6 – Fuzzy Sets and Systems

Computational Intelligence for the Internet of Things (2019-20)

João Paulo Carvalho

joao.carvalho@inesc-id.pt

INESC-ID
Instituto Superior Técnico, Universidade de Lisboa

inesc id lisboa

TÉCNICO LISBOA

Fuzzy Sets and Systems

- Introduction
- Fuzzy Sets
- Fuzzy Logic and Fuzzy Systems
- Fuzzy Modeling
- Conclusions

Introduction

inesc id lisboa

TÉCNICO LISBOA

Fuzzy Logic

- Fuzzy Logic is basically a multivalued logic that allows intermediate values to be defined between conventional evaluations like yes/no, true/false, black/white, etc.
  - Notions like “rather warm” or “pretty cold” can be formulated mathematically and processed by computers
- In Fuzzy systems, an attempt is made to apply a more human-like way of thinking in the programming of computers
- The term Fuzzy Logic is a misnomer:
  - It implies that in some way the methodology is vague or ill-defined. This is far from true!
  - Fuzzy logic just evolved from the need to model the type of vague or ill-defined systems that are difficult to handle using conventional binary valued logic. The methodology itself is based on a strong mathematical background

Historical Background

- Fuzzy Logic was initiated in 1965 by Lotfi A. Zadeh, professor for computer science at the University of California in Berkeley
- Japan and other far east countries quickly adopted this new approach to think about systems. Fuzzy systems become much more popular there than in Europe or USA
  - Fuzzy became a keyword for marketing in Asia in the late 90's. It was used everywhere. Electronic articles without Fuzzy components gradually turn out to be dead stock
  - As a gag that shows the popularity of Fuzzy Logic, it was possible to find even toiletpaper with "Fuzzy Logic" printed on it, and scooters named Fuzzy although they didn't have any Fuzzy Logic

Lotfi Zadeh
1921-2017

Historical Background

Lotfi Zadeh 1921-2017

Historical Background (II)

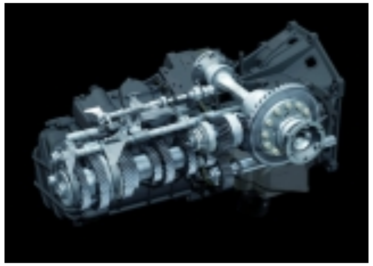
- In the Western world "Fuzzy" was definitely much less popular and in many countries it was met with suspicion, especially from the fields of classical AI (Osherson and Smith vs. Zadeh papers, 80's) and classic control theory:
  - Mostly their issues arose from the fact that Fuzzy Logic allowed simple and efficient solutions to hard to solve problems in those areas (e.g. Fuzzy controllers can be implemented by anyone and can equal or beat sophisticated controllers)
- Most of the attacks focused (and still do) on the lack of mathematical background, which doesn't make any sense since there is a strong mathematical support behind fuzzy sets theory!

Lotfi Zadeh
1921-2017

Historical Background (III)

- Starting mid 90's, efforts were made in Europe and the Americas to catch up with the tremendous japanese success. Examples:
  - The VAG, PSA and BMW adaptive automatic gearboxes;
  - Siemens intelligent washing machines;
  - NASA applying Fuzzy Logic for complex docking-maneuvers;
  - The European Centre for Fuzzy Logic (created in Spain in the mid 00's)

Lotfi Zadeh
1921-2017



Historical Background (III)

NASA Technical Memorandum 106599
AIAA–94–3163

13475
18 P

Fuzzy Logic Approaches to Multi-Objective Decision-Making in Aerospace Applications

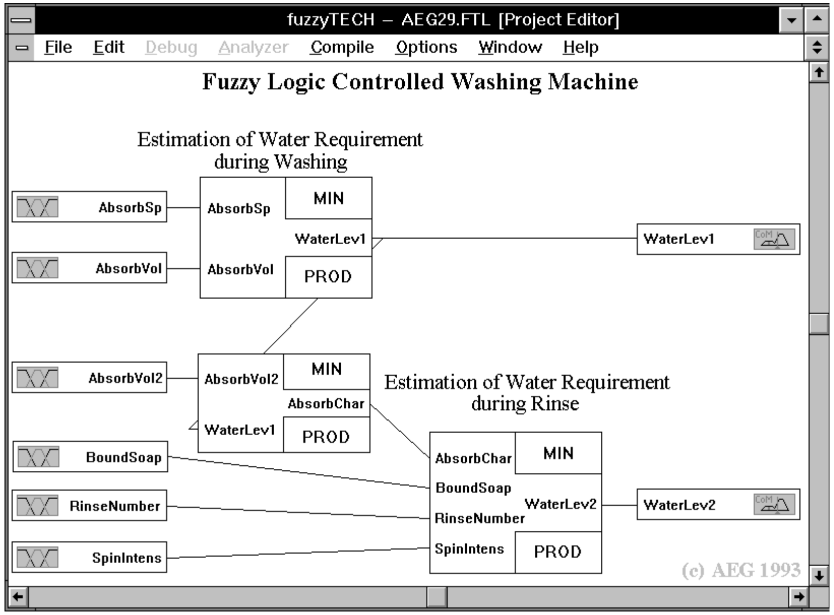
(NASA-TM-106599) FUZZY LOGIC APPROACHES TO MULTI-OBJECTIVE DECISION-MAKING IN AEROSPACE APPLICATIONS (NASA. Lewis Research Center) 18 p

N94-33953

Unclas

G3/20 0013475

Terry L. Hardy
Lewis Research Center
Cleveland, Ohio



Historical Background (IV)

- "Fuzzy" has been slowly decaying in the XXI century
  - Mostly due to the lack of applicational impact of novel developments
  - Still a lot of research but mostly in mathematics and other theoretical developments
  - Machine Learning and Deep Learning popularity had a large negative impact in Fuzzy Systems attractiveness
- However Fuzzy Systems are still big in Japan and Taiwan, with a very high number of patents applied for every year
  - Most consist of rather simple applications of Fuzzy Control, but Fuzzy research is widely supported with a huge budget

Lotfi Zadeh
1921-2017

Fuzzy Logic

“It is better to be vaguely right than exactly wrong”

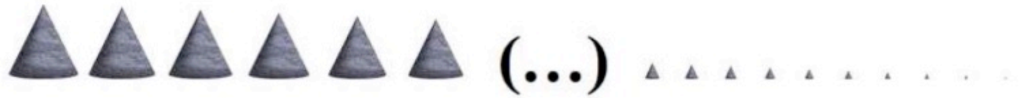
Carveth Read, “Logic, deductive and inductive”, p.35 (1898)

Motivation

- Consider the following questions:
  - "Among all the customers of a cellphone company, which ones have a large income?"
  - "Will this customer purchase service S1 if his plan includes large selection of I?"
  - "How much will this customer use the service?"
  - "What is the typical cellphone usage of this customer segment?"
- What kind of answers would you like to have?

The Sorites Paradox

- If you remove sand grains form a sand dune one by one, when does the sand dune turns into a sand Hill, into a sand pile?



- When is the sky cloudy? How many clouds does it take to make a clear sky not clear?

  – c.f. mathematical induction

Graduality

- Humans conceptualize the world based on concepts of similarity, gradualness, fuzziness
  - Excerpt of speech from the United Nations Conference on Trade and Development:

"However, it is safe to say that the rapid expansion of electronic transactions constitutes a major opportunity for trade and development: it can be the source of a significant number of success stories by which developing countries and their enterprises can reach new levels of international competitiveness and participate more actively in the emerging global information economy."

Fuzziness/graduality in language

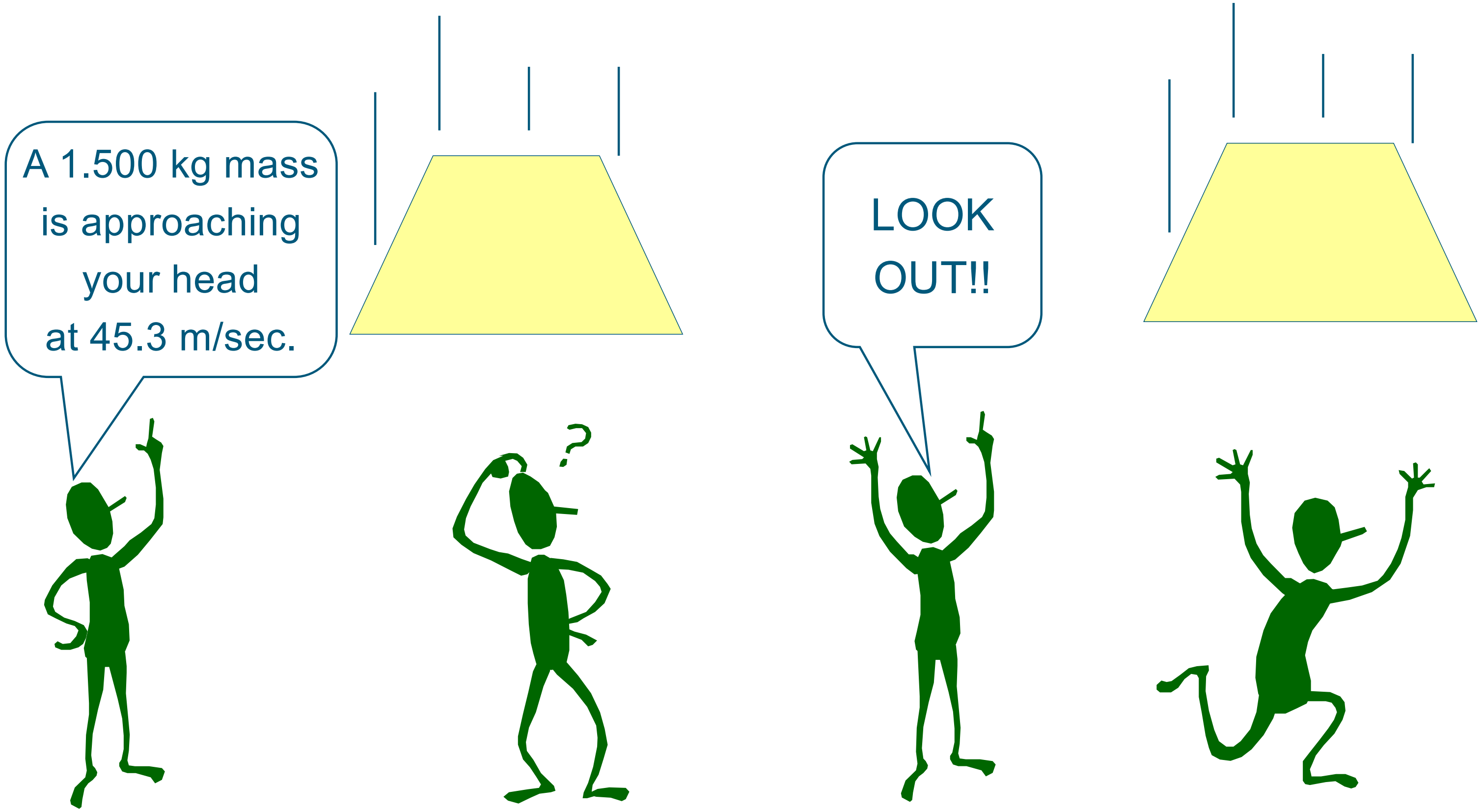
"... the high levels of taxation on petroleum products in most consuming countries are greatly amplifying the effects of rises in the price of crude, to the detriment of the consumer. OPEC expresses the hope, once again, that the governments of these countries will reduce their high taxes on a barrel of oil - which is much more than producers themselves receive - in the interests of market stability. In addition, speculation in the oil market has become a key factor that has distorted realities and has artificially influenced prices far beyond what the fundamentals indicate."

(From Opening Address to the 111th Meeting of the OPEC Conference).

Motivation – revisited

- Consider the following questions:
  - “Among all the customers of a cellphone company, which ones have a large income?”
  - “Will this customer purchase service S1 if his plan includes large selection of I?”
  - “How much will this customer use the service?”
  - “What is the typical cellphone usage of this customer segment?”
- What kind of information would you like to have?
  - In this presentation we discuss a mathematical formalism for these types of imprecise/vague/fuzzy statements

Precision and Relevance



Fuzzy Sets

Fuzzy Sets

- Definition
- Interpretation
- Properties
- Operations
- Linguistic Terms

Incompatibility Principle

"As the complexity of a system increases, our ability to make precise and yet relevant (significant) statements about the system diminishes, until a threshold is reached beyond which precision and relevance (significance) become mutually exclusive characteristics" (Zadeh 1973)

Examples:

- What is the circumference of a circle?
- How long are Portuguese borders?
- Can you describe the motion of a pendulum?
- What is the creditworthiness of a company?

Lotfi Zadeh
1921-2017

Crisp Sets

- Collection of definite, well-definable objects (elements) to form a whole

Representation of sets:

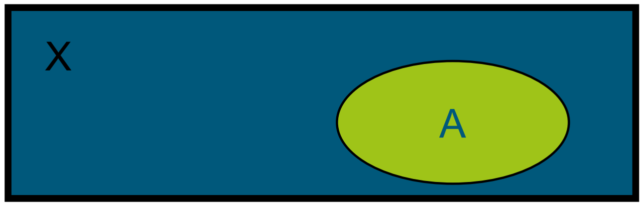
- list of all elements

  $A=\{x_1, \ldots, x_n\}, x_j \in X$

- elements with property P

  $A=\{x | x \text{ satisfies } P\}, x \in X$

- Venn diagram



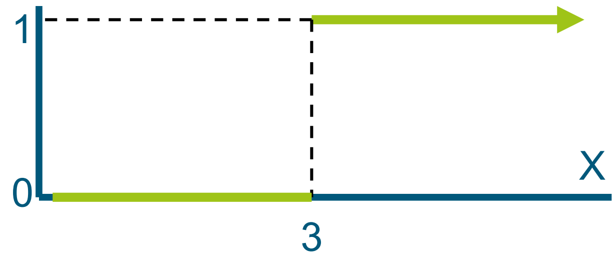
- characteristic function

  $f_A: X \to \{0,1\},$

  $f_A(x) = 1, \Leftrightarrow x \in A$

  $f_A(x) = 0, \Leftrightarrow x \notin A$

Real numbers larger than 3:



Fuzzy Sets

- Sets with fuzzy, gradual boundaries (Zadeh 1965)
- A fuzzy set A in X is characterized by its membership function:

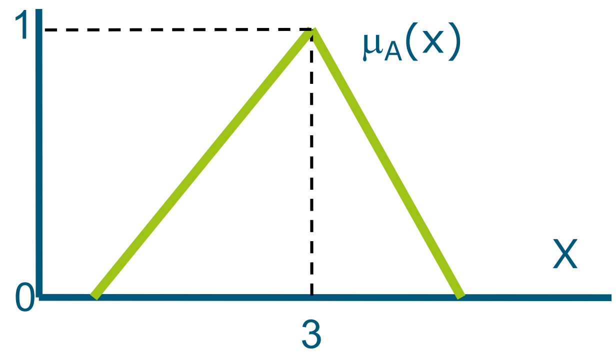
$$\mu_A : X \rightarrow [0,1]$$

A fuzzy set A is completely determined by the set of ordered pairs:

$$A = \{(x, \mu_A(x)) | x \in X\}$$

X is called the *domain* or *universe of discourse*

Real numbers about 3:



Crisp vs. Fuzzy Sets

- Integers larger than 3
- Families without children
- People with job description "manager"

- Tall people
- Fast cars
- Cold beer
- Bold men
- Tall and blond Swedes
- Comfortable care

Interpretation

- Fuzzy sets are usually related to vagueness
- This vagueness is not defined as uncertainty of meaning but instead as the standard definition of vagueness with the possession of borderline cases
- Fuzzy sets are used to represent three different concepts:
  - gradualness (original idea of Zadeh-1965)
  - epistemic uncertainty* (not discussed)
  - bipolarity* (not discussed)

Interpretation – Gradualness

- Gradualness refers to the idea that many categories in natural language are a matter of degree, including truth
- The fuzzy set is used as representing some precise gradual entity consisting of a collection of items (sets)
- Gradualness is indicated through membership. The transition between membership and non-membership is “gradual rather than abrupt”

Interpretation – Gradualness (II)

- The gradualness can be linked to different situations:
  - Example: Forest zone in a grey level image
    - The boundary of this zone is inherently gradual
    - The boundary of the set is precisely known, but it is not possible to measure it (or indicate it) precisely.

Interpretation – Gradualness (III)

- The gradualness can be linked to different situations:
  - Example: Define the boundaries of a forest when the density of trees is slowly decreasing in peripheral zones
  - It is possible to measure each element of the set precisely (e.g. position of the trees), the boundaries of the set are known, but a (crisp) definition of its boundaries is not precise


Interpretation – Gradualness (IV)

- The gradualness can be linked to different situations:
  - Example: Define dense forest zone
  - The uncertainty is linked to a fuzzy predicate referring to a gradual concept (e.g. “dense” forest zone)
  - In this case the boundaries are known, the measure of each element is precise, but the fuzzy predicate indicates gradualness.

Degree of membership

- The degree of membership $\mu_A(x)$ of an element x in a fuzzy set A can be used to express:
  - Degree of similarity - related to gradualness
  - Degree of preferences (in utility functions) - related to gradualness
  - Degree of uncertainty* (not discussed)

Degree of Similarity

- The membership degree $\mu_A(x)$ represents the degree of proximity of x to prototype elements of A
- This view is used in clustering analysis and regression analysis, where the problem is representing a set of data by the proximity between pieces of information
  - Example: classification of cars of known dimensions in categories of A = {big cars, regular cars, small cars}
    - If the prototype of the category big cars is a Mercedes Class S, then we can construct a measure of distance between any car to this prototype, where the distance is a measure of similarity

Degree of Preference

- The membership degree $\mu_A(x)$ represents an intensity of preference of object x, to a set A of preferred objects
- Alternatively, A represents a set of values of a decision variable x' and $\mu_A(x)$ represents the feasibility of selecting x as a value of x'
- This view of fuzzy sets as criteria or flexible constraints is used in fuzzy optimization and decision analysis
  - Example: an agent buying a big car. In this case the membership degree will reflect the degree of satisfaction of cars chosen by the agent to the class of big cars, according to the criterion size
    - The membership indicates the preference of the agent.

Fuzzy Sets vs. Probabilities

- Probabilities are related to randomness – uncertainty described by tendency or frequency of a random variable to take on a value in a specific region. Mathematical formulation:
  - A probabilistic measure Pr of an experiment ε yet to be performed, is a mapping $2^U \rightarrow [0,1]$ that assigns a number $Pr(A)$ of event A to each subset of U, satisfying the Kolmogorov axioms
  - $Pr(A)$ is the probability that a generic outcome of ε, an ill-known single-valued variable x, hits set A
  - If the outcome of ε is such that $x \epsilon A$, then we say that event A has occurred (uncertainty about the occurrence of any particular x and consequently of event A

Fuzzy Sets vs. Probabilities (II)

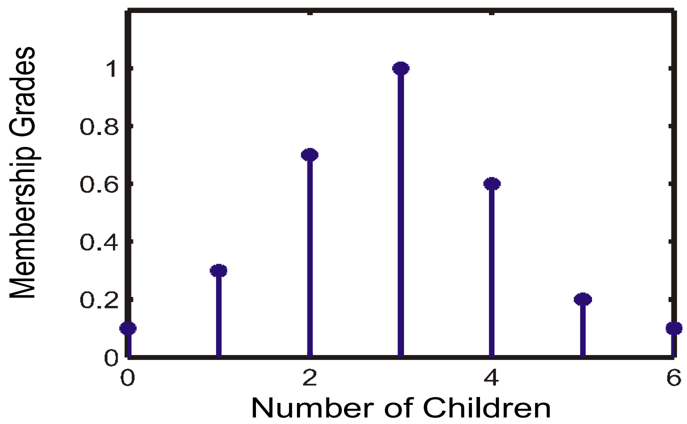
- Interpretations of Probability:
  - Symmetry (e.g. six outcomes in a 6-sided dice)
  - Frequency – physical random phenomenon over long-run frequencies and accounts for variability of (precise) observations
  - Subjective probability (exchangeable betting rates) – the epistemic state of an agent, assuming degrees of belief, related to a status of uncertainty (Bayes rule)
  - "Probability does not exist" (in an objective sense) - de Finetti (Theory of Probability 1970/1974)
- Fuzzy sets are related to gradualness

Fuzzy Sets vs. Probabilities (Example)

- Suppose I have two boxes of 10 bottles filled with a clear liquid. You are super thirsty, and need to pick up a bottle from one of the boxes. Ask yourself which box you should choose from:
  - Box 1 is labeled with “membership (in the set) of poison is 0.1”
  - Box 2 is labeled with “probability of poison is 0.1”
- Which one to choose?
  - Box 1: Each bottle contains 10% poison and 90% water (taste funky)
  - Box 2: 1 of the bottles is pure poison, and the remaining 9 bottles are pure water

Fuzzy Sets on Discrete Universes

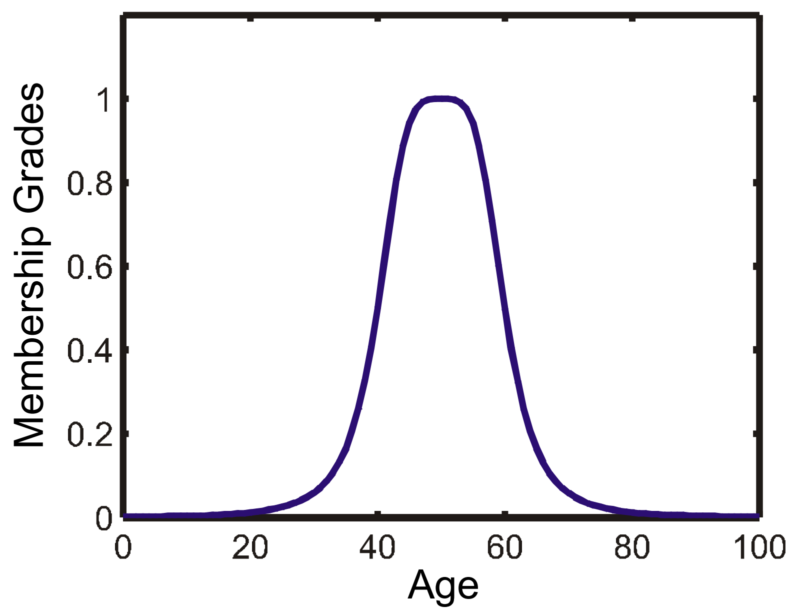
- Fuzzy set C = "desirable city to live in"
  - X = {Braga, Lisboa, Beja} (discrete and non-ordered)
  - C = {(Braga, 0.6), (Lisboa, 0.9), (Beja, 0.2)}
- Fuzzy set A = "sensible number of children"
  - X = {0, 1, 2, 3, 4, 5, 6} (discrete universe)
  - A = {(0, 0.1), (1, 0.3), (2, 0.7), (3, 1.0), (4, 0.6), (5, 0.2), (6, 0.1)}



