Fundamentals of Control Systems

Elec 372

Lab Experiment #3

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"I certify that this submission is my original work and meets the Faculty's Expectations of Originality."

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1) Objectives

The objective of the third experiment of the course Elec 372 is to understand the principles behind improving system performance through PB, PI and PID control. Students will also perform experiment to learn about the elimination of a steady state error due to a disturbance input.

2) Theory

PID-Control: Proportional Integral Derivative Control serves to integrate an error signal in order to reduce or eliminate steady state output offset. It improves performance and maintains stability.

Steady State Error: This analysis can be examined based on the final value theorem and error coefficients which depends on the type of system as well as the input signals (step, ramp, parabolic, etc.). All systems have different requirements of control to reduce steady state error.

Disturbance Control: The disturbance input is reduced based on PI control. We are able to analyse the disturbance transfer function to observe the impact of loop gain on disturbance rejection.

3) Tasks / Results / Discussions

3.1 PID Control Tasks

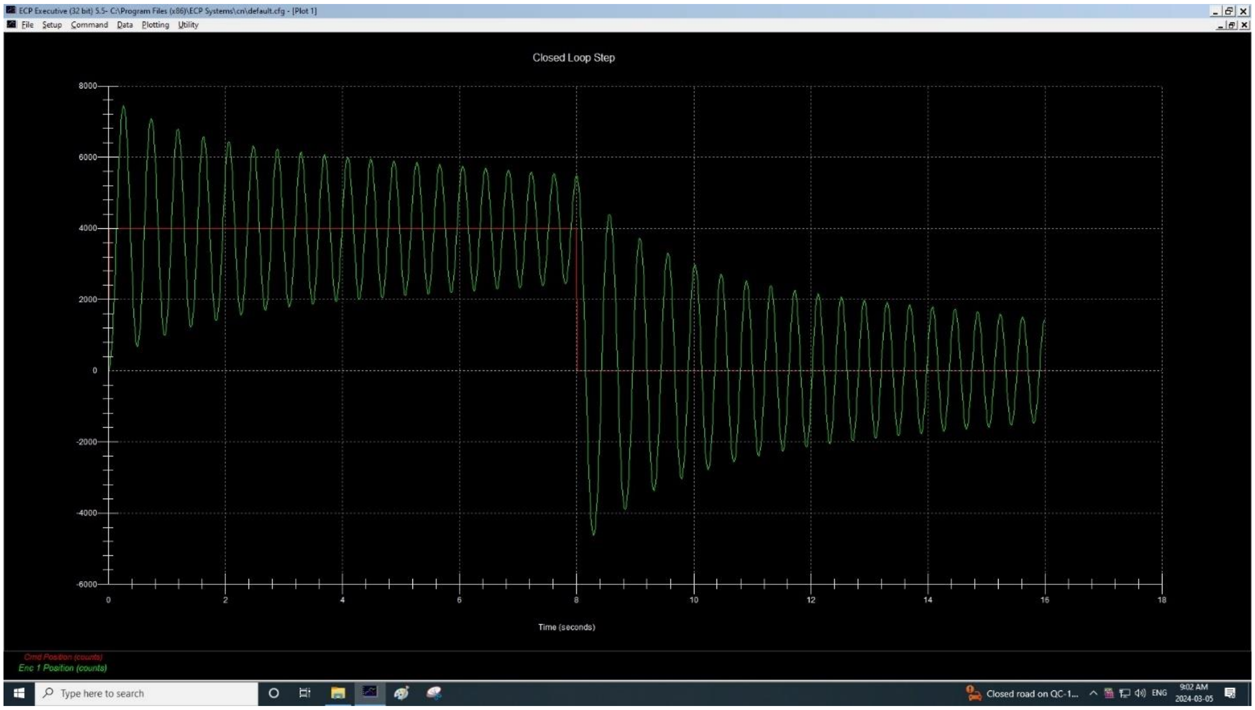

Figure 3.1.1 PI-Control

Percentage Overshoot = $(y_{max}-y_{ss})/y_{ss}=(7500-4000)/4000=87.5\%$

$T_p \approx 0.15s$

$\omega_d = \pi/T_p = 20.94$ rad/s

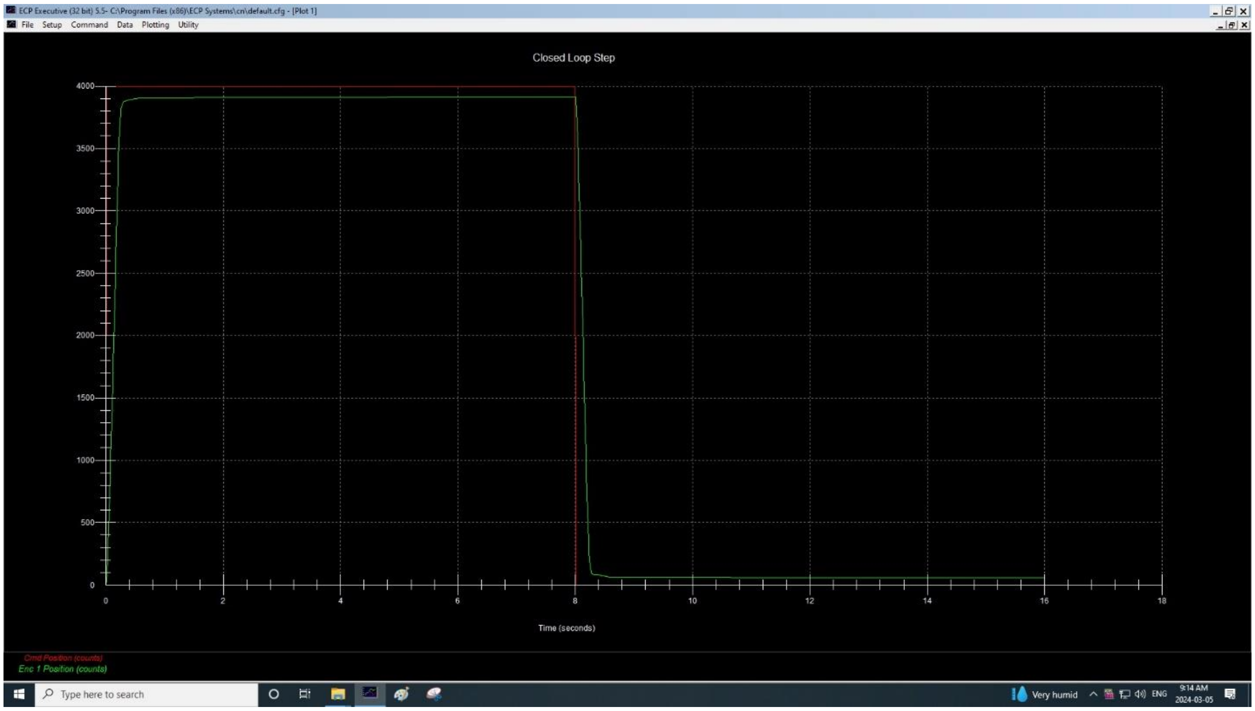

Figure 3.1.2 PI-Control with Viscous Friction

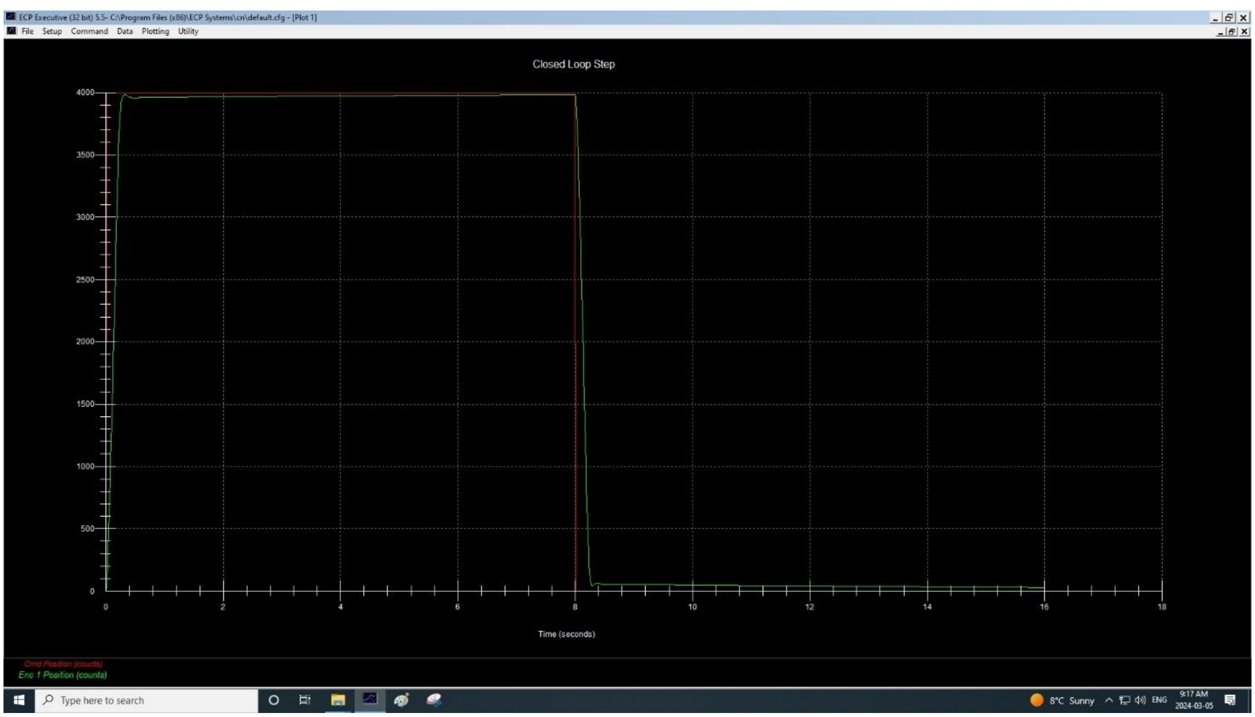

Figure 3.1.3 ($K_i = 0.02$) PI-Control with adjusted K_i

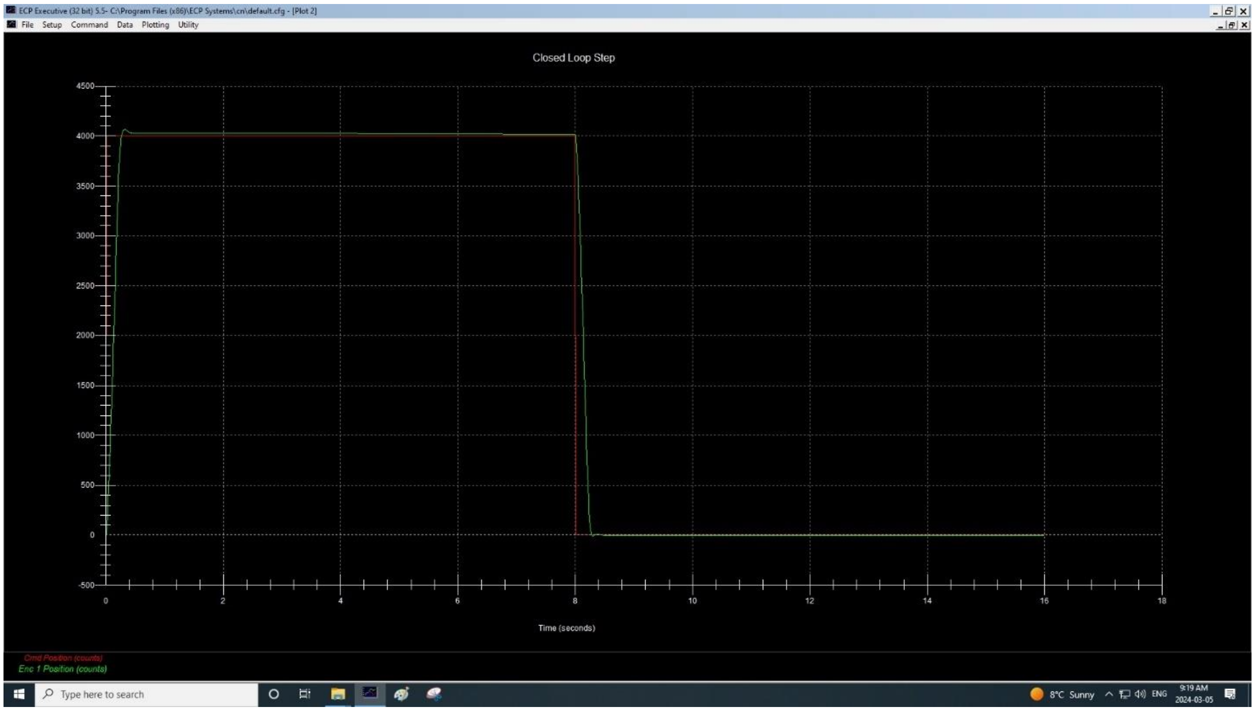

Figure 3.1.4 (K_i = 0.04) PI-Control with adjusted K_i

