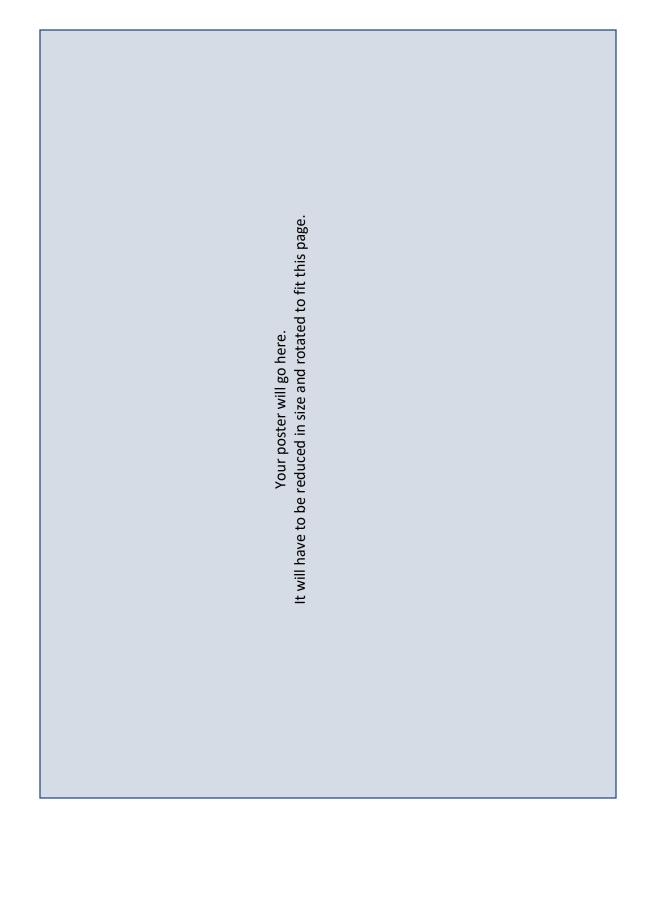


# Analyzing Global Navigation Satellite Systems (GNSS) Fit Intervals

Adelaide Johnson Vandegrift High School Austin, Texas

Supervisors Brent Renfro and Miquela Stein Space and Geophysics Laboratory This page intentionally left blank.



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# Analyzing Global Navigation Satellite Systems (GNSS) fit intervals

# Adelaide Johnson

Vandegrift High School, Austin, Texas

Supervisor: Brent Renfro and Miquela Stein, Space and Geophysics Laboratory

Global Navigation Satellite Systems (GNSS) are used around the world to assist in navigation. After the success of the United States' Global Positioning System (GPS), the European Union launched its own constellation known as Galileo, China has created BeiDou, and other systems are currently in development. As more countries plan to create and expand their satellite positioning systems, it is important to determine how long the signals of each satellite maintain their highest level of accuracy. The purpose of this project was to create a statistical plotter to analyze and determine the time period for each system that has the lowest User Range Error (URE), which is a measure of accuracy of the satellite's navigation message. This time period is known as the fit interval. In addition, we determined the usable time, or length of time the fit interval is actually available to users. Using this plotter, we confirmed the GPS fit interval is 4 hours. After plots for Galileo were generated, it became apparent that while they generally have lower UREs during their fit interval than GPS, their nav messages have a shorter usable time. To explain the observed differences between GPS and Galileo, a separate plotting script was created to show the frequency of navigation message transmissions. The plots of the BeiDou satellites that were analyzed revealed that on average, they have higher URE than GPS and Galileo and a usable time similar to Galileo. In the future, this plotting tool can be expanded to accommodate GLONASS and other GNSS.

# I. Background

Global Navigation Satellite Systems (GNSS) compromise of all international navigation satellite constellations. In this analysis, we considered the Global Positioning System (United States), Galileo (European Union), and BeiDou (China). Within the Global Positioning System (GPS), we considered the Legacy Navigation Message (LNAV) and the Civilian Navigation Message (CNAV). Within BeiDou we focused our analysis on the Inclined Geosynchronous Orbit (IGSO) satellites.

Satellites broadcast a signal containing the navigation (nav) message, which provides the receiver with the information needed to derive the satellite location at a given time (Broadcast Orbit). The distance from the receiver to the satellite can be determined by knowing how long it took the signal to reach the receiver. This identifies a sphere around the satellite where the individual or vehicle must be located, as seen in Fig. 1. The center of the spheres are satellite locations derived from the nav message. The intersection of four or more of these spheres defines a specific location in space and time.