Fuzzy Sets on Continuous Universes

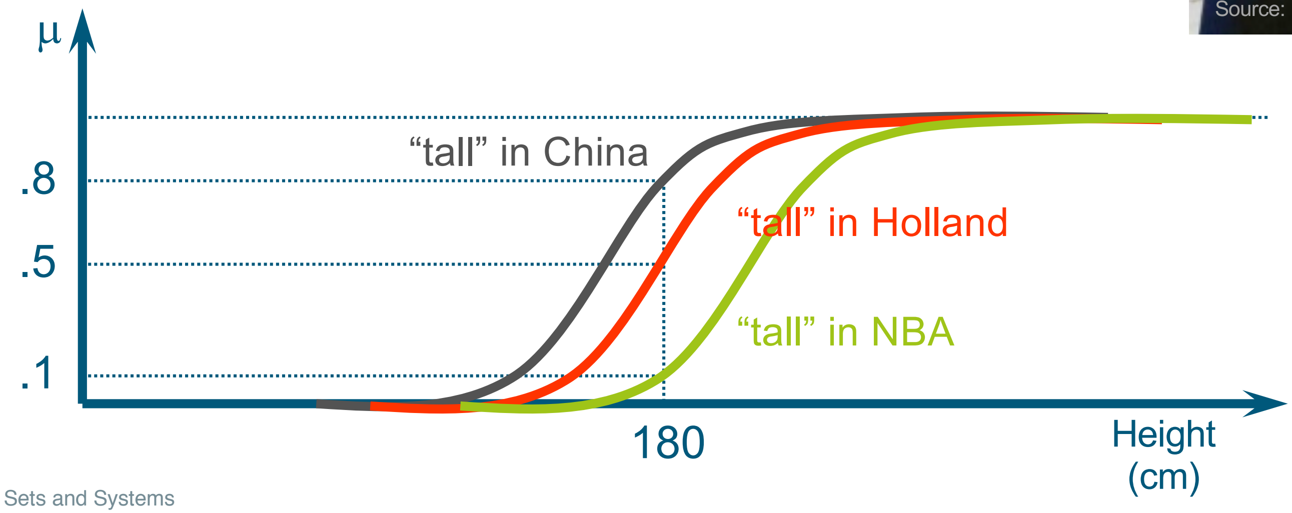
- Fuzzy set B = "around 50 years old"
  - X = Set of positive real numbers (continuous)
  - $B = \{(x, \mu_B(x)) | x \in X\}$
  - B is defined by a membership function
  - E.g:

$$\mu_B(x) = \frac{1}{1 + \left(\frac{x-50}{10}\right)^2}$$



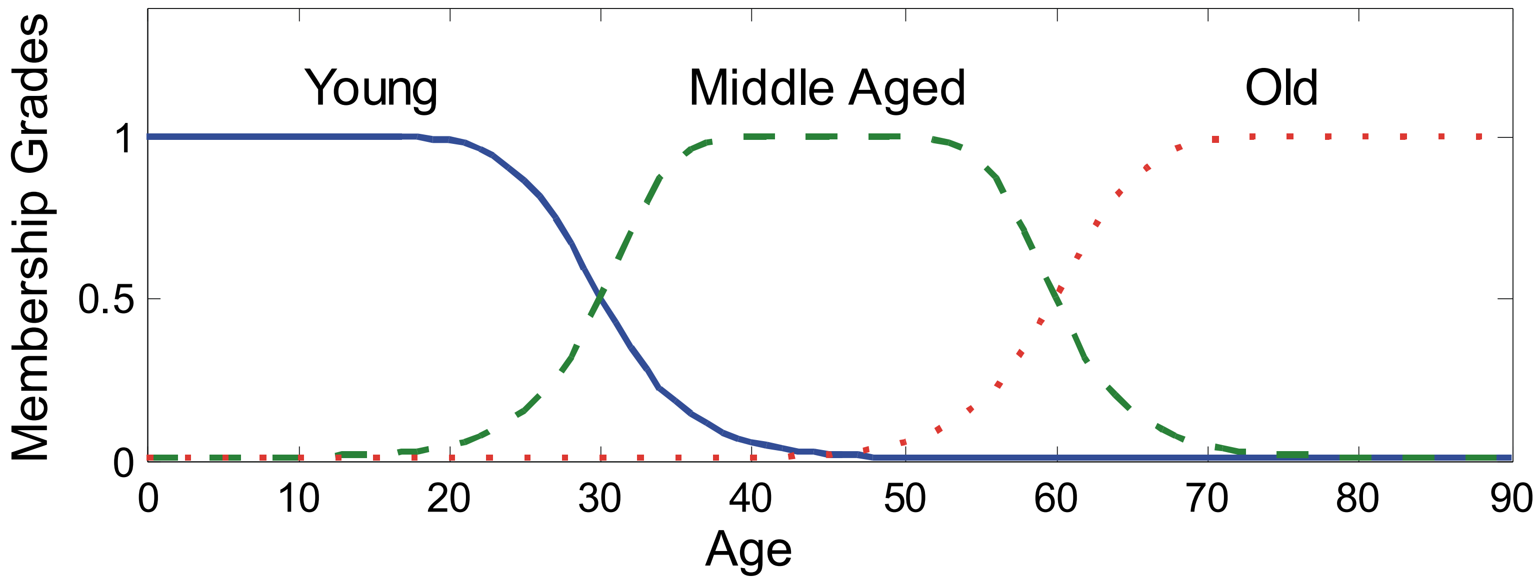
About Membership Functions (MBF)...

- Subjective measures
- Context dependent
- Not probability functions
  - E.g: Tall men

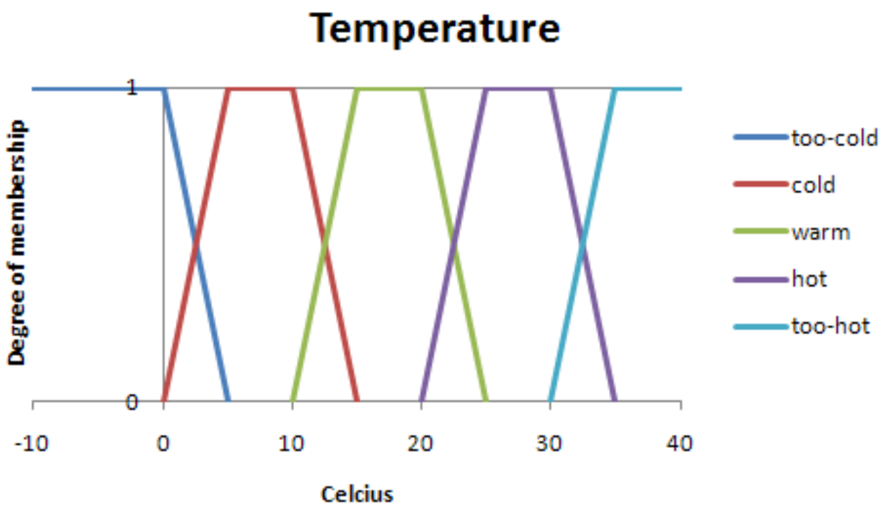


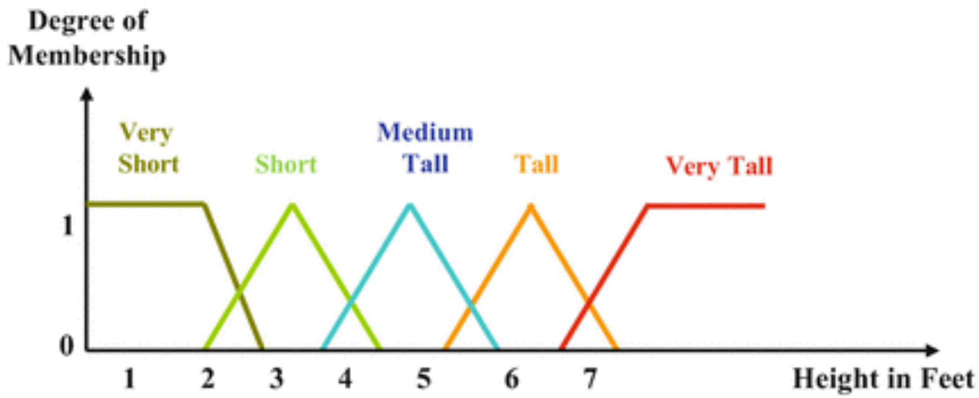
Partition

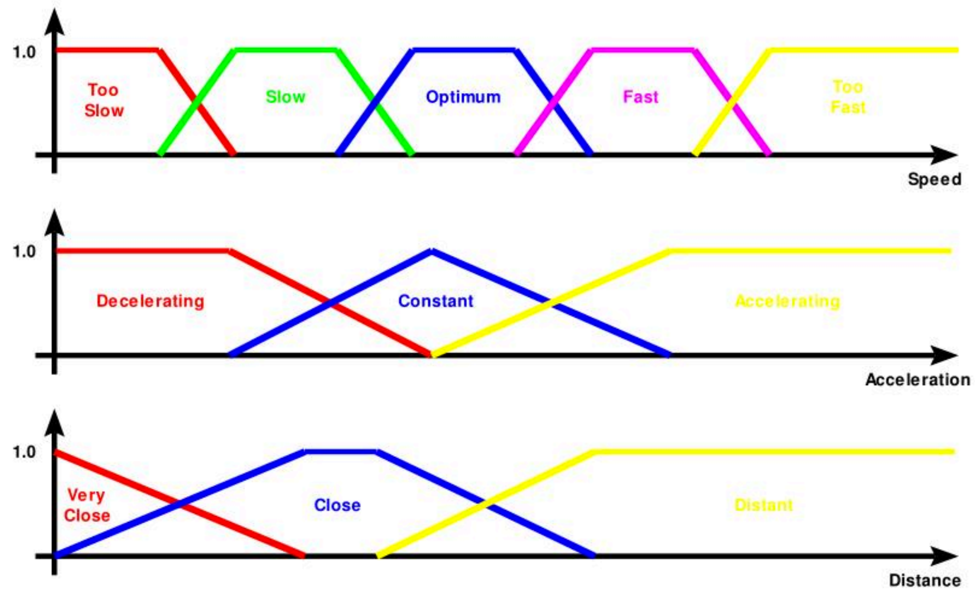
- Example: Fuzzy sets partition formed by the term set {“Young”, “Middle aged”, and “Old” } with the following membership functions:



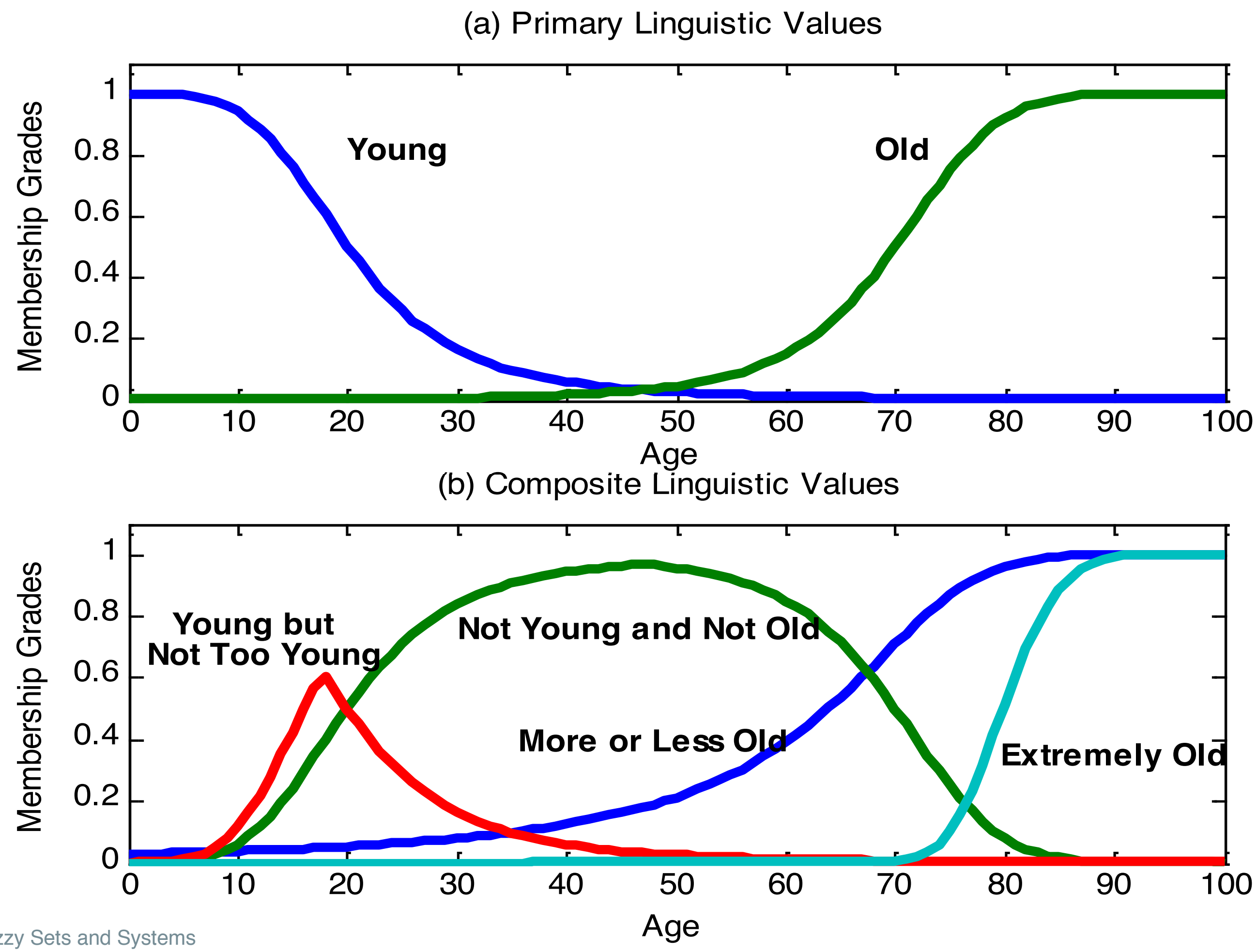
Linguistic Terms Examples







Linguistic Terms: Age Example



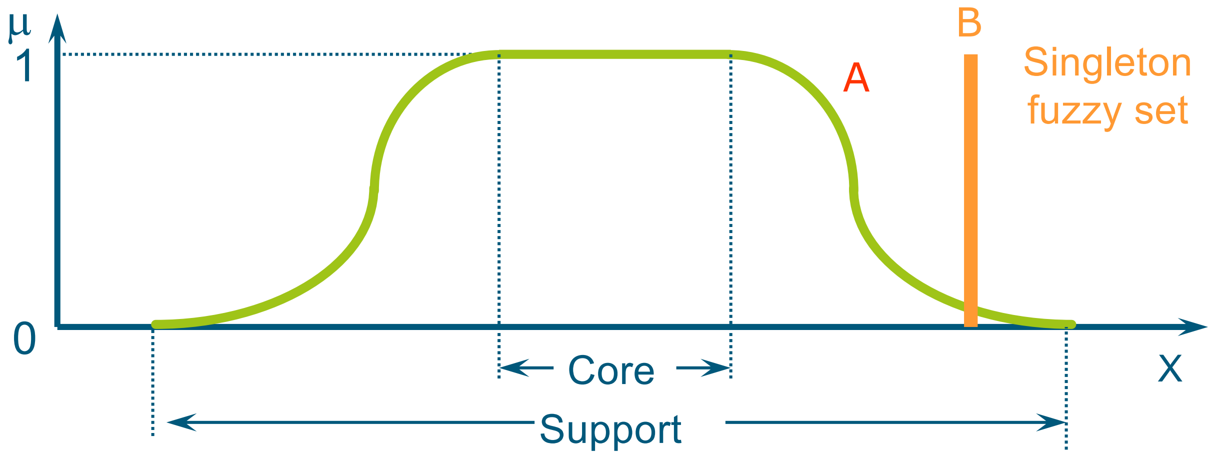
Fuzzy Sets: Support, Core, Singleton

- The support of a fuzzy set A in X is the crisp subset of X whose elements have non-zero membership in A:

$$supp(A) = \{x \in X | \mu_A(x) > 0\}$$

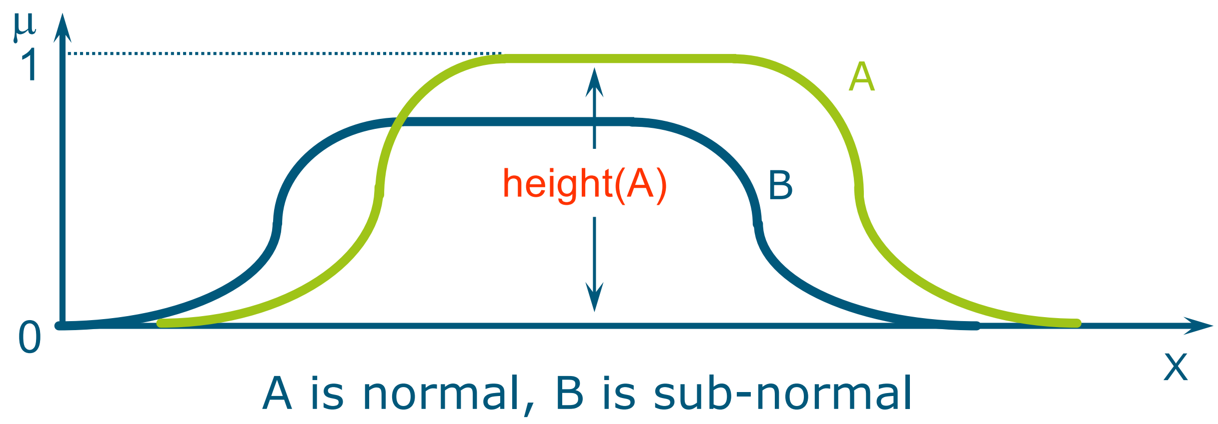
- The core of a fuzzy set A in X is the crisp subset of X whose elements have membership 1 in A:

$$core(A) = \{x \in X | \mu_A(x) = 1\}$$



Normal Fuzzy Sets

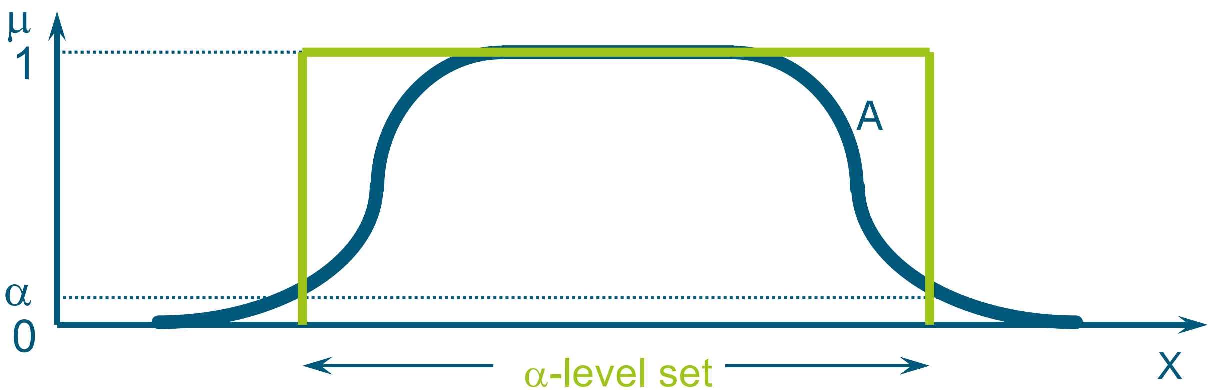
- The height of a fuzzy set A is the maximum value of $\mu_A(x)$
- A fuzzy set is called normal if its height is 1, otherwise it is called sub-normal



α-cut of a Fuzzy Set (Level Set)

- An α-level set of a fuzzy set A of X is a crisp set denoted by A_α and defined by:

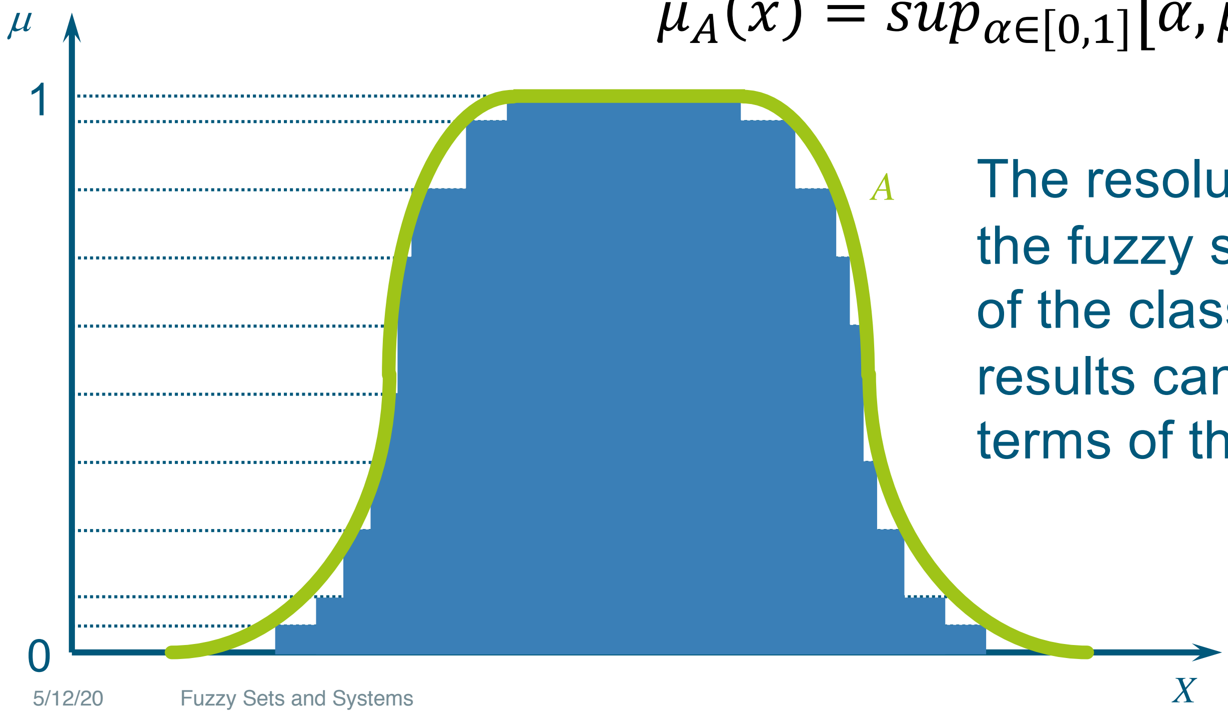
$$A_\alpha = \begin{cases} \{x \in X | \mu_A(x) \geq \alpha\}, \alpha > 0 \\ cl\big(supp(A)\big), \alpha = 0 \end{cases}$$



