Fundamentals of Control Systems

Elec 372

Lab Experiment #2

Andre Hei Wang Law

4017 5600

Section UJ-X

TA: Saba Sanami

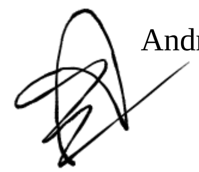
TA Email: sabasanami272@gmail.com

Professor: Amir Aghdam

Performed on February 13, 2024

Due on March 5, 2024

“I certify that this submission is my original work and meets the Faculty’s Expectations of Originality.”



Andre Hei Wang Law

4017 5600

05/03/2024

1) Objectives

For the second lab experiment of the course Elec 372, students are tasked to perform system identification on a mechanical first-order system to determine its inertia, damping and gain (J, B, K) parameters. The experiment will involve open-loop and closed-loop response on a DC motor. They will also perform calculations on the system model and verifying the model using MATLAB. As such, these theoretical predictions and the experimental results will allow for students to compare data which deepens their understanding on the fundamentals of control systems.

2) Theory

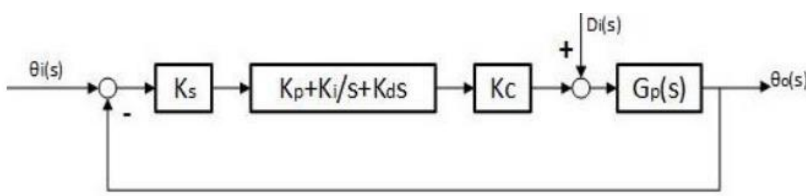


Figure 2.1

The above figure represents a ECP 220 system block diagram of a closed-loop configuration. A PI+V controller configuration is considered as the “default” configuration as it allows for a good control of closed-loop system parameters.

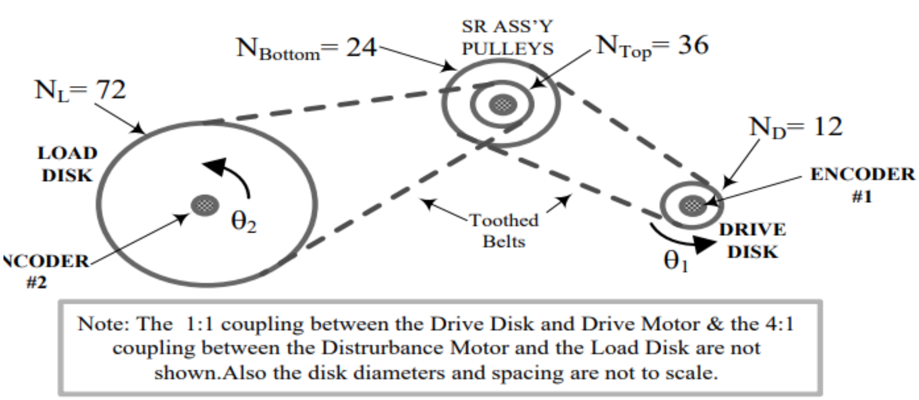


Figure 2.2

The figure 2.2 demonstrates the rotation inertia ratio between disks. The letter of the above figure represents:

