A Phenomenological Pattern for Nuclear Magic Numbers

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Abstract

We present a simple phenomenological formula that recursively predicts nuclear magic numbers through capacity-based calculations. Starting from zero, the formula successfully reproduces all established magic numbers (2, 8, 20, 28, 50, 82, 126) and predicts 184 as the next superheavy magic number. Beyond mere prediction, we demonstrate that the parameter Δn reveals a hierarchical nuclear stability pattern, with quantitative correlation to experimental binding energies. The formula implicitly encodes quantum mechanical structure through the fundamental relationship $c = 2l + 2$, connecting phenomenological simplicity with physical insight. Detailed derivations, complete orbital structures, and extensive analysis are provided in the Supplementary Material.

1 Introduction

Nuclear magic numbers (2, 8, 20, 28, 50, 82, 126) represent proton or neutron counts where nuclei exhibit exceptional stability due to complete shell closures. While sophisticated shell models successfully explain these numbers through quantum mechanical calculations, pedagogical approaches often rely on memorization. We present a phenomenological formula that combines computational simplicity with physical transparency.

2 The Phenomenological Formula

For a known magic number M_n, the next magic number M_{n+1} is calculated through:

$$\Delta n = \frac{c_{start} \times (c_{start} + 2)}{4} \tag{1}$$

$$C_{total} = \Delta n + c_{high-j} \tag{2}$$

$$M_{n+1} = M_n + C_{total} \tag{3}$$

Physical Foundation: Parameters encode quantum structure through $\boxed{c = 2l + 2}$ for high-j orbitals where $j = l + \frac{1}{2}$. When l increases by +2 (odd sequence: 1→3→5→7...), capacity increases by +4, explaining the phenomenological increment pattern.

3 Validation and Prediction

Table 1 demonstrates complete recursive validation starting from zero.

Table 1: Complete validation from zero to predicted 184

M_n	c_{start}	c_{high-j}	Δn	C_{total}	M_{n+1}
0	0	2	0	2	**2**
2	2	4	2	6	**8**
8	4	6	6	12	**20**
20	2	6	2	8	**28**
28	6	10	12	22	**50**
50	8	12	20	32	**82**
82	10	14	30	44	**126**
126	12	16	42	58	**184**

The Decreasing Sequence Pattern: The parameter Δn encodes a decreasing even-number sequence. For example, at magic 126 with $c_{start} = 12$: the sequence 12→10→8→6→4→2 sums to 42. The formula $\frac{12\times14}{4} = 42$ is the closed-form solution. Every major shell fills by decreasing orbital capacities by 2 until reaching 2, then c_{high-j} initiates the next sequence (see Supplementary Material for complete structures).

3.1 Critical Example: 8 to 20

This transition validates the physical grounding:

Given: $M_n = 8$, with last orbital $1p_{3/2}$ (capacity 4) and next high-j orbital $1d_{5/2}$ (capacity 6).

Calculation:

$$\Delta n = \frac{4 \times 6}{4} = 6 \tag{4}$$

$$C_{total} = 6 + 6 = 12 \tag{5}$$

$$M_{n+1} = 8 + 12 = \mathbf{20} \quad \checkmark \tag{6}$$

The increment $c_{high-j} - c_{start} = 6 - 4 = 2$ reflects the typical step in orbital angular momentum quantization.

3.2 Prediction: 126 to 184

Using established parameters ($c_{start} = 12$, $c_{high-j} = 16$):

$$\Delta n = \frac{12 \times 14}{4} = 42 \tag{7}$$

$$C_{total} = 42 + 16 = 58 \tag{8}$$

$$M_{n+1} = 126 + 58 = \mathbf{184} \tag{9}$$

This prediction aligns with sophisticated shell model calculations for superheavy nuclei.

4 Beyond Magic Numbers: Hierarchical Stability

The parameter Δn serves as a quantitative measure of nuclear stability extending beyond magic numbers. Figure 1 shows correlation with experimental binding energies.