Resolution Principle

- Every fuzzy set A can be uniquely represented as a collection of α-level sets according to:

$$\mu_A(x) = sup_{\alpha \in [0,1]}\left[\alpha, \mu_{A_\alpha}(x)\right]$$



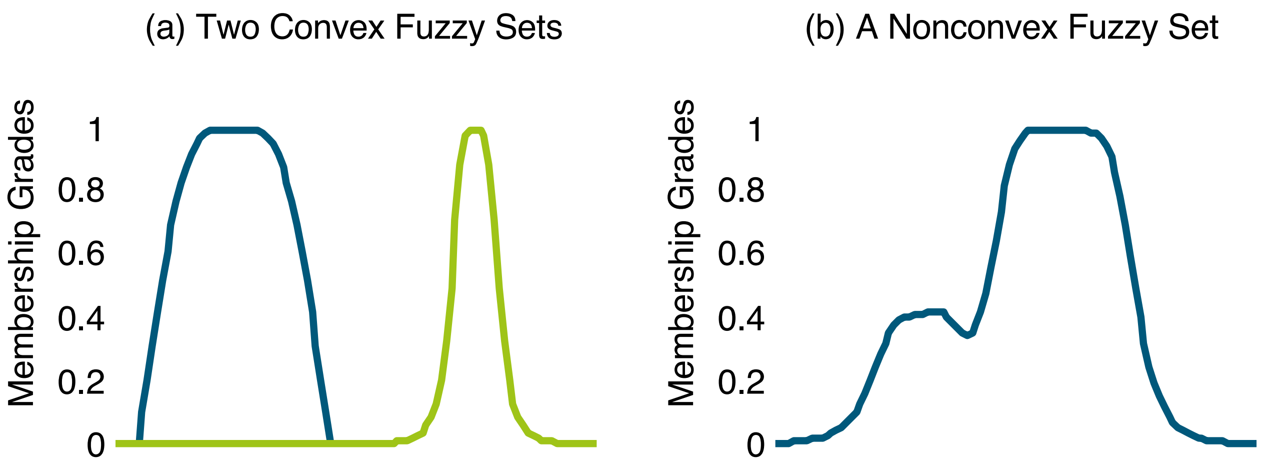
The resolution principle implies that the fuzzy set theory is a generalization of the classical set theory, and that its results can also be represented in terms of the classical set theory

Convexity of Fuzzy Sets

- A fuzzy set A is convex if for any λ in [0, 1],

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\big(\mu_A(x_1), \mu_A(x_2)\big)$$

- Alternatively, A is convex if all its α-cuts are convex



Set Theoretic Operations

- Subset:

$$A \subseteq B \Leftrightarrow \mu_A \leq \mu_B$$

- Complement:

$$\bar{A} = X - A \Leftrightarrow \mu_{\bar{A}}(x) = 1 - \mu_B(x)$$

- Union:

$$C = A \cup B \Leftrightarrow \mu_C(x) = max\big(\mu_A(x), \mu_B\,(x)\big) = \mu_A(x) \vee \mu_B(x)$$

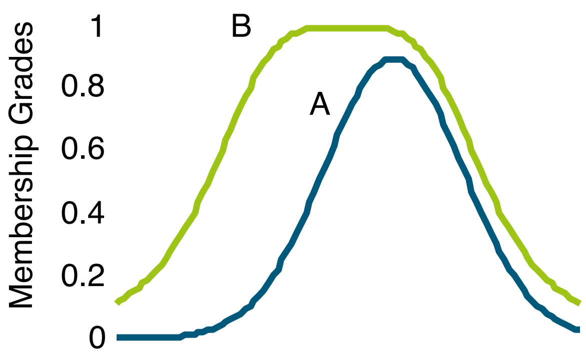
- Intersection:

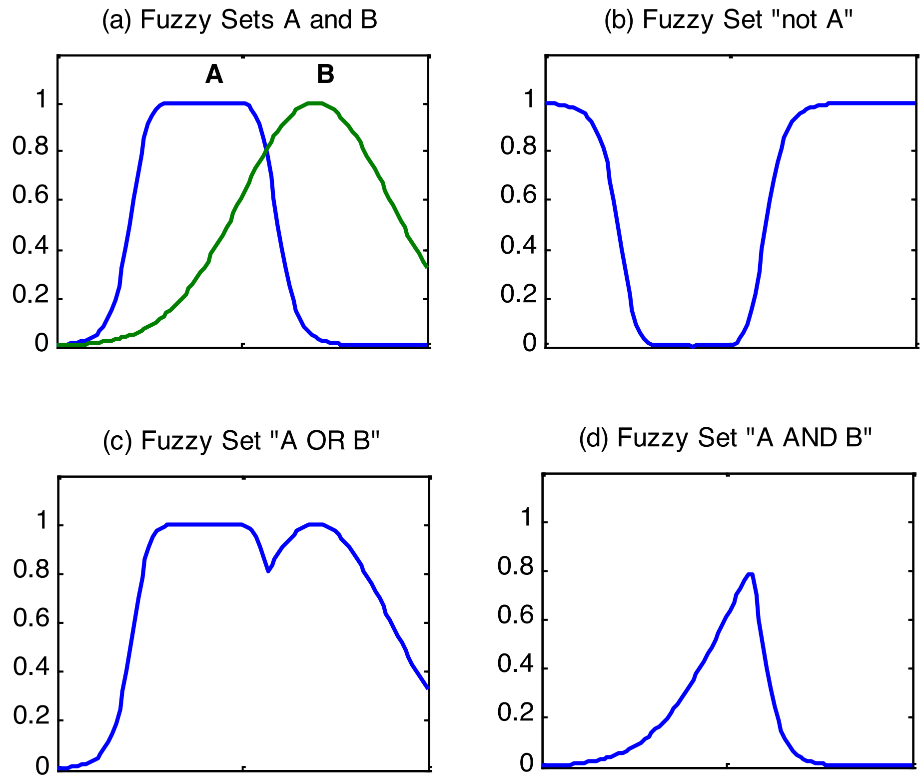
$$C = A \cap B \Leftrightarrow \mu_C(x) = min\big(\mu_A(x), \mu_B\,(x)\big) = \mu_A(x) \wedge \mu_B(x)$$

Set Theoretic Operations (cont.)

$$A \subseteq B \Leftrightarrow \mu_A \leq \mu_B$$

A is Contained in B





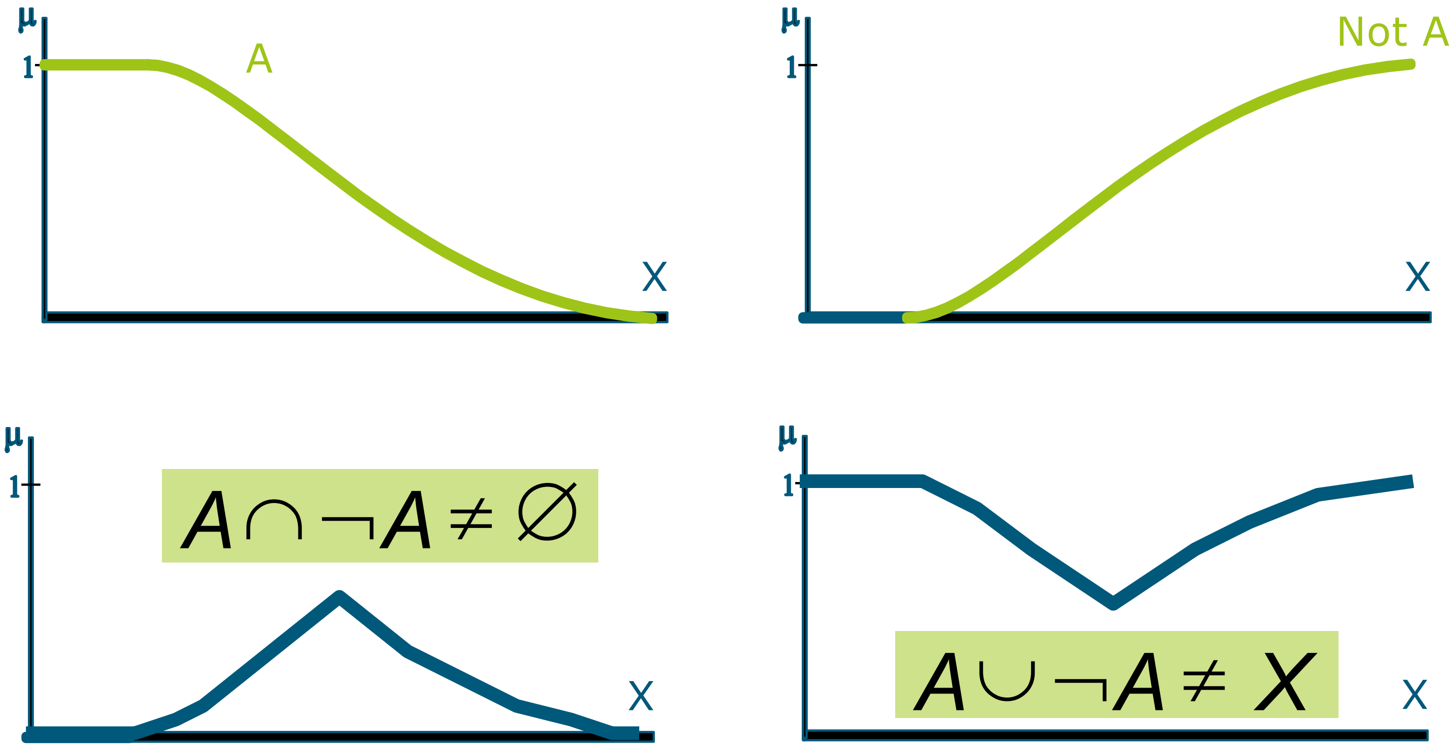
Average

- The average of fuzzy sets A and B in X is defined by:

$$\mu_{(A+B)/2}(x) = \frac{\mu_A(x) + \mu_B(x)}{2}$$

  - Note that the classical set theory does not have averaging as a set operation. This is an extension provided by the fuzzy set approach.

Combinations with Negation



– Note: The De Morgan Laws hold in Fuzzy Set theory

Membership Function Formulation

Triangular MF:

$$trimf(x;a,b,c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$

Trapezoidal MF:

$$trapmf(x;a,b,c,d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

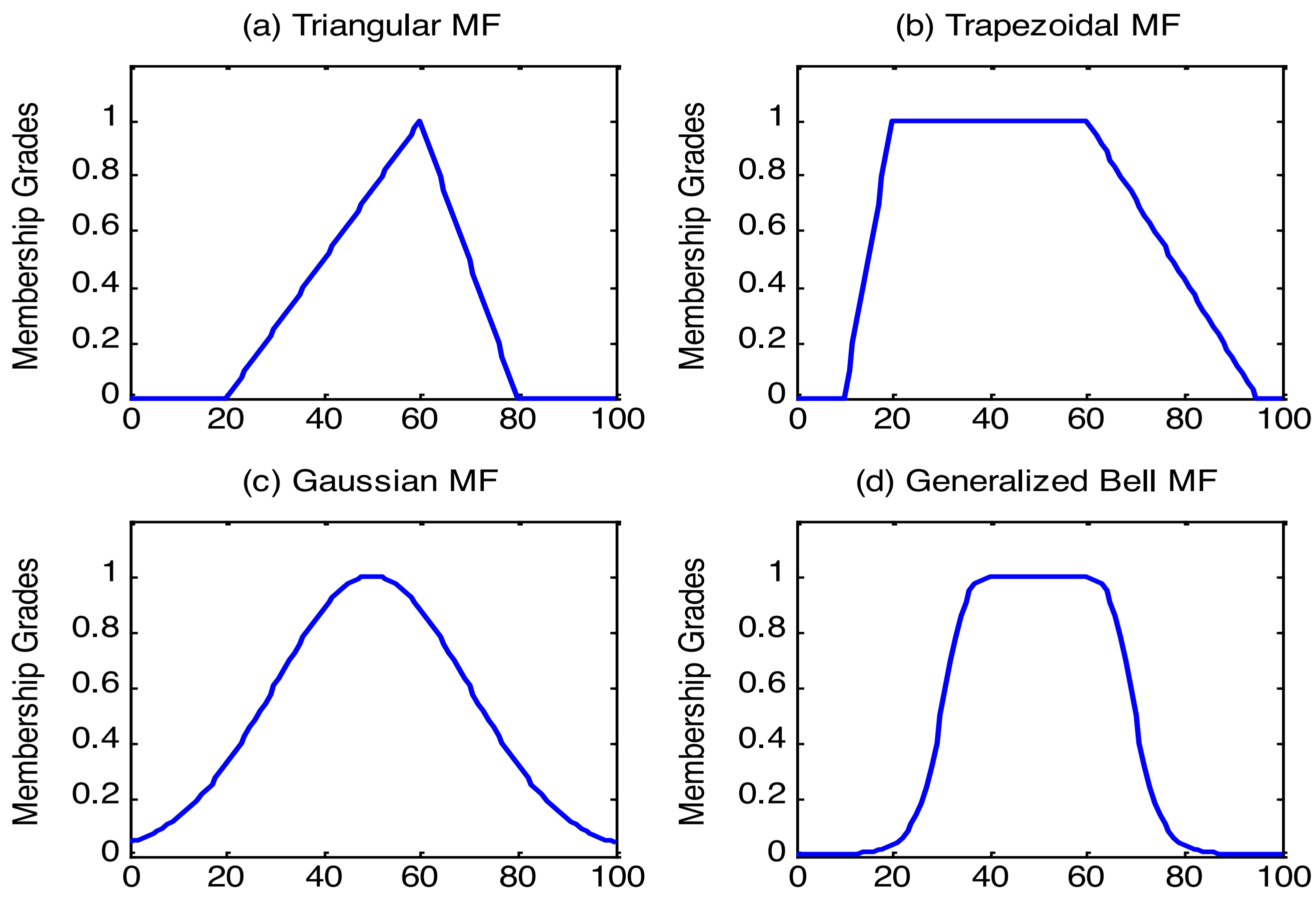
Gaussian MF:

$$gaussmf(x;a,b,c) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$

Generalized bell MF:

$$gbellmf(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{b}\right|^{2a}}$$

Membership Function Formulation (exemples)



Cartesian Product

- Cartesian product of fuzzy sets A and B is a fuzzy set in the product space $X \times Y$ with membership:

$$\mu_{A\times B}(x,y) = min\left(\mu_A(x), \mu_B(y)\right)$$

- Cartesian co-product of fuzzy sets A and B is a fuzzy set in the product space $X \times Y$ with membership:

$$\mu_{A+B}(x,y) = max\left(\mu_A(x), \mu_B(y)\right)$$

Fuzzy Systems

inesc id lisboa

TÉCNICO LISBOA

Fuzzy Systems

- Fuzzy inference and fuzzy rules
- Fuzzy inference systems (Mamdani)
- Fuzzy inference systems (Takagi-Sugeno)
- Implementation
- Complex Fuzzy systems

Fuzzy Sets and Fuzzy Inference

- Fuzzy sets enable us to handle imprecision (vagueness) in a very intuitive and natural manner:
  - Formalize imprecise (vague) data;
  - Imprecise arithmetic and logic operations;
  - Describe inference systems based on fuzzy rules.

- Fuzzy inference systems are composed of fuzzy rules and fuzzy-reasoning processes, all based on fuzzy set theory

Fuzzy Sets and Fuzzy Inference (II)

- Fuzzy inference is a process of mapping from a given input to an output, using the theory of fuzzy sets

- Fuzzy rules are commonly of the form:

  IF . . . THEN . . .

- Fuzzy rules encode the general relation between the inputs and the outputs
  - Rules can be examples, rules of thumb, encoded experience, qualitative relations between variables, etc.

Fuzzy Inference Systems

- Fuzzy systems manipulate fuzzy sets to model the world
- Most fuzzy systems are rule based:

linguistic variable　　fuzzy set (linguistic term)

antecedents

IF *water temperature* is low AND *hot water flow rate* is medium,
THEN open the warm water tap slightly

consequent

- Antecedents describe a condition in which a rule is valid locally
- Consequents can be fuzzy sets, numbers or another model (e.g. a linear model)

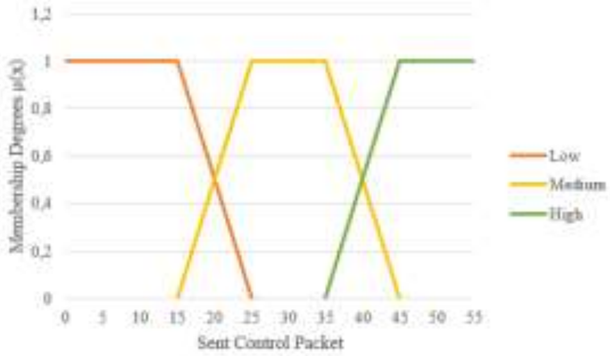
Fuzzy If-Then Rules

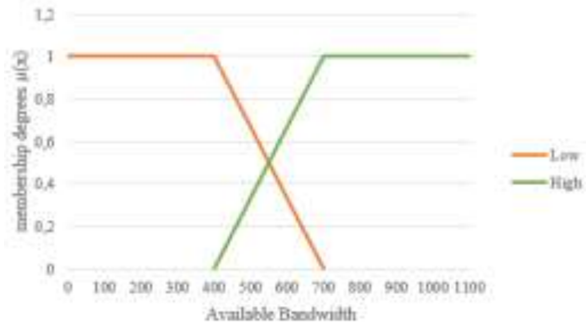
- General format:

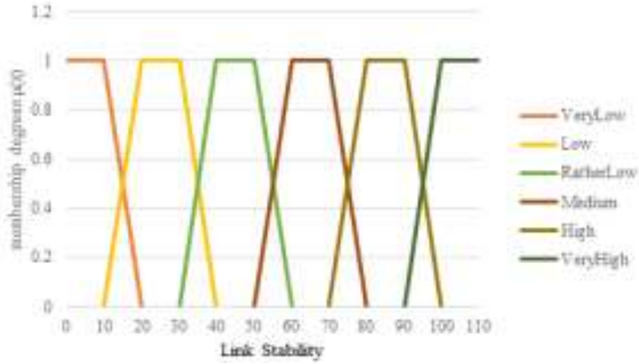
  If x is A then y is B

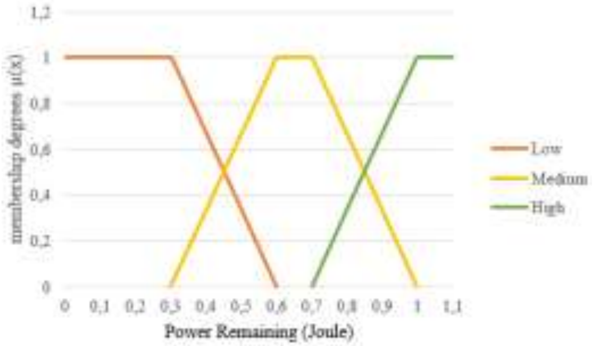
- Examples:
  - If pressure is high, then volume is small
  - If temperature is hot, then fan speed is fast
  - If the speed is high, then apply brake slightly
  - If demand is rising fast and quantity-on-hold is low, then order amount is large
  - If bandwidth is good and battery is high, then frame rate is high

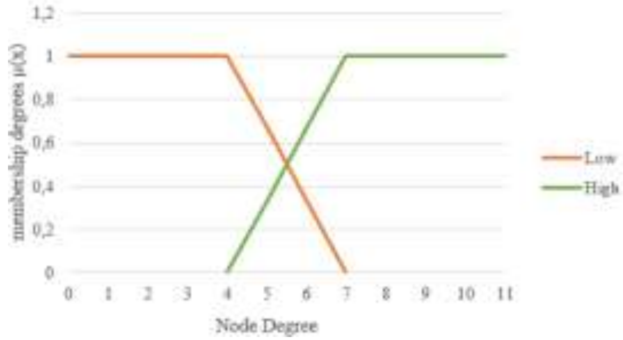
Rule Base Example (Linguistic Terms)









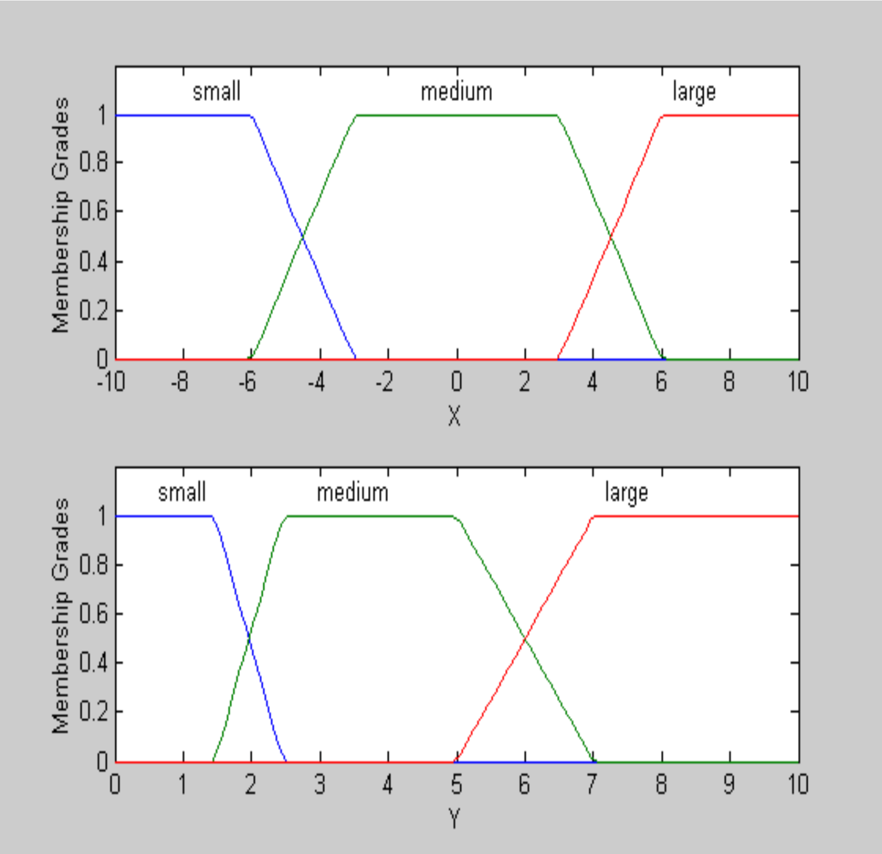


Rule Base Example (II)

No. i	Input				Output
	Available Bandwidth	Power Remaining	Node Degree	Sent Control Packet	Link Stability
1	Low	Low	Low	Low	Very Low
2	Low	Low	Low	Medium	Very Low
3	Low	Low	Low	High	Very Low
4	Low	Low	High	Low	Low
5	Low	Low	High	Medium	Rather Low
6	Low	Low	High	High	Rather Low
7	Low	Medium	Low	Low	Low
8	Low	Medium	Low	Medium	Rather Low
9	Low	Medium	Low	High	Rather Low
10	Low	Medium	High	Low	Medium
11	Low	Medium	High	Medium	Medium
12	Low	Medium	High	High	Medium
13	Low	High	Low	Low	Low
14	Low	High	Low	Medium	Rather Low
15	Low	High	Low	High	Rather Low
16	Low	High	High	Low	Medium
17	Low	High	High	Medium	High
18	Low	High	High	High	High
19	High	Low	Low	Low	Very Low
20	High	Low	Low	Medium	Low
21	High	Low	Low	High	Low
22	High	Low	High	Low	Rather Low
23	High	Low	High	Medium	Rather Low
24	High	Low	High	High	Rather Low
25	High	Medium	Low	Low	Medium

No. i	Input				Output
	Available Bandwidth	Power Remaining	Node Degree	Sent Control Packet	Link Stability
26	High	Medium	Low	Medium	Medium
27	High	Medium	Low	High	Medium
28	High	Medium	High	Low	High
29	High	Medium	High	Medium	High
30	High	Medium	High	High	Very High
31	High	High	Low	Low	Medium
32	High	High	Low	Medium	High
33	High	High	Low	High	High
34	High	High	High	Low	High
35	High	High	High	Medium	Very High
36	High	High	High	High	Very High

Why Fuzzy Rules?



