

3)

Queremos mostrar que $Mw'' + Kw = 0$, tendo $w(t) = \sum_{k=1}^n \Phi^{(k)} c_k \sin(\omega_k t + \phi_k)$

i)

$$\cdot w'(t) = \sum_{k=1}^n \Phi^{(k)} c_k \omega_k \cos(\omega_k t + \phi_k)$$

$$\cdot w''(t) = - \sum_{k=1}^n \Phi^{(k)} c_k \omega_k^2 \sin(\omega_k t + \phi_k)$$

ii)

$$Mw'' + Kw = M \cdot \left[- \sum_{k=1}^n \Phi^{(k)} c_k \omega_k^2 \sin(\omega_k t + \phi_k) \right] + K \left[\sum_{k=1}^n \Phi^{(k)} c_k \sin(\omega_k t + \phi_k) \right] =$$

$$= - \sum_{k=1}^n M \Phi^{(k)} c_k \omega_k^2 \sin(\omega_k t + \phi_k) + \sum_{k=1}^n K \Phi^{(k)} c_k \sin(\omega_k t + \phi_k)$$

$$\cdot \text{Mas } K \Phi^{(k)} = \omega_k^2 M \Phi^{(k)}, \text{ logo:}$$

$$- \sum_{k=1}^n M \Phi^{(k)} c_k \omega_k^2 \sin(\omega_k t + \phi_k) + \sum_{k=1}^n K \Phi^{(k)} c_k \sin(\omega_k t + \phi_k) =$$

$$= - \sum_{k=1}^n M \Phi^{(k)} c_k \omega_k^2 \sin(\omega_k t + \phi_k) + \sum_{k=1}^n M \Phi^{(k)} c_k \omega_k^2 \sin(\omega_k t + \phi_k) = 0$$

$$\therefore Mw'' + Kw = 0$$