Initial Value Problems and Stability

Andreas Hegseth

March 24, 2022

Abstract

1 problem

we are given a IVP y'=-10y, o¡=t¡=10, y(0)=1, with an exact solution y(t)=e**(-10y). To which we will use forward and backward Euler's methods. As well take $\Delta t = .05$, .1, .25, .5 and take these values and put them under the conditions of absolute stability. When all that is done show what conditions are need to make it stable.

2 Mathematics

With IVP we a have many tools to use to solve them. We will focus on Forward Euler and Backwards Euler. First we should mention that these are first order differential equations. Now let focus on Forward Euler, the generalized equation is $y_{n+1} = y_n + \Delta t * f(y_n, t_n)$ as seen it is explicit. Explicit methods calculate the state of the system at a later time from the state of the system at the current time without the need to solve algebraic equations. Since it is this we can choose a $\Delta t < 1$. Know we turn our focus on Backward Euler's method which it's generalized equation is $y_{n+1} = y_n + \Delta t * f(y_{n+1}, t_{n+1})$ unlike forward Euler, Backward Euler is implicit. Which contrary to explicit methods finds the solution by solving an equation involving the current state of the system and the later one. Know to show that these two methods are stable we need to use absolute stability. Both methods have different conditions to show stability. For Forward Euler we have $y' = \lambda y$, with $y(t_0) = y_0$ where $y_{n+1} = (1+\lambda\Delta t)^{(n+1)} * y_0$ and is stable when $|1 - \lambda\Delta t| \leq 1$. With Backwards Euler we have $y' = \lambda y$ with $y(t_0) = y_0$ where $y_{n+1} = (1/(1 - \Delta t)^{n+1} * y_0$ and is stable if $|1/(1 - \Delta t)| \leq 1$ or $|1 - \Delta t| \geq 1$. If these conditions are met then the IVP is stable.

3 algorithm

step 1 import packages numpy and mathplotlib

step 2 set variables ti, yi, tf,h

step 3 set t exact and y exact

step 4 plot the exact answer

step 5 set up Forward Euler approximation and plot it

step 6 set up Backwards Euler approximation and plot it

4 results

With the given result of our codes it show that in the beginning the exact and approximation are not exact. Between $0 \leq t \leq 3$ the difference in answer are of notable but after that the exact and approximation merge.

