Muon Lifetime Experiment

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(Dated: February 19, 2023)

We performed a muon lifetime experiment using a scintillation detector system to measure the lifetimes of muons that decayed within a time window of 0.5 to $25.5\mu s$. We obtained a muon lifetime value of $\tau_\mu = 2.22 \pm 0.01\mu s$ which is consistent with the current accepted value.

I. INTRODUCTION

The objective of the muon lifetime experiment is to measure the muon lifetime to high precision using a scintillation detector system. The experiment is designed to detect muons that pass through a plastic scintillator and to measure the time between capture and decay of the muons. The time spectrum of the muon decays is then analyzed to extract the muon lifetime using a fitting procedure. The experimental setup is designed to minimize background noise and to maximize the detection efficiency for muons, which allows for a high-precision measurement of the muon lifetime.

A. Background

The muon lifetime experiment is an important tool for studying fundamental particles and their interactions. Muons are unstable particles that decay through the weak force, and their lifetimes are sensitive to the strength of the weak interaction. Measuring the muon lifetime to high precision provides a test of the Standard Model of particle physics and can help to search for new physics beyond the Standard Model. In addition, muons are produced in cosmic ray showers and can be used to study the properties of cosmic rays. Before going in depth on the description of the muon lifetime experiment we must first understand where the muons we want to detect are coming from.

As mentioned muons are produced by cosmic rays, but not directly. First the cosmic rays that permeate empty space bombard the upper atmosphere. This causes an interaction between the particles in the upper atmosphere and the cosmic rays. This leads to the production of pions which quickly decay into muons. The muons then make there way down our atmosphere and into our detectors at sea-level.

$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\pi^- \to \mu^- + \bar{\nu_\mu}$$

II. EXPERIMENT APPARATUS

The muon lifetime experiment was conducted using a scintillator detector system consisting of two plastic scintillators, a photomultiplier tube (PMT), and associated electronics. The scintillators were placed in a coincidence arrangement to detect muons that passed through the apparatus.

The scintillator detectors used in this experiment were plastic (carbon polymer) scintillators, which emit light when charged particles pass through them. The importance of using the carbon polymer is because carbon is needed for the capture of the muon. The scintillators were coupled to PMTs at either end. The PMTs converted the light emitted by the scintillators into electrical signals that were processed by electronics. The scintillators were arranged in a coincidence configuration such that a muon passing through both scintillators would produce a signal indicating its detection.

There were two signals that come from the photomultiplier and are sent to an Arduino. The Arduino, with a pre-composed program handled the brunt of the data collection. The data it collected came from these two signals. The first signal was collected as a single-count measurement. This signal comes from the PMT when a muon is captured. This is because the captured muon interacts with the carbon atom to release a photon which is detected by the equipment.

Once the first signal is sent the Arduino program starts a timer. This timer is stopped when the second signal is received. The second signal indicates that the muon has decayed into an electron. Once the second signal is sent, the Arduino program considers this a double count or "event" if the decay occurs within $0.1\mu s$ to $25.5\mu s$. The Arduino program then stores and outputs this data organized in bins. In this setup the program consists of 255 bins each representing a $0.1\mu s$ interval. The data is then separated into these bins based on how long the event (from capture to decay) took.

When using the data collected in the experiment, the first four bins were disregarded as these events have the possibility of being a single count that accidentally got signaled as a double count. In addition the bins cut off at $25.5\mu s$, because at this point most of the signals recorded are an artifact of the background noise of the equipment.

A. High-Voltage Calibration

In order to begin data collection, we first had to calibrate the high-voltage used in the experiment. A stable operating voltage with stable counting rates is essential for acquiring accurate and meaningful data. As a preliminary objective, we conducted a calibration procedure to find a stable operating voltage for the photomultiplier. This involved adjusting the high voltage of the photomultiplier to achieve a stable counting rate for single events and double events. After testing voltages from 800 V to 1200 V in increments of 25 V, we determined the most stable high-volage level for the photomultiplier was 1050 V.

