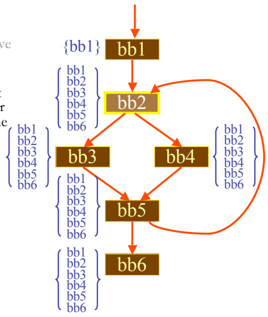

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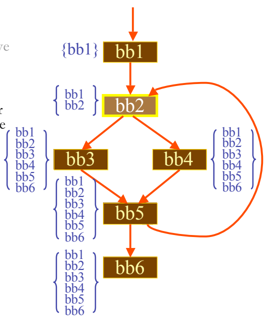

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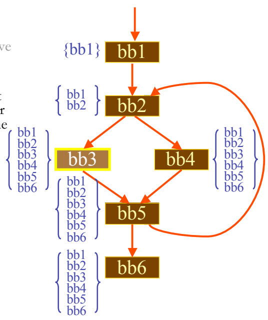

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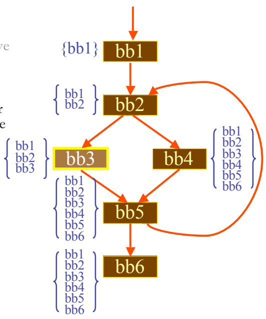

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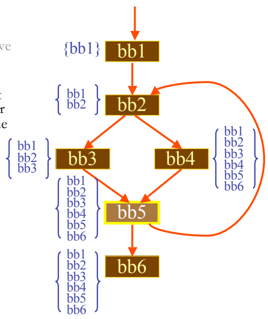

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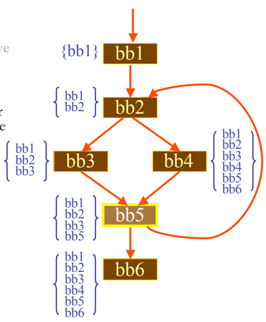

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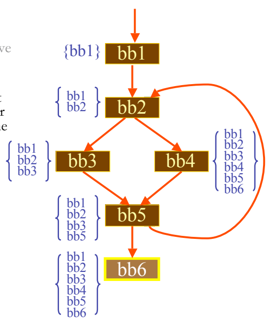

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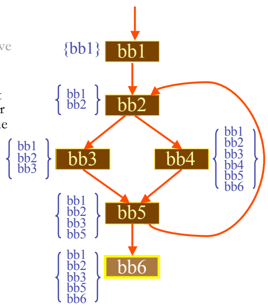

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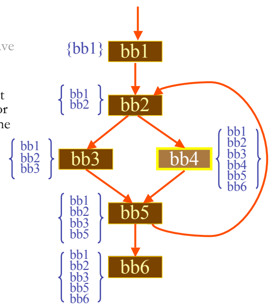

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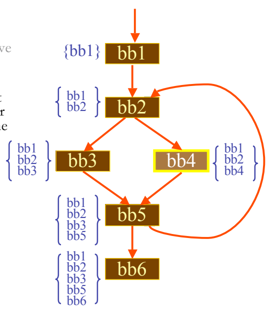

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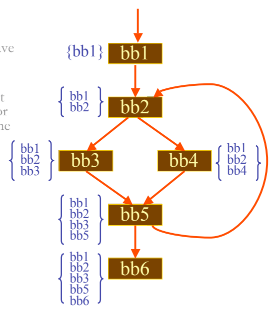

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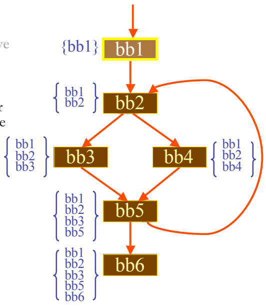

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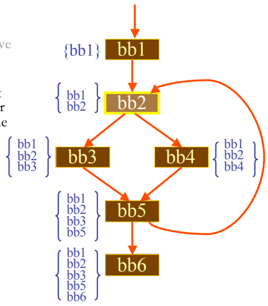