D: The drive, inertia specified by the manufacturer is $4(10)^{-4}kgm^2$

L: The load, inertia specified by the manufacturer is $65(10)^{-4}kgm^2$

SR: The SR disks, inertia specified by the manufacturer is $78(10)^{-6}kgm^2$

w: the additional inertia of the weights fixed to the driver and the load disks

3) Tasks / Results / Discussions

3.1) EVALUATION OF J AND K USING OPEN-LOOP VELOCITY OUTPUT

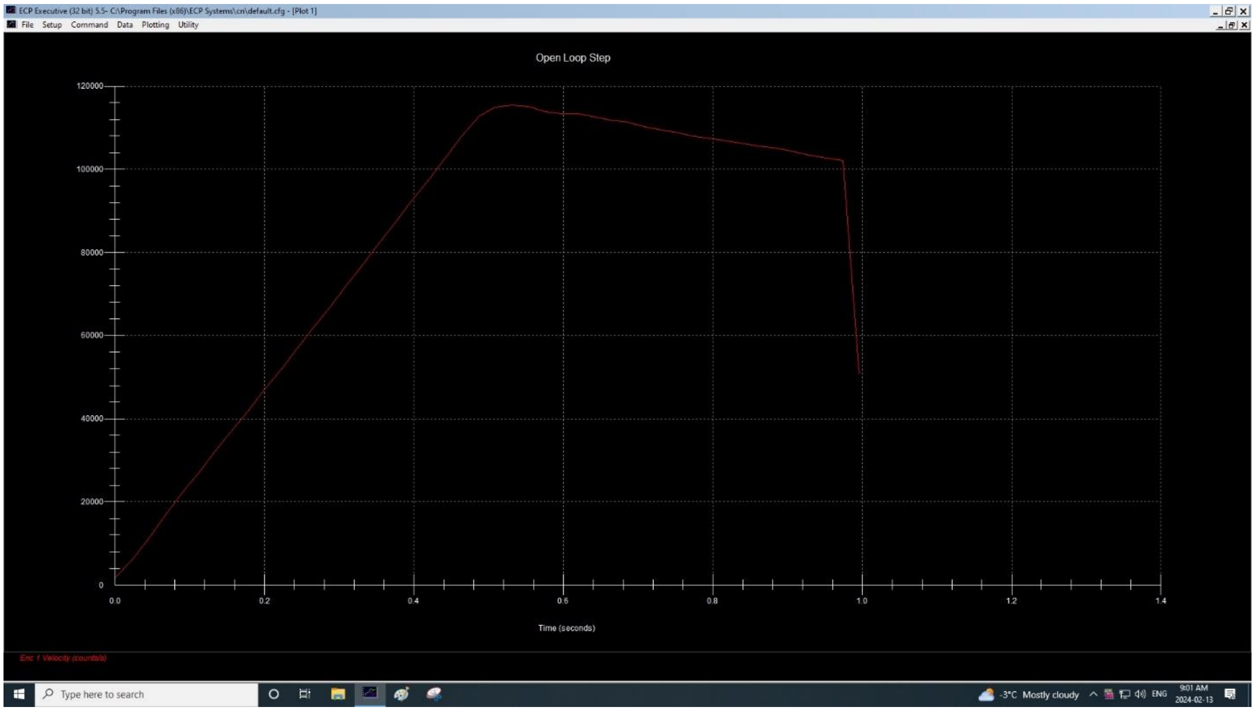


Figure 3.1.1

The left axis is Encoder #1 velocity and the x axis is time in seconds. Notice that the large linear increase of velocity on the y axis at the start represents a constant acceleration. Around time 0.5 seconds, we reach the maximum and the velocity decreases. This occurs because the step reverse and the "overload" circuit disable the system at this point.

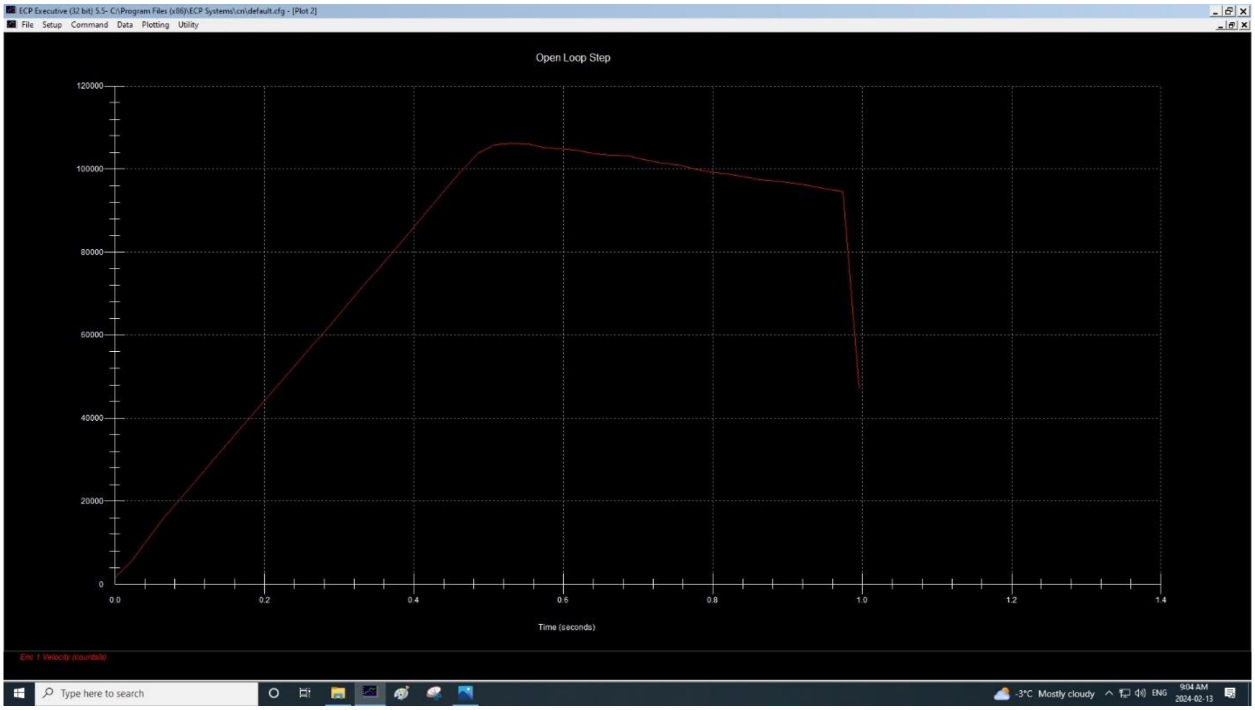


Figure 3.1.2

This is similar is obtain by performing the exact same procedure as the one before it, however we added an additional weight to the system. Notice how the shape of the graph of figure 3.2 is very similar to figure 3.1. Both increases linearly and both stops around 0.5 seconds. However, a major difference is the peak. For the non-weighted system, the peak reached around 116000, while the weighted system could only reach 106000. This means that had a higher velocity or acceleration rate compared to the weighted system.