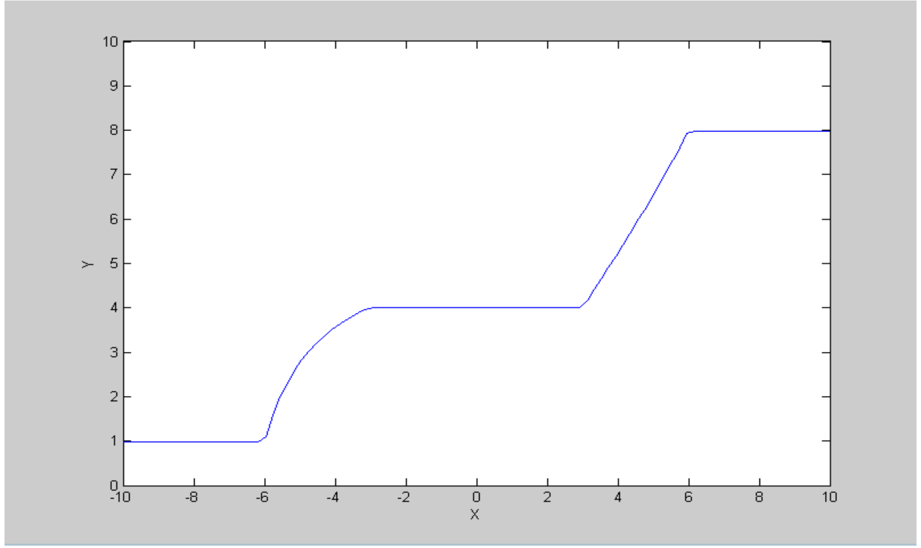
Rules:

If x is Small then y is Small

If x is Medium then y is Medium

If x is Large then y is Large

Instead of 3 discontinuous steps, we have a smoother continuous output. Imagine you are controlling a plane…



Fuzzy Inference System

- A.K.A.:
  - Fuzzy rule-based system
  - Fuzzy expert system
  - Fuzzy model
  - Fuzzy associative memory
  - Fuzzy logic controller
  - Fuzzy system (simply)

Fuzzy Systems

Mamdani Fuzzy Systems

Mamdani Fuzzy Models

- Composed by a set of if-then rules
  - Antecedents are fuzzy sets
  - Consequents are fuzzy sets

- Example of a Mamdani fuzzy rule:

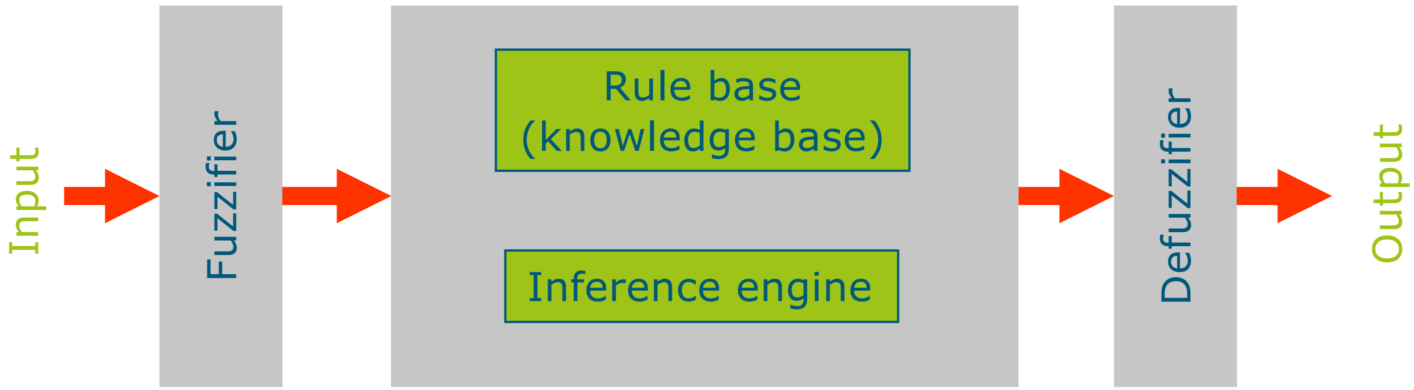
Antecedent fuzzy sets

If age is young and call frequency is high
then call length is long

Consequent fuzzy set

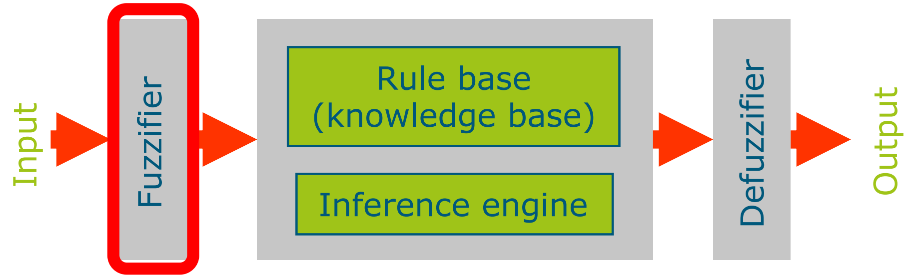
Building Blocks

- Fuzzifier
- Rule base
- Inference engine
- Defuzzifier



Fuzzifier

- Interface between the outside world and the fuzzy system (input side)
- Determines the match between a given input and the linguistic terms
  - Calculate the membership degrees of the (measured) inputs to the linguistic terms in the fuzzifier

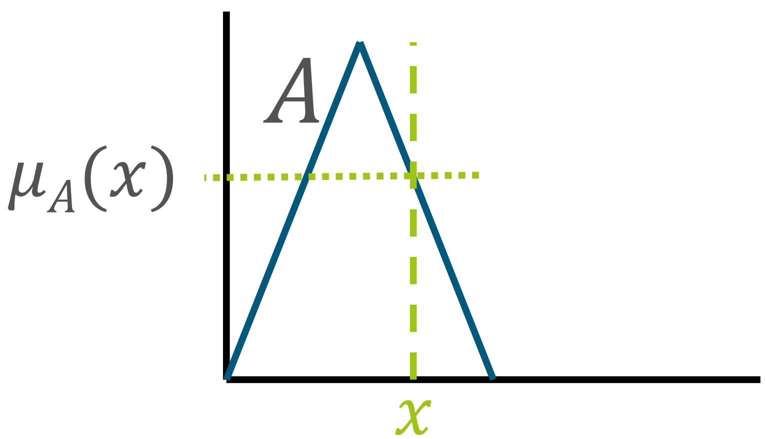


Fuzzification

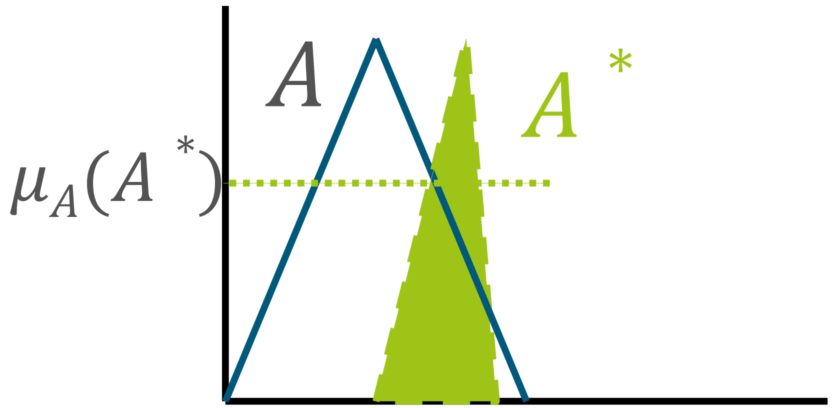
If bandwidth is good then frame rate is high

- For crisp inputs: compute the membership to linguistic terms
- For fuzzy inputs: compute the maximum membership of the fuzzy input in the linguistic terms

Crisp inputs

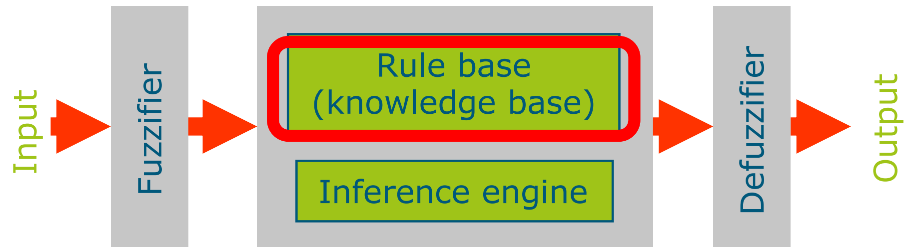


Fuzzy inputs



Rulebase

- Heart of the knowledge base of the fuzzy system
- Encodes the general relation between the inputs and the outputs
- Rules can be examples, rules of thumb, encoded experience, qualitative relations between variables, etc.
- Rules are often represented as if-then statements



Rulebase – Examples

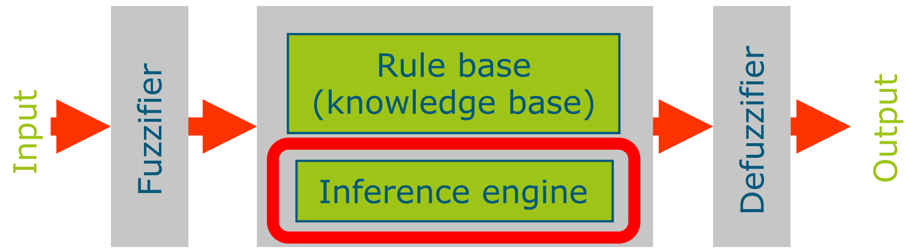
	Input				Output
No. i	Available Bandwidth	Power Remaining	Node Degree	Sent Control Packet	Link Stability
1	Low	Low	Low	Low	Very Low
2	Low	Low	Low	Medium	Very Low
3	Low	Low	Low	High	Very Low
4	Low	Low	High	Low	Low
5	Low	Low	High	Medium	Rather Low
6	Low	Low	High	High	Rather Low
7	Low	Medium	Low	Low	Low
8	Low	Medium	Low	Medium	Rather Low
9	Low	Medium	Low	High	Rather Low
10	Low	Medium	High	Low	Medium
11	Low	Medium	High	Medium	Medium
12	Low	Medium	High	High	Medium
13	Low	High	Low	Low	Low
14	Low	High	Low	Medium	Rather Low
15	Low	High	Low	High	Rather Low
16	Low	High	High	Low	Medium
17	Low	High	High	Medium	High
18	Low	High	High	High	High
19	High	Low	Low	Low	Very Low
20	High	Low	Low	Medium	Low
21	High	Low	Low	High	Low
22	High	Low	High	Low	Rather Low
23	High	Low	High	Medium	Rather Low
24	High	Low	High	High	Rather Low
25	High	Medium	Low	Low	Medium

	Input				Output
No. i	Available Bandwidth	Power Remaining	Node Degree	Sent Control Packet	Link Stability
26	High	Medium	Low	Medium	Medium
27	High	Medium	Low	High	Medium
28	High	Medium	High	Low	High
29	High	Medium	High	Medium	High
30	High	Medium	High	High	Very High
31	High	High	Low	Low	Medium
32	High	High	Low	Medium	High
33	High	High	Low	High	High
34	High	High	High	Low	High
35	High	High	High	Medium	Very High
36	High	High	High	High	Very High

Fan Speed		CPU Speed		
		Low	Normal	Turbo
Core Temp	Cold	Slow	Slow	Fast
	Warm	Slow	Medium	Fast
	Hot	Fast	Fast	Fast

Inference Engine

- The reasoning mechanism of the fuzzy system
  - to infer: to reason/deduce
- Combines actual inputs with the information encoded in the rule base to compute the fuzzy output of the system
- Not as context-independent as the inference engine of an expert system: context-dependent choices are usually necessary



Inference Engine: Degree of Fulfillment

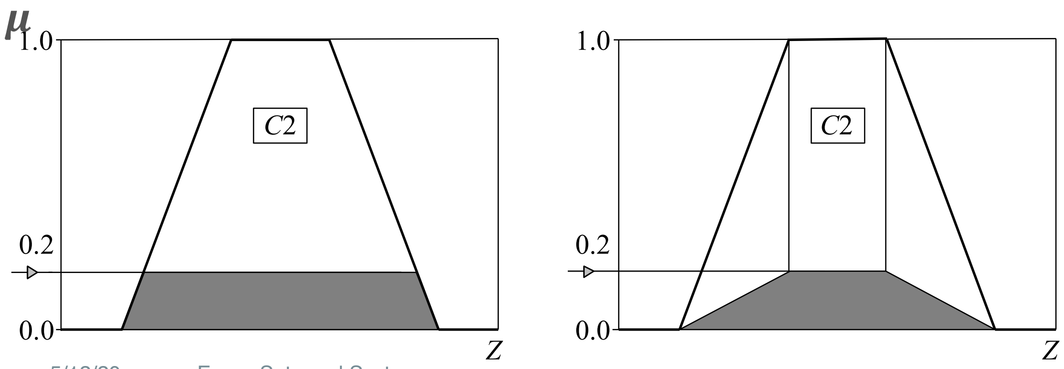
If bandwidth is good AND battery is high,
Then frame rate is high

- When the antecedent (if-part) of a rule contains multiple variables connected with an "AND", the total match between all inputs and the rule antecedent must be determined
- Degree of fulfilment determines to what degree the rule is valid:
  - Take minimum of all fuzzified values in the rule antecedente (if minimum is used as t-norm)
  - Take product of all fuzzified values in the rule antecedente (if product is used as t-norm)

Inference Engine: Output Inference

If bandwidth is good and battery is high, Then frame rate is high

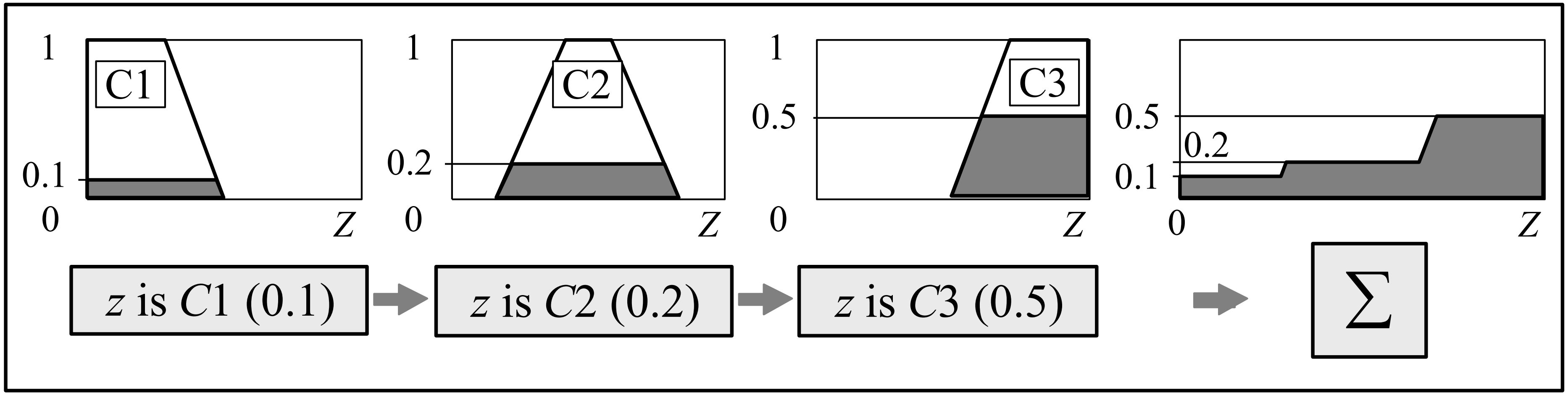
- Computes the output of each fuzzy rule in the rule base, given the degree of fulfilment of the rule base
- Modifies the output fuzzy set depending on the degree of fulfilment
  - Clip the output fuzzy set at the height corresponding to the degree of fulfilment (max-min composition)
  - Scale the output fuzzy set at the height corresponding to the degree of fulfilment (max-product composition)



- Clipping might lose information but is computationally easier and allows for a simpler aggregation and defuzification

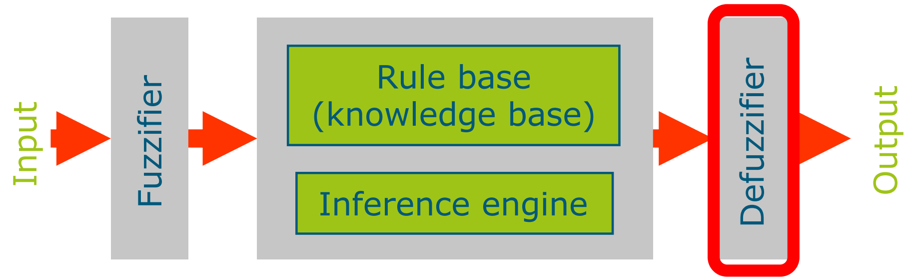
Inference Engine: Aggregation

- Multiple rules may become active in a fuzzy system
- The output of the fuzzy system is the combined output of all active rules
- Corresponds to calculating the total relation from the relations of individual rules
  - E.g.: Maximum (OR) of the fuzzy sets resulting from the individual rules



Defuzzifier

- Interface between the fuzzy systems and the outside world (output side)
- Needed when a crisp output is required (e.g. a final decision, a control action, a final advice, etc.)
- Computes a crisp number that represents the output fuzzy set (defuzzification)
- Enhances the interpolation properties of the fuzzy system

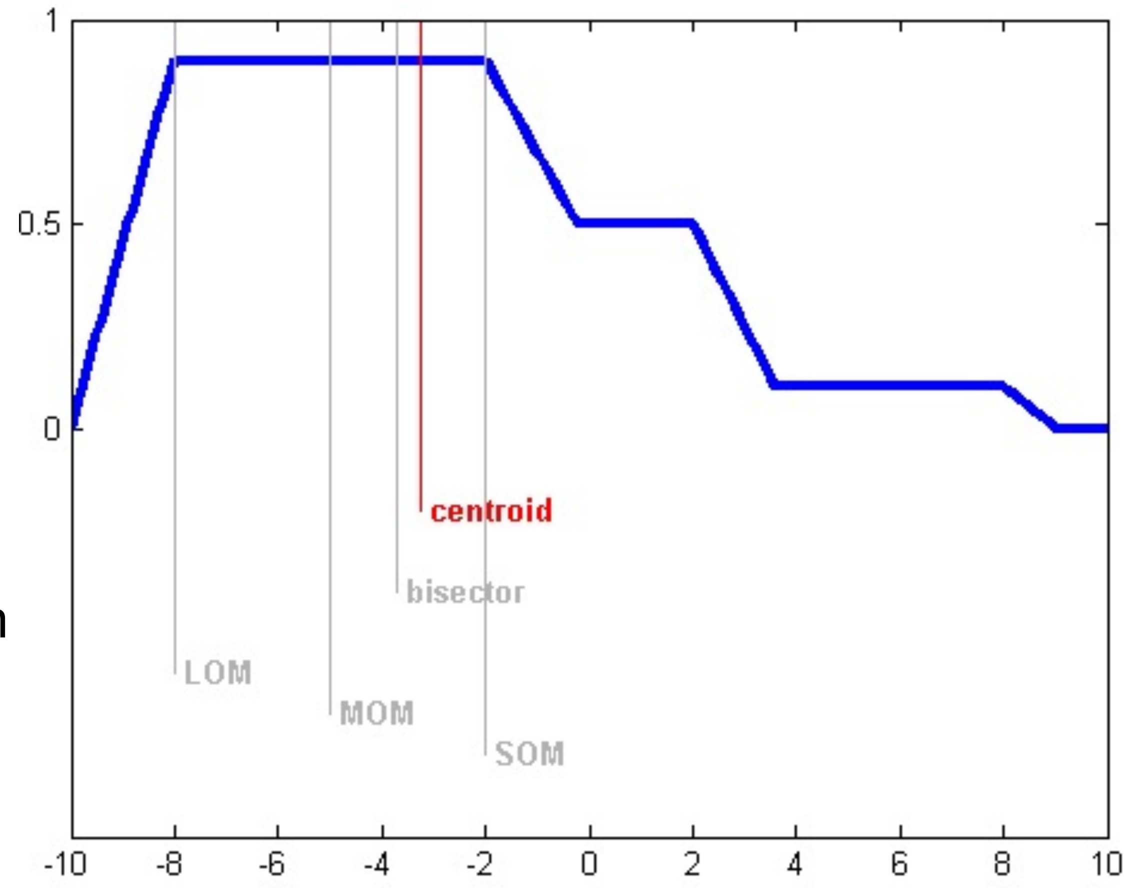


Defuzzification Methods

- Centroid-of-area $$z^* = \frac{\int_Z \mu_A(z)\, z\, dz}{\int_Z \mu_A(z)\, dz}$$
- Bisector of area $$\int_{-\infty}^{z^*} \mu_A(z) dz = \int_{z^*}^{\infty} \mu_A(z) dz$$
- Mean of maximum $$z^* = \frac{\int_{Z'} z\, dz}{\int_{Z'} dz}, \quad Z' = \{z \mid \mu_A(z) = \mu^*\}$$
- Smallest of maximum $$\min_{z \in Z'} z$$
- Largest of maximum $$\max_{z \in Z'} z$$

Defuzzification Methods (II)