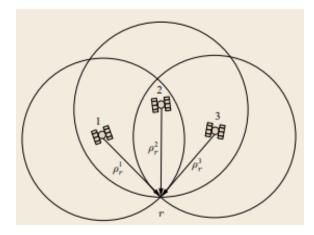


Figure 1. The distance from point r to the satellites is known. This creates a sphere around the satellite where r could be located. The intersection of the 3 spheres identifies an exact location in space. Reprinted from Springer Handbook of Global Navigation Satellite Systems (p. 7), by P. J. G. Teunissen, O. Montenbruck, 2017, Springer. Copyright 2017 by Springer International Publishing AG.

The nav message is generated by the ground control segment. To generate the nav message, a model is made to project the orbital path the satellite will travel the next day. Due to this model being an imperfect ellipse, it is cut into multiple different sections, each of which is approximated by a smaller

ellipse, as shown in Fig. 2. Sections of these approximated ellipses are transmitted to users.

Each of the approximated ellipses that makes up the nav message can only model a small portion of the projected orbit. Outside of that portion, the accuracy decreases. Because of this the nav message consists of sections of the smaller ellipses layered over one another. The interval of time for which these smaller ellipses can be used is known as the fit interval.

The Precise Orbit is the exact orbital path the satellite travels on, this is calculated after the satellite has orbited. User Range Error is the difference between the Broadcast Orbit and the Precise Orbit. The larger the URE, the less accurate a user's calculated location is. Each nav message is based around a set reference time, or epoch time ( $t_{oe}$ ). In general, as distance in time from this set reference increases, the URE increases.

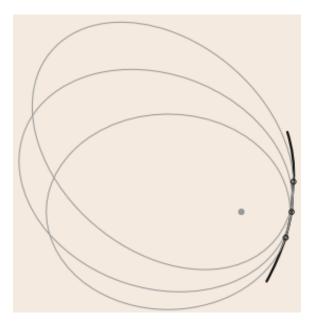


Figure 2. The orbital path of the satellite is approximated by a smaller curve. The fit interval is the length of time when this curve is most accurate.

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# **II. Project Description**

## A. Project Objective

The objective of this project is to develop a statistical plotter to determine the fit interval for GNSS and identify patterns of performance within different systems, satellite blocks, and navigation (nav) messages, ensuring the use of accurate satellite data.

### B. Approach

To determine when a nav message is most accurate, the User Range Error (URE) is analyzed. The URE has two components: the URE ephemeris and the URE clock. The URE total is the root sum square of the URE clock and URE ephemeris.

Each unique nav message has a reference time. URE data was generated for each unique nav message starting 10 hours before the reference time and ending 10 hours after at a 5 minute step size. This 20 hour span is five times the size of the defined GPS fit interval, providing a greater look at URE behavior at the extremities of the nav message. There are typically several unique nav messages each day for a given satellite. The statistical plotter allows the user to specify a date range and satellites to analyze. The window of time from which data is pulled can also be altered; instead of plotting URE data 10 hours before the reference the plotter can start 4 hours before the reference time to allow for a closer look at UREs around the t<sub>oe</sub>. After pulling URE data for the selected window, the plotter then stacks the URE data for each unique nav message on top of one another aligned with the toe. This provides a large number of samples at each 5 minute step. For example, for GPS, there are typically 12-13 sets of nav messages per day per satellite. Therefore, over a three month period, there are roughly 35,000 samples at each step (32 satellites x 90 days x 12 nav messages/satellite/day).

The plots generated to compare URE between systems have time from the  $t_{\text{oe}}$  on the x-axis and the y-axis indicates URE components in meters. Initially, plots compiling 95th percentile, mean, median, and minimum statistics were generated for URE data for a single satellite on a single day. After this was successful, the plotting tool was expanded to accommodate multiple satellites, to accept data from a wider range of days, and to accommodate Galileo and BeiDou.

After comparing the plots across the three systems, it was noted that the center of the fit interval was not aligned with the  $t_{\text{oe}}$  for systems other than GPS. Therefore, we analyzed the time of occurrence of nav message transmissions to understand the relationship between the  $t_{\text{oe}}$ , the UREs, and fit intervals. This is displayed in the histogram plots comparing number of nav message transmissions to time from  $t_{\text{oe}}$ .

Finally, a plot was generated to compare the start of transmission to the interval that the 95<sup>th</sup> percentile URE total remained under 2, 5, and 10 m for each system. Using this plot we could determine not only the fit interval, but how long the nav message was available to users at varying accuracies.