<u>Using Equation (4.4), determine the value of J and compare the value obtained with the calculated nominal value given in sections (4.2.1) and (4.2.2) and find the % error.</u>

$$J = \left(\frac{a_2}{a_1 - a_2}\right) \Delta J \; kgm^2$$

$$a_1 = \frac{y_2 - y_2}{x_2 - x_2} = \frac{98000 - 20000}{0.4 - 0.085} = 247619.05 \; counts/sec^2$$

$$a_2 = \frac{y_2 - y_2}{x_2 - x_2} = \frac{80000 - 40000}{0.37 - 0.18} = 210526.32 \; counts/sec^2$$

$$\Delta J = 0.000494$$

The experimental value of J is calculated as such:

$$J = \left(\frac{a_2}{a_1 - a_2}\right)\Delta J = \left(\frac{210526.32}{247619.05 - 210526.32}\right) * 0.000494 = 2.8 * 10^{-3}$$

The % error is calculated as such:

$$\%\ error = \left|\frac{Actual\ Value - Experimental\ Value}{Experimental\ Value}\right| = \left|\frac{4.23 * 10^{-3} - 2.8 * 10^{-3}}{2.8 * 10^{-3}}\right| = 0.51$$

The percent error is around 51%. It is unavoidable that we obtain a high percent error since the values of y and x used to calculated a_1 and a_2 are rough estimates.

<u>Using the determined value of J, find $K_aK_tK_e$ using Equation (4.3). Multiply this value by K_sK_c to obtain K (see section 4.2.1) for the equipment at your lab station.</u>

$$Ja_1 = 2K_aK_tK_e$$

$$K_aK_tK_e = \frac{Ja_1}{2} = \frac{2.8 * 10^{-3} * 247619.05}{2} = 346.67$$

$$K = K_sK_cK_eK_aK_t = 346.67 * 0.009766 = 3.39$$

The value of K obtained by using the determined value of J is 3.39.