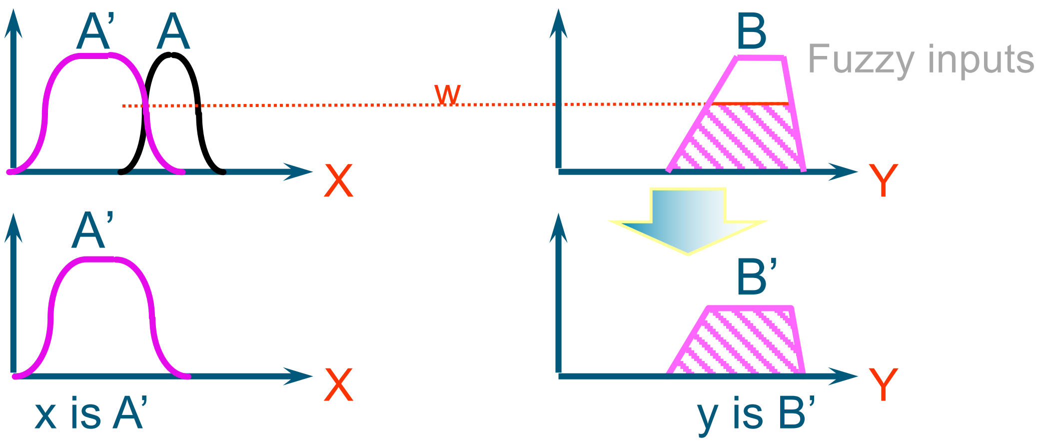
- Centroid-of-area
- Bisector of area
- Mean of maximum
- Smallest of maximum
- Largest of maximum



Example: Single Rule, Single Antecedent

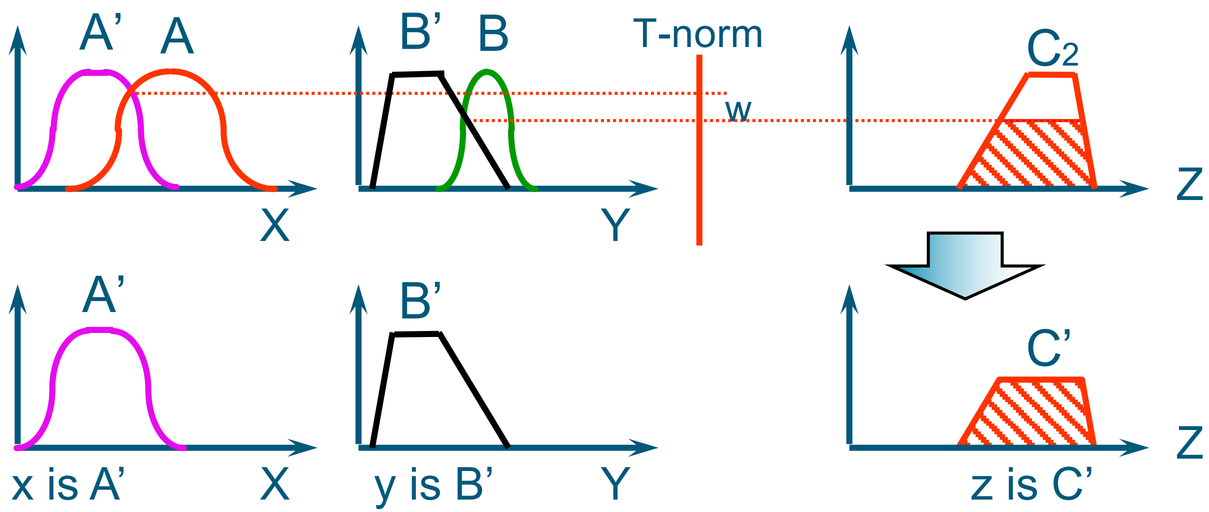
- Rule: If x is A then y is B
- Fact: x is A′ (fuzzy input)
- Conclusion: y is B′

$$\mu_{B'}(y) = [\vee_x(\mu_{A'}(x) \wedge \mu_A(x))] \wedge \mu_B(y)$$
$$= w \wedge \mu_B(y)$$



Example: Single Rule, Multiple Antecedents

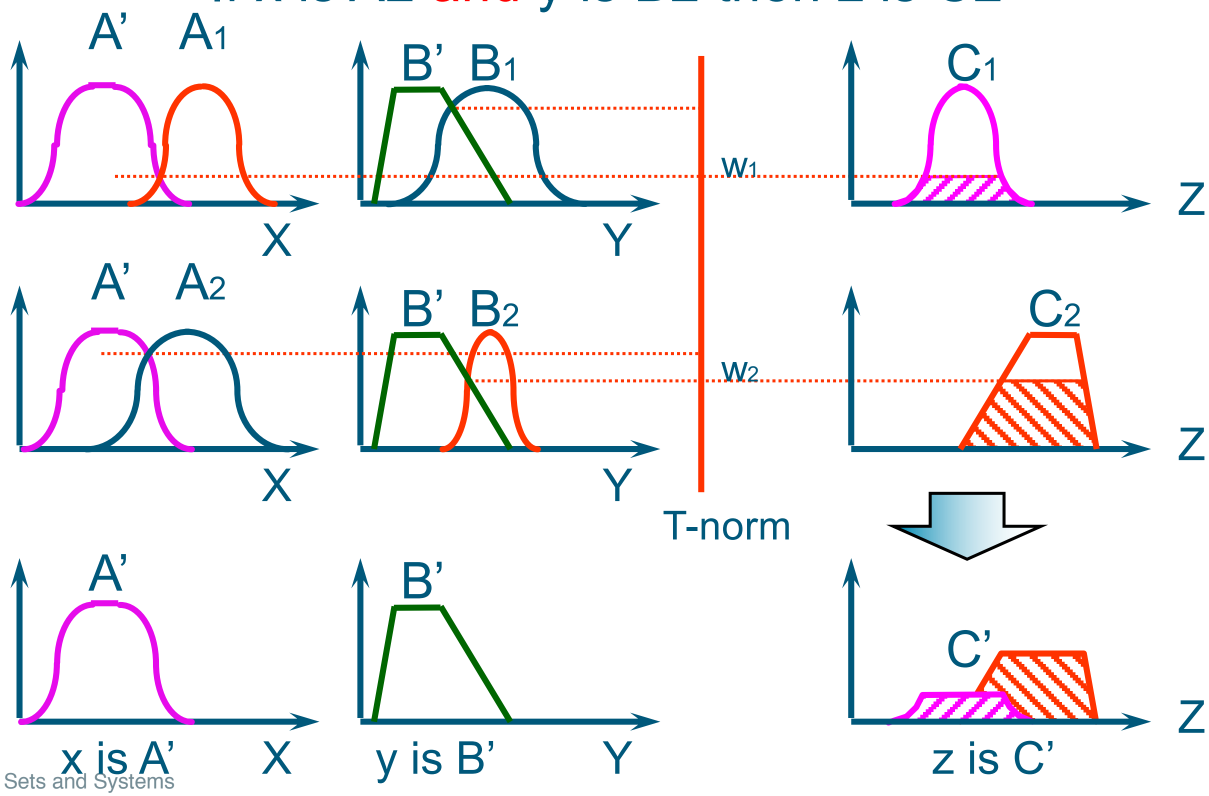
- Rule: If x is A and y is B then z is C2
- Fact: x is A′ and y is B′ (fuzzy inputs)
- Conclusion: z is c′



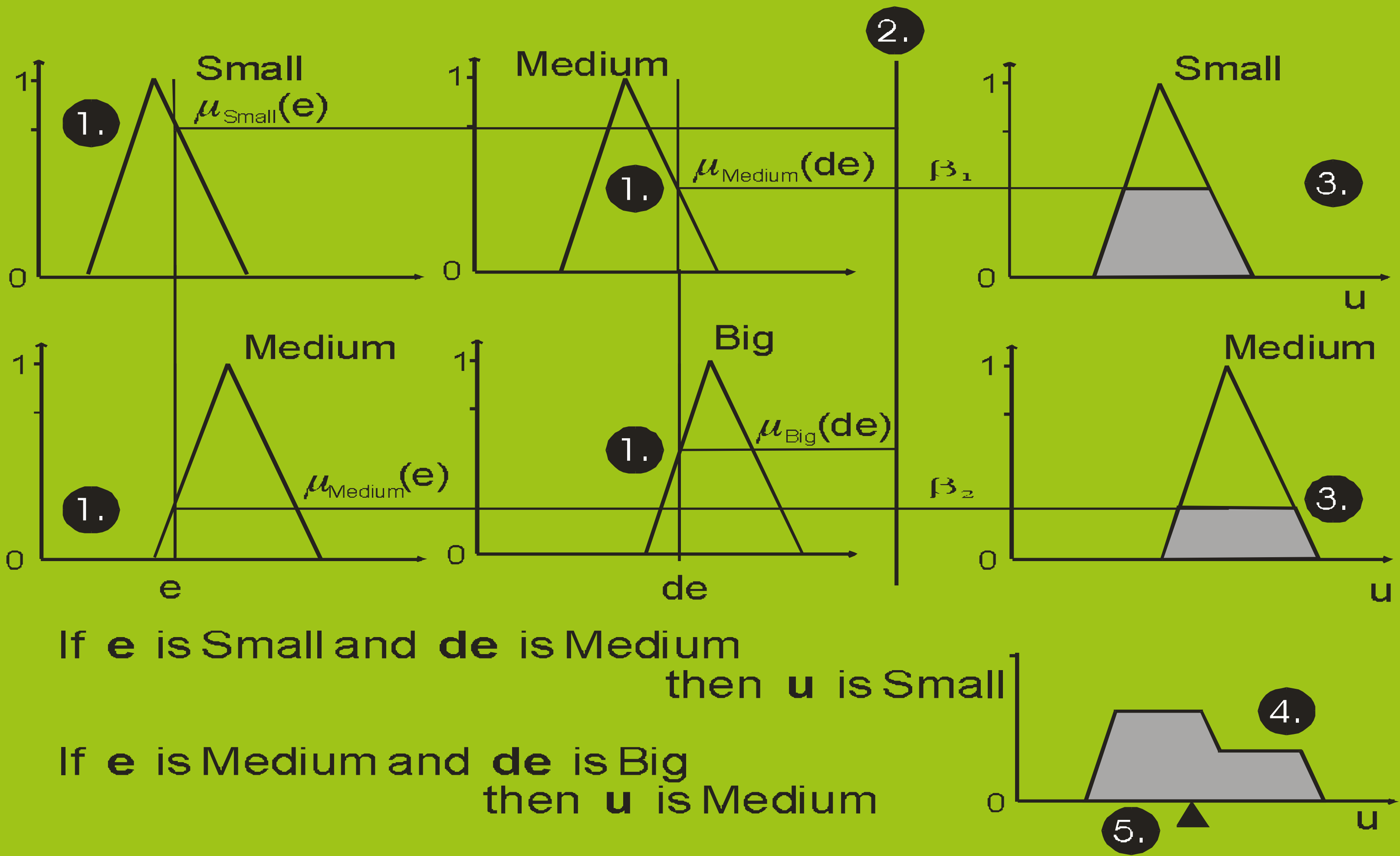
Example: Multiple Rules, Multiple Antecedents

- Rules: If x is A1 and y is B1 then z is C1

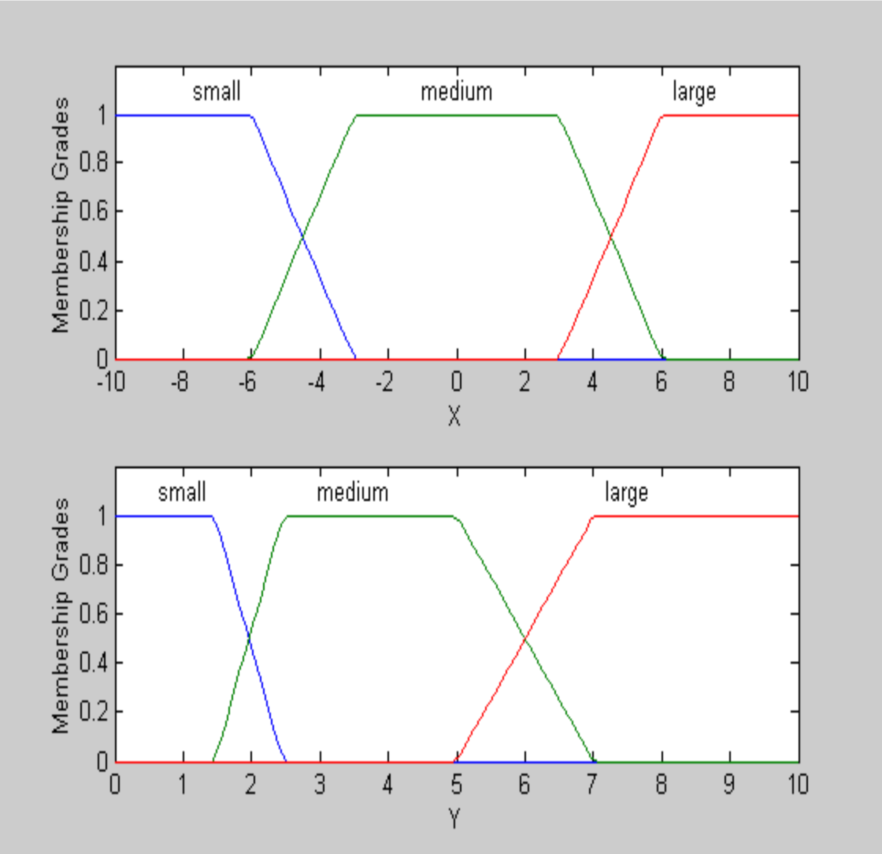
  If x is A2 and y is B2 then z is C2



Mamdani system - example



Mamdani – Single Input (x), Single Output (y)

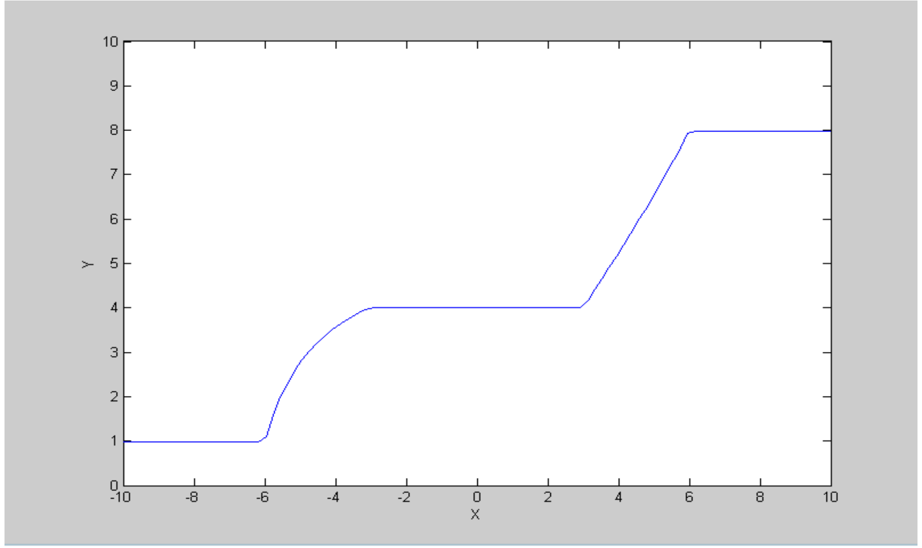


Rules:

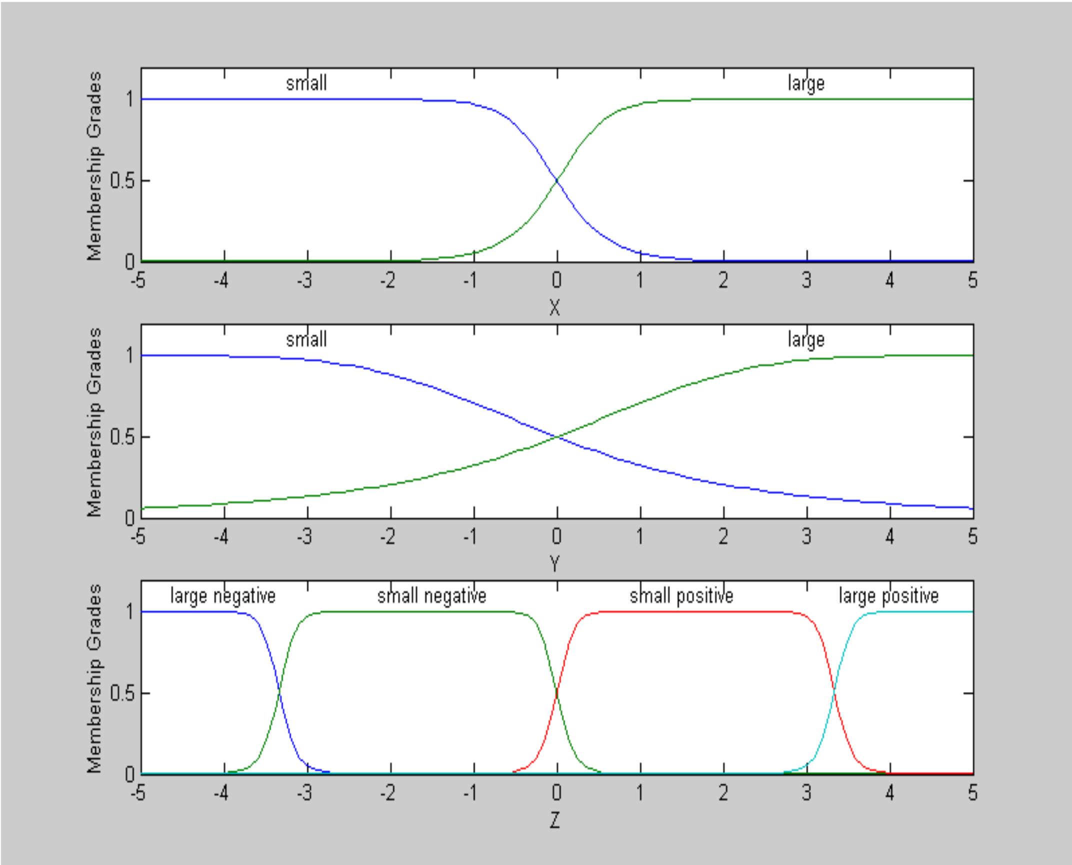
If x is Small then y is Small

If x is Medium then y is Medium

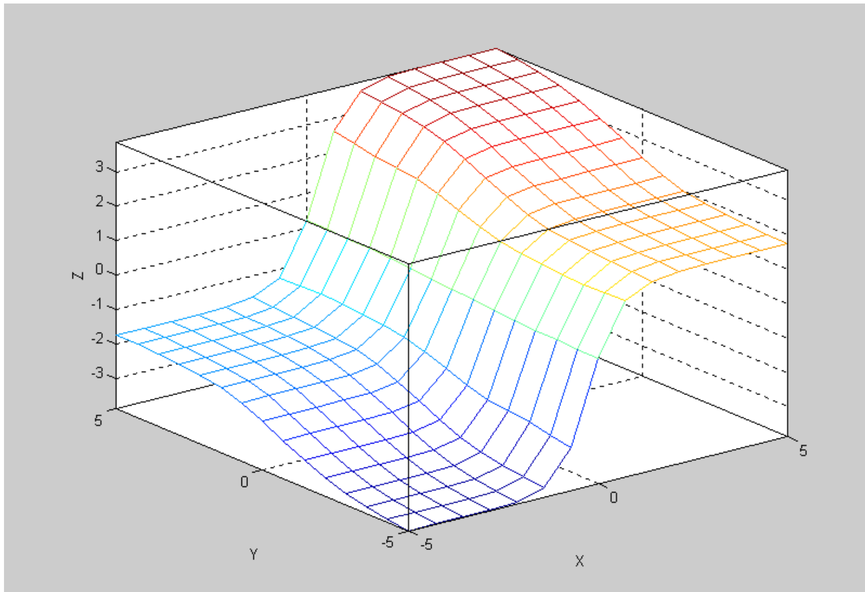
If x is Large then y is Large



Mamdani - Double Input



Rules:
If X is small & Y is small then Z is negative large
If X is small & Y is large then Z is negative small
If X is large & Y is small then Z is positive small
If X is large & Y is large then Z is positive large



Fuzzy Systems

Takagi-Sugeno Fuzzy Systems

inesc id lisboa

TÉCNICO LISBOA

Takagi-Sugeno (TS) Fuzzy Models

- Contain a set of if-then rules
- Antecedents are fuzzy sets
- Consequents are mathematical functions (typically of the inputs)

- Example TS fuzzy rule:

Antecedent fuzzy sets

If age is young and call frequency is high
then call length is 5*age – 0.2*call frequency

Common TS Fuzzy Models

- Zero-order Sugeno: constant consequent

  if x is A and y is B then z = c

- First-order Sugeno: linear consequent

  if x is A and y is B then z = ax+by+c

- Reasoning: Overall output is a weighted average of individual rule outputs

$$z^* = \frac{\sum_{i=1}^{N} \beta_i \, z_i}{\sum_{i=1}^{N} \beta_i}$$

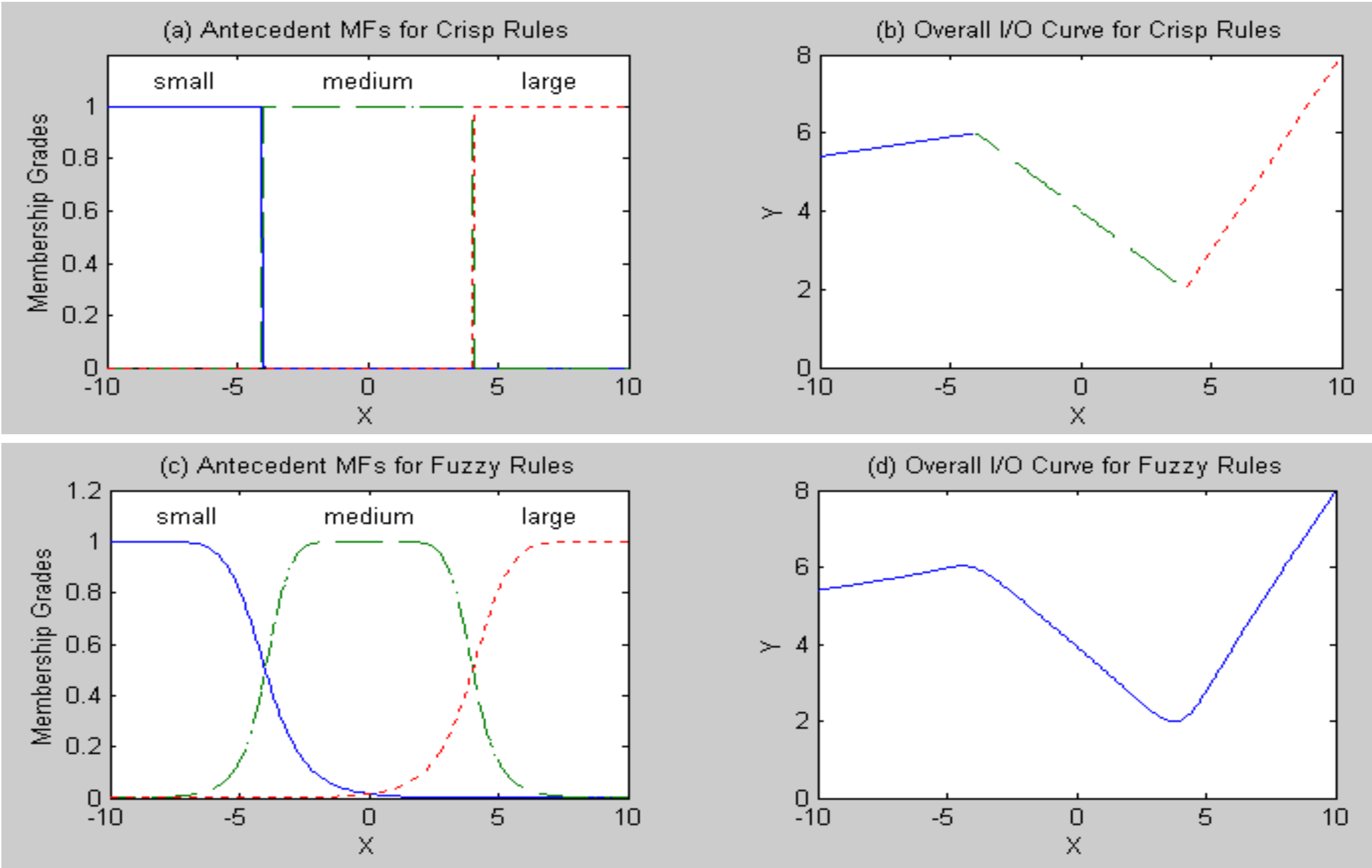
Consequent (output) of rule i

Degree of fulfillment of rule i (same as with Mamdani)

