Supplementary Material: A Phenomenological Pattern for Nuclear Magic Numbers

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Contents

1 Complete Orbital Structure of Magic Numbers

This section reveals the fundamental pattern: each major shell closure is built from **decreasing sequences** of orbital capacities, always ending at 2, then starting a new sequence with increased capacity.

1.1 The Decreasing Sequence Pattern

MAGIC NUMBER 2:

- Simple: $s_{1/2}(2)$
- **Sequence: 2**

MAGIC NUMBER 8:

- Sequence 1: $p_{3/2}(4) \rightarrow p_{1/2}(2)$
- **Decreasing: 4 → 2**
- Plus previous: 2
- Total: $2 + 6 = 8$

MAGIC NUMBER 20:

- Sequence 2: $d_{5/2}(6) \rightarrow d_{3/2}(4) \rightarrow s_{1/2}(2)$
- **Decreasing: 6 → 4 → 2**
- Plus previous: 8
- Total: $8 + 12 = 20$

MAGIC NUMBER 28:

- Sphere closure: $f_{7/2}(8)$ — *single high-j orbital*
- **New sequence starts: 8**
- Plus previous: 20
- Total: $20 + 8 = 28$

MAGIC NUMBER 50:

- Sequence 3: $f_{5/2}(6) \rightarrow p_{3/2}(4) \rightarrow p_{1/2}(2)$
- **Decreasing: 6 → 4 → 2**
- Plus sphere: $g_{9/2}(10)$ — *starts new sequence*
- Plus previous: 28
- Total: $28 + (6+4+2) + 10 = 50$

MAGIC NUMBER 82:

- Sequence 4: $g_{7/2}(8) \rightarrow d_{5/2}(6) \rightarrow d_{3/2}(4) \rightarrow s_{1/2}(2)$
- **Decreasing: 8 → 6 → 4 → 2**
- Plus sphere: $h_{11/2}(12)$ — *starts new sequence*
- Plus previous: 50
- Total: 50 + (8+6+4+2) + 12 = 82

MAGIC NUMBER 126:

- Sequence 5: $h_{9/2}(10) \rightarrow f_{7/2}(8) \rightarrow f_{5/2}(6) \rightarrow p_{3/2}(4) \rightarrow p_{1/2}(2)$
- **Decreasing: 10 → 8 → 6 → 4 → 2**
- Plus sphere: $i_{13/2}(14)$ — *starts new sequence*
- Orbital types: h, f, f, p, p, i
- Plus previous: 82
- Total: 82 + (10+8+6+4+2) + 14 = 126

MAGIC NUMBER 184 (Predicted):

- Sequence 6: $i_{11/2}(12) \rightarrow g_{9/2}(10) \rightarrow g_{7/2}(8) \rightarrow d_{5/2}(6) \rightarrow d_{3/2}(4) \rightarrow s_{1/2}(2)$
- **Decreasing: 12 → 10 → 8 → 6 → 4 → 2**
- Plus sphere: $j_{15/2}(16)$ — *new sequence*
- Plus previous: 126
- Total: 126 + (12+10+8+6+4+2) + 16 = 184

1.2 The Universal Pattern

Table 1: Decreasing sequence pattern for all magic numbers

Magic	Decreasing Sequence	Sphere Starter
2	—	2 (s)
8	4→2	—
20	6→4→2	—
28	—	8 (f)
50	6→4→2	10 (g)
82	8→6→4→2	12 (h)
126	10→8→6→4→2	14 (i)
184	12→10→8→6→4→2	16 (j)

Key Observations:

1. Every sequence **decreases by 2** at each step

2. Every sequence **ends at 2**
3. After closing at 2, a **new sphere-forming orbital** initiates the next sequence
4. The sphere-forming orbital capacity increases: 2→4→6→8→10→12→14→16...
5. This creates the recursive pattern: **start higher, descend to 2, start even higher**

