Multi-Level Monte Carlo Markov Chain

Bayesian Statistics Project

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1 Introduction

We decided to structure our work exploring two different paths: studying the MLMCMC method both on a PDE and on an ODE, to understand its potential in different frameworks.

2 Multi-Level MCMC Method

PDE: the COMET equation

3 PDE problem: Comet Equation

3.1 Mathematical Model

The **COMET equation** is a **linear advection-diffusion PDE**, featuring 2 parameters (μ and θ), each with a clear physical interpretation:

$$\begin{cases} -\mu\Delta u + 10(\cos\theta, \sin\theta) \cdot \nabla u = 10e^{-50\|\underline{x}-\underline{x}_0\|_2} & \underline{x} \in \Omega = [0,1]^2 \\ u = 0 & \underline{x} \in \partial\Omega \end{cases}$$

- $\mu \in (0, \infty)$ is the **diffusion** parameter;
- $\theta \in (0, 2\pi)$ is the **angle of advection** term;
- $\underline{x}_0 = (0.5, 0.5)$ is the **centre of the forcing bump**.

3.2 Approach 1: Coarse and fine model differ in the mesh refinement

3.2.1 Variance for the levels

Best result with (0.00075, 0.00050).

Problem: computationally unfeasible We obtain this results with nsubs=2 and it takes a lot of time: we can't even think of changing nsubs, since the computation would soon become unfeasible and very time consuming.

3.3 Approach 2: Coarse level features a NN as a surrogate model

So we decided to build a Neural Network to substitute at coarse level.

3.3.1 Subsampling rate

3.3.2 Data collection grid

3.3.3 Prior for the parameters

gamma(10,5) is the best one

3.4 Conclusion

4 ODE problem: Epidemic Model

4.1 Mathematical Model

Epidemic models are used to model the spread of infectious disease, but they can be useful also to analyze phenomena in widely different fields, like the adoption of a product or the diffusion of opinions/information.
SIR is the simplest compartmental model, composed by 3 compartments:

- **Susceptible (S)**: number of people who are still unaffected by the disease;
- **Infectious (I)**: number of people who are ill at time t (and thus can contribute to the diffusion of the epidemics);
- **Recovered (R)**: number of the people who have recovered or died because of the infection.

SEIR is another compartmental model, often used as a backbone in the analysis of infectious diseases, where we add a fourth compartment, the one of the:

- **Exposed (E)**: number of people who have contracted the infection but are still not infective (i.e. people that are in their incubation period for the disease);

The system of equations describing SEIR model is:

$$\begin{cases} S'(t) = -\beta SI \\ E'(t) = \beta SI - \sigma E \\ I'(t) = \sigma E - \gamma I \\ R'(t) = \gamma I \\ S(0) = S_0, E(0) = E_0, I(0) = I_0, R(0) = R_0 \end{cases}$$

where:

- $\beta \in (0, \infty)$: **Infection** rate
- $\sigma \in (0, \infty)$: **Incubation** rate
- $\gamma \in (0, \infty)$: **Recovery** rate

These 3 rates are the 3 parameters we want to estimate.

4.2 Approach 1: Coarse and fine model differ in the solver time-step

4.2.1 Subsampling rate

nsubs=10 is the best

4.2.2 Time grid refinement

4.2.3 Prior for the parameters

We chose the prior after having read different scientific papers on the subjects.

4.2.4 Prior informed on historical data

4.3 Approach 2: Coarse level is the SIR model

We used the SIR model as the coarse level for the SEIR

4.3.1 Applicability range

Analysis of the range of parameters for which the ODE system is functioning

4.3.2 SIR-SEIR compatibility

Analysis of the range of compatibility of SIR and SEIR models

4.4 Conclusion

Conclusions?/Bibliography?

5 Conclusions