MAT352 Initial Value Problems and Stability Homework

Instructions

Use the format described in the Write Up Form document to present your work for this assignment. Your report must be written with LaTeX. Include your answers to specific questions posed in the assignment as part of your report, as well as the content listed in the write-up format description. You will need to write some computer code to solve the problems in this homework, but you are free to use any language of your choice.

1 Background

For this assignment, you will be using the Forward Euler and Backward Euler numerical methods to solve the IVP:

$$y' = -10y\,, \qquad 0 \le t \le 10\,, \qquad y(0) = 1 \tag{1}$$

This is a stiff equation, with exact solution $y(t) = e^{-10t}$.

2 Numerical Experiments

- Write code to solve the IVP using both the Forward Euler method and the Backward Euler method. (Since the right-hand-side of the IVP is linear in y, you should be able to write the Backward Euler increment explicitly.)

- Use your code to construct numerical solutions to the IVP using both methods, with $h = 0.05$, $h = 0.1$, $h = 0.25$, $h = 0.5$.

- Describe the behavior you observe with each method. Use the theory of the absolute stability of IVP methods to explain what you observe.

- Compute the theoretical stepsizes required for each method to be stable. Do your results agree with the analysis? (Hint: This looks like the model test problem with $\lambda = -10$.)

To study the accuracy of your approximations, you may find it helpful to create plots of the real solution and your approximation at each step along the t axis.