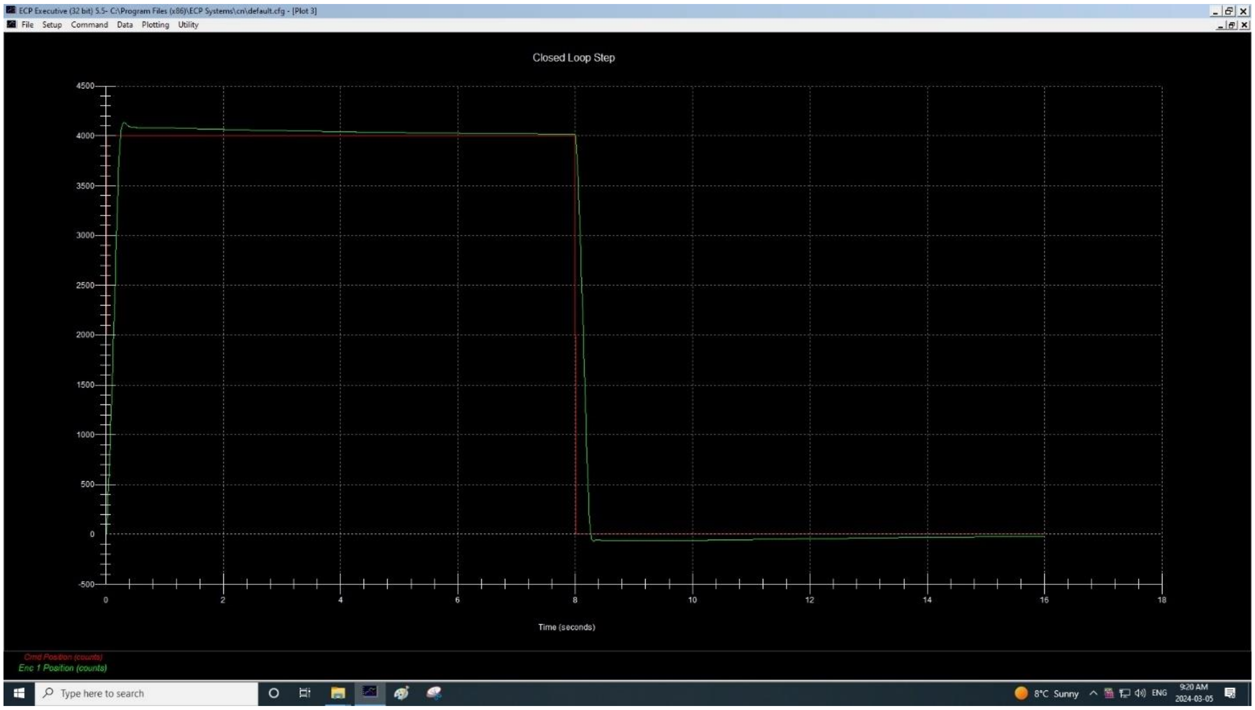

Figure 3.1.5 (K_i = 0.06) PI-Control with adjusted K_i

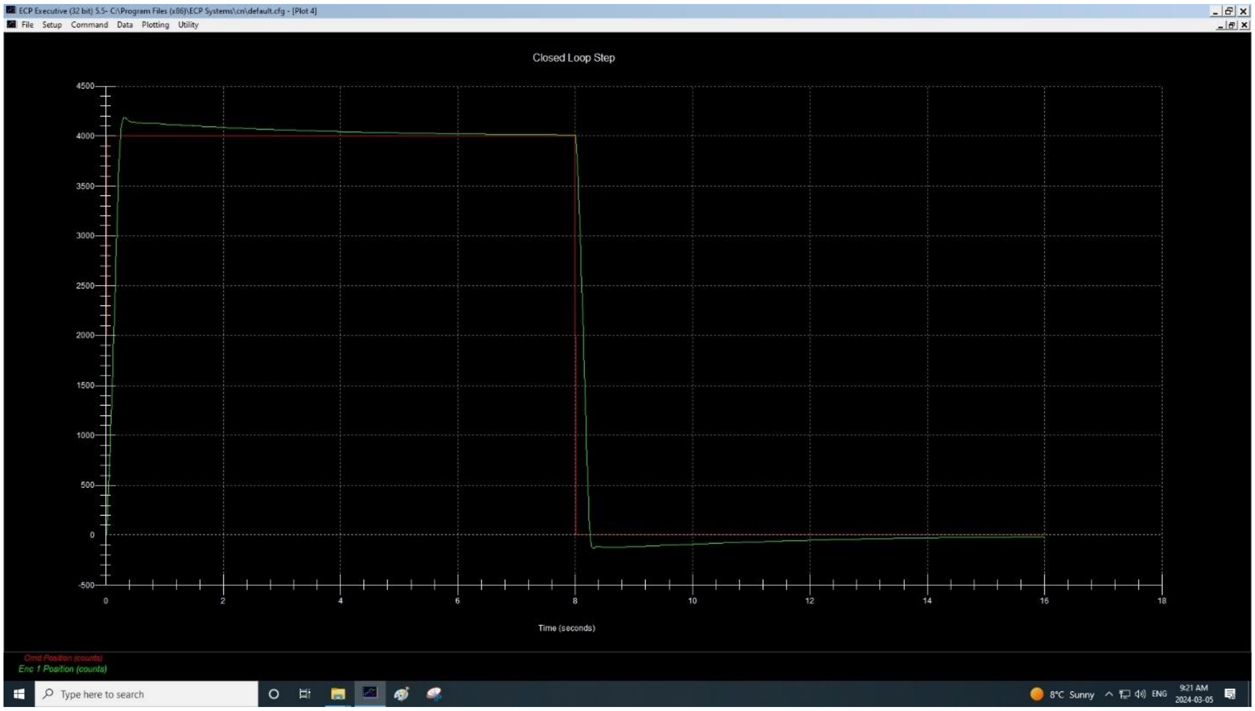

Figure 3.1.6 (K_i = 0.08) PI-Control with adjusted K_i

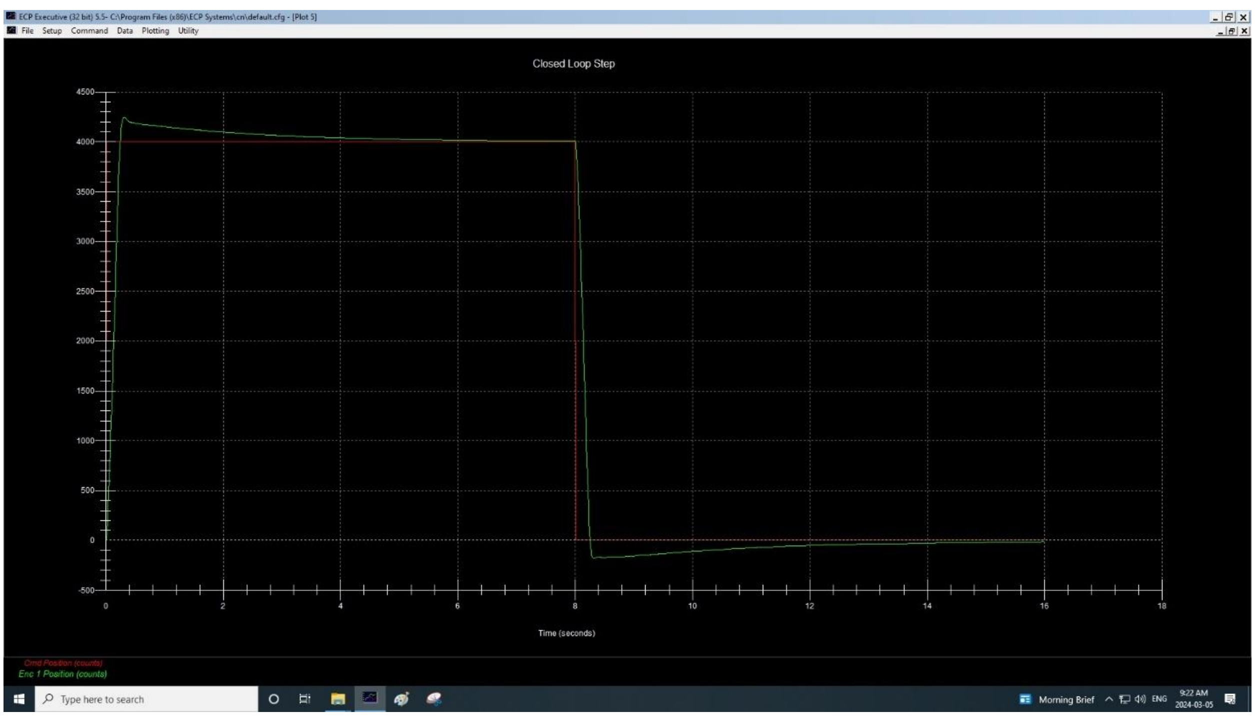

Figure 3.1.7 (K_i = 0.1) PI-Control with adjusted K_i

For the above 7 figures, they represent the results obtained by performing task 5.3.1 PI-CONTROL of the lab manual. This section investigates the effectiveness of PI control in reducing steady state error for a closed loop system. We tested a system with proportional control, followed by integral feedback. Then, Figure 3.1.3 through Figure 3.1.7, we increased the integral gain K_i to find the optimal value which minimizes steady state error. So, value of $K_i = 0.04$ minimizes the steady state error.

Results

Describe the effect of increase in Ki on the offset error and on the overshoot.

