3)

Queremos mostrar que $M\ddot{w} + Kw = 0$, tendo $w(t) = \sum_{k=1}^{n} \Phi^{(k)} c_k \, sen(\omega_k t + \phi_k)$

i)

- $\dot{w}(t) = \sum_{k=1}^{n} \Phi^{(k)} c_k \omega_k \cos(\omega_k t + \phi_k)$

- $\ddot{w}(t) = -\sum_{k=1}^{n} \Phi^{(k)} c_k \omega_k^2 \, sen(\omega_k t + \phi_k)$

ii)

$$M\ddot{w} + Kw = M \cdot \left[-\sum_{k=1}^{n} \Phi^{(k)} c_k \omega_k^2 \, sen(\omega_k t + \phi_k) \right] + K \left[\sum_{k=1}^{n} \Phi^{(k)} c_k \, sen(\omega_k t + \phi_k) \right] =$$

$$= -\sum_{k=1}^{n} M \Phi^{(k)} c_k \omega_k^2 \, sen(\omega_k t + \phi_k) + \sum_{k=1}^{n} \boxed{K \Phi^{(k)}} c_k \, sen(\omega_k t + \phi_k)$$

- Mas $\boxed{K \Phi^{(k)}} = \omega_k^2 M \Phi^{(k)}$, logo:

$$-\sum_{k=1}^{n} M \Phi^{(k)} c_k \omega_k^2 \, sen(\omega_k t + \phi_k) + \sum_{k=1}^{n} \boxed{K \Phi^{(k)}} c_k \, sen(\omega_k t + \phi_k) =$$

$$= -\sum_{k=1}^{n} M \Phi^{(k)} c_k \omega_k^2 \, sen(\omega_k t + \phi_k) + \sum_{k=1}^{n} M \Phi^{(k)} c_k \omega_k^2 \, sen(\omega_k t + \phi_k) = 0$$

$\therefore \boxed{M\ddot{w} + Kw = 0}$