

Ordinary Differential Equations and Dynamical Systems

Part I

Modeling

Fundamentals

Quality and Process Control

Definition of Quality

- Quality means fitness for use. and
- Quality is inversely proportional to variability.
- My definition: *Is* and *should* are the same

Statistical Process Control (SPC)

- Statistical process control is, first and foremost, a way of thinking which happens to have some tools attached.

The Magnificent Seven

1. histogram
2. check sheet
3. Pareto chart
4. defect concentration diagram
5. cause-and-effect diagram
6. control chart
7. scatter diagram

Classification

Differential equations can be classified according to various criteria. Besides the order of an ODE we are also interested in whether an ODE is linear, homogeneous, separable or autonomous.

Linearity

An n -th order ODE is *linear*, if it is of the form:

$$a_n(x) \cdot y^{(n)} + \dots + a_1(x) \cdot y' + a_0(x) \cdot y = g(x)$$

where $a_n(x), \dots, a_1(x), a_0(x)$ are $g(x)$ fixed functions.

Homogeneity

A linear ODE is *homogeneous*, if $g(x) = 0$ for all x ; otherwise the ODE is *inhomogeneous*, and $g(x)$ is the *inhomogeneity* or *source* term.

Constant coefficient

A linear ODE has constant coefficients, if it is of the form

$$a_n \cdot y^{(n)} + \dots + a_1 \cdot y' + a_0 \cdot y = g(x),$$

with $a_n \neq 0$.

Examples

$$y' = f(x)$$

Inhomogeneous linear ODE for $y(x)$ with source term $f(x)$

$$m \cdot \dot{v} = m \cdot g - k \cdot v^2$$

Nonlinear ODE for $v(t)$

$$l \cdot \ddot{\Phi} + g \cdot \sin(\Phi) = 0$$

Nonlinear ODE for $\Phi(t)$

$$l \cdot \ddot{\Phi} + g \cdot \phi$$

Homogeneous linear ODE for $\Phi(t)$

$$l \cdot \ddot{\Phi} + g \cdot \phi = \sin(\omega t)$$

Inhomogeneous linear ODE for $\Phi(t)$ with source term $\sin(\omega t)$

$$i'' + \frac{R}{L}i' + \frac{1}{LC}i = 0$$

Homogeneous linear ODE for $i(t)$

Separability