Increasing K_i will reduce system offset error, but it also increases overshoot. Observing Figure 3.1.3 (Ki = 0.02), we can notice that it has a slight offset error. However, by increasing Ki to 0.04 such as Figure 3.1.4 (Ki = 0.04), notice how the offset error is reduced. As for the overshoot, observe Figure 3.1.7 (Ki = 0.1) which has a significant higher overshoot compared to Figure 3.1.4 due to its higher Ki value.

Derive the CLTF for the PID system (#1 in Figure 5.1) and obtain the relation between coefficients required for stability.

Using the values of B and K from page 7 of my own lab experiment 2, we have:

J: $3.1825*10^{-3}$ kgm^2 or 0.0031825

B: $1.7825*10^{-3}$ Nm/radian or 0.0017825

K: 3.39 Nm/radian

$$CLTF = \frac{KK_p}{Js^2 + (B + KK_d)s + KK_p}$$

$$CLTF = \frac{3.39 * 0.04}{3.1825 * 10^{-3}s^2 + (1.7825 * 10^{-3} + 0)s + 3.39 * 0.04}$$

$$CLTF = \frac{0.1356}{3.1825 * 10^{-3}s^2 + 1.7825 * 10^{-3}s + 0.1356}$$

$(3.1825*10^{-3})s^2 + (1.7825*10^{-3})s + 0.1356 = as^3 + bs^2 + cs + d$

Know that no root will have a positive real part if the condition bc>ad is satisfied. In this case, bc is equal to a positive number while ad is equal to zero, thus this condition is satisfied. As such, no root will be a positive real part.

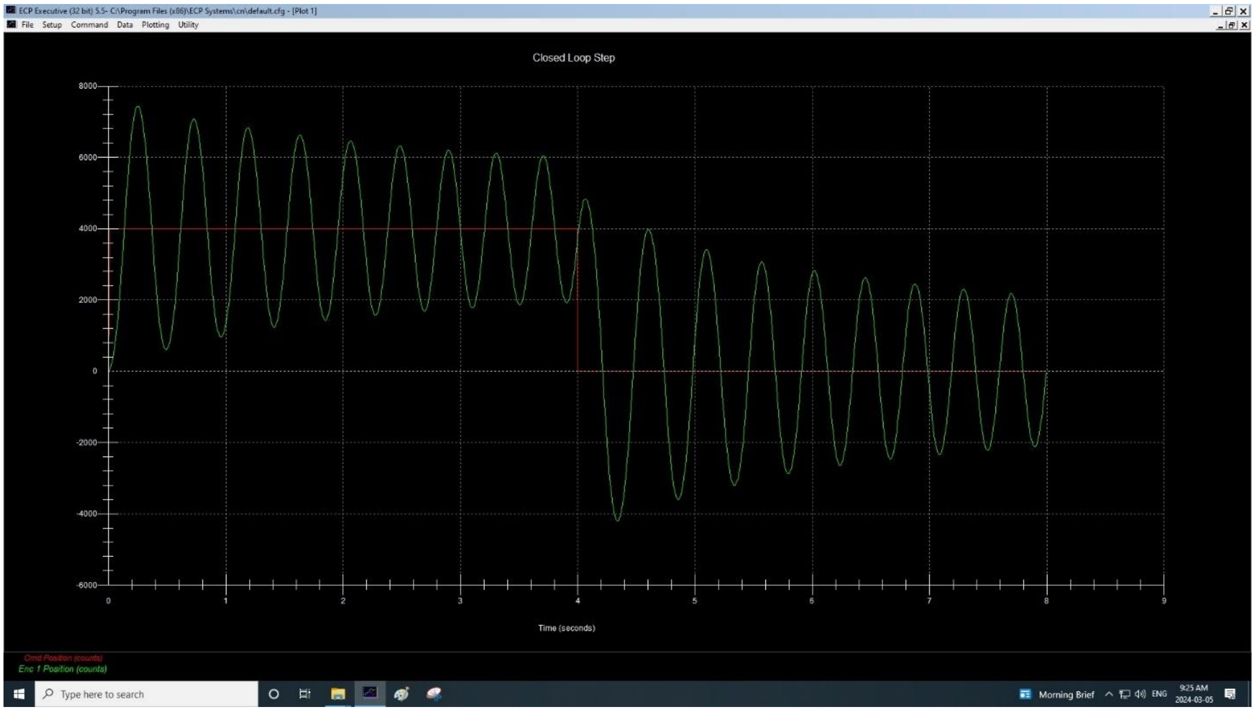

Figure 3.1.8 PID-Control

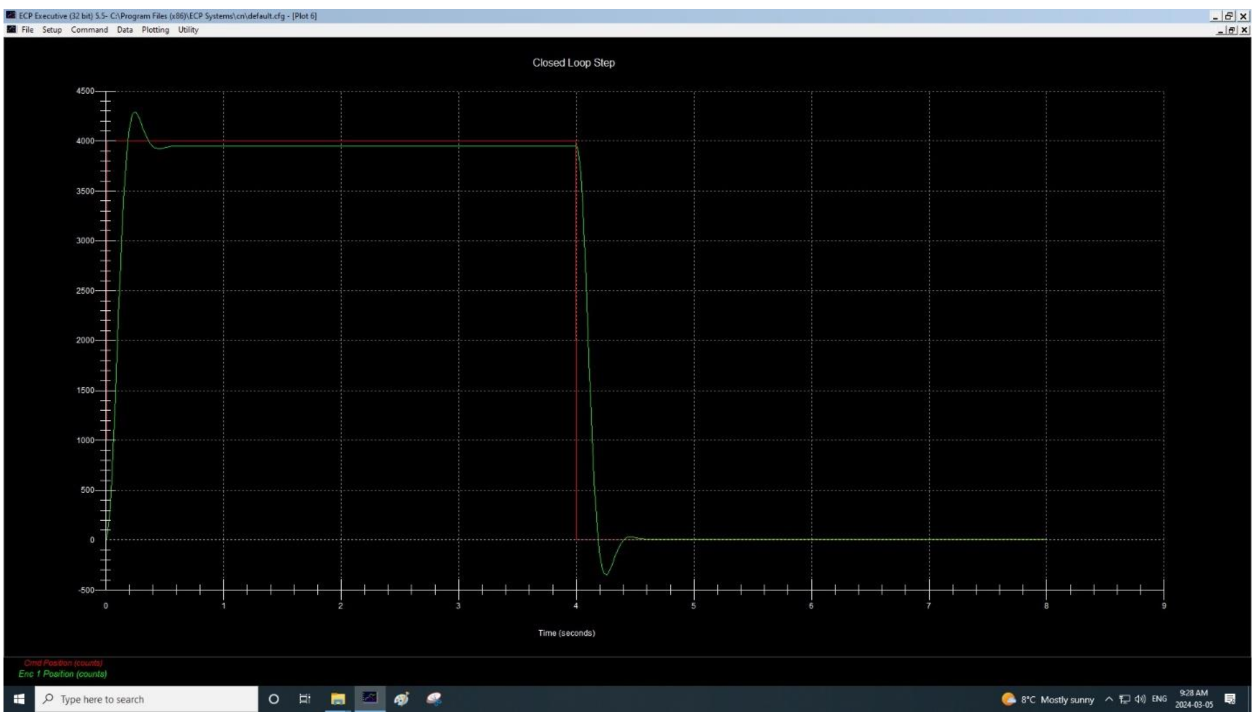

Figure 3.1.9 (K_d = 0.014) PID-Control with increase in K_d (desired overshoot)

