A. Comet Equation

It's a linear advection-diffusion PDE, featuring 2 parameters, each with a clear physical interpretation:

$$\begin{cases} -\mu\Delta u + 10(\cos\theta, \sin\theta) \cdot \nabla u = 10e^{-50\|\underline{x}-\underline{x}_0\|_2} & \underline{x} \in \Omega = [0,1]^2 \\ u = 0 & \underline{x} \in \partial\Omega \end{cases}$$

- $\mu \in (0, \infty)$: diffusion parameter
- $\theta \in (0, 2\pi)$: angle of advection term
- $\underline{x}_0 = (0.5, 0.5)$: centre of the forcing bump

Data is produced through simulation with fixed and known values of parameters and error scale:

$$\mu^* = 2 \qquad \theta^* = \pi \qquad \tau^* = 10^{-4}$$

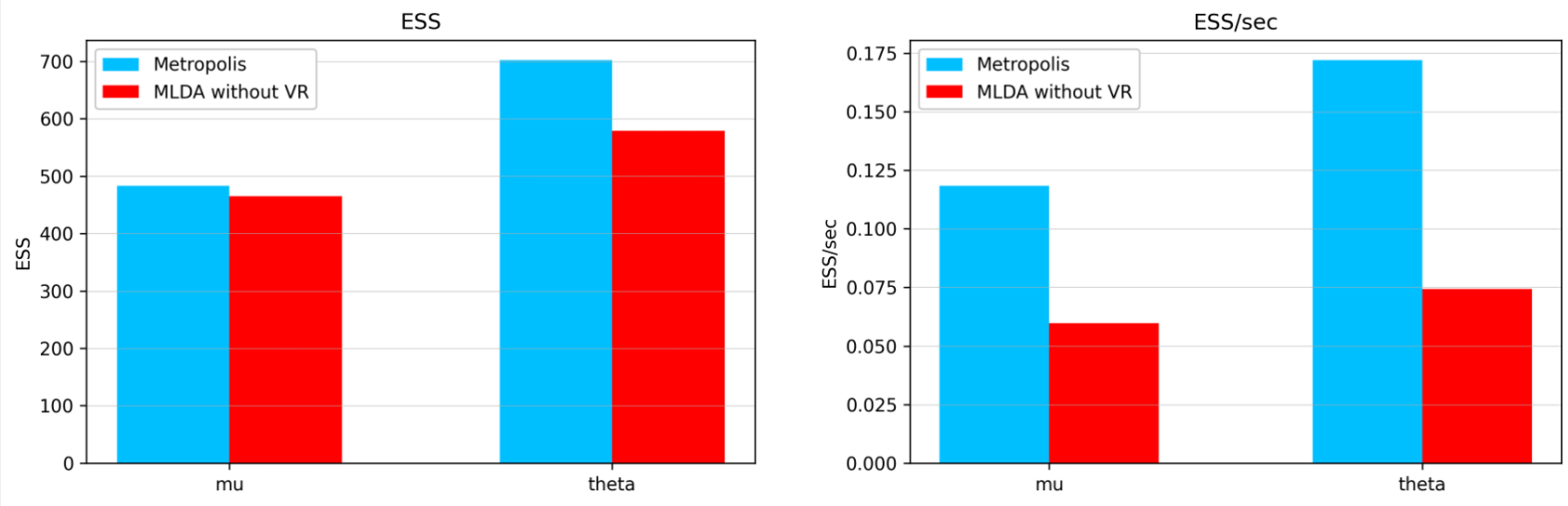
A1. Comet with Different Grids

$u(\underline{\theta}, \underline{x})$ is approximated through Finite Element Method (FEM). The mesh refinement affects the accuracy of the numerical solution.

In MLDA, coarse and fine model differ for the mesh refinement.