3.2) EVALUATION OF J AND K USING OPEN-LOOP POSITION OUTPUT

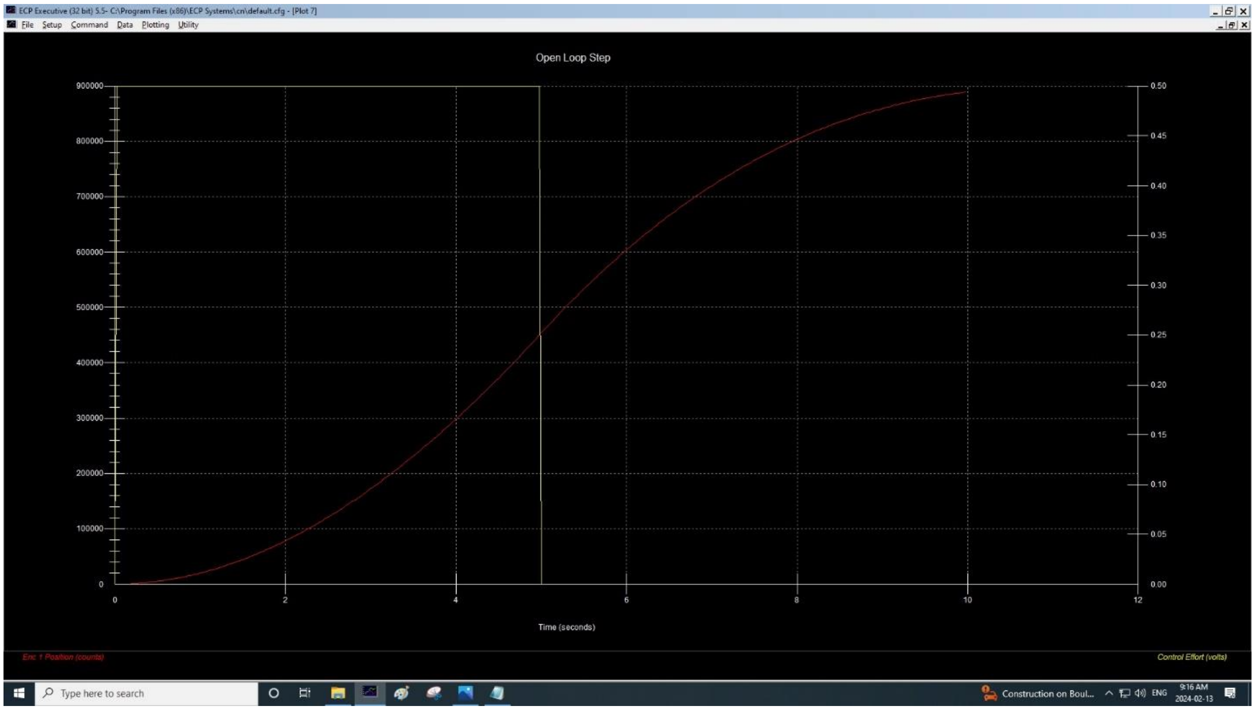


Figure 3.2.1

The left axis is encoder #1 position while CE is the right axis with both in relation to time on the x axis. This can be visually seen with the red line (position) and the yellow line (CE).

<u>Determine the slope S of the line (in Counts/sec) and the time-axis intercept τ (sec).</u>

$$S = \frac{y_2 - y_2}{x_2 - x_2} = \frac{700000 - 300000}{6.8 - 4} = 142857.14\ count/sec$$

Convert from counts to radians:

$$S(rad/sec) = \frac{S(count/sec)}{K_e} = \frac{142857.14}{2546.5} = 56.099\ radian/sec$$

$$B = \frac{0.1}{S(radians/sec)} = 1.7825 * 10^{-3}$$

<u>From the displayed time-axis intercept, determine the time-constant τ = J/B from x-axis intercept.</u>

Based on figure 3.2.1, the intercept is at 2 seconds.

Determine J:

$$J = \tau * B = 2 * 1.7825 * 10^{-3} = 3.565 * 10^{-3}$$

As such, the experimental value of J is $3.565*10^{-3}$ and B is $1.7825*10^{-3}$.

Compare with the determined value of 4.3.1 and obtain the average value:

The value of J in 4.3.1 is $2.8*10^{-3}$.

The value earlier is J is 3.565*10-3.

As such, the average value of J is $3.1825*10^{-3}$.

Percentage Error:

$$\% \, error = \left|\frac{Actual\ Value - Experimental\ Value}{Experimental\ Value}\right| = \left|\frac{4.23 * 10^{-3} - 3.1825 * 10^{-3}}{3.1825 * 10^{-3}}\right| = 0.33$$

The percentage error is around 33%.

Record the average value of J, and the values of B and K obtained, these values are to be used in subsequent lab work.