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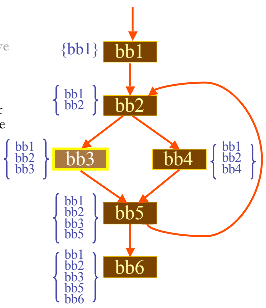

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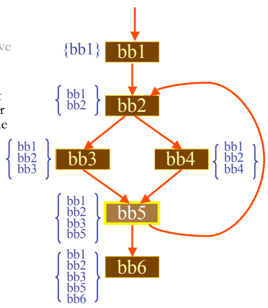

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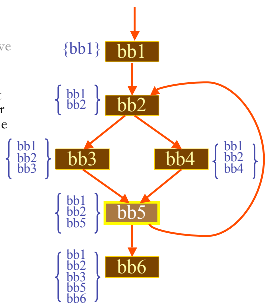

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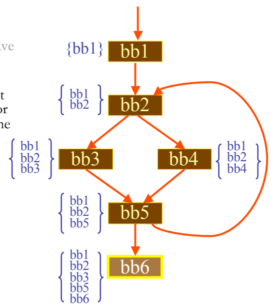

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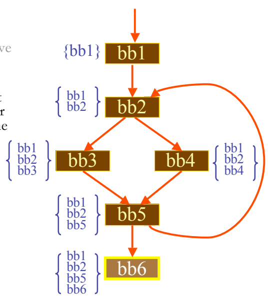

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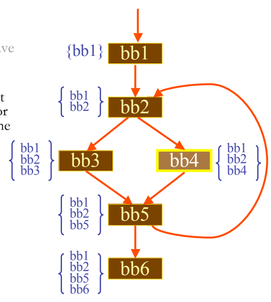

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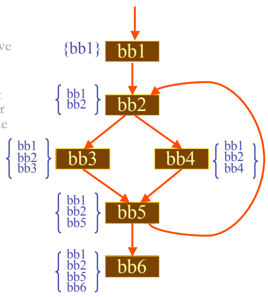

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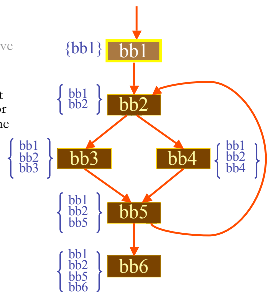

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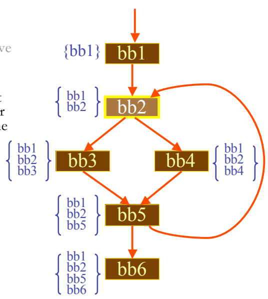

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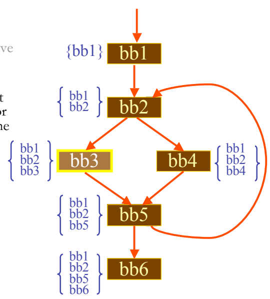

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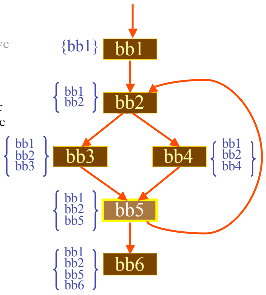

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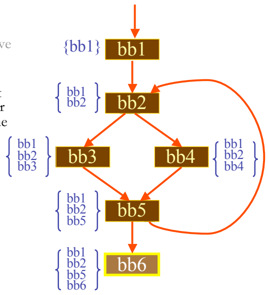
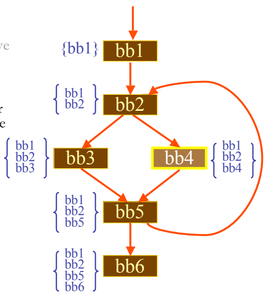


Computing Dominators

- Algorithm
  - Make dominator set of the entry node has itself
  - Make dominator set of the rest have all the nodes
  - Visit the nodes in any order
  - Make dominator set of the current node intersection of the dominator sets of the predecessor nodes + the current node
  - Repeat until no change



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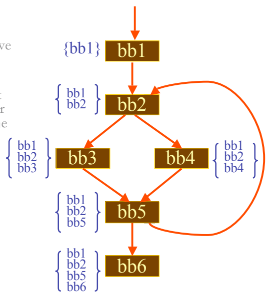

Computing Dominators

- What we just witness was an iterative data-flow analysis algorithm in action
  - Initialize all the nodes to a given value
  - Visit nodes in some order
  - Calculate the node's value
  - Repeat until no value changes
- Will talk about this in the coming lectures

What is a Back Edge?