Comparison of:

- Metropolis: 32x32 elements
- MLDA
  - Coarse model: 16x16 elements
  - Fine model: 32x32 elements



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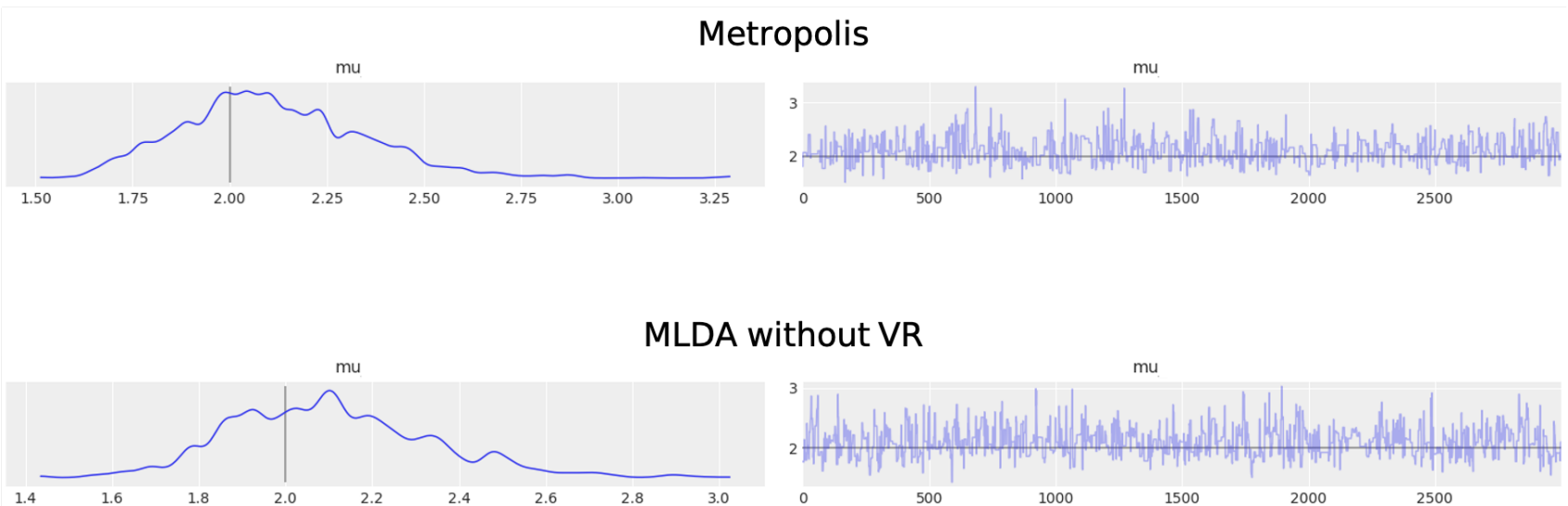
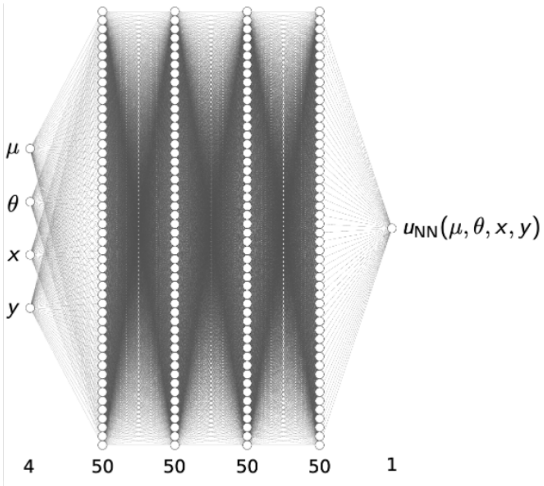
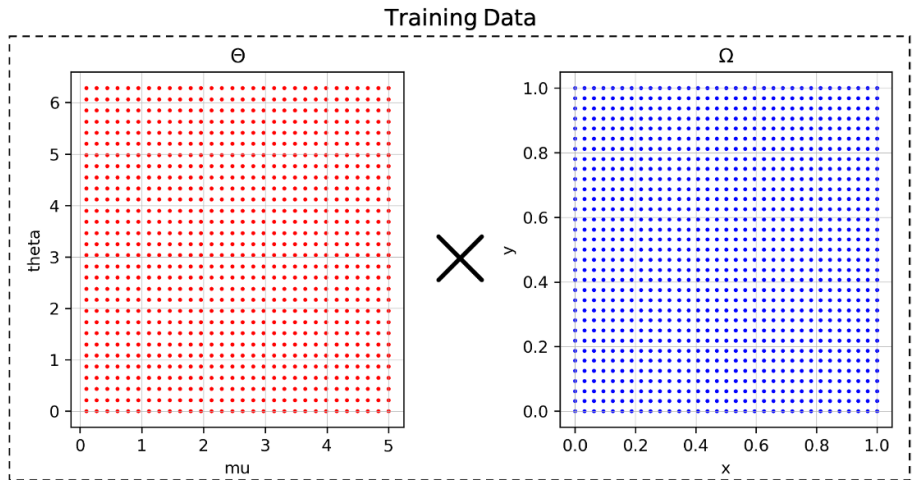