## III. Results

All of the following plots were generated by pulling data from January-March of 2019 (90 days of data).

#### A. GPS LNAV

The Legacy Navigation signal was the first signal developed for GPS. The GPS signal interface specification states that GPS LNAV has a fit interval of 4 hours during normal operations.

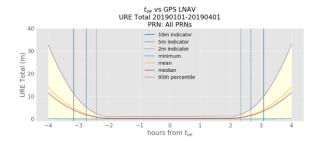


Figure 3. URE total data vs. time from toe. The x-axis shows 4 hours from toe in order to provide a closer look at the URE closest to the toe. Vertical lines indicate the interval for which the 95th percentile URE is less than 10, 5, and 2 m..

	10 m	5 m	2 m
Interval	6 hrs. 15	5 hrs. 25	4 hrs. 45
length	min.	min.	min.

Table 1. Length of time for which GPS LNAV URE remains below 10, 5, and 2 m. Based upon 95th percentile data determined in Fig. 3.

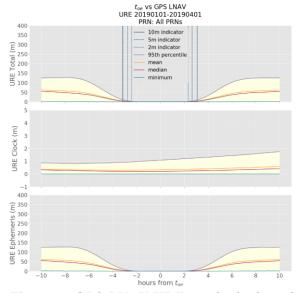


Figure 4. GPS LNAV URE total, clock and ephemeris data for a 10 hour period before and after the t<sub>oe</sub>.

Figures 3 and 4 illustrate how, as time from the t<sub>oe</sub> increases, the URE increases, decreasing the accuracy in navigation.

However, compared to Fig. 3 on a four hour time scale, Fig. 4 gives a better perspective on the behavior of the URE farther out from the fit interval. Note that the maximum on the y-scale in Fig. 3 is 40 m, while the limit on the y-scale for Fig. 4 is 400 m. It is interesting to observe that the URE clock increases throughout the time period. As shown by Table 1, GPS LNAV is performing just as well --if not better-- than expected. The stated fit interval of GPS is 4 hours. As seen in Table 1, the URE remains below 2 m for 4 hrs. 45 min, which is 45 min longer than the fit interval.

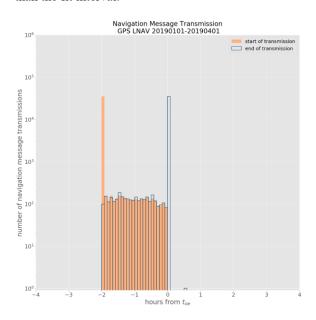


Figure 5. Time from  $t_{oe}$  vs. number of navigation message transmissions. The time on the x-axis is 4 hours before and after the  $t_{oe}$ . The y-axis is on a log scale.

Figure 5 shows that GPS LNAV begins transmission 2 hours before the  $t_{oe}$  and transmissions normally end at the  $t_{oe}$ . Fig. 3 shows us that the URE between 2 hours before the  $t_{oe}$  and the  $t_{oe}$  is within the 2 m cutoff. This indicates that data used during the normal transmission time will be most accurate.

#### 1. GPS LNAV Block IIF

In the following plots we will be looking specifically at GPS Block IIF satellites to show how the URE plotter can be used to reveal patterns in performance within a system. Block IIF is a group of satellites launched within a similar time period. Some satellites in Block IIF run on cesium clocks, while the majority run on rubidium.

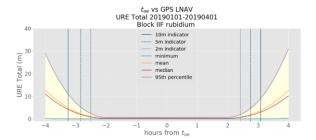


Figure 6. URE total data for the Block IIF satellites running on rubidium clocks for 4 hours before and after the toe.

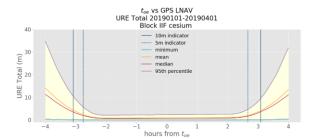


Figure 7. URE total data for the Block IIF satellites running on cesium clocks for 4 hours before and after the toe.

The vertical lines in Fig. 6 and Fig. 7. indicate the region where the 95<sup>th</sup> percentile URE remains below 2, 5, and 10 m. It is important to note that for the Block IIF satellites operating on cesium clocks, there are no 2 m indicators. This is because the 95<sup>th</sup> percentile URE never falls below 2 m. The URE closest to the t<sub>oe</sub> is noticeably lower for the PRNs operating on rubidium clocks. Table 2 shows how satellites operating with rubidium clocks maintain a higher level of accuracy longer than cesium. Satellites operating on rubidium maintain a 95<sup>th</sup> percentile URE below 2 m for 4 hrs. 55 min. while the 95<sup>th</sup> percentile of those using cesium never falls below 2 m.