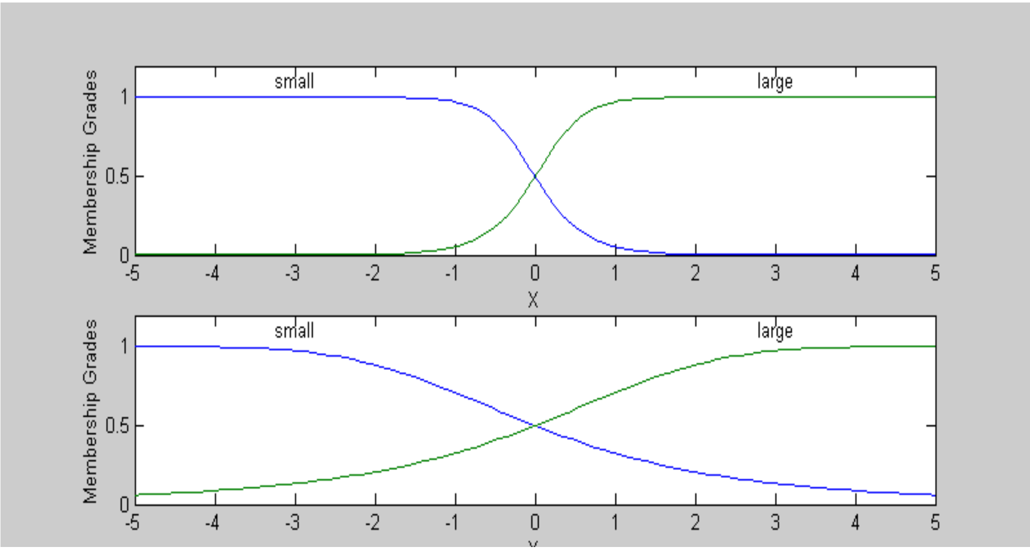
Single Input / Single Output TS Example

- Rulebase:
  - If x is small then $y = 0.1x + 6.4$
  - If x is medium then $y = -0.5x + 4$
  - If x is large then $y = x - 2$
- If small, medium and large are crisp sets, the output is a piecewise linear function
- If small, medium and large are fuzzy sets, the output is a smooth non-linear function

Single Input, Single Output TS: Fuzzy vs. Crisp Rule Set



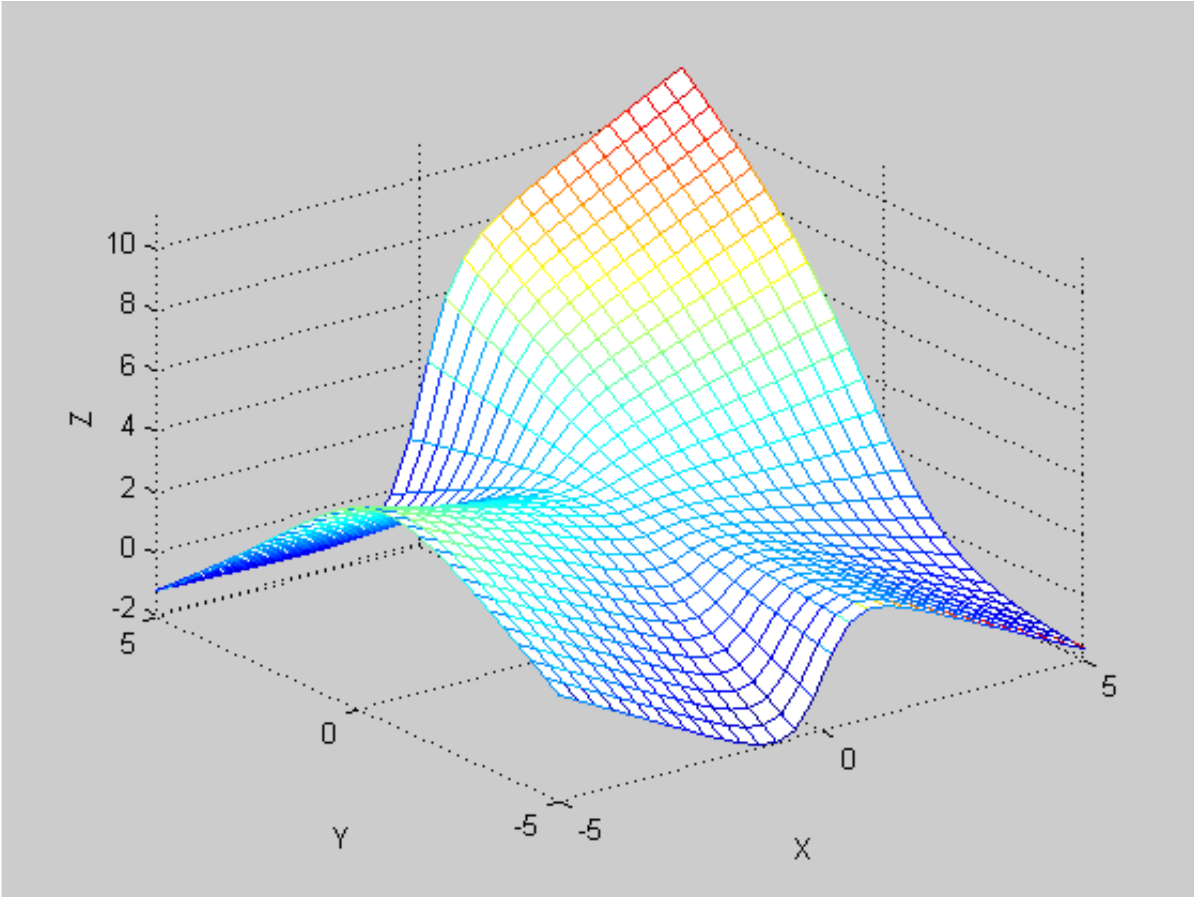
Two input / Single Output TS Example



Rulebase:

- If x is Small and y is Small then $z = -x+y+1$
- If x is Small and y is Large then $z = -y+2$
- If x is Large and y is Small then $z = -x+3$
- If x is Large and y is Large then $z = x+y+2$

Two Input / Single Output TS Example: Output



Fuzzy Systems Implementation

(using a case study)

inesc id lisboa

TÉCNICO LISBOA

Case Study: CPU Fan Speed Controler

- The aim is to control the speed of a CPU fan based on the:
  - Core temperature (in degrees Celsius)
  - Clock speed (in GHz)
- Compromise:
  - Higher speeds use more energy and decrease battery life
  - High core temperatures harm the processor and should be avoided (by increasing the fan speed)

Variables	Units	Range
Core Temperature	Degrees Celsius (ºC)	0 - 100
Clock Speed Frequency	Gigahertz (GHz)	0 - 4
Fan Speed	Revolutions per min (RPM)	0 - 6000

Case Study: CPU Fan Speed Controler (II)

- The aim is to control the CPU Fan Speed

Variables	Units	Range
Core Temperature	Degrees Celsius (ºC)	0 - 100
Clock Speed Frequency	Gigahertz (GHz)	0 - 4
Fan Speed	Revolutions per min (RPM)	0 - 6000

- How to do it?
  - There are obviously many options
  - Any ideas?

Developing a "Simple" Fuzzy Expert System

1. Specify the problem ✓
2. Define linguistic variables and respective linguistic terms
3. Determine fuzzy sets
4. Elicit and construct fuzzy rules
5. Encode the fuzzy sets, fuzzy rules and procedures in order to perform fuzzy inference
6. Evaluate and tune the system

Step 2: Define Linguistic Terms

Variables	Units	Range
Core Temperature	Degrees Celsius (ºC)	0 - 100
Clock Speed Frequency	Gigahertz (GHz)	0 - 4
Fan Speed	Revolutions per min (RPM)	0 - 6000

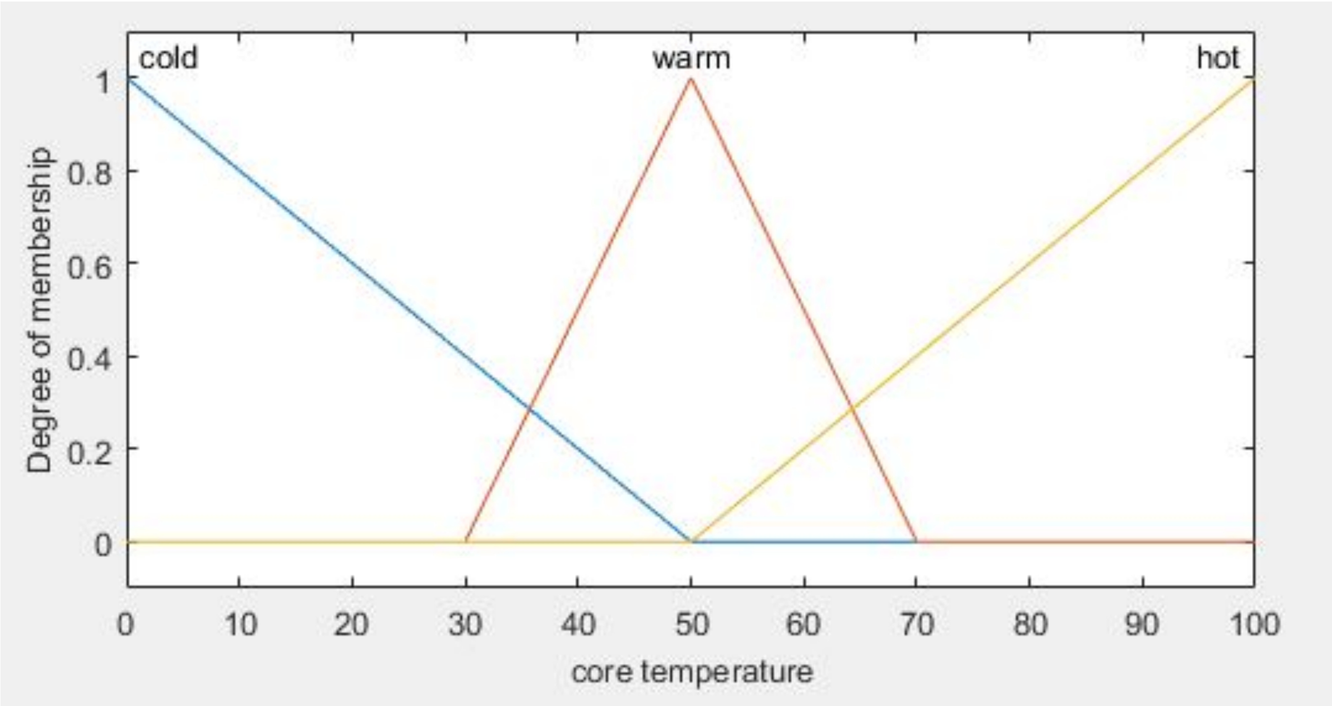
- Core Temperature (input)
  - Cold, Warm, Hot
- Clock Speed (input)
  - Low, Normal, Turbo
- Fan Speed (output)
  - Slow, Fast

Step 3: Determine Fuzzy Sets (Membership Functions)

- Fuzzy sets can have a variety of shapes
- Triangles or trapezoids can often provide an adequate representation of the expert knowledge, and at the same time, significantly simplify the process of computation
- For this example we will use triangular membership functions

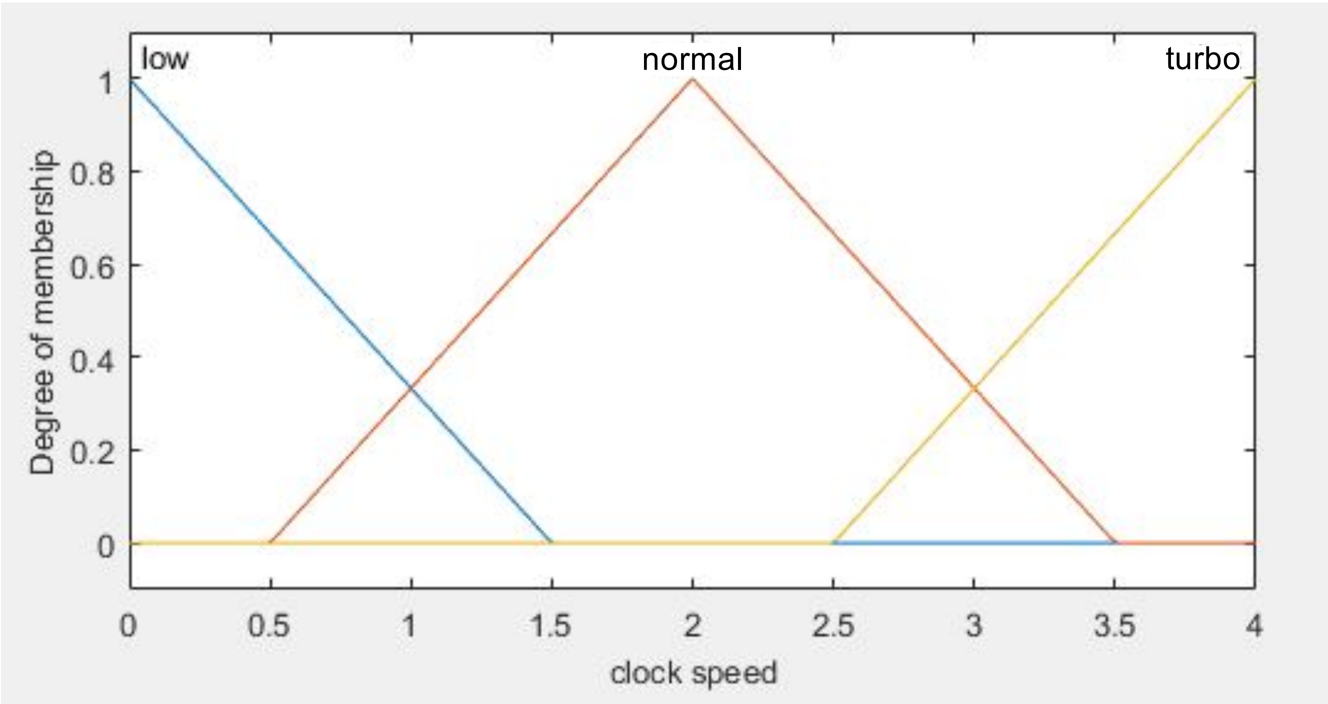
Step 3: Core Temperature Fuzzy Sets (Input)

- Core Temperature; [0,100] ºC; {“Cold”, “Warm”, “Hot” }



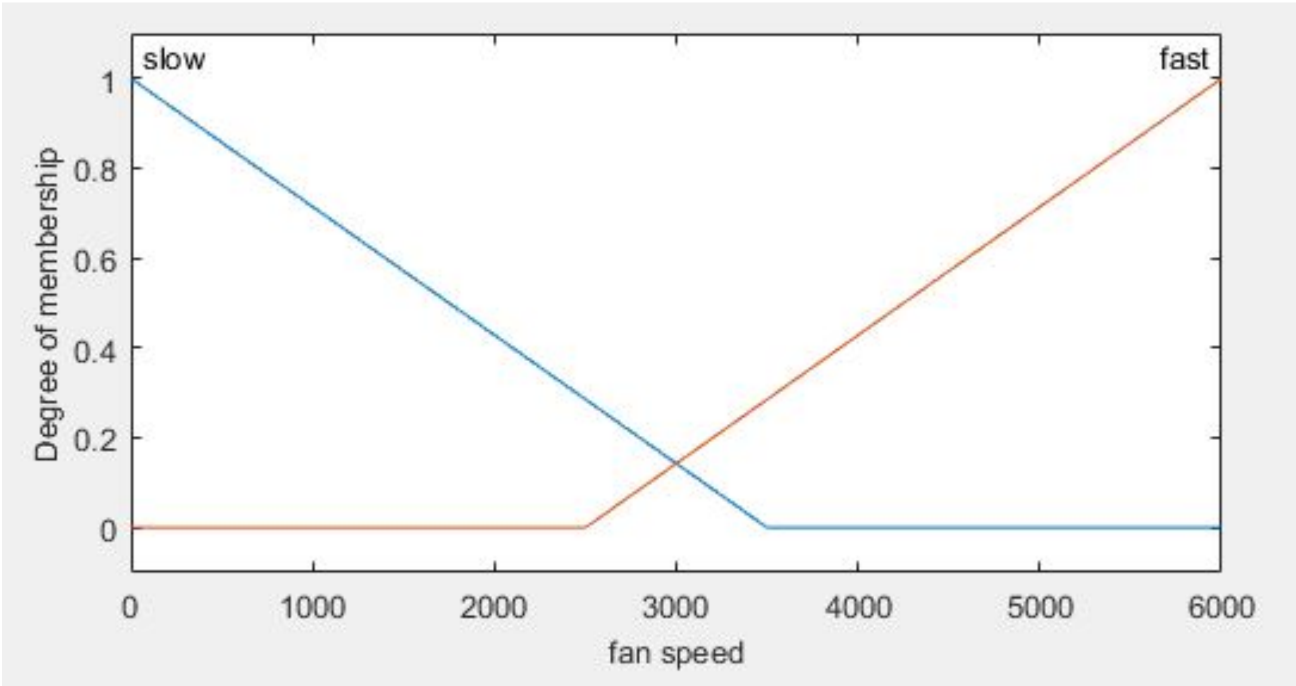
Step 3: Clock Speed Fuzzy Sets (Input)

- Clock Speed; [0,4] Ghz; {“Low”, “Normal”, “Turbo” }



Step 3: Fan Speed Fuzzy Sets (Output)

- Fan Speed; [0,6000] RPM; {“Slow”, “Fast” }



Step 4: Elicit and Construct Fuzzy Rules

- Expert knowledge is used to describe how the problem can be solved using the fuzzy linguistic variables
- Required knowledge also can be collected from other sources such as books, computer databases, flow diagrams and observed human behaviour (or directly from data)
- Rule bases can be represented in many different ways:
  - Rules, Table, Matrix, etc.
  - A matrix form of representing fuzzy rules is often called fuzzy associative memory (FAM)

Step 4: Rule Base Construction (using a FAM)

Fan Speed		CPU Speed		
		Low	Normal	Turbo
Core Temp	Cold	Slow	Slow	Fast
	Warm	Slow	Slow	Fast
	Hot	Fast	Fast	Fast

Step 4: Rule Base

- If Core Temp is cold AND Clk Speed is low THEN Fan Speed is slow
- If Core Temp is cold AND Clk Speed is normal THEN Fan Speed is slow
- If Core Temp is cold AND Clk Seed is turbo THEN Fan Speed is fast
- If Core Temp is warm AND Clk Speed is low THEN Fan Speed is slow
- If Core Temp is warm AND Clk Speed is normal THEN Fan Speed is slow
- If Core Temp is warm AND Clk Seed is turbo THEN Fan Speed is fast
- If Core Temp is hot AND Clk Speed is low THEN Fan Speed is fast
- If Core Temp is hot AND Clk Speed is normal THEN Fan Speed is fast
- If Core Temp is hot AND Clk Seed is turbo THEN Fan Speed is fast

Step 5: Encode the Fuzzy Sets, Fuzzy Rules and Procedures to Perform Fuzzy Inference

- To accomplish this task, we may choose one of two options:
  1. Apply a fuzzy logic development tool such as MATLAB Fuzzy Logic Toolbox, Fuzzy Clips, or Fuzzy Knowledge Builder
  2. Build the system using a programming language such as Python, C/C++, Java, etc.
  - ”Fuzzy Systems Made Easy”:
    - Several libraries are available for Python: FuzzyPy; PyFuzzy; Gfuzzy; Scikit Fuzzy; etc.
    - https://pythonhosted.org/scikit-fuzzy/

Step 6: Evaluate and Tune the System

- Does the modeled fuzzy system meet the requirements specified at the beginning?
- Fuzzy Logic Toolbox, or Maplotlib, can be used to generate surfaces that help analyzing the system's performance
  - https://pythonhosted.org/scikit-fuzzy/auto_examples/plot_control_system_advanced.html
- If the system does not perform as expected, then it should be tuned

Tuning Fuzzy Systems

1. Review model input and output variables, and if required redefine their ranges
2. Review the fuzzy sets, and if required redefine the partition of the universe of discourse
   - The use of wide fuzzy sets may cause the fuzzy system to perform roughly ⇒ Increase the number of linguistic terms
   - E.g., add a third linguistic term “Medium” to Fan Speed
3. Provide sufficient overlap between neighbouring sets
   - It is suggested that triangle-to-triangle and trapezoid-to-triangle fuzzy sets should overlap between 25% to 50% of their bases

Tuning Fuzzy Systems (Cont.)