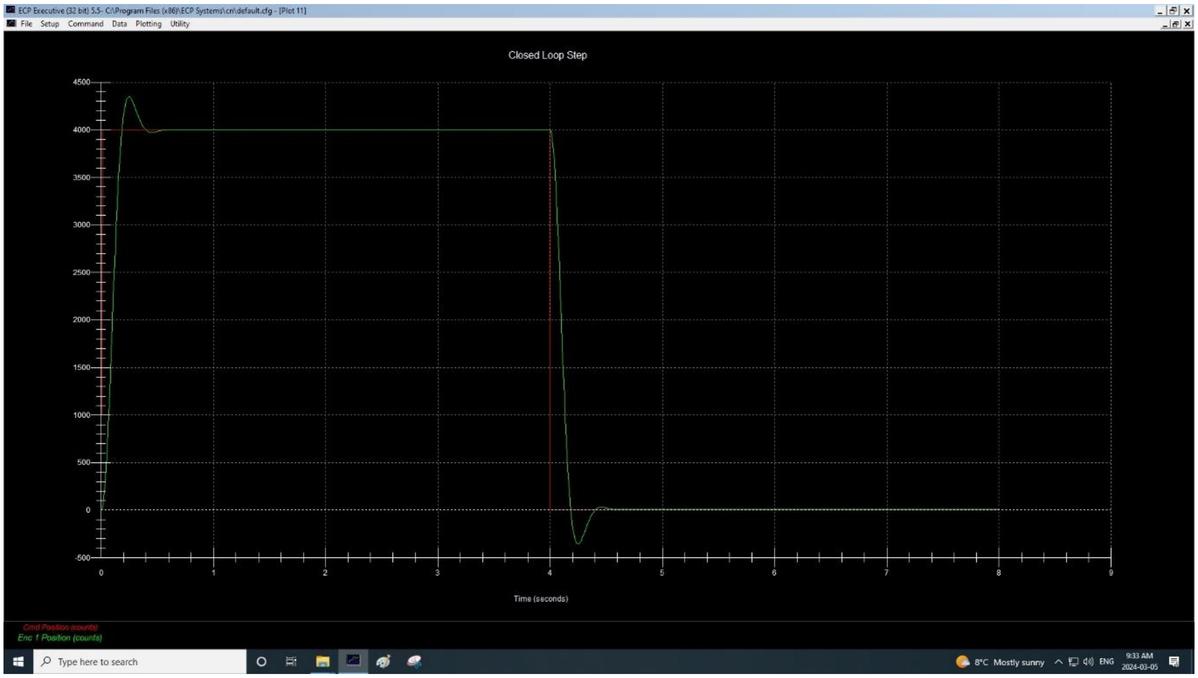

Figure 3.1.10 (K_i = 0.03) PID-Control with increase in K_i (desired offset)

<u>Tabulate your choice of controller coefficients for PID control.</u>

	K_p	K_i	K_d	Step Size	Dwell Time	Rep
Fig 3.1.8	0.2	0	0	4000	4000	1
Fig 3.1.9	0.2	0	0.014	4000	4000	1
Fig 3.1.10	0.2	0.03	0.014	4000	4000	1