- An edge (x, y) ∈ E is a back edge iff y dom x
  - is node y in the dominator set of node x

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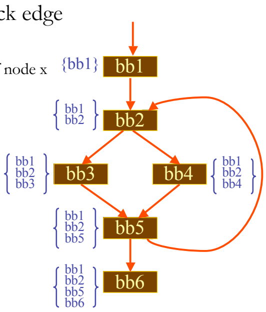

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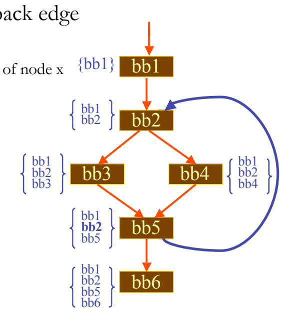


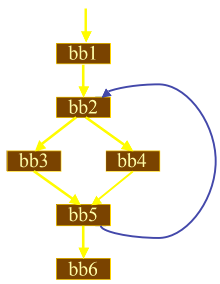
Outline

- Overview of Optimizations
- Control-Flow Analysis
- Dominators
- Graph Traversal
- Reducible Graphs
- Interval Analysis
- Few Definitions

Traversing the CFG

- Depth-First Traversal
  - Visit all the descendants of a node before visiting any siblings
- Depth-first spanning tree
  - a set of edges corresponding to a depth-first visitation of CFG

Depth-First Spanning Tree



Preorder and Postorder

- In preorder traversal, each node is processed before its descendants in the depth-first tree
- In postorder traversal, each node is processed after its descendants in the depth-first tree

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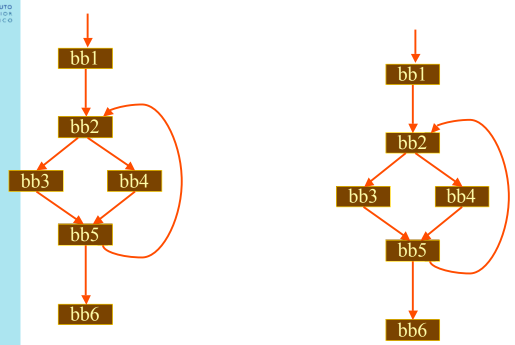
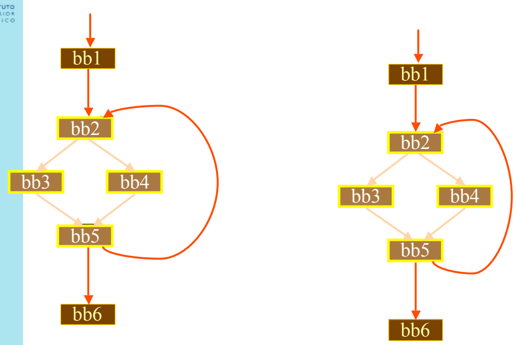
Reducible CFGs

- Reducibility formalizes well structuredness of a program
- A graph is reducible iff repeated application of the following two actions yields a graph with only one node
  - replace self loop by a single node
  - Replace a sequence of nodes such that all the incoming edges are to the first node and all the outgoing edges are to the last node