	10 m	5 m	2 m
Rubidium	6 hrs.	5 hrs.	4 hrs. 55 min.
interval length	20 min.	35 min.	
Cesium	6 hrs.	5 hrs.	
interval length	10 min.	25 min.	

Table 2. Interval of time 95th percentile of the URE total remains below the 2, 5, and 10 m 95th percentile cutoffs.

## B. GPS CNAV

The following plots compare the URE for the GPS Civilian Navigation Message. CNAV is a newer nav message type and was developed after LNAV.

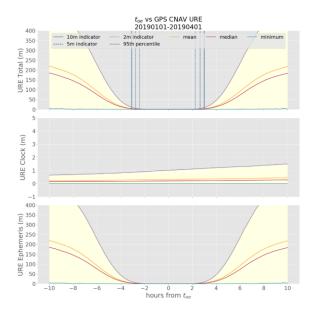


Figure 8. GPS CNAV URE total, clock, and ephemeris data for a 10 hour period before and after the  $t_{\rm se}$ .

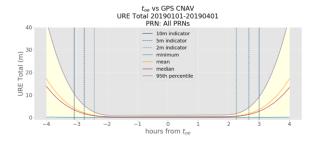


Figure 9. URE total data vs. time from toe. The x-axis shows only 4 hours from toe in order to provide a closer look at the URE closest to the toe. Vertical lines indicate the interval for which the 95th percentile of the URE total is less than 10, 5, and 2

	10 m	5 m	2 m
Interval	6 hrs. 5	5 hrs. 25	4 hrs. 40
length	min.	min.	min.

Table 3. Length of time for which GPS CNAV URE remains below 10, 5, and 2 m. Based upon 95th percentile data determined in Fig. 9.

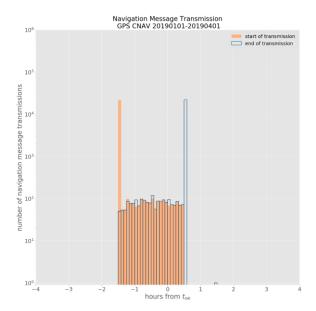


Figure 10. Time from  $t_{oe}$  vs. number of navigation message transmissions. The time on the x-axis is 4 hours before and after the  $t_{oe}$ . The y-axis is on a log scale.

CNAV shows very similar behavior to LNAV: within 2 hours of the t<sub>oe</sub>, the URE dramatically decreases, as seen in Fig. 8 and Fig. 9. However, the URE of CNAV is much greater towards the extremities of Fig 8, which shows URE up to 10 hours from the t<sub>oe</sub>. The impact of this is, as distance in time from the t<sub>oe</sub> increase, the accuracy users can expect dramatically decreases.

Figure 10 shows that CNAV begins its transmission 1.5 hours before t<sub>oe</sub>, instead of 2 hours before like LNAV. According to Table 3, CNAV has similar interval lengths to LNAV, indicating their nav messages maintain their highest level of accuracy for a similar length of time.

## C. Galileo

The following plots show URE data and time of nav message transmissions for Galileo, the European satellite system.

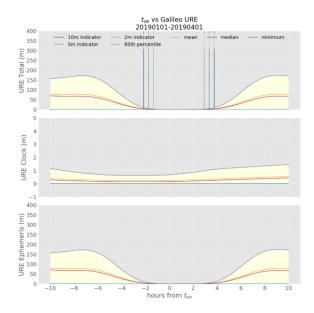


Figure 11. Galileo URE total, clock, and ephemeris data for a 10 hour period before and after the  $t_{oe}$ .

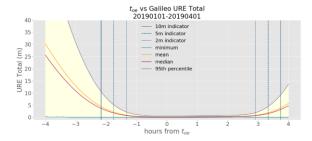


Figure 12. URE total data vs. time from toe. The x-axis shows only 4 hours from toe in order to provide a closer look at the URE closest to the toe. Vertical lines indicate the interval for which the 95th percentile URE is less than 10, 5, and 2 m.

	10 m	5 m	2 m
Interval	5 hrs. 55	5 hrs. 5	4 hrs. 15
length	min.	min.	min.

Table 4. Length of time for which Galileo URE total remains below 10, 5, and 2 m. Based upon 95th percentile data determined in Fig. 12.