3.2 Steady State Error Analysis

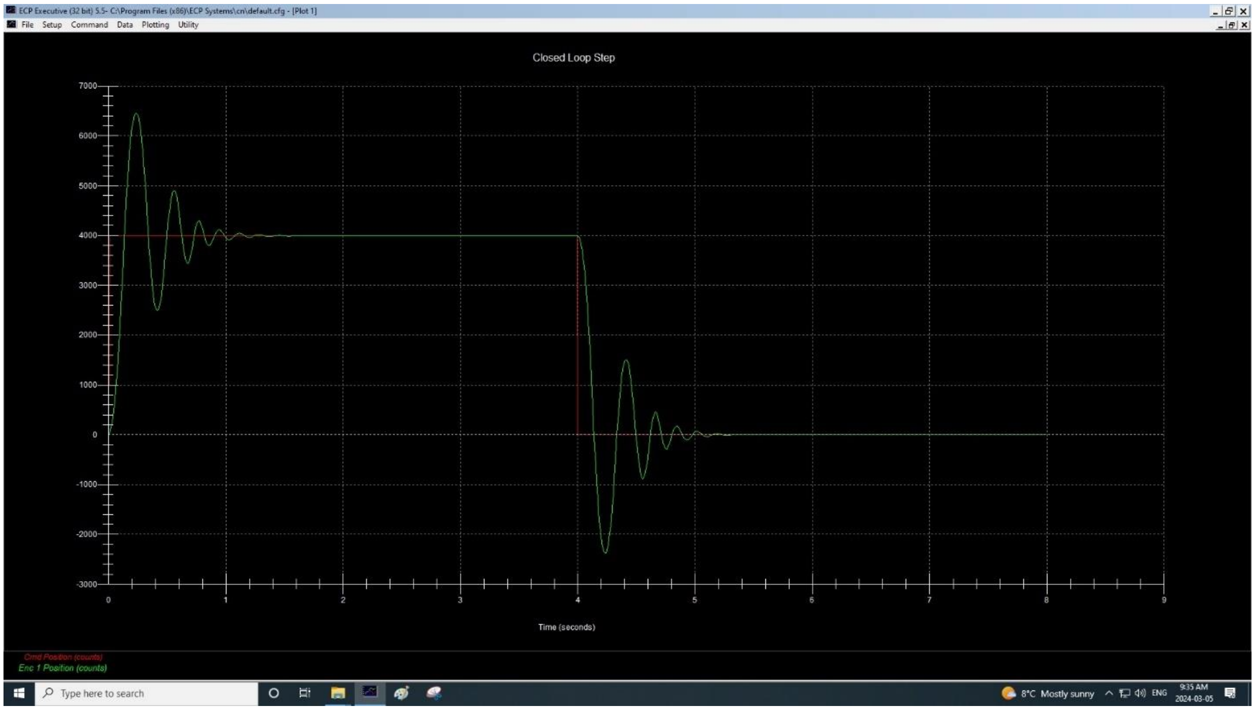

Figure 3.2.1 Step Signal with PD

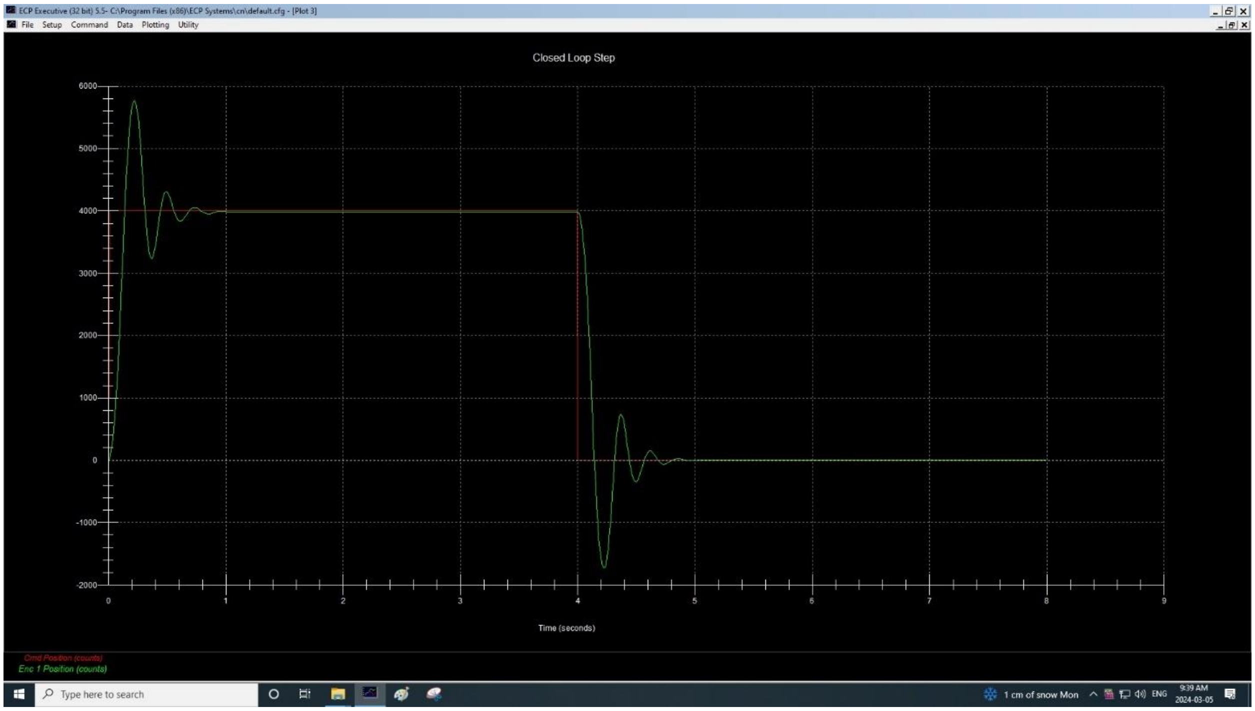

Figure 3.2.2 ($K_p = 0.5$) Step Signal with PD

Percentage Overshoot = $(y_{max}-y_{ss})/y_{ss}$=(5800-4000)/4000=45%

$T_p \approx 0.2s$

$\omega_d = \pi/T_p = 5\pi$ rad/s

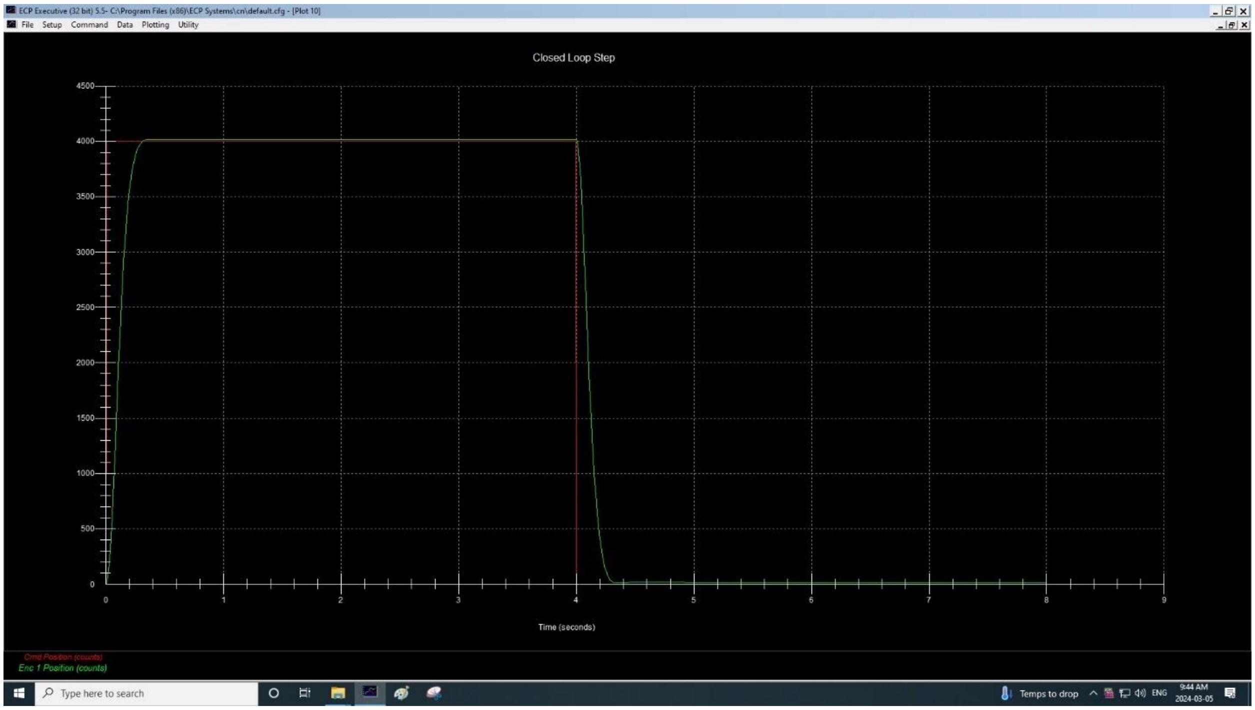

Figure 3.2.3 (K_p = 0.2, K_d = 0.02, K_i = 0.05) Step Signal with Stable PD

This first and second figure have a high percentage offset which can be seen by how unstable it looks. For figure 3.2.3, we changed the input values until the system is stable such that the values are K_p = 0.2, K_d = 0.02, K_i = 0.05. This can be observed by how similar its pattern is to the step signal that rises to 4000 and drops to 0 every 4 seconds.

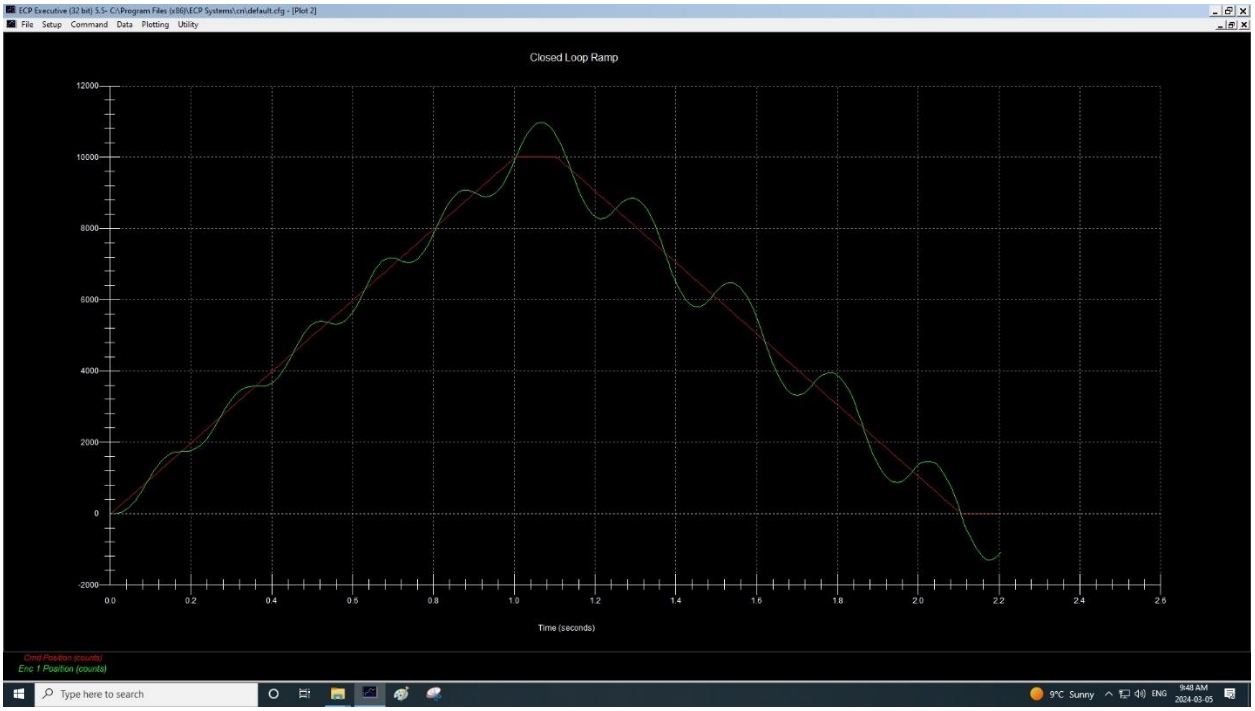

Figure 3.2.4 Ramp Signal (PI + Velocity Feedback)