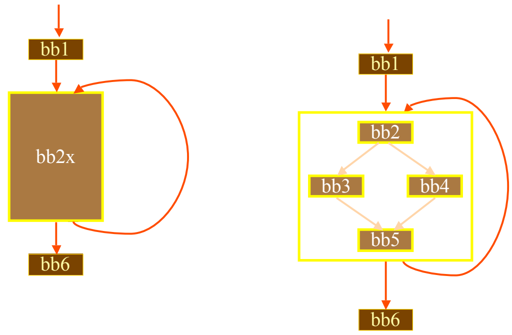
Reducible CFGs



Reducible CFGs



Reducible CFGs



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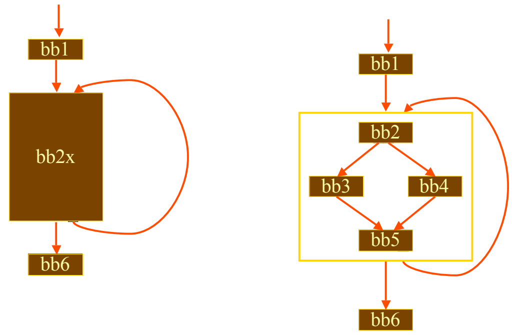
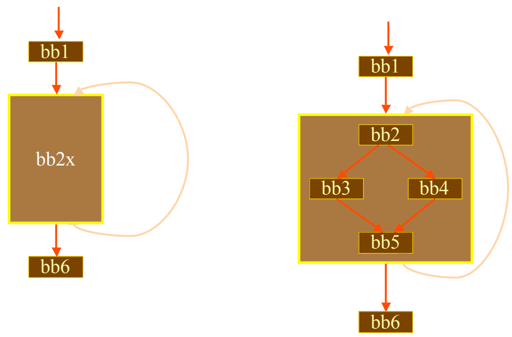
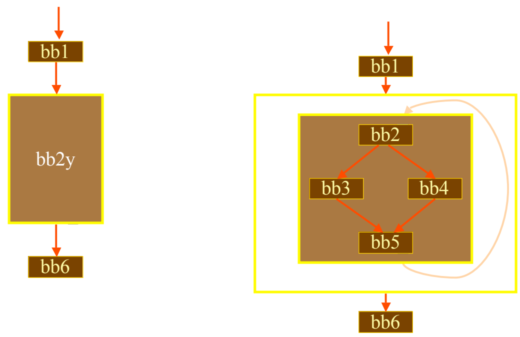


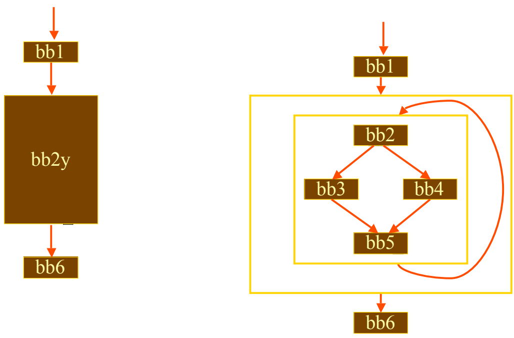
Reducible CFGs



Reducible CFGs



Reducible CFGs



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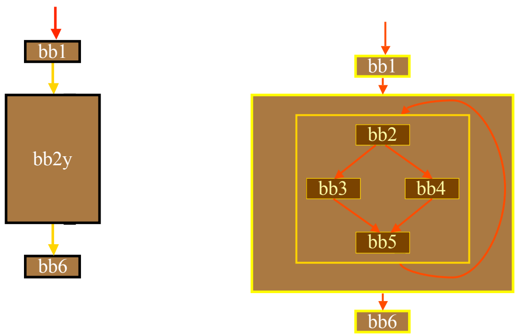

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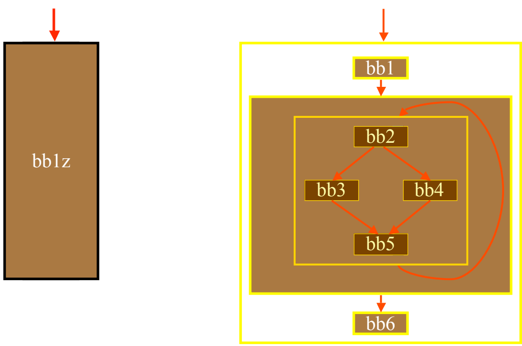