Average of J: $3.1825*10^{-3}$ kgm^2

B: $1.7825*10^{-3}$ Nm/radian

K: 3.39 Nm/radian

3.3) SYSTEM IDENTIFICATION VIA THE OPEN-LOOP RESPONSE TO A SINE SWEEP INPUT

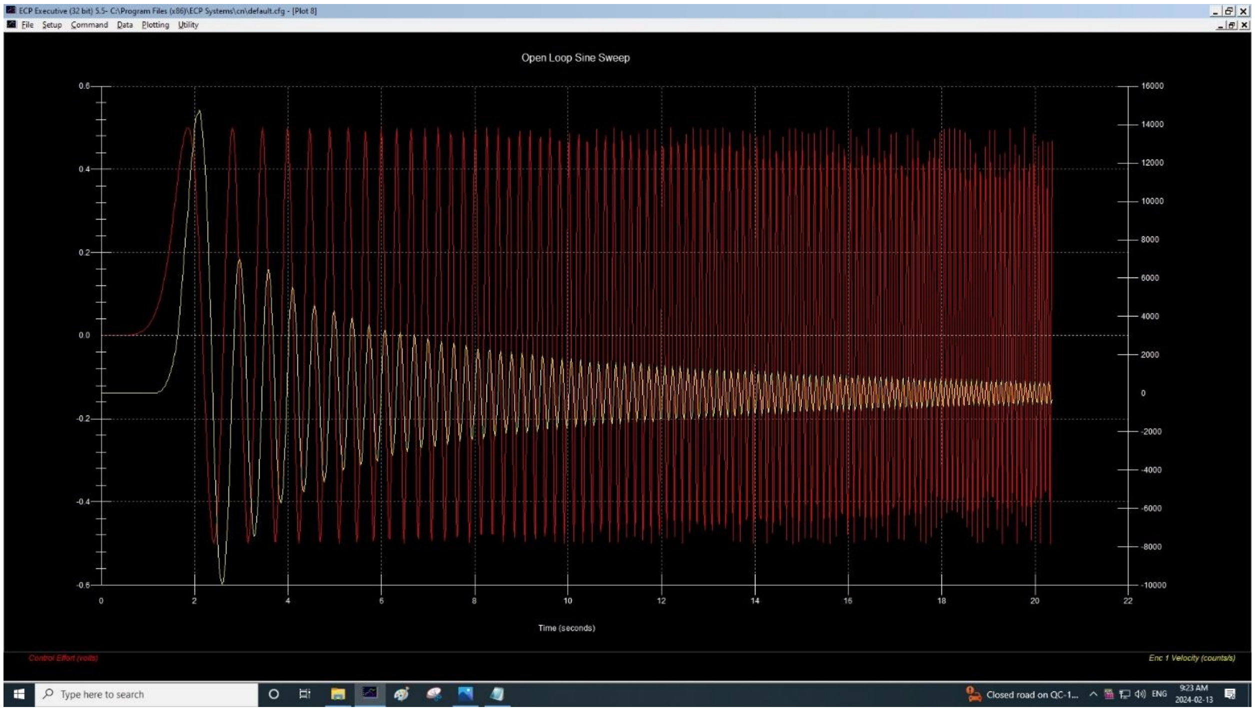


Figure 3.3.1

The above figure has CE displayed on the left axis and encoder #1 velocity on the right axis with both dependent on times in seconds. This is an open-loop system with sweep input of 0.1 to 10Hz and 0.5 volt with sweep time of 20 seconds.

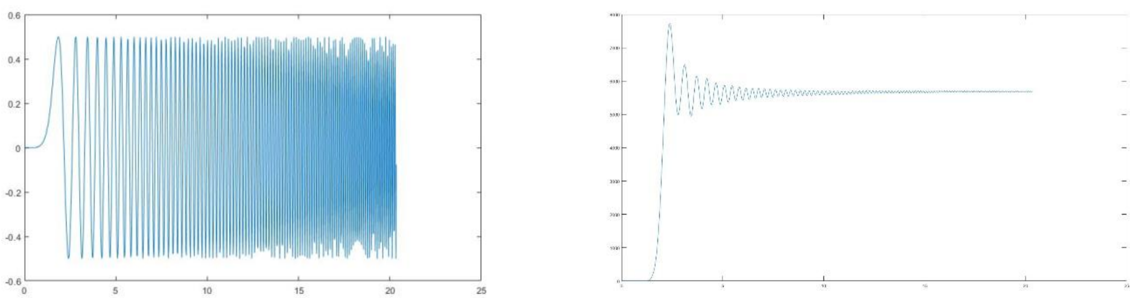


Figure 3.3.2

The above figure represents the MATLAB model obtained when running the MATLAB code found in the appendix. We can observe that figure 3.3.1 and 3.3.2 are the exact same. The left MATLAB figure is the red line CE, while the right figure is the yellow velocity.

3.4) SYSTEM IDENTIFICATION VIA CLOSED-LOOP SYSTEM RESPONSE

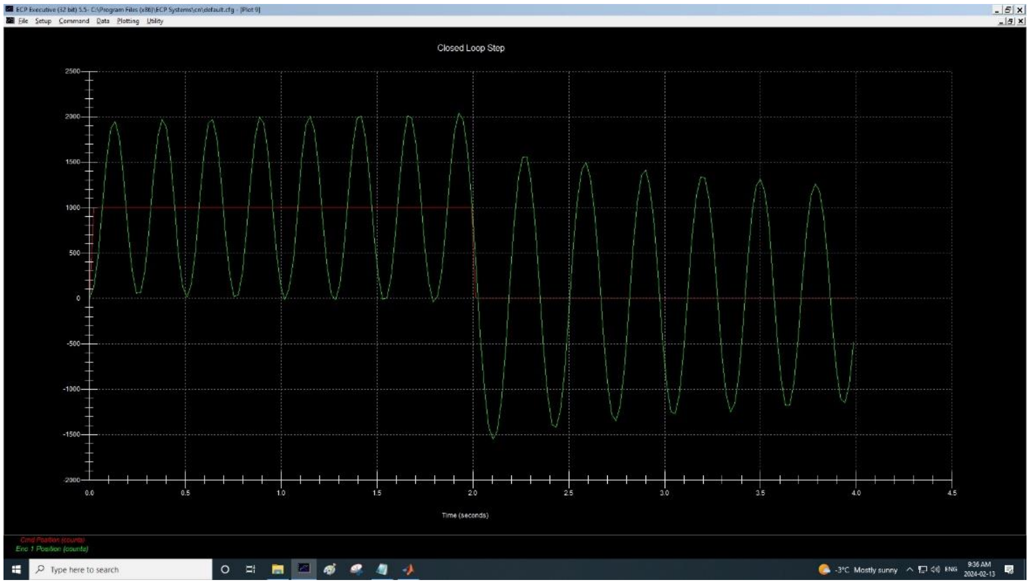


Figure 3.4.1

The commanded position and encoder #1 position are both on the left axis. The algorithm input is $K_p = 1$, $K_d = 0$ and $K_i = 0$.