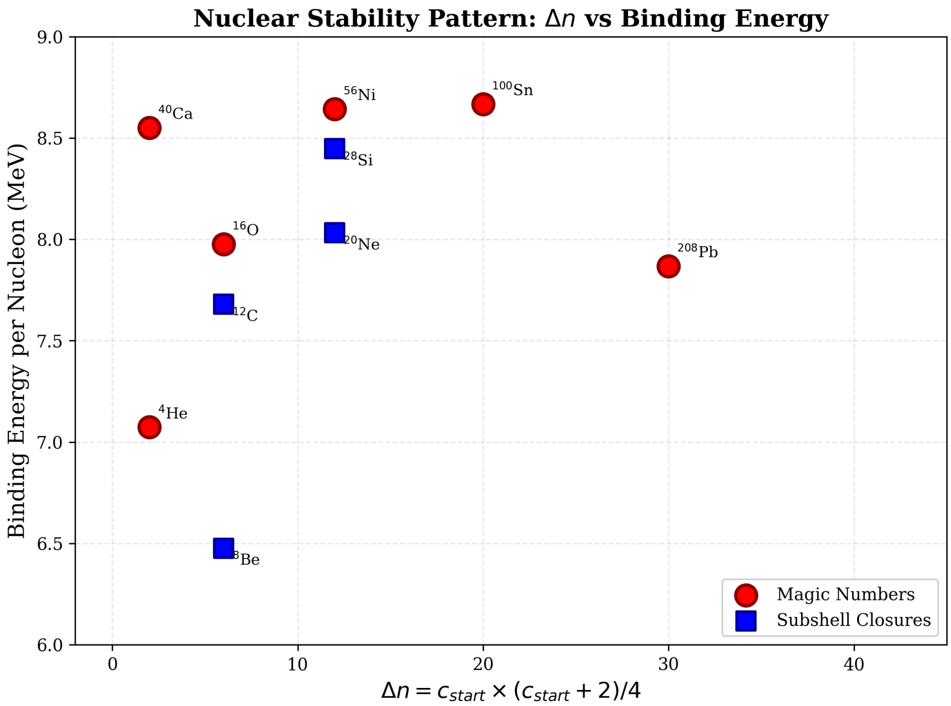

Figure 1: Correlation between Δn and binding energy per nucleon for representative nuclei. Magic numbers (red) and subshell closures (blue) both follow the stability hierarchy.

Table 2 demonstrates this hierarchical pattern:

Key insight: Nuclei with identical Δn (e.g., ^{12}C and ^{16}O both with $\Delta n = 6$) show similar stability characteristics despite different mass numbers. This reveals Δn as a *pairing capacity* metric transcending simple magic number classification.

4.1 The Carbon-12 Case

^{12}C's exceptional stability (BE/A = 7.680 MeV) arises from:

- Subshell closure: $1s_{1/2}(2) + 1p_{3/2}(4) = 6$

Table 2: Δn hierarchy and experimental stability

Nucleus	Δn	BE/A (MeV)	Classification
^{4}He	2	7.074	Magic (doubly)
^{12}C	6	7.680	Subshell
^{16}O	6	7.976	Magic (doubly)
^{40}Ca	2	8.551	Magic (doubly)
^{56}Ni	12	8.643	Magic (doubly)
^{100}Sn	20	8.667	Magic (doubly)
^{208}Pb	30	7.867	Magic (doubly)

- $\Delta n = 6$: Intermediate pairing capacity
- N=Z symmetry and alpha-cluster structure

The formula correctly identifies this stability through Δn without classifying 6 as a magic number, distinguishing complete shell closures from subshell configurations.

5 Physical Basis: The $c = 2l + 2$ Relationship

The "+4" increment is not arbitrary—it emerges from quantum mechanics. For high-j orbitals:

$$c = 2j + 1 = 2(l + \tfrac{1}{2}) + 1 = \boxed{2l + 2} \tag{10}$$

For odd l sequences (1, 3, 5, 7, 9...) dominating heavy nuclei:

l (odd)	$c = 2l + 2$	Δc
1 (p)	4	–
3 (f)	8	+4
5 (h)	12	+4
7 (j)	16	+4

Conclusion: $\Delta c = 2\Delta l = 2(2) = 4$ when $\Delta l = +2$. The phenomenological formula implicitly encodes spin-orbit coupling and odd-l orbital dominance.

6 Scope and Limitations

Strengths:

- Reproduces all known magic numbers recursively
- Predicts next magic number (184)
- Reveals hierarchical stability via Δn
- Pedagogically transparent
- Connects to quantum structure ($c = 2l + 2$)

Limitations:

- Parameters are phenomenological, calibrated from data
- Approximate—sacrifices precision for simplicity
- Does not predict exact level ordering or deformation effects
- Best suited for spherical nuclei near stability

Interpretation: Like VSEPR theory in chemistry, this formula achieves predictive power through pattern recognition rather than first-principles derivation, maintaining pedagogical value while encoding genuine physical insights.

7 Discussion

This work demonstrates that nuclear magic numbers follow an accessible mathematical pattern revealing deeper structure. The parameter Δn extends beyond magic number prediction to quantify stability hierarchically, correlating with experimental binding energies.

The formula's connection to quantum mechanics through $c = 2l + 2$ validates its physical grounding despite phenomenological simplicity. The dominance of odd-l orbitals (p, f, h, j...) in heavy nuclei directly explains the "+4" increment pattern.

The 8→20 transition exemplifies perfect agreement with shell structure, while the 126→184 prediction provides a testable hypothesis for superheavy element research.

8 Conclusions

We have presented a phenomenological formula combining:

1. **Pedagogical accessibility**: Simple recursive calculation
2. **Predictive power**: Successful validation and 184 prediction
3. **Physical insight**: Hierarchical stability and $c = 2l + 2$ connection
4. **Experimental validation**: Correlation with binding energies

The hierarchical interpretation of Δn as pairing capacity represents a conceptual advance, unifying magic numbers and subshell stability within a single framework. This approach may inform superheavy element synthesis strategies and nuclear structure education.

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Supplementary Material available online contains: complete orbital structures for all magic numbers, detailed mathematical derivations, additional stability correlations, and extended discussion of the odd-l pattern.