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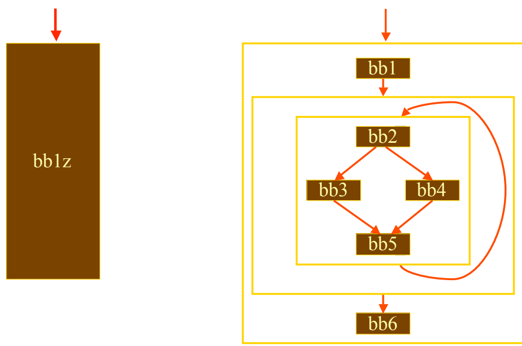
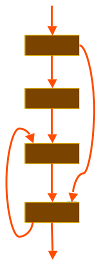
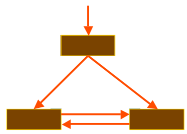


Irreducible graphs



Irreducible graphs



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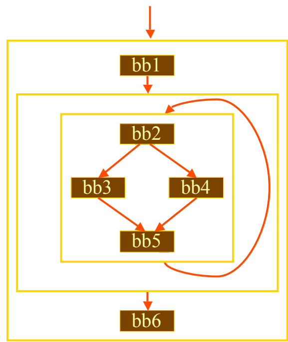
Approaches of Control-Flow Analysis

- Iterative Analysis
  - Use a CFG
  - Propagate values
  - Iterate until no change
- Interval Based Analysis
  - Use a reducible CFG
  - Calculate in hierarchical graphs
  - No iterations (faster)

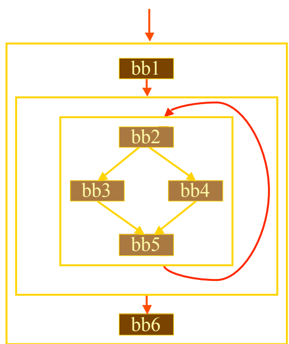
Interval Based Analysis

- If a node does not include a graph:
  - calculate value
- If a node includes a graph
  - Calculate values of the nodes in that graph
  - Propagate values (no back edges ⇒ no iteration)
  - Use entry (or exit) value as the value of the enclosing node

Interval Analysis



Interval Analysis



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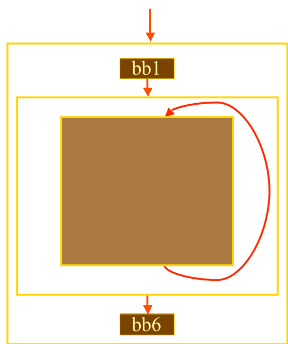

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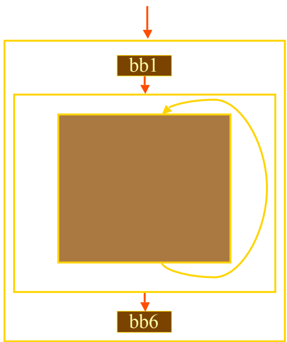
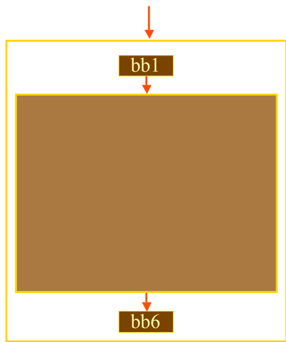


Interval Analysis



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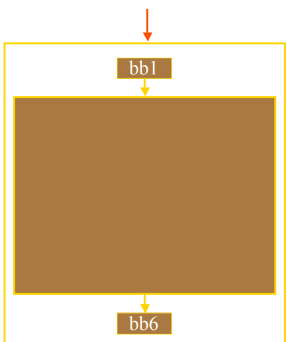

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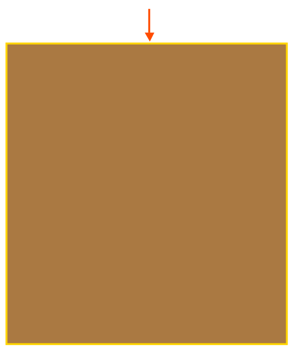