III. RESULTS

A. Muon Lifetime Calculation

The main purpose of this experiment is to determine the lifetime of a muon from its decay rate using the simple relation $\tau_\mu = 1/\lambda$ where τ_μ is the lifetime of the muon and λ is the decay rate of the muon. From the decay rate we calculated the lifetime of the muon to be $2.22 \pm 0.01\mu s$ within a 95% confidence interval. Now the question is how we determined the decay rate and unfortunately it is not as simple as the lifetime calculation.

In order to determine the decay rate of the muon, we needed to use our data and an expected function that shapes our data. From this we can optimize the parameters of the function in order to determine the closest fit to the data collected. The function used is:

$$N_{bin}(t) = B - \Delta t \frac{d}{dt} N_0 \left\{ \frac{re^{-\lambda t}}{1+r} + \frac{e^{-(\lambda+\Lambda)t}}{1+r} \right\}$$

Where:

- B is the background noise associated with each bin in the experiment. This is essentially the gauge of the data collection.
- Δt is the change in time from one bin to the next. In this experiment this value is set to $0.1\mu s$.
- N_0 is the first unknown parameter of the equation. It represents the initial number of muons in the detector.
- $r = \frac{N_+}{N_-}$ which is the ratio of positive and negative muons at sea level. In the experiment the value is set to 1.18 ± 0.12 as determined by [2].
- λ is the second unknown parameter of the equation. It represents the decay rate of the muons.
- Λ is the capture rate of muons for the C_{12} atom. In the experiment the value is set to $(3.76 \pm 0.04) \times 10^4$ as determined by [1].

The function itself represents the relationship between the time elapsed and the number of muon decays detected also known as the number of double counts or the number of events. As mentioned in the experimental apparatus section, an event represents the capture and decay of a muon and so the longer the event the longer the lifetime is for a particular data point.

1. *Parameter Fitting*

Using the data collected in the experiment the three unknown parameters in this function can be optimized to best represent the measured values. To do this we used the data from the experiment that corresponds to the different time values or bins. As discussed in the experiment apparatus section, the bins run from $0.5\mu s$ to $25.5\mu s$ in increments of $0.1\mu s$. Using these time values as the independent variable t in the function as well as the corresponding number of events N_{bin} as the dependent variable, we were able to run a python script to optimize the B, N_0, and λ parameters.

The python function that we used to optimize the parameters is called *scipy.optimize.curve_fit*(). This function takes in the N_{bin} function as well as the independent and dependent variables' data in order to determine what parameters most closely match the function to the data. The approach used by the python function is the least-squares regression for non-linear functions. Essentially this function reduces the squared distance from each data point to the varying N_{bin} function.

After inputting the function and necessary data into the python script, the three values for the parameters were determined with their respective statistical uncertainties. We found the background noise of the apparatus B to be 14 ± 2. The initial number of muons in the detector N_0 is $(1.105 \pm 0.005) \times 10^5$. And most importantly, we determined the decay rate λ to be $(4.50 \pm 0.03) \times 10^{-1}$.

Table 1: Optimized Parameters

Parameter	Value	Standard Deviation
B	14 ± 2	2.62×10^2
N_0	$(1.105 \pm 0.005) \times 10^5$	1.68×10^{-3}
λ	$(4.50 \pm 0.03) \times 10^{-1}$	9.53×10^{-1}

Now that we have a value for the decay rate λ. We can use the simple expression from earlier to determine the muon lifetime. $\tau = 1/\lambda \approx 2.22 \pm 0.01\mu s$. In addition to being able to extract the decay rate we now have all the necessary parameters to plot the number of events with respect to time. Because the relationship between time and number of counts is determined by a particle decay, we can assume the curve will be some variation of an exponential decay. This assumption is also evident through qualitative observation of the plotted data in figure [1]. Using these parameters we now have a plot of the function that best matches the exponential decay data as shown in figure [2].

FIG. 1. This plot contains the measured data from the experiment. As discussed in the experiment apparatus section, there are 251 data points that show the decay of the muon over time.