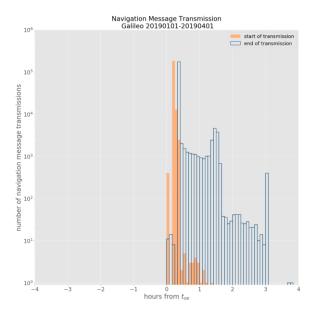


Figure 13. Time from  $t_{oe}$  vs. number of navigation message transmissions. The time on the x-axis is 4 hours before and after the  $t_{oe}$ . The y-axis is on a log scale.

Galileo's URE plots appear very similar to LNAV and CNAV. However, by looking at Fig. 11 and Fig. 12 it can be observed that the interval of time with the lowest URE is no longer centered on the t<sub>oe</sub>. Instead, the URE curve is centered around the 1 hour mark.

A likely cause of this can be discovered when looking at Fig. 13, the plot showing the time of nav message transmissions. This plot shows how, unlike GPS, which begins transmission well before the too, Galileo begins most of it nav message transmissions a few minutes after. For this reason their fit interval is set to better align with their time of transmissions. According to Table 4, Galileo also has a slightly shorter interval length then GPS CNAV and LNAV. Galileo has stated nav messages older than 4 hours should not be used; Table 4 supports that data within this time period is indeed the most accurate.

#### D. BeiDou IGSO

For our analysis of BeiDou, we chose to focus on the IGSO (Inclined Geosynchronous Orbit) satellites. This was because we had the most accurate URE data for these satellites and did not wish to have one set of statistics stand for satellites in Medium Earth Orbit (MEO), Geostationary Orbit (GEO), and IGSO.

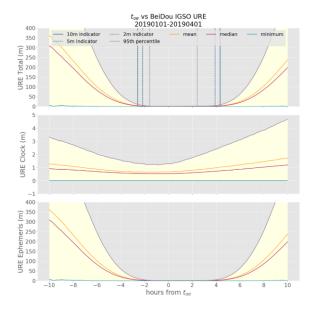


Figure 14. BeiDou URE total, clock, and ephemeris data for a 10 hour period before and after the  $t_{oe}$ .

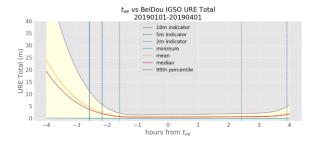


Figure 15. URE total data vs. time from  $t_{oe}$ . The x-axis shows only 4 hours from  $t_{oe}$  in order to provide a closer look at the URE closest to the toe. Vertical lines indicate the interval for which the 95th percentile URE is less than 10, 5, and 2 m.

	10 m	5 m	2 m
Interval	6 hrs. 55	6 hrs. 5	4 hrs. 0
length	min.	min.	min.

Table 5. Length of time for which BeiDou URE total remains below 10, 5, and 2 m. Based upon 95th percentile data determined in Fig. 15.

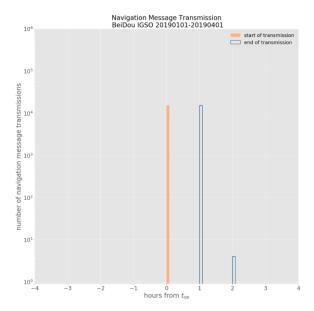


Figure 16. Time from  $t_{oe}$  vs. number of navigation message transmissions. The time on the x-axis is 4 hours before and after the  $t_{oe}$ . The y-axis is on a log scale.

The IGSO satellites exhibit the same pattern as Galileo. As seen in Fig. 14 and 15, the center of fit interval is not aligned with the t<sub>oe</sub>. This is again explained by looking at the time of nav message transmissions as seen in Fig. 16. BeiDou also begins transmission at the t<sub>oe</sub>, so their fit interval is aligned with their time of transmission. According to Table 5, BeiDou remains accurate to 10 meters for a longer period than any of the previous systems, but maintains a URE less than 2 m for a shorter period.

# **IV.** Conclusion

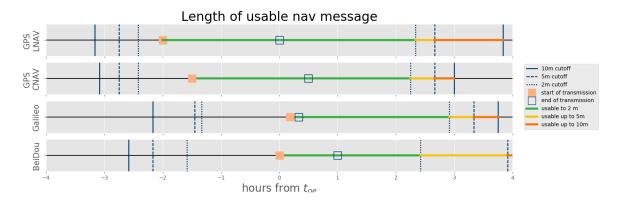


Figure 17. Length of time from the start of nav message transmission to the cutoff of varying levels of accuracy. The orange squares indicate typical start of transmission while the blue square indicates the typical end.