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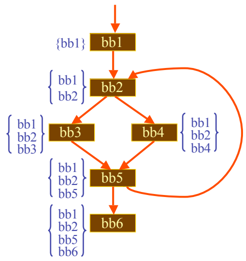
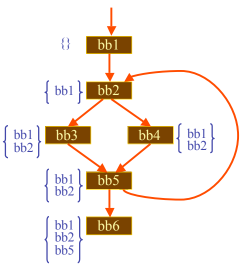
Dominators

- Node x **dominates** node y (x dom y) if every possible execution path from entry to node y includes node x



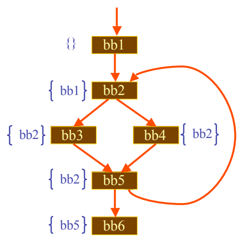
Dominators

- Node x **strictly dominates** node y (x sdom y) if
  - x dom y
  - x ≠ y



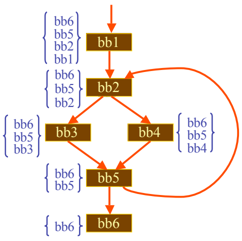
Dominators

- Node x **immediately dominates** node y (x idom y) if
  - x dom y
  - x ≠ y
  - ∃ c ∈ N such that c ≠ x and c ≠ y and x dom c and c dom y

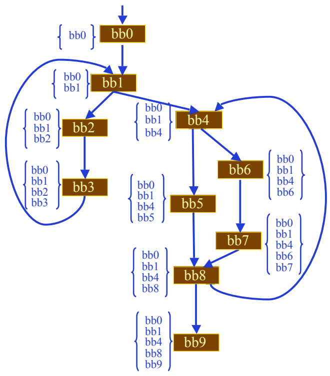


Dominators

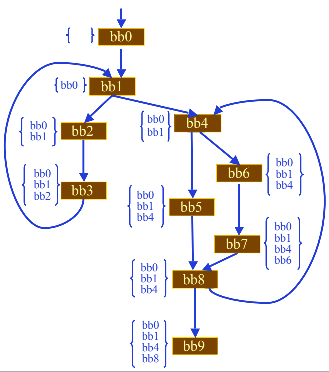
- Node x **post dominates** node y (x pdom y) if every possible execution path from node x to the exit node includes node y



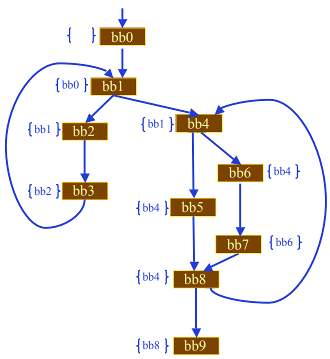
Dominators (dom)



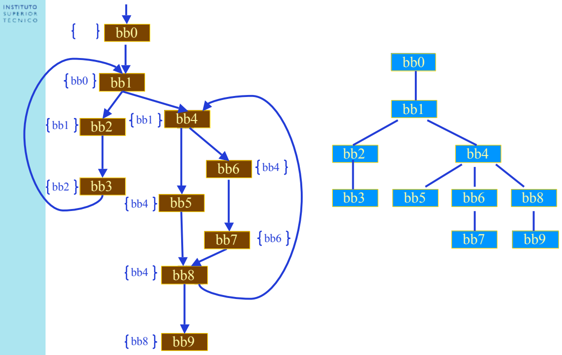
Strictly Dominates (sdom)



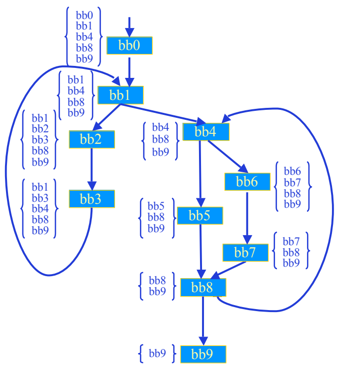
Immediately Dominates (idom)



Dominator Tree



Post Dominators (pdom)



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