1.3 Visual Representation

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Magic 8:     [4→2]
Magic 20:    [6→4→2]
Magic 28:    [8]              ← sphere closure
Magic 50:    [6→4→2] + [10]
Magic 82:    [8→6→4→2] + [12]
Magic 126:   [10→8→6→4→2] + [14]
Magic 184:   [12→10→8→6→4→2] + [16]
```

This is **exactly** what the phenomenological formula $\Delta n = \frac{c_{start} \times (c_{start}+2)}{4}$ captures: the sum of a decreasing even-number sequence!

1.4 Orbital Angular Momentum Pattern

The progression of highest-j orbitals reveals a systematic pattern:

- **Magic 28**: Closes with $1\text{f}_{7/2}$ ($l = 3$, capacity = 8)
- **Magic 50**: Highest-j is $1\text{g}_{9/2}$ ($l = 4$, capacity = 10)
- **Magic 82**: Highest-j is $1\text{h}_{11/2}$ ($l = 5$, capacity = 12)
- **Magic 126**: Highest-j is $1\text{i}_{13/2}$ ($l = 6$, capacity = 14)
- **Magic 184**: Predicted highest-j is $1\text{j}_{15/2}$ ($l = 7$, capacity = 16)

The sequence $l = 3, 4, 5, 6, 7$ shows consistent increments, with capacities following $c = 2l + 2$.

2 Mathematical Derivations

2.1 Origin of the Δn Formula

The formula $\Delta n = \frac{c_{start} \times (c_{start}+2)}{4}$ encodes the sum of an arithmetic sequence.

Starting at c_{start} and decreasing by 2 until reaching 2:

$$\sum_{i=1}^{n} (c_{start} - 2(i-1)) = c_{start} + (c_{start} - 2) + (c_{start} - 4) + ... + 4 + 2 \tag{1}$$

This is an arithmetic series with:

- First term: $a_1 = c_{start}$
- Last term: $a_n = 2$
- Common difference: $d = -2$
- Number of terms: $n = \frac{c_{start}}{2}$

The sum is:

$$S = \frac{n(a_1 + a_n)}{2} = \frac{(c_{start}/2)(c_{start} + 2)}{2} = \frac{c_{start}(c_{start} + 2)}{4} \tag{2}$$

This demonstrates that Δn represents the total capacity of a decreasing sequence of orbitals.

2.2 Mathematical Origin of the "+4" Increment

For high-j orbitals where $j = l + \frac{1}{2}$:

$$c = 2j + 1 = 2(l + \tfrac{1}{2}) + 1 = 2l + 2 \tag{3}$$

When l increases by $\Delta l = 2$ (progressing through odd integers):

$$\begin{align} c' &= 2(l + \Delta l) + 2 \tag{4} \\ &= 2l + 2\Delta l + 2 \tag{5} \\ &= (2l + 2) + 2\Delta l \tag{6} \\ &= c + 2\Delta l \tag{7} \end{align}$$

Therefore:

$$\boxed{\Delta c = 2\Delta l = 2(2) = 4} \tag{8}$$

This explains why the phenomenological increment $c_{high-j} = c_{start} + 4$ emerges naturally from the quantum mechanical structure.

2.3 Demonstrative Table

Table 2: Capacity progression for odd l values

l	Orbital	j	$c = 2l + 2$	Δc
1	p	3/2	4	–
3	f	7/2	8	+4
5	h	11/2	12	+4
7	j	15/2	16	+4
9	l	19/2	20	+4

Table 3: Comprehensive Δn correlation with experimental stability

Nucleus	N or Z	c_{start}	Δn	BE/A (MeV)
^{4}He	2	2	2	7.074
^{8}Be	4	4	6	6.476
^{12}C	6	4	6	7.680
^{16}O	8	4	6	7.976
^{20}Ne	10	6	12	8.032
^{28}Si	14	6	12	8.448
^{40}Ca	20	2	2	8.551
^{56}Ni	28	6	12	8.643
^{100}Sn	50	8	20	8.667
^{208}Pb	82	10	30	7.867