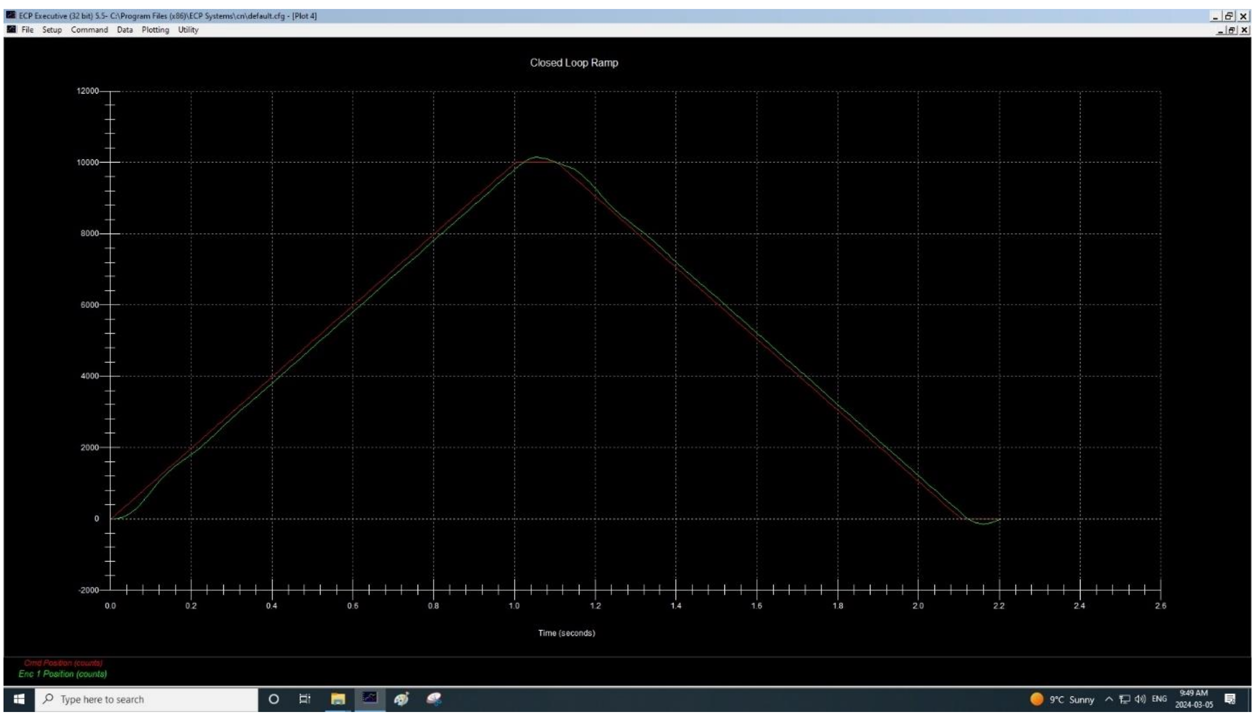

Figure 3.2.5 Ramp Signal (PID)

Based on figure 3.2.5, we have two points for the red line, P1(0.4, 4000) and P2(0.6, 6000) and two other points for the green line, P3(0.5, 4800) and P4(0.42, 4000).

Equation of rising part of the red line:

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6000 - 4000}{0.6 - 0.4} = 10000$$
$$y = 10000x + b$$
$$Choosing\ P1, 4000 = 10000 * 0.4 + b$$
$$4000 = 4000 + b$$
$$b = 0$$
$$Slope\ Eq1 = y = 10000x$$

Equation of rising part of the green line:

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4800 - 4000}{0.5 - 0.42} = 10000$$
$$y = 10000x + b$$
$$Choosing\ P3, 4800 = 10000 * 0.5 + b$$
$$4800 = 5000 + b$$
$$b = -200$$
$$Slope\ Eq2 = y = 10000x - 200$$

We can confirm that both lines are parallel. The steady state step error is then the difference between the intercept of the red and green line which is 200.

RESULTS

Is there any change in the step response between the two cases?

Both Figure 3.1.10 and Figure 3.2.3 are step response which are set to have the least amount of offset. However, notice that Figure 3.1.10 which is the PID configuration has a higher overshoot than Figure 3.2.3 which is the PI + velocity feedback configuration.

What is the difference in the ramp response between the two cases?

As for the difference in the ramp response, it can be analysed that the ramp response with PID controller has better performance than PI + velocity feedback controller. This is due to the fact that it has a better reduced steady state error. This can be seen from Figure 3.2.4 and Figure 3.2.5 where Figure 3.2.5 has a clearer rising and falling slope.

Show velocity error-coefficients for the configurations #1 (PID) and #2 (PI+V).

PID - Closed loop transfer function:

$$T(s)=((KK_p+KK_d s)G(s))/(1+(KK_p+KK_d s)G(s))$$

PI+V - Closed loop transfer function:

$$T(s)=((KK_p+KK_i/s)G(s))/(1+(KK_p+KK_i/s)G(s))$$

To find K_{vel}, we use the steady state error equation:

$$e_{ss}(ramp)=1/k_{vel}$$

Thus,

$$K_{vel(PID)} = KK_p/B$$

$$K_{vel(PI+V)} = KK_p/(B+KK_d)$$

3.3 Disturbance Attenuation

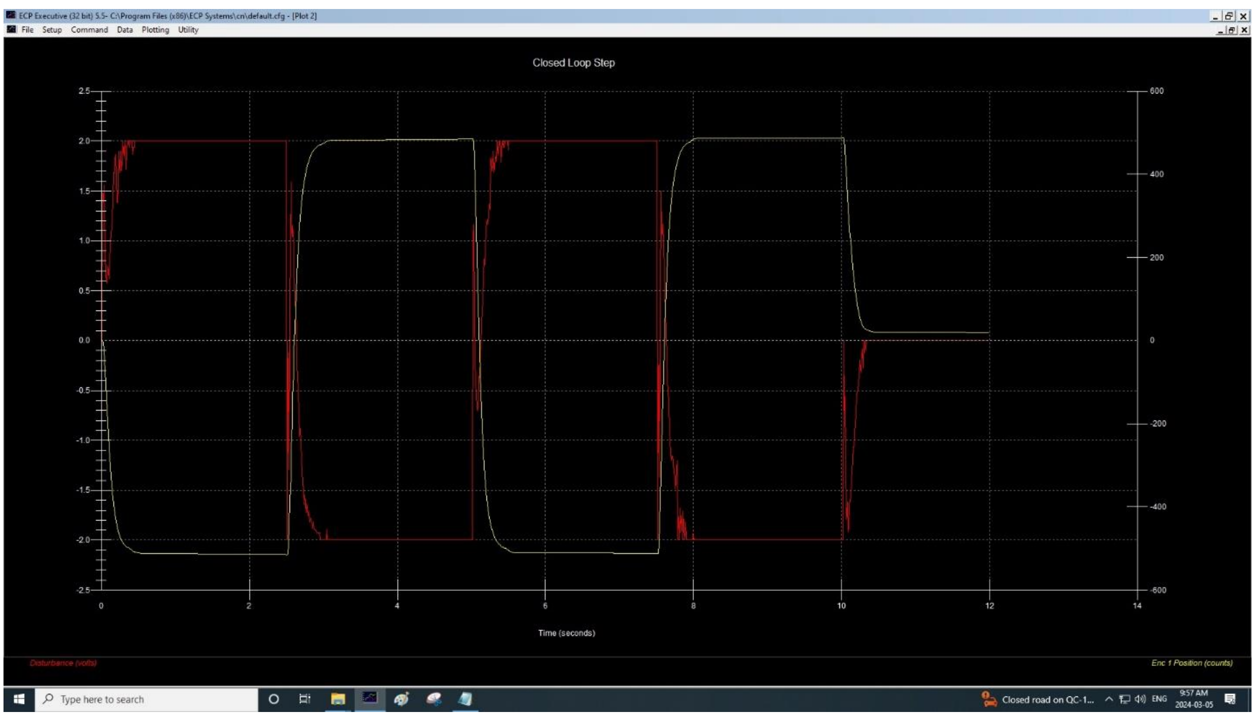

Figure 3.3.1 Disturbance Step

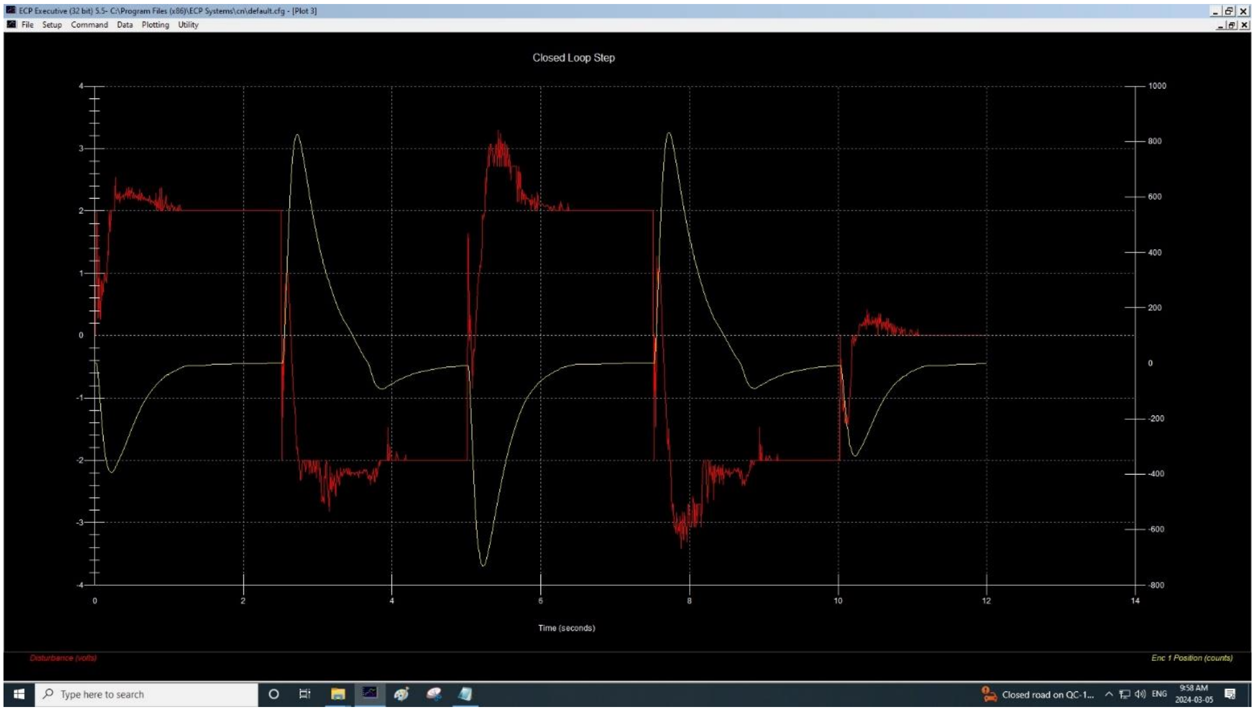

Figure 3.3.2 Disturbance Step with $K_i = 0.6$

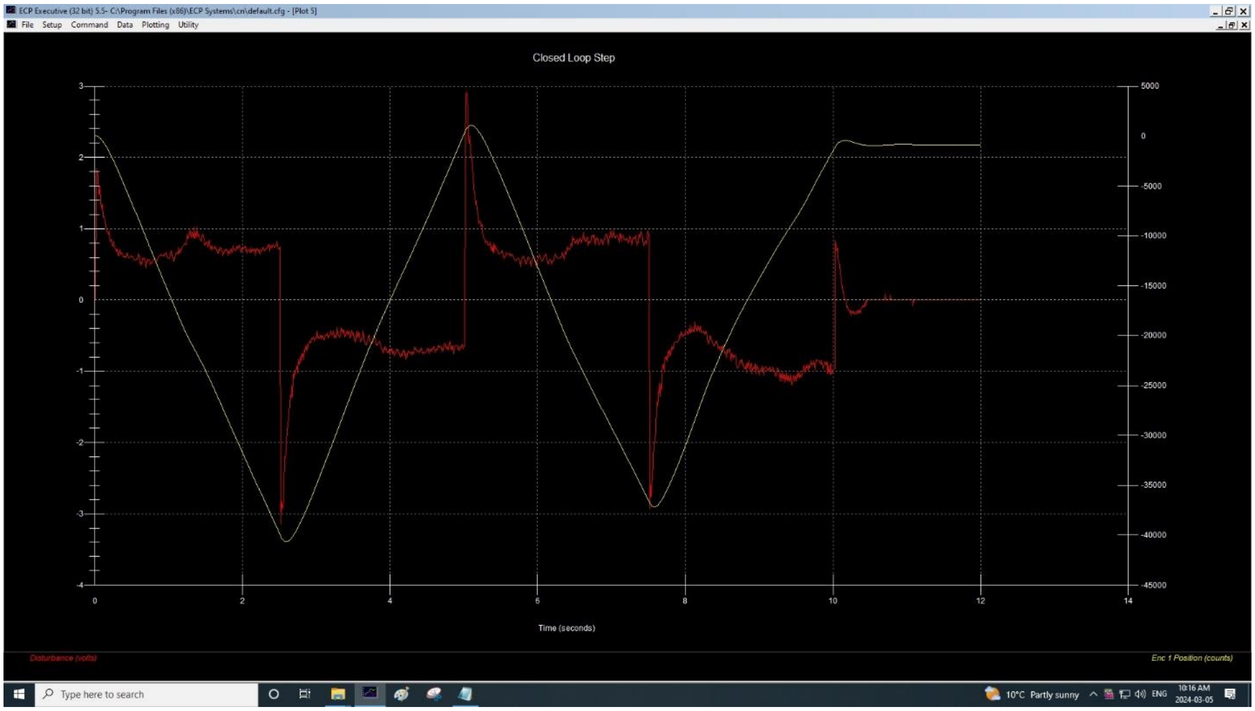

Figure 3.3.3 Disturbance Step with $K_i = 0.3$

Figure 3.3.4 Disturbance Step with $K_i = 0.9$

4) Conclusions

In conclusion, the third Elec 372 experiment was performed to explore what kind of effect PID Control had on system performance. Mainly, it can be used to reduce steady state errors while also effectively attenuating disturbance. Then, it was observed that the integral and derivative terms of a controller coefficients is used to achieve optimal system response and improve stability. As such, the analysis of steady state errors under various input conditions allowed the students to better grasp design optimization and methods to reduce disturbance.

5) Appendix

None