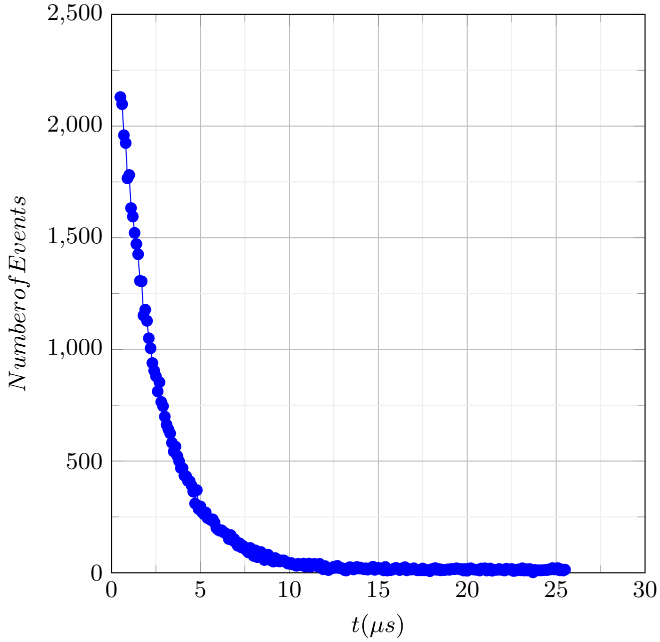
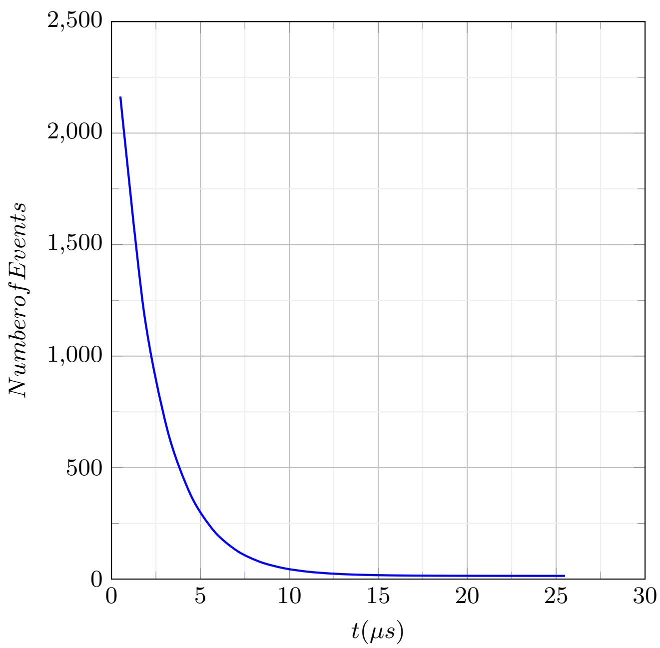


FIG. 2. This plot shows the decay rate of the muon. Essentially, the number of events (which is proportional to the number of muons detected) decreases exponentially as time progresses. We can see that the curve of this plot matches the shape of the data in figure [1]



B. Muon Lifetime Uncertainty

Within the results provided each measured value and calculated value is accompanied by an uncertainty. In this experiment there are two main sources of uncertainty. One is the statistical uncertainty that is present with any analysis of data. The other is the systematic uncertainty that accounts for the accuracy of the equipment used.

1. *Systematic Uncertainty*

As mentioned in the experimental apparatus section, most of the experiment came set up before we began data collection. Regardless of this fact, there are still uncertainties that come about due to imperfect measurement instruments. These uncertainties were accounted for in the N_{bin} equation from earlier. For instance, the background value B as well as the present muon count N_0 both correct the fitting of the data and account for the noise present in the apparatus which could cause uncertainty in the measurement.

2. *Statistical Uncertainty*

Most of the statistical uncertainty measurements were done by the python program itself. Before the parameter fitting was done, we had two values with known uncertainties: Λ and r. The uncertainty of these values came from the referenced experiments and therefore were assumed to be the correct uncertainties. Using the values from those experiments we then found the optimized parameters B, N_0, and λ with their respect uncertainties.

The uncertainties of the fitted parameters were determined from the $scipy.optimize.curve\_fit()$ python function. This function not only returned the optimized parameters, but the corresponding covariance matrix. From the covariance matrix, the respective standard deviations can be extracted by taking a square-root of the diagonal elements. Once we have the standard deviation we can determine the uncertainty.