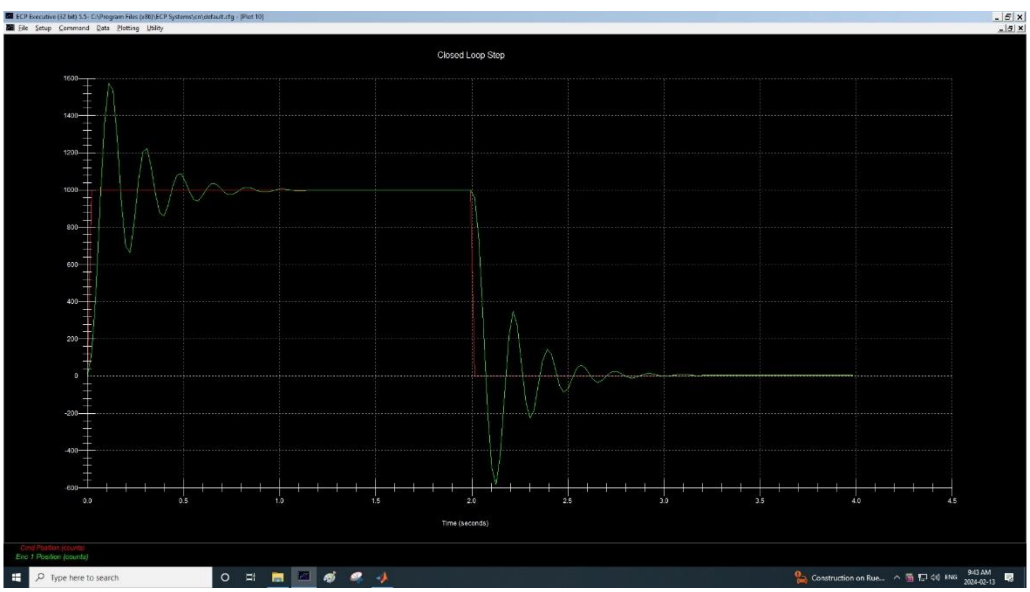


Figure 3.4.2

The system was running exactly the same except that we changed the algorithm input to $K_p = 1$, $K_d = 0.01$ and $K_i = 0$.

Their MATLAB simulation counterparts are as follows:

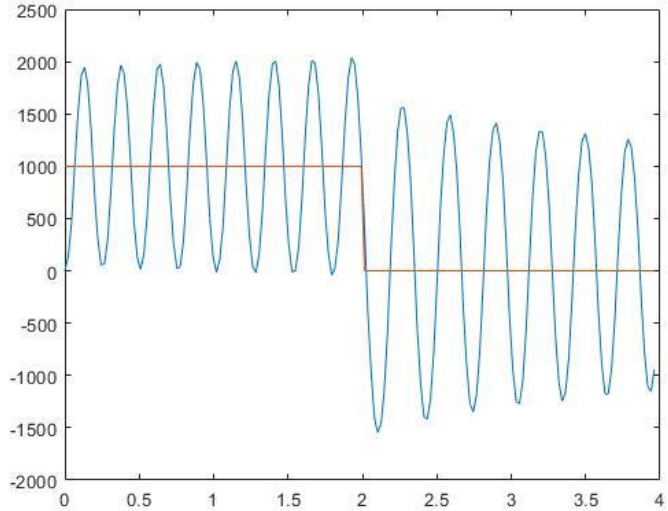


Figure 3.4.3

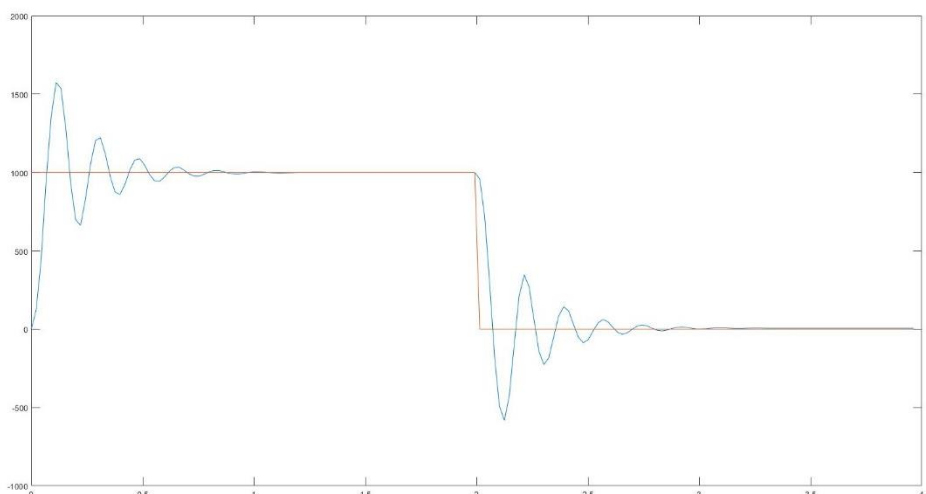


Figure 4.3.4

It can be observed that their MATLAB counterparts are very similar, thus proving that our experimental and theoretical systems are behaving accordingly. Given these sounds results, we can calculate and find the first-order transfer functions.