A2. Comet with Surrogate Model

Metropolis overcomes MLDA evidently. Indeed, coarse solution in MLDA is too time consuming.
Hence, we place at coarse level a surrogate model, implemented through a Neural Network:

$$u_{\text{NN}} : (\mu, \theta, x, y) \mapsto u_{\text{NN}}(\mu, \theta, x, y)$$





u_{NN} is trained on a dataset of 900 PDE solutions (each corresponding to a different (μ_i, θ_i)), evaluated on a grid of Ω

\+ Long training time

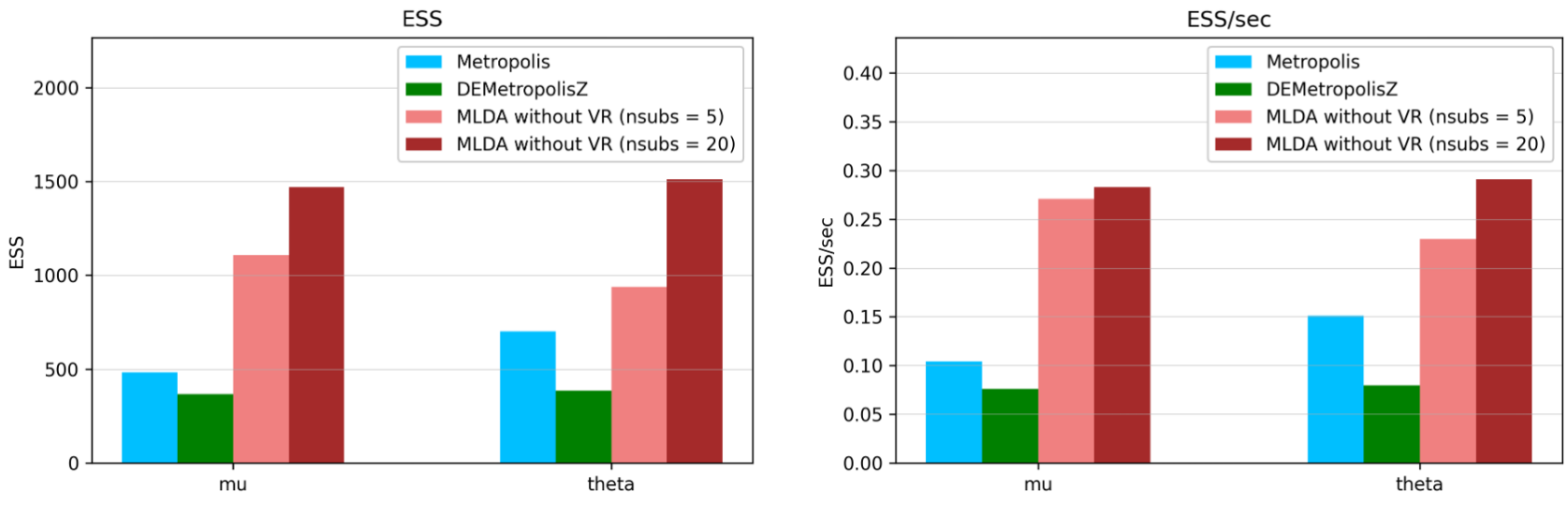
\- Small execution time

A2. Comet with Surrogate Model

Within this framework, we investigated the performances of Metropolis, DEMetropolisZ and MLDA for different:

- frequencies of subsampling (nsubs) of proposed samples from coarse level to fine level in MLDA
- grids of physical points where data is available
- choices of the priors for (μ, θ)

Comparison of Metropolis, DEMetropolisZ and MLDA (nsubs = 5, 20):



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