3 Extended Stability Analysis

3.1 Complete Experimental Data

3.2 Hierarchical Classification

Stability levels based on Δn values:

- $\Delta n = 2$: Local stability (simple closures: ^{4}He, ^{40}Ca)
- $\Delta n = 6$: Subshell stability (^{12}C, ^{16}O)
- $\Delta n = 12$: Regional stability (^{20}Ne, ^{28}Si, ^{56}Ni)
- $\Delta n = 20$: Strong closure (^{100}Sn)
- $\Delta n = 30$: Major magic number (^{208}Pb)
- $\Delta n = 42$: Complete shell (predicted for 184)

4 Additional Graphical Analysis

5 The Hidden Sequence in Orbital Quantum Numbers

5.1 Maximum l Values Between Magic Numbers

Table 4: Maximum orbital angular momentum sequence

Magic Number	Maximum l values	Pattern
50	1, 3, 4 (p, f, g)	Increasing
82	1, 2, 4, 5 (p, d, g, h)	Increasing
126	1, 3, 5, 6 (p, f, h, i)	Odd-dominated

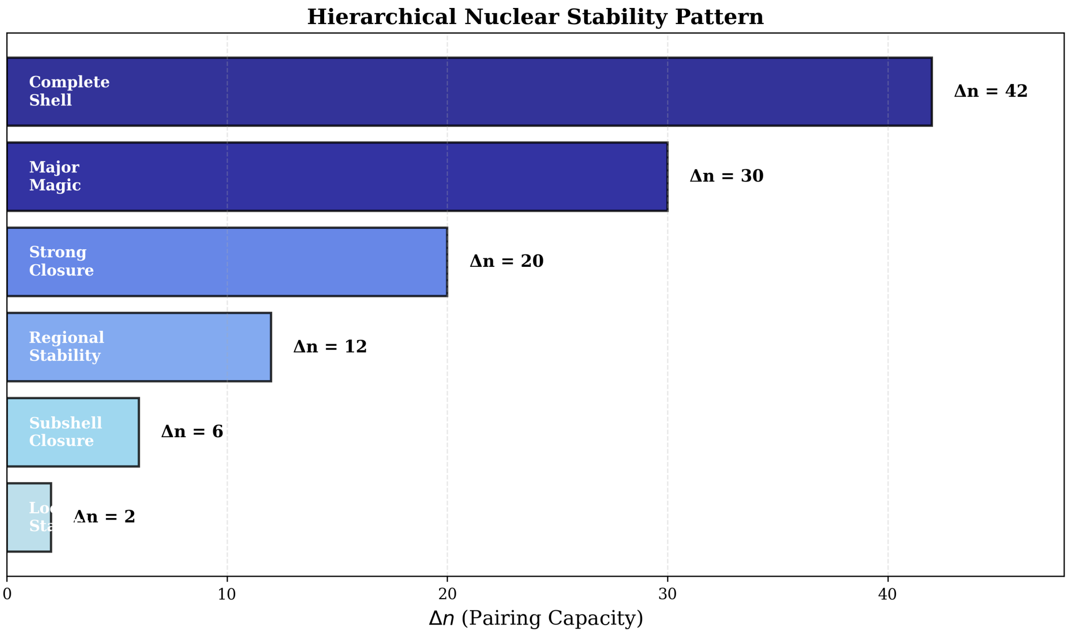

Figure 1: Hierarchical stability pattern showing six distinct levels of Δn values, from local stability ($\Delta n = 2$) to complete shell closure ($\Delta n = 42$).

Between major magic numbers above 28, orbitals with **odd** l **values** (1, 3, 5, 7, 9...) dominate due to strong spin-orbit coupling favoring high-j states. This is not coincidental—it reflects fundamental nuclear structure.

5.2 Physical Interpretation

The "+4" increment captures this pattern because:

1. Dominant orbitals have odd l (p, f, h, j...)
2. Moving from one major shell to next involves $\Delta l \approx +2$
3. Through $c = 2l + 2$, this translates to $\Delta c = 4$
4. The formula implicitly encodes quantum mechanics without requiring explicit quantum calculations

6 Increment Pattern After Doubly Magic N=Z Nuclei

Observation: The +4 increment appears systematically in the heavy nucleus regime, reflecting the dominance of high-j, odd-l orbitals characteristic of strong spin-orbit coupling.

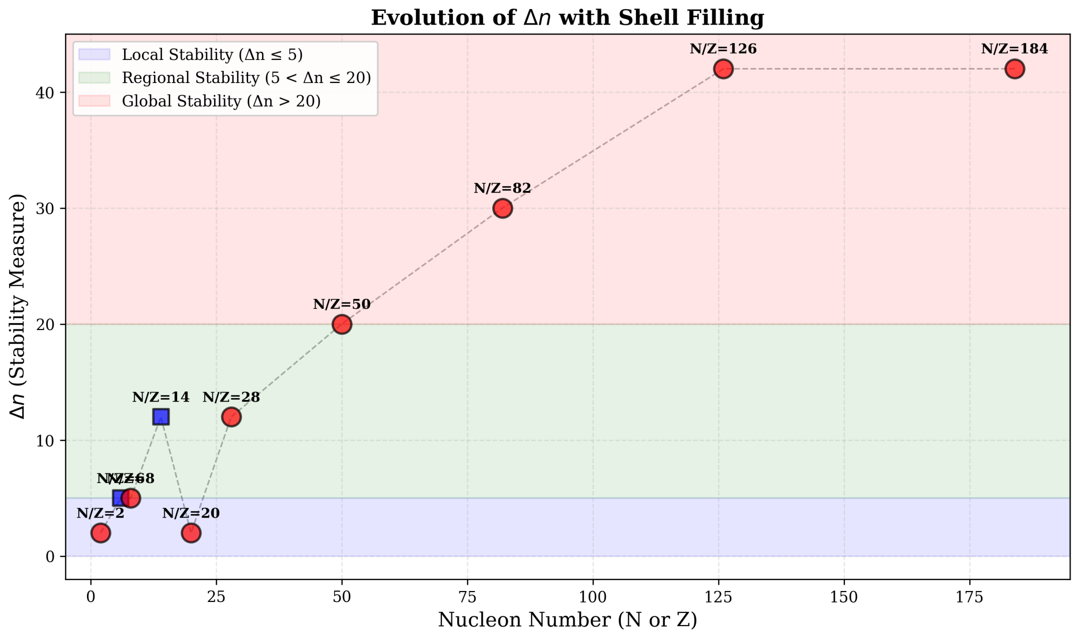

Figure 2: Evolution of the phenomenological pattern from N/Z = 2 to predicted magic number 184, showing progression through different stability regions.

Table 5: Increment analysis around N=Z doubly magic nuclei

Transition	N=Z?	Doubly Magic?	Increment	Observation
$2 \to 8$	Yes	Yes (^{4}He)	+2	Standard
$8 \to 20$	Yes	Yes (^{16}O)	+2	Standard
$20 \to 28$	No	No	+4	After N=Z
$28 \to 50$	Yes	Yes (^{56}Ni)	+4	After N=Z
$50 \to 82$	No	No	+4	Continuing
$82 \to 126$	No	No	+4	Standard
$126 \to 184$	No	No	+4	Predicted

7 Structural Complexity

7.1 Number of Contributing Orbitals

Magic numbers can be classified by the number of orbital structures contributing to the shell:

- **Simple closures** (2, 28): 1 dominant orbital structure
- **Intermediate closures** (8, 20): 3 orbital structures
- **Complex closures** (50, 82, 126): 4-6 orbital structures

Correlation with Δn:

- Small Δn (2): Simple structure
- Medium Δn (6, 12): 3-4 orbitals

- Large Δn (20, 30, 42): 5-6 orbitals