Calculate system the open-loop transfer function (OLTF) with the determined values of K, J and B from open-loop tests in section 4.3.2 and 4.3.3.

$$OLFT = \frac{K}{s(B+Js)} = \frac{3.39}{s(1.7825 * 10^{-3} + 3.1825 * 10^{-3} * s)}$$

Obtain the OLTF in MATLAB by the data of the closed-loop test., then calculate K, B and J.

$$tfd2 = \frac{6623}{s^2 + 28.07s + 6611} = \frac{K}{s(B+Js)}$$

Given this equation, we can find K = 6623, B = 28.07 and J = 1.

Compare these models

For this experiment, we have obtained models from the lab equipment as well as the simulation done through MATLAB. It can be observed from those figures that both types of models are similar. While we might encounter slight differences when observing specific data points, the general tendencies are similar.

Write a brief summary about how the lab equipment might facilitate a better understanding of control systems

Theory learned during lectures can only go so far. The best method to encapsulates theory is through practice. As such, by having students perform the experiment by themselves, they are able to grasp the topic and understand it better. In other words, lab equipment allows for experimentations which in turn facilitate a better understanding. In the end, this approach bridges the theory with practical application as it provides hands-on experience.

4) Questions

None

5) Conclusions

In conclusion, the students performed experiments on system identification and control system analysis of an open-loop and closed-loop response systems. We observed the effect of inertia, damping coefficient and gain. Then, by performing both the experiment with the lab equipment and a simulation through MATLAB, students are able to compare plots and analyse the effect due to changes in parameters.

6) Appendix

MATLAB Code used for this lab:

```
time = data(:,2);
y = data(:,4);
u = data(:,6);
dy = diff(y);
dy(end + 1) = dy(end);
zf = iddata(dy,u,0.00884);
tfl = tfest(zf,1,0);
plot(time, y);
plot(time, u);
```

```
time = data(:,2);
y = data(:,4);
u = data(:,6);
dy = diff(y);
dy(end + 1) = dy(end);
zf = iddata(dy,u,0.00884);
tfl = tfest(zf,1,0);
plot(time, y);
plot(time, u);
```

```
time = data(:,2);
y = data(:,4);
u = data(:,3);
zf = iddata(y,u,0.00884);
Tfd2 = tfest(zf,2,0);

plot(time, y);
hold on;
plot(time, u);
```