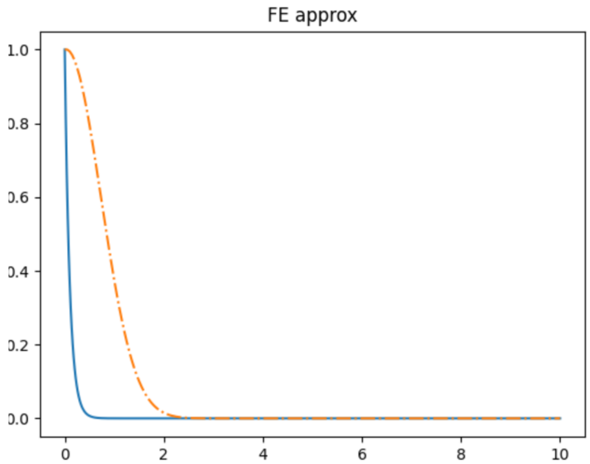

Figure 1: FE approx and exact for h=.05

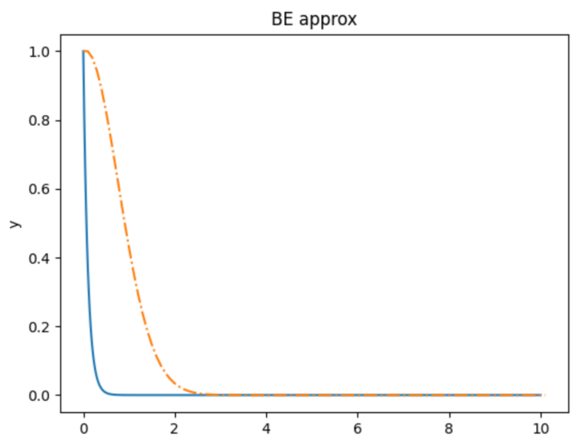

Figure 2: BE approx and exact for h=.05

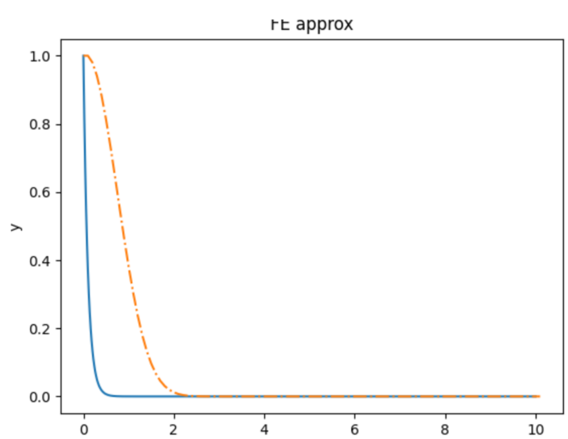

Figure 3: FE approx and exact for h=0.1

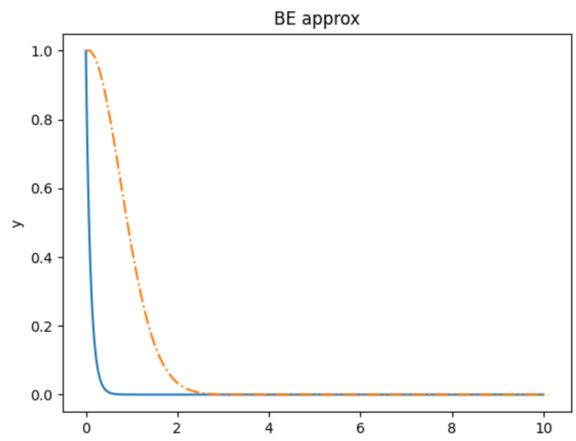

Figure 4: BE approx and exact for h=0.1

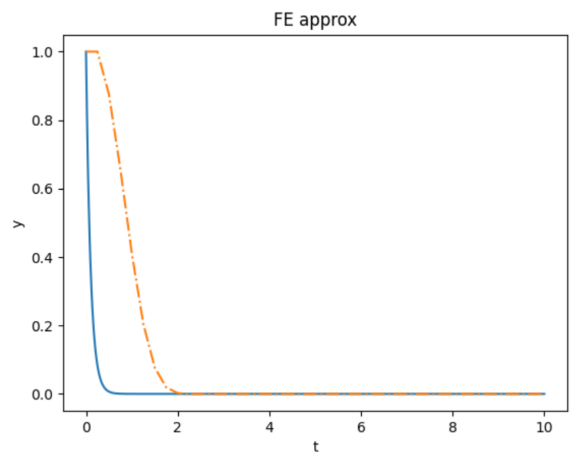

Figure 5: FE approx and exact for h=.25

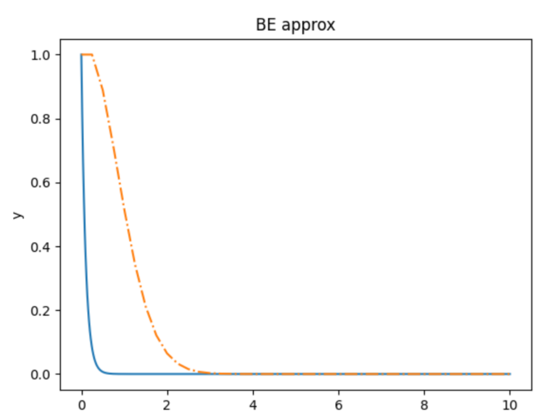

Figure 6: BE approx and exact for h=.25

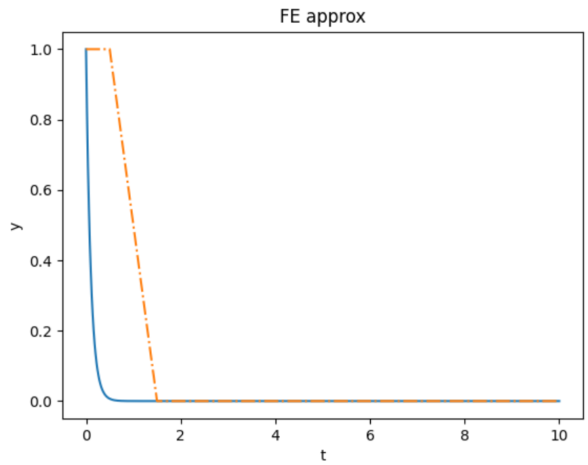

Figure 7: FE approx and exact for h=.5

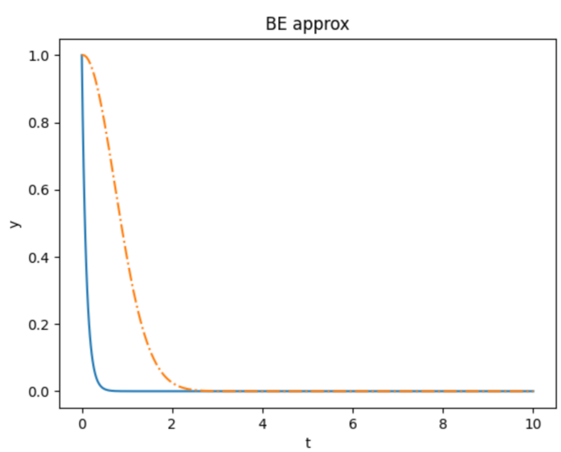

Figure 8: BE approx and exact for h=.5

5 conclusion

With our given IVP we see that it is stable and since all our t's are less than 1 we can say this as well with our absolute stability is stable. These two method are great show cases that we can approximate an IVP that is stable really accurately. Though at the start it was a little of but it got more accurate as t increase. It should also be stated that the conditions stated in mathamatics for absolute stability need to be met for these method to work. As well it is good to use IVP when have collect data.