Duration from start of transmission to URE total 95th percentile cutoff					
URE (m)	GPS LNAV	GPS CNAV	Galileo	BeiDou	
10	5 hrs. 5 min.	4 hrs. 30 min.	3 hrs. 55 min.	4 hrs. 20 min.	
5	4 hrs. 40 min.	4 hrs. 10 min.	3 hrs. 30 min.	3 hrs. 55 min.	
2	4 hrs. 20 min.	3 hrs. 45 min.	3 hrs. 5 min.	2 hrs. 25 min.	

Table 6. The amount of time from the start of transmission to the cutoff period in which the nav message maintains a certain level of accuracy. This table shows the numerical data represented visually in Figure 21. The 2 m row shows the length of the green line, the 5 m row shows the length of the green line plus the yellow, the 10 m row shows the length of the green line, the yellow and the orange.

The initial purpose of this study was to determine the fit interval of GPS, Galileo, and BeiDou. However, it became apparent that another important factor was the length of time the data within the fit interval is available to users. This is shown in Fig. 16 and Table 6, which show the duration of time between when nav message transmissions typically begin and the 2, 5, and 10 meter cutoffs.

As illustrated by Fig. 17, Galileo and BeiDou begin transmitting towards the center of their fit interval. Therefore, the actual usable period within the fit interval is less than GPS. This means BeiDou and Galileo must change nav messages more frequently. The blue squares in Fig. 17 show that nav message transmission typically ends well within the 2 m interval for all three systems. However, during periods when the nav message is not updated, GPS nav messages will maintain a higher degree of accuracy for longer.

GPS begins transmission towards the beginning of its fit interval, so for 4 hr. 20 min. the transmitted nav message has a 95<sup>th</sup> percentile URE total less than 2 m. Therefore the GPS fit interval is

longer than published. Galileo does not provide a fit interval, but users are told not to use data beyond 4 hrs. from the start of transmission. From start of transmission to the 2 m cutoff Galileo has a period

Minimum URE Total based on 95th Percentile					
	GPS	GPS	Galileo	BeiDou	
	LNAV	CNAV			
Minimum	1.011	0.947	0.583	1.370	
(m)					

Table 7. Minimum 95<sup>th</sup> percentile value for each GNSS system.

of 3 hr. 5 min. So while the URE is below 2 m for 3 hr. 55 min, 50 min of this is unusable because it is prior to the typical start of transmission. This 3 hr. 5 min. period is shorter than the recommended 4 hr. period. BeiDou UREs remain below 2 m for about 4 hours, but only 2 hrs. and 25 min. of this period is after the start of transmission. As seen in Table 7, Galileo has the lowest minimum URE total (0.583 m)

However, GPS has the longest recommended usable transmission period (4 hrs. 20 min.).

# **Acknowledgments**

Thank you so much to Brent Renfro and Miquela Stein, without whom this report would not have been possible. This has been an amazing opportunity and I have learned so much this summer.

## References

<sup>1</sup>An Introduction to GNSS. 2nd ed., NovAtel, 2015.

- <sup>2</sup>Misra, Pratap, and Per Enge. *Global Positioning System*. 2nd ed., Ganga-Jamuna Press, 2001.
- <sup>3</sup> Springer Handbook of Global Navigation Satellite Systems (p. 7), by P. J. G. Teunissen, O. Montenbruck, 2017, Springer. Copyright 2017 by, Springer International Publishing AG.
- <sup>4</sup>Springer Handbook of Global Navigation Satellite Systems (p. 66), by P. J. G. Teunissen, O. Montenbruck, 2017, Springer. Copyright 2017 by Springer International Publishing AG.
- <sup>5</sup>U.S. Department of Defense. Standard Positioning Service Performance Standard, 4th Edition. http://www.gps.gov/technical/ps/2008-SPSperformance-standard.pdf, 2008
- <sup>6</sup>U.S. Department of Defense. Navstar GPS Space Segment/Navigation UserInterfaces, IS-GPS-200, Revision G. https://www.gps.gov/technical/icwg/ IS-GPS-200G.pdf, September 2012.
- <sup>7</sup>European GNSS (Galileo) Open Service Signal-In-Space Interface Control Document (OS-SIS-ICD), Issue 1.3, European Union, December 2016
- <sup>8</sup>European GNSS (Galileo) Open Service Definition Document (OS-SDD), Issue 1.0, European Union, December 2016
- <sup>9</sup>BeiDou Navigation Satellite System Open Service Performance Standard (Version 2.0), China Satellite Navigation Office, December 2018.