4. Review the existing rules, and if required add new rules to the rulebase
   4. Note: Fuzzy rulebases are not necessarily complete (as were in the previous case study)
5. Examine the rulebase for opportunities to write hedge rules to capture the pathological behaviour of the system
6. Adjust the rule execution weights
   – Most fuzzy logic tools allow control of the importance of rules by changing a weight multiplier
7. Revise shapes of the fuzzy sets (Note: in most cases, fuzzy systems are highly tolerant of a shape approximation)

Fuzzy Systems

Combinatorial Rule Explosion

Number of Rules and Approximation Capability

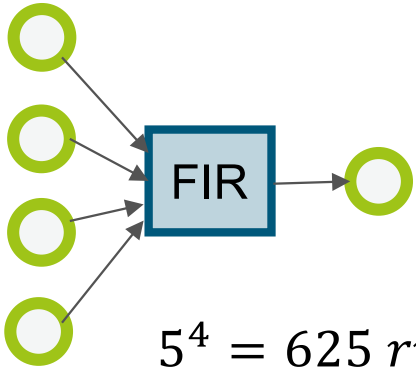
- Fuzzy systems are general function approximators (c.f. neural networks)
- You can increase the accuracy of a mapping by increasing the number of rules (examples) in the rule base
- ☺ Best results are obtained when the number of linguistic terms in the input and the output are increased (a finer partition)
- ☹ However, using too many linguistic terms diminishes the transparency of fuzzy systems…
  - ☹ …and leads to combinatorial rule explosion

Number of Inputs

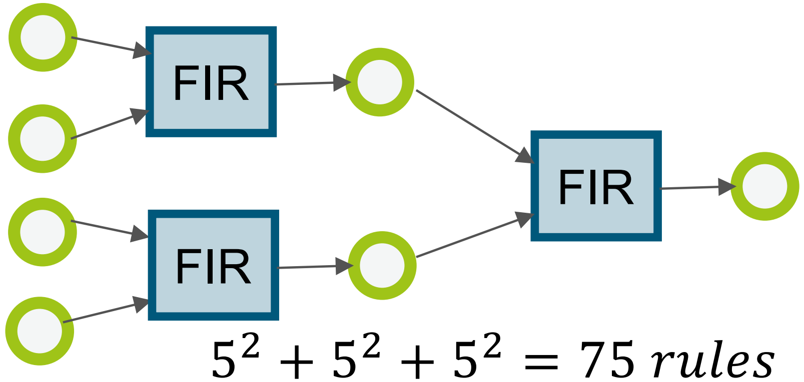
- Fuzzy systems' rulebases become exponentially harder to implement with the increase of the number of inputs:
  - Visualization of the rulebase becomes much less intuitive with more than 3 inputs
  - The number of rules increases exponentially (combinatorial rule explosion):
    - A complete rulebase of a fuzzy system with n inputs of m term sets has m^n rules
    - E.g: 3 inputs with 5 linguistic terms each = 5^3 = 125 rules
- Note: a fuzzy rulebase doesn't need to be complete (i.e., not all input combinations are always needed)

Number of Inputs (II)

- It is possible to prevent combinatorial rule explosion using the Combs method [Combs 1997]
- Another alternative to prevent combinatorial rule explosion is to separate the several inputs into smaller more manageable groups that are further combined in a subsequent layer (losing some information)
  - E.g: 4 inputs with 5 linguistic terms



$5^4 = 625\ rules$



$5^2 + 5^2 + 5^2 = 75\ rules$

Fuzzy Modeling

inesc id lisboa
20 YEARS
DEFINING TECHNOLOGY

How to Obtain Fuzzy Models?

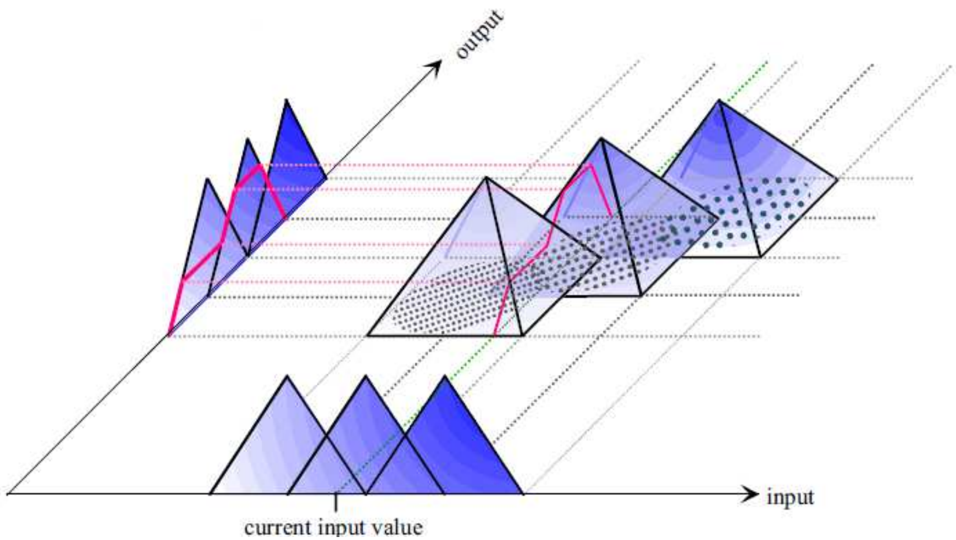
- Expert knowledge driven
  - Initialize FS using expert knowledge
  - Optimize parameters with expert knowledge
- Data driven
  - Apply fuzzy clustering with projection of fuzzy clusters or
  - Partition input/output spaces and use available data to induce the rules
  - Refine the parameters (e.g., neuro-fuzzy approach)
- Combinations of above
  - Data-driven estimation of optimal parameters after a suitable expert-driven initialization

Obtaining Fuzzy Systems from Data

- There are several different methods to learn fuzzy systems from data:
  - **Cluster** oriented approaches find clusters in data where each cluster corresponds to one rule
  - **Hyperbox** oriented approaches find clusters in form of hyperboxes that are used to generate rules
  - **Structure** oriented approaches use predefined fuzzy sets to structure the data space and pick rules from grid cells
  - **Neuro-fuzzy systems (NFS)** combine artificial neural networks with fuzzy rules in order to automatically generate optimal fuzzy inference systems

Learning Fuzzy Rules from Clusters

- Perform a fuzzy cluster analysis of the input-output data
- Then project the clusters
- Finally, obtain the fuzzy rules, e.g. “if x is small, then y is medium”



Learning Fuzzy Rules from Clusters (II)

- Given data X, $\mathbf{x}_k = [x_{1k}, x_{2k}, \ldots, x_{nk}]^T \in \Re^n, \quad k = 1, \ldots, N$

- Apply fuzzy clustering to X
- Obtain fuzzy partition matrix $\mathbf{U} = \begin{bmatrix} \mu_{11} & \ldots & \mu_{1N} \\ \vdots & \ddots & \vdots \\ \mu_{C1} & \ldots & \mu_{CN} \end{bmatrix}, \quad \mu_{ij} \in [0,1]$
- Use obtained U to define membership functions
  - Since X is usually multidimensional, how to specify meaningful labels for multidimensional membership functions?

Extending μ_{ij} to Continuous Membership Functions

- Assign labels for one-dimensional domains:
  - project U down to all *n* axis
  - only consider upper envelope of membership degrees
  - linearly interpolate membership values membership functions
  - cylindrically extend membership functions
- Original clusters are interpreted as conjunction of cyl. extensions
  - e.g., cylindrical extensions “x_1 is low”, “x_2 is high”
  - multidimensional cluster label “x_1 is low and x_2 is high”
- Labelled clusters = classes characterized by labels
- Each cluster = one fuzzy rule

Convex Completion

- Problem of this approach: non-convex fuzzy sets
- Solution: Having upper envelope, compute convex completion [Höppner et al., 1999]
  - we denote $p_1,\ldots,p_n$, $p_1 \leq \ldots,p_k$ as ordered projections of $x_1,\ldots,x_n$ and $\mu_{i1},\ldots,\mu_{in}$ as respective membership values
  - eliminate each point $(p_t\ ,\ \mu_{it})$, $t = 1,\ldots,n$, for which two limit indices $t_l,t_r = 1,\ldots,n$, $t_l < t < t_r$, exist such that

$$\mu_{it} < min\{\mu_{it_l}, \mu_{it_r}\}$$

  - after: apply linear interpolation of remaining points

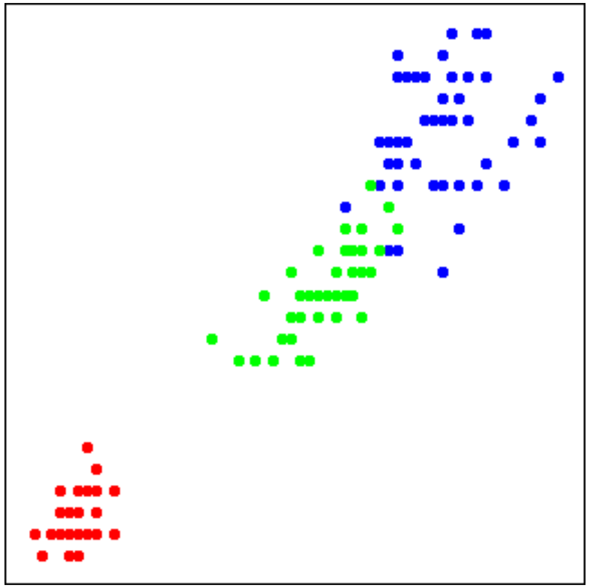
Example: The Iris Data

- © Iris Species Database http://www.badbear.com/signa/
  - Collected by Ronald Aylmer Fischer (famous statistician)
  - 150 cases in total, 50 cases per Iris flower type
  - Measurements: sepal length/width, petal length/width (cm)
- Probably the most famous dataset in pattern recognition and data analysis

Iris setosa

Iris versicolor

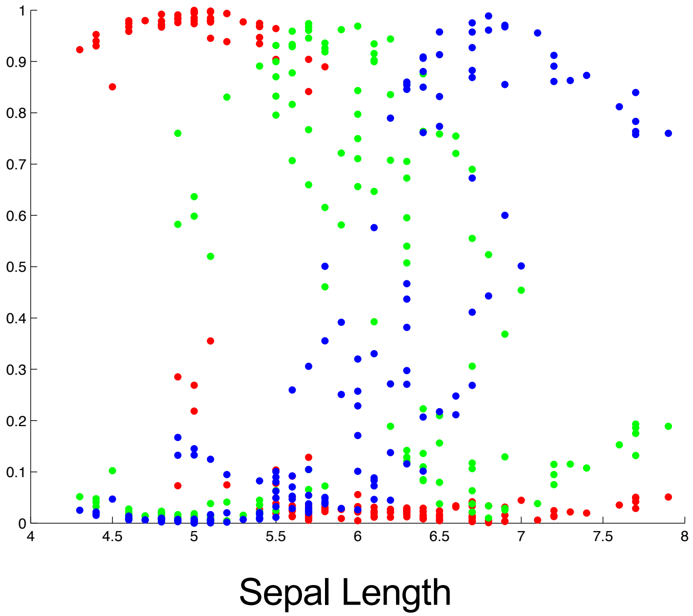
Iris virginica

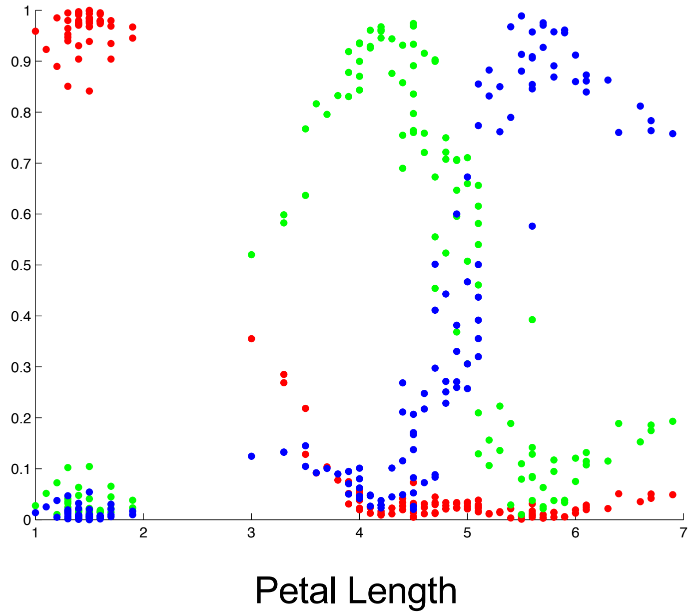


Iris setosa (red)
Iris versicolor (green)
Iris virginica (blue)
(Shown: sepal length and petal length)

Example: The Iris Data – Rules from Fuzzy Clusters (I)

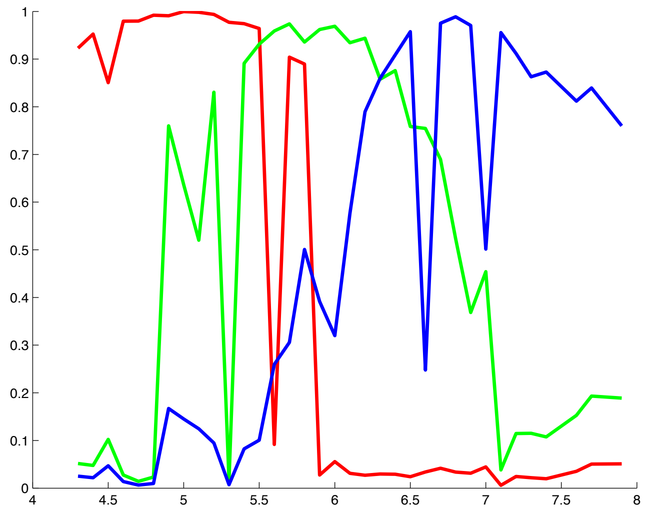
1. Use membership Degrees from FCM
   – Raw, unmodified membership degrees:



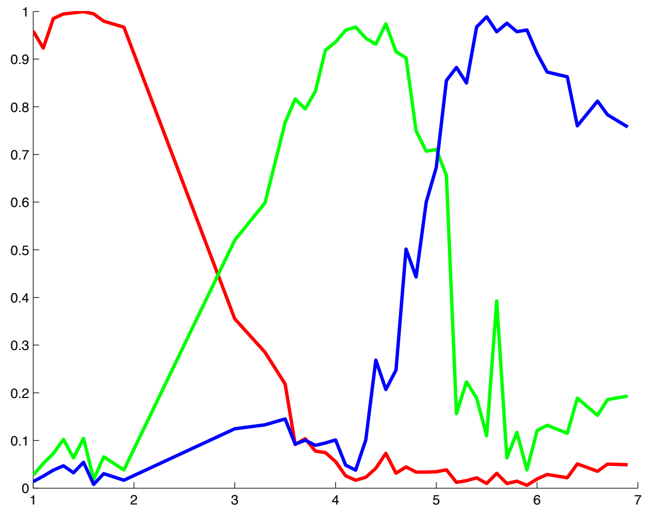


Example: The Iris Data – Rules from Fuzzy Clusters (II)

2. Upper envelope
   – For every attribute value and cluster centre, only consider maximum membership degree



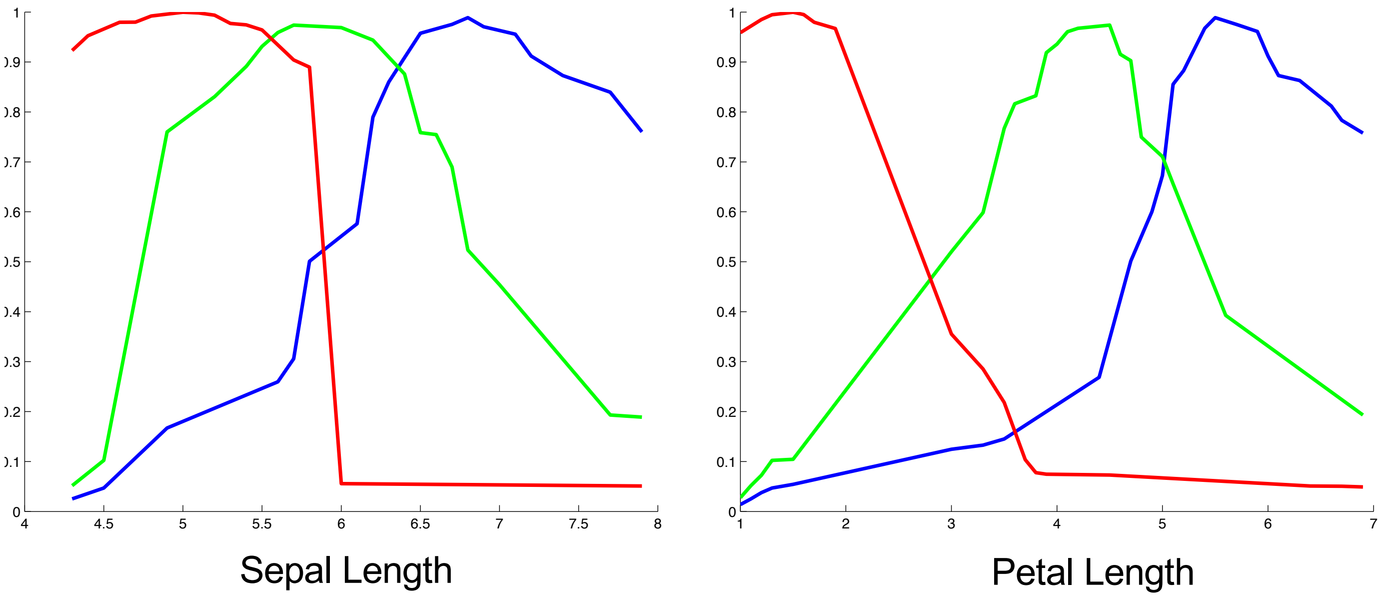
Sepal Length



Petal Length

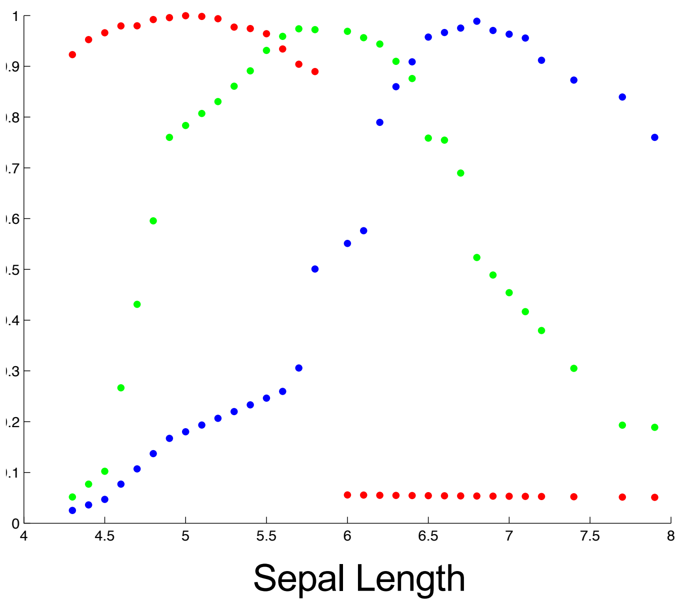
Example: The Iris Data – Rules from Fuzzy Clusters (III)

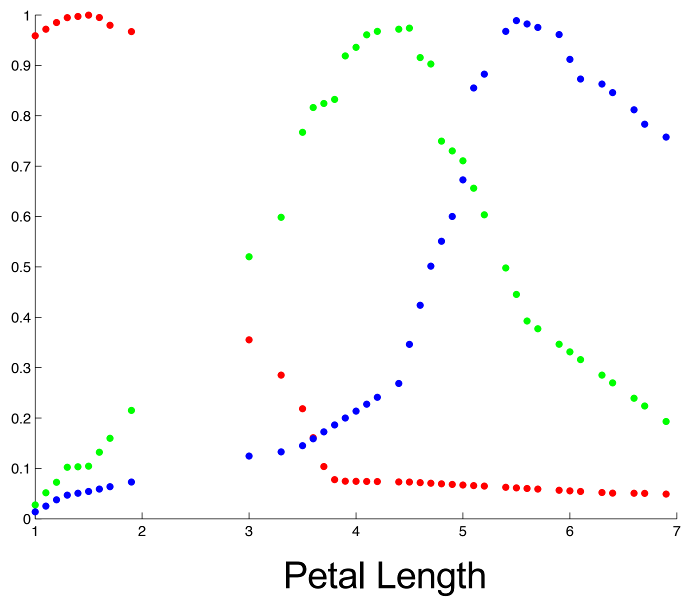
3. Convex Completion
   – Convex completion removes spikes [Höppner et al., 1999]



Example: The Iris Data – Rules from Fuzzy Clusters (IV)

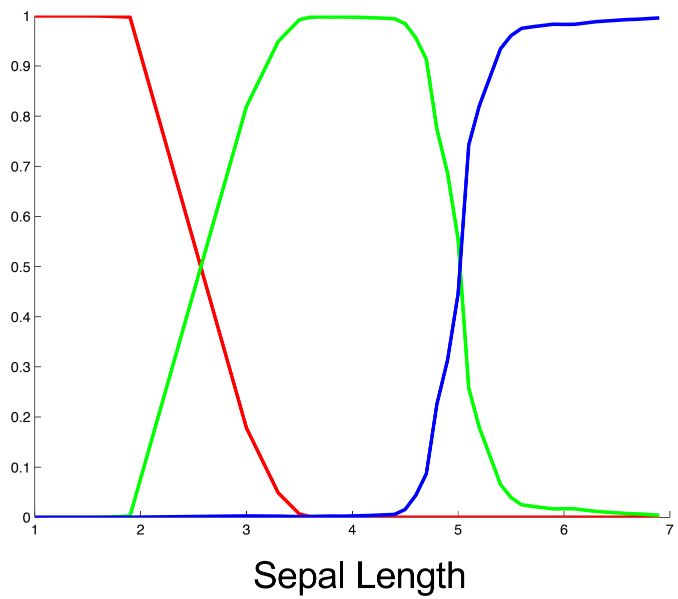
4. Linear Interpolation
   – Interpolation for missing values (needed for normalization)

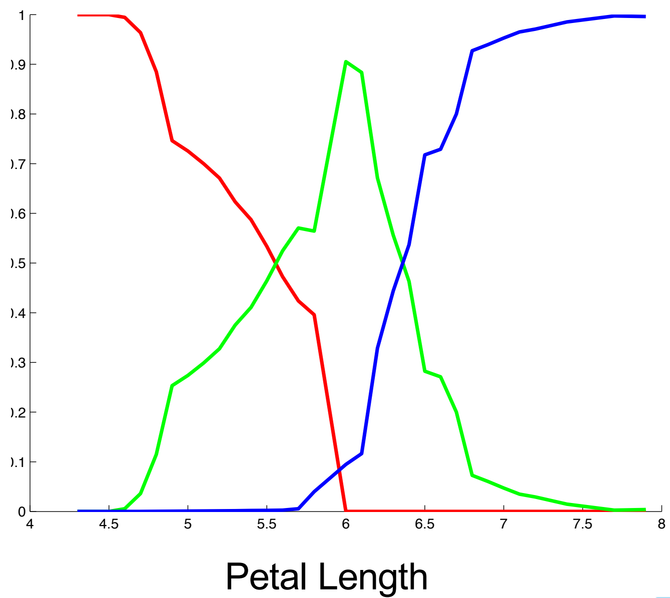




Example: The Iris Data – Rules from Fuzzy Clusters (V)