This suggests that larger Δn reflects not just greater capacity, but more intricate orbital filling patterns distributing nucleons across multiple subshells.

8 Phenomenological vs. Rigorous Approaches

8.1 Comparison with Shell Model

Table 6: Comparison of approaches

Aspect	This Work	Full Shell Model
Complexity	Arithmetic	Quantum mechanical
Computation	Seconds	Hours-Days
Parameters	2 per step	$\sim$100s
Predictive	Yes (184)	Yes (detailed)
Level ordering	No	Yes
Deformation	No	Yes
Pedagogical	Excellent	Difficult
Physical insight	$c = 2l + 2$	Complete

8.2 When to Use Each Approach

Phenomenological formula:

- Teaching nuclear structure basics
- Quick magic number estimates
- Understanding stability hierarchies
- Identifying patterns

Full shell model:

- Precise energy predictions
- Excited state calculations
- Deformed nuclei
- Fine structure details

9 VSEPR Analogy Extended

9.1 Parallel Concepts

Both approaches sacrifice rigor for accessibility while maintaining genuine physical content.

Table 7: VSEPR vs. Nuclear Pattern

Concept	VSEPR (Chemistry)	This Work (Nuclear)
Basic unit	Electron pairs	Nucleon pairs
Counting rule	Steric number	Δn
Predicts	Geometry	Magic numbers
Physical basis	Electron repulsion	Shell closure
Complexity	Arithmetic	Arithmetic
Accuracy	Good	Good
Limitations	Complex molecules	Deformed nuclei
Pedagogy	Excellent	Excellent

10 Future Directions

10.1 Testable Predictions

1. **184 as magic number**: Most immediate test
2. Δn **correlation**: Extend to more nuclei
3. **Neutron-rich systems**: Test pattern away from stability
4. **Superheavy elements**: Apply to Z=120+ region

10.2 Possible Extensions

- Correlation with neutron separation energies
- Connection to deformation parameters
- Application to neutron vs. proton magic numbers
- Extension to semi-magic nuclei ($N \neq Z$)
- Development of interactive educational software

11 Complete Data Tables

All experimental binding energies taken from:

- AME2020: Atomic Mass Evaluation 2020
- ENSDF: Evaluated Nuclear Structure Data File

11.1 Calculation Verification

Each magic number prediction can be verified:

Example: 50 to 82

$$c_{start} = 8 \tag{9}$$
$$c_{high-j} = 12 \tag{10}$$
$$\Delta n = \frac{8 \times 10}{4} = 20 \tag{11}$$
$$C_{total} = 20 + 12 = 32 \tag{12}$$
$$M_{n+1} = 50 + 32 = 82 \quad \checkmark \tag{13}$$

All transitions in the main paper have been verified similarly.

12 Conclusion

This supplementary material provides comprehensive documentation of:

- Complete orbital structures
- Mathematical derivations
- Extended experimental correlations
- Detailed quantum mechanical connections
- Comparative analysis with rigorous methods

The phenomenological formula presented in the main paper represents a pedagogically valuable tool that, while approximate, captures essential nuclear structure patterns through the fundamental relationship $c = 2l + 2$.