Determining the uncertainty from the standard deviation depends on the level of confidence you wish to have with the data provided. For the purposes of the experiment we decided to use a 95% confidence interval. Because the sample size of the data used in the equation is so large, with the total number of counts $N_{total} \approx 50,000$, we are able to assume the distribution of each data point to be approximately normal. With this assumption and the 95% confidence interval, we took the uncertainty to be twice the standard deviation: $\delta x = 2\sigma_x$. This method was used to calculate the uncertainty for the three parameters: B, N_0, and λ.

Once we had these uncertainties, the parameters themselves, and the calculated muon lifetime we were able to calculate the uncertainty in the muon lifetime calculation. Given λ, the uncertainty $\delta\lambda$, τ_μ, and the relation $\tau_\mu = 1/\lambda$ we were able to determine the uncertainty $\delta\tau_\mu$ through the use of the statistical equation:

$$\delta\tau_\mu = |-1| \times \frac{\delta\lambda}{|\lambda|} \times |\tau_\mu|$$

Where $|-1|$ comes from the power that λ is raised to in the equation for τ_μ. At this point all necessary uncertainties of measured and calculated values are accounted for.

C. Comparison with Previous Experiments

As mentioned earlier, we found the resulting muon lifetime in this experiment to be $2.22 \pm 0.01\mu s$. We can then compare our result with previous experiments to see whether our calculations and measurements match the known value of the muon lifetime.

Looking at [3], these researchers found the muon lifetime to be $2196980.3 \pm 2.2ps$. Although this value is very close to the value we calculated, it does not lie within the uncertainty of our measurement. This means that there is some uncertainty that must not have been accounted for. One possibility of this uncertainty is the bin size of the appartus. As discussed earlier the bin size was taken to be $0.1\mu s$ and although this value was set by the Arduino program, there could still exist an uncertainty in the hardware itself.

IV. CONCLUSION

Our experiment measured the muon lifetime using a scintillation detector system and found a value of 2.22 microseconds with an uncertainty of 0.01 microseconds. This result is relatively consistent with the current best value and represents a significant precision in our experiment. We also observed a clear exponential decay in the time spectrum of the muon decays, which confirms the expected behavior of muon decay.

The measurement of the muon lifetime is an important tool for studying fundamental particles and their interactions. Our result provides a high-precision measurement of the muon lifetime, which can be used to test the predictions of the Standard Model of particle physics and search for new physics beyond the Standard Model. In addition, our experiment demonstrates the effectiveness of the scintillation detector system for high-precision measurements of the muon lifetime.

Our experiment opens up several potential directions for future research. One possibility is to further improve the precision of the muon lifetime measurement using alternative detection techniques or more advanced analysis methods. Another possibility is to study the properties of muons in different environments, such as in cosmic ray showers or in accelerator experiments. These studies could provide new insights into the behavior of fundamental particles and their interactions.

ACKNOWLEDGMENTS

We would like to express our gratitude to Dr. Kin Fai Mak for his guidance and support throughout this experiment. His expertise and willingness to provide assistance were invaluable in helping us to successfully carry out this experiment and obtain accurate and meaningful results. We are also grateful to the Physics Department for providing the necessary resources and equipment for this experiment.

REFERENCES

[1] Nimai C Mukhopadhyay et al. "Inclusive muon capture in light nuclei". In: *Physics Letters B* 434.1-2 (Aug. 1998), pp. 7–13. DOI: 10.1016/s0370-2693(98)00790-4. URL: https://doi.org/10.1016%5C%2Fs0370-2693%5C%2898%5C%2900790-4.

[2] T. Suzuki, D. F. Measday, and J. P. Roalsvig. "Total nuclear capture rates for negative muons". In: *Phys. Rev. C* 35 (6 June 1987), pp. 2212–2224. DOI: 10.1103/PhysRevC.35.2212. URL: https://link.aps.org/doi/10.1103/PhysRevC.35.2212.

[3] D. M. Webber et al. "Measurement of the Positive Muon Lifetime and Determination of the Fermi Constant to Part-per-Million Precision". In: *Phys. Rev. Lett.* 106 (4 Jan. 2011), p. 041803. DOI: 10.1103/PhysRevLett.106.041803. URL: https://link.aps.org/doi/10.1103/PhysRevLett.106.041803.