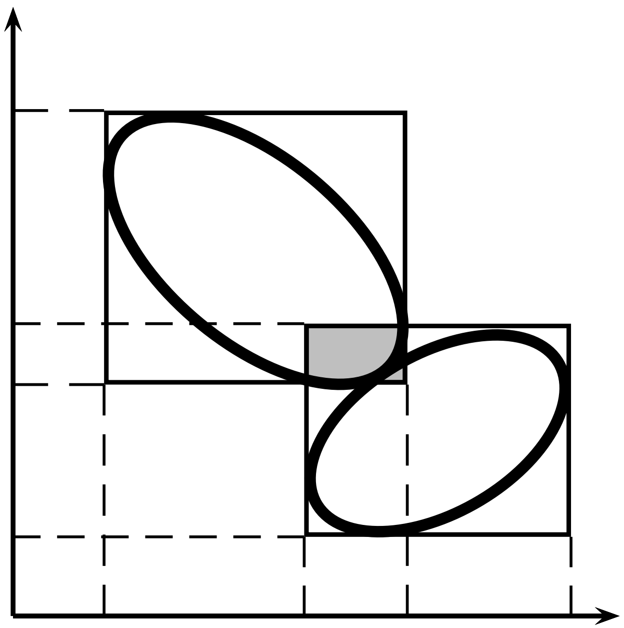
4. Stretch and normalize to obtain the final Fuzzy Sets
   – Extend core and support for normalization (sum of all μ =1 at every point of the UoD)





Information Loss from Projection

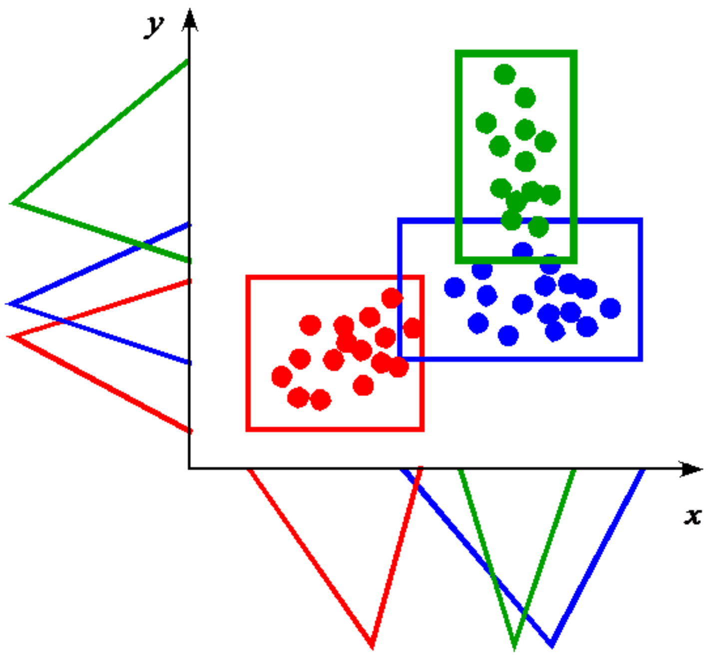
- Each rule derived from fuzzy cluster represents an approximation of the cluster
- Information gets lost by projection
  - Cluster shape of FCM is spherical
  - Cluster projection leads to hypercube
  - Hypercube contains hypersphere
- Loss of information can be kept small using axes-parallel clusters



Rule Learning Based on Input Space Partition

- In input space partition based methods, rule induction partitions the input space into a number of fuzzy (overlapping) regions
- The type of partition generated depends on chosen algorithm
- Optimization modifies membership parameters to define a partition that minimizes output error
- Outline:
  - Hyperbox oriented rule induction methods
  - Structure oriented rule induction methods

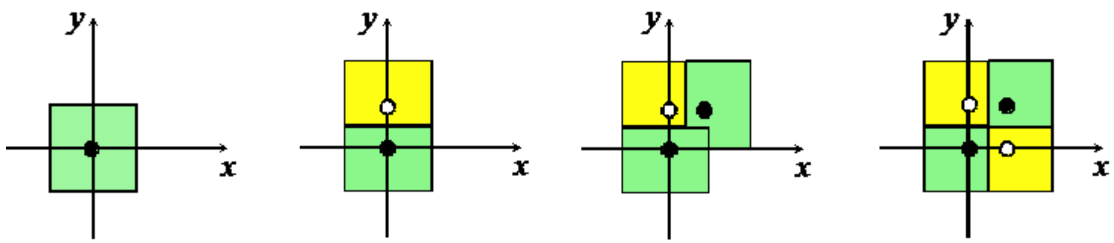
Hyperbox-Oriented Rule Learning



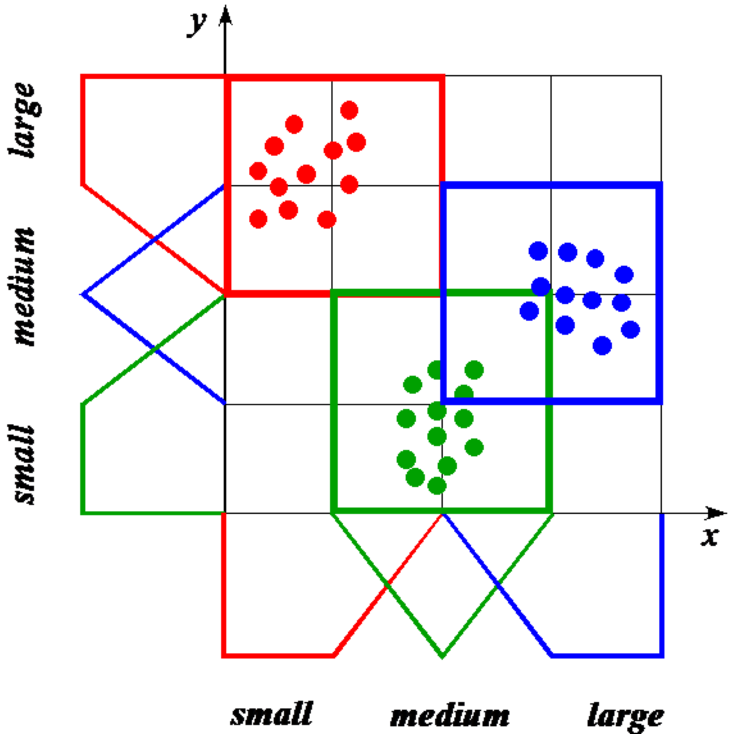
- Search for hyperboxes in the data space
- Create fuzzy rules by projecting hyperboxes
- Fuzzy rules and fuzzy sets are created at the same time
- These algorithms are usually very fast

Hyperbox-Oriented Rule Learning (II)

- ☺ Advantage over fuzzy cluster analysis:
  - There is no loss of information when hyperboxes are represented as fuzzy rules;
  - Not all variables need to be used, don't care variables can be discovered.
- ☹ Disadvantage: The rule base might be unnecessarily complex
- Example: Hiperboxes in XOR Data



Structure-Oriented Rule Learning



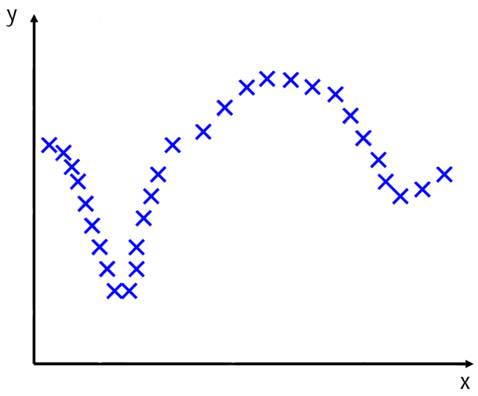
- We must provide the initial fuzzy sets for all variables
- This partitions the data space by a fuzzy grid
- Then we detect all grid cells that contain data [Wang and Mendel, 1992]
- Finally we compute the best consequents and select the best rules, e.g. using NFS [Nauck and Kruse, 1997]

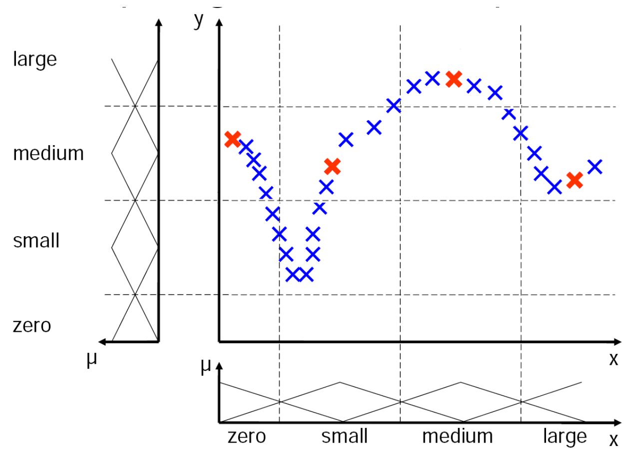
Structure-Oriented Rule Learning (II)

- ☺ Simple – The rule base is available after 2 steps through the training data:
  1. Discover all antecedents;
  2. Determine the best consequents.
- ☺ Missing values can be handled
- ☺ Numeric and symbolic attributes can be processed at the same time (mixed fuzzy rules)
- ☺ All rules share the same fuzzy sets
- ☹ Fuzzy sets must be given in advance

Structure-Oriented Rule Learning (III)

- Wang and Mendel algorithm:
  - Step 1: Given data points, granulate the data space
    - Single input / Single output example:

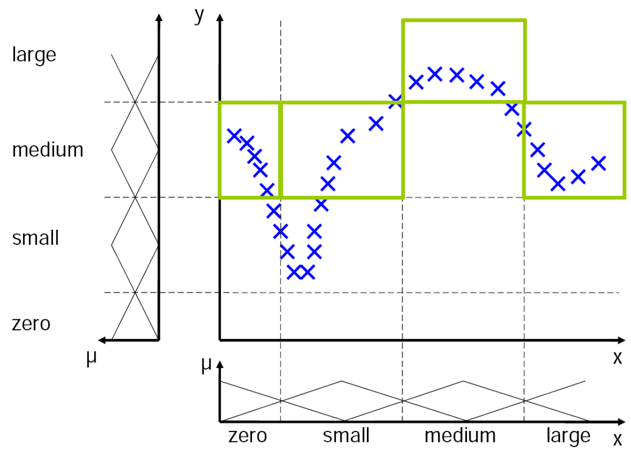


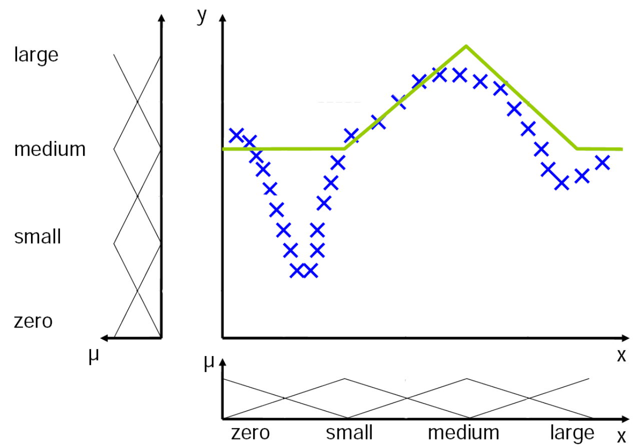


    - The points that are closer to the corresponding rule are represented in red

Structure-Oriented Rule Learning (IV)

- Wang and Mendel algorithm (cont.):
  - Step 2: Generate rules





- Fuzzy rules are shown by their $\alpha = 0.5$-cuts
- The learned model misses extrema far away from the rule centers
  - Note the resulting crisp approximation in green

Structure-Oriented Rule Learning (V)

- Wang and Mendel algorithm example (cont.):
  - Generated rulebase:

$$
\begin{array}{llll}
R_1: & \textbf{if } x \textbf{ is } \mathrm{zero}_x & \textbf{then } y \textbf{ is } \mathrm{medium}_y \\
R_2: & \textbf{if } x \textbf{ is } \mathrm{small}_x & \textbf{then } y \textbf{ is } \mathrm{medium}_y \\
R_3: & \textbf{if } x \textbf{ is } \mathrm{medium}_x & \textbf{then } y \textbf{ is } \mathrm{large}_y \\
R_4: & \textbf{if } x \textbf{ is } \mathrm{large}_x & \textbf{then } y \textbf{ is } \mathrm{medium}_y
\end{array}
$$

  - Intuitively, rule R2 should describe the minimum of the data instead of a medium value:

$$
R_2': \quad \textbf{if } x \textbf{ is } \mathrm{small}_x \quad \textbf{then } y \textbf{ is } \mathrm{small}_y
$$

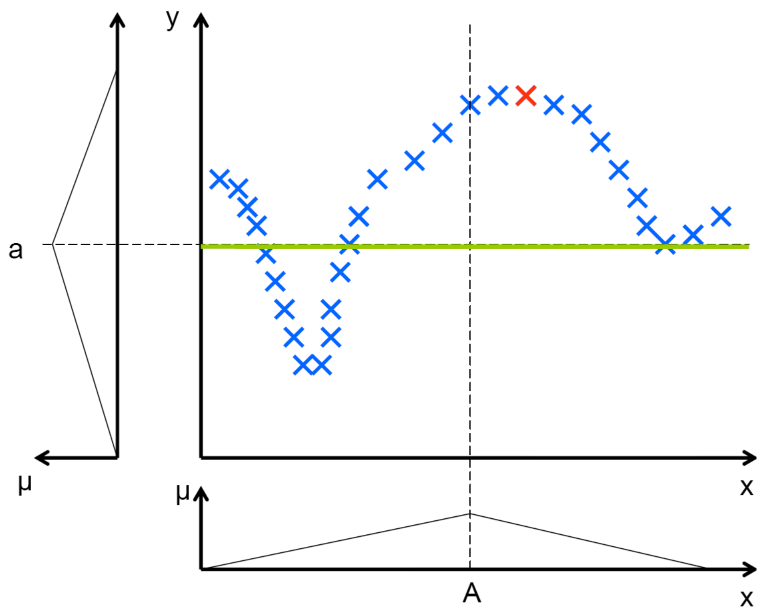
Structure-Oriented Rule Learning (VI)

- Higgins & Goodman Algorithm [Higgins and Goodman, 1993]
  - An extension of Wang and Mendel
  - Step 1: Only one membership function is used for each X_j and Y
    - Initially, one large rule covers the entire feature space
  - Step 2: Any new membership function is placed at the points of maximum error
  - Both steps are repeated until:
    - a maximum number of divisions is reached, or
    - the approximation error remains below a certain threshold.

Structure-Oriented Rule Learning (VII)

- Higgins & Goodman Algorithm – Example:

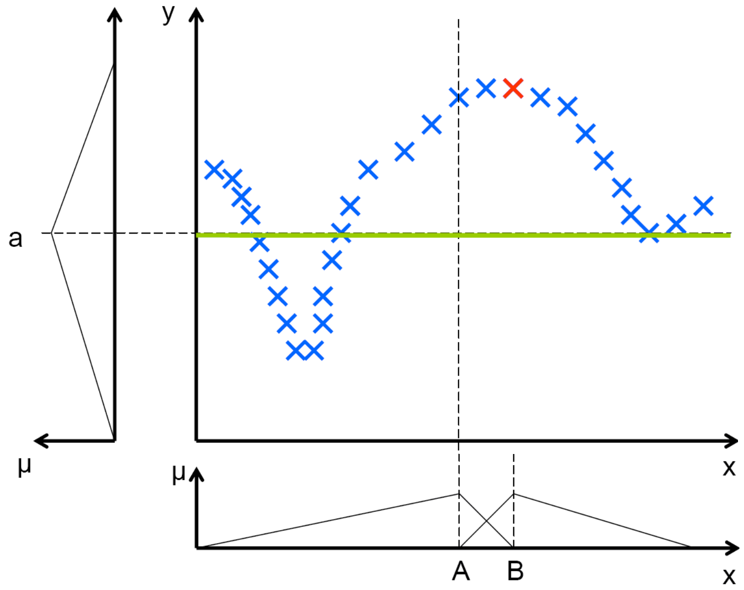
1. Initialization



- Create a membership function for each input covering the entire domain range
- Create a membership function for the output at the corner points of the input
- At the corner point, each input is maximal or minimal of its domain range
- For each corner point, the closest example from the data is used to add a membership function at its output value

Structure-Oriented Rule Learning (VIII)

- Higgins & Goodman Algorithm – Example (cont.):
  2. Adding new membership functions

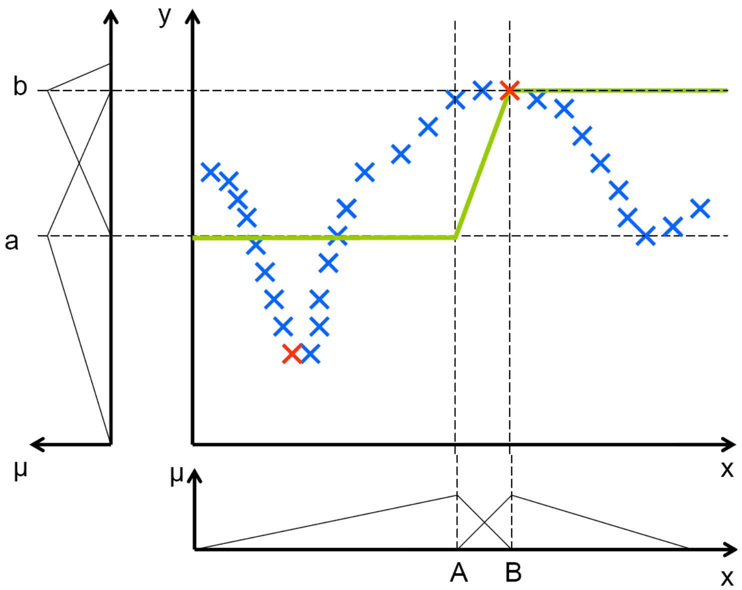


- Find the point within the data with maximum error
  – The defuzzification equals Wang and Mendel
- For each X_j, add a new membership function at the corresponding value of “maximal error point”
  – The point is perfectly described by the model

Structure-Oriented Rule Learning (IX)

- Higgins & Goodman Algorithm – Example (cont.):

3. Create New Cell-Based Rule Set

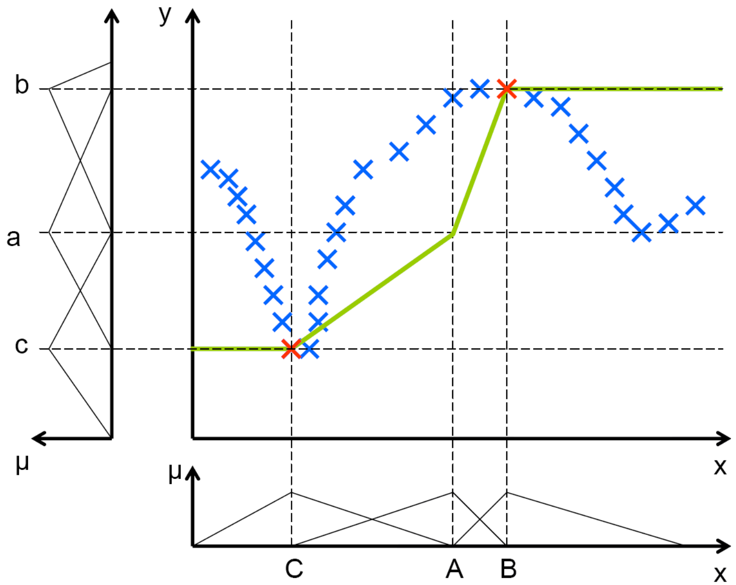


- New rules: Associate the output membership functions with the newly created cells
  - Take the closest point to all membership functions of the input (as in Wang and Mendel)
  - The associated output membership function is the closest one to the output value of the "closest point"
  - If the output value of the "closest point" is "far away", then a new output function is created

Structure-Oriented Rule Learning (X)

- Higgins & Goodman Algorithm – Example (cont.):

4. Termination Detection



- If the error is below a certain threshold (or if a certain number of iterations has been performed), then the algorithm stop
- Otherwise jump to step 2

Input Space Partition Based Methods: Conclusions

- Partition-based fuzzy rule learning methods are usually very fast
  - This is due to their greedy strategies to select rules
- However, for some applications, these strategies are too simple in terms of accuracy
- In such situations more sophisticated rule learning methods should be used, e.g. neuro-fuzzy systems (to be taught later)

FIS Generation from Data "Made Easy"

- In Mathworks' Fuzzy Logic Toolbox (FLT), the functions genfis1, genfis2 and genfis3 (generate fuzzy inference system) automatically induce a fuzzy initial partition and rule base from the available data
  - genfis1 uses a Grid Partitioning approach
  - genfis2 uses Subtractive Clustering
  - genfis3 uses Fuzzy c-Means Clustering
- The generated FIS can be further improved by using optimization methods such as neural-fuzzy systems (e.g. anfis function in FLT) or evolutionary fuzzy systems

Interplay with Expert Knowledge

- When designing FIS, use expert knowledge whenever available
- Multiple strategies possible, e.g.
  - initialize a rulebase with expert knowledge and optimize with data
  - design a rulebase from data and optimize with expert knowledge
  - use experts to validate final rulebase
  - use experts to provide boundary conditions for rulebase optimization

Conclusions

inesc id lisboa

TÉCNICO LISBOA

Conclusions

- We presented a short introduction to fuzzy sets and rule based fuzzy systems
  - It barely touches the surface of the field known as Fuzzy Sets and Systems
- There are unaccountable interpretations and uses of fuzzy sets. Examples:
  - Fuzzy clustering (next chapter); Fuzzy information retrieval; Fuzzy similarity; Fuzzy arithmetics and mathematics (an whole field!); Fuzzy aggregation; Fuzzy classification; etc.

☺ Advantages of Fuzzy Systems

- Considerable skill for little investment
  - Fuzzy modeling replicates human analysis:
    - Humans encode rules after intelligent analysis of lots of data;
    - Verbal rules generated by humans are robust.
  - Simple to create:
    - Not much need for data or ground truth;
    - Logic tends to be easy to program
- Interpretability: Fuzzy rules and fuzzy systems are (usually) understandable by humans

☹ Disadvantages of Fuzzy Systems

- Avoid fuzzy rule based systems if:
  - Humans do not understand the system;
  - Different experts disagree (although some systems can conciliate different opinions);
  - Knowledge cannot be expressed with verbal rules.

Some Notes on Fuzzy Systems’ Applicability

- Fuzzy logic is not a cure-all. Fuzzy logic is a convenient way to map an input space to an output space
  - If you find it's not convenient, try something else. If a simpler solution already exists, use it!
- Fuzzy logic is the codification of common sense — use common sense when you implement it and you will probably make the right decision
- If you take the time to become familiar with fuzzy logic, you'll see it can be a very powerful tool for dealing quickly and efficiently with imprecision and nonlinearity

Some Notes on Fuzzy Systems' Applicability (II)

- (IMHO) The biggest obstacle towards Fuzzy Systems implementation:
  - The modeler has to believe in Fuzzy Systems
  - The modeler must be able to use a coherent qualitative oriented reasoning when building the system
- (Still MHO) Not every person is a “fuzzy” person, and those that aren't should definitely stay away...

Fuzzy Modeling

inesc id lisboa
20 YEARS
DEFINING TECHNOLOGY