Sample example of gather raw data used in MATLAB

```
Sample    Time     Commanded Pos    Encoder 1 Pos    Encoder 2 Pos    Control Effort

[     0    0.000            1000                0                0           0.0000;
      1    0.022            1000              125               12           4.8840;
      2    0.044            1000              445               95           4.0611;
      3    0.066            1000              942              229          -1.3022;
      4    0.089            1000             1354              355          -4.7595;
      5    0.111            1000             1574              409          -4.8840;
      6    0.133            1000             1537              405          -4.8565;
      7    0.155            1000             1278              334          -1.5201;
      8    0.177            1000              929              234           1.9359;
```

```
    9   0.199          1000          698          160         3.6044;
   10   0.221          1000          663          152         3.1746;
   11   0.243          1000          829          198         0.9225;
   12   0.266          1000         1053          262        -1.3352;
   13   0.288          1000         1204          307        -2.4066;
   14   0.310          1000         1222          315        -2.0977;
   15   0.332          1000         1121          286        -0.7070;
   16   0.354          1000          976          246         0.7552;
   17   0.376          1000          875          217         1.5079;
   18   0.398          1000          860          211         1.3272;
   19   0.421          1000          926          230         0.4164;
   20   0.443          1000         1018          255        -0.5055;
   21   0.465          1000         1080          272        -0.9542;
   22   0.487          1000         1088          275        -0.8474;
   23   0.509          1000         1046          263        -0.2521;
   24   0.531          1000          987          248         0.3358;
   25   0.553          1000          948          237         0.6215;
   26   0.576          1000          943          235         0.5714;
   27   0.598          1000          971          242         0.1551;
   28   0.620          1000         1008          252        -0.2125;
   29   0.642          1000         1032          259        -0.3822;
   30   0.664          1000         1034          260        -0.3462;
   31   0.686          1000         1015          255        -0.0714;
   32   0.708          1000          992          249         0.1593;
   33   0.730          1000          977          245         0.2637;
   34   0.753          1000          976          244         0.2393;
   35   0.775          1000          988          247         0.0562;
   36   0.797          1000         1003          251        -0.0836;
   37   0.819          1000         1012          253        -0.1477;
   38   0.841          1000         1013          254        -0.1313;
   39   0.863          1000         1006          253        -0.0220;
   40   0.885          1000          996          250         0.0659;
   41   0.908          1000          991          248         0.1136;
   42   0.930          1000          990          248         0.1026;
   43   0.952          1000          994          248         0.0379;
   44   0.974          1000         1000          250        -0.0214;
   45   0.996          1000         1004          251        -0.0476;
   46   1.018          1000         1004          251        -0.0433;
   47   1.040          1000         1003          251         0.0024;
   48   1.063          1000         1000          251         0.0147;
   49   1.085          1000          998          251         0.0311;
   50   1.107          1000          997          251         0.0385;
   51   1.129          1000          997          251         0.0342;
   52   1.151          1000          998          251         0.0195;
   53   1.173          1000          999          251         0.0012;
   54   1.195          1000         1000          251        -0.0079;
   55   1.217          1000         1000          251        -0.0043;
   56   1.240          1000         1000          251        -0.0043;
   57   1.262          1000         1000          251        -0.0043;
   58   1.284          1000         1000          251        -0.0043;
   59   1.306          1000         1000          251        -0.0043;
   60   1.328          1000         1000          251        -0.0043;
   61   1.350          1000         1000          251        -0.0043;
   62   1.372          1000         1000          251        -0.0043;
   63   1.395          1000         1000          251        -0.0043;
   64   1.417          1000         1000          251        -0.0043;
   65   1.439          1000         1000          251        -0.0043;
   66   1.461          1000         1000          251        -0.0043;
```

```
  67    1.483          1000          1000           251        -0.0043;
  68    1.505          1000          1000           251        -0.0043;
  69    1.527          1000          1000           251        -0.0043;
  70    1.549          1000          1000           251        -0.0043;
  71    1.572          1000          1000           251        -0.0043;
  72    1.594          1000          1000           251        -0.0043;
  73    1.616          1000          1000           251        -0.0043;
  74    1.638          1000          1000           251        -0.0043;
  75    1.660          1000          1000           251        -0.0043;
  76    1.682          1000          1000           251        -0.0043;
  77    1.704          1000          1000           251        -0.0043;
  78    1.727          1000          1000           251        -0.0043;
  79    1.749          1000          1000           251        -0.0043;
  80    1.771          1000          1000           251        -0.0043;
  81    1.793          1000          1000           251        -0.0043;
  82    1.815          1000          1000           251        -0.0043;
  83    1.837          1000          1000           251        -0.0043;
  84    1.859          1000          1000           251        -0.0043;
  85    1.882          1000          1000           251        -0.0043;
  86    1.904          1000          1000           251        -0.0043;
  87    1.926          1000          1000           251        -0.0043;
  88    1.948          1000          1000           251        -0.0043;
  89    1.970          1000          1000           251        -0.0043;
  90    1.992          1000          1000           251        -0.0043;
  91    2.014             0           958           250        -4.8840;
  92    2.036             0           724           199        -4.8840;
  93    2.059             0           286            86        -0.9280;
  94    2.081             0          -182           -54         3.3529;
  95    2.103             0          -494          -143         4.8840;
  96    2.125             0          -583          -163         4.8840;
  97    2.147             0          -423          -124         3.3303;
  98    2.169             0           -98           -30        -0.3254;
  99    2.191             0           211            62        -3.0531;
 100    2.214             0           347           102        -3.6783;
 101    2.236             0           269            81        -2.0940;
 102    2.258             0            58            20         0.2411;
 103    2.280             0          -141           -36         2.0238;
 104    2.302             0          -226           -63         2.3748;
 105    2.324             0          -184           -51         1.4927;
 106    2.346             0           -52           -15        -0.0183;
 107    2.368             0            81            22        -1.2289;
 108    2.391             0           143            40        -1.5263;
 109    2.413             0           117            34        -0.9420;
 110    2.435             0            32            10         0.0293;
 111    2.457             0           -51           -12         0.7668;
 112    2.479             0           -87           -22         0.9274;
 113    2.501             0           -68           -18         0.5281;
 114    2.523             0           -12            -3        -0.1007;
 115    2.546             0            40            12        -0.5574;
 116    2.568             0            62            18        -0.6380;
 117    2.590             0            47            15        -0.3700;
 118    2.612             0            11             5         0.0421;
 119    2.634             0           -21            -4         0.3053;
 120    2.656             0           -33            -7         0.3407;
 121    2.678             0           -22            -6         0.1538;
 122    2.701             0             1             1        -0.1062;
 123    2.723             0            21             6        -0.2741;
 124    2.745             0            27             8        -0.2680;
```

125	2.767	0	20	8	-0.1526;
126	2.789	0	5	3	0.0110;
127	2.811	0	-7	0	0.1068;
128	2.833	0	-11	-1	0.1026;
129	2.855	0	-6	-1	0.0354;
130	2.878	0	4	1	-0.0769;
131	2.900	0	11	4	-0.1386;
132	2.922	0	13	5	-0.1313;
133	2.944	0	11	5	-0.0958;
134	2.966	0	5	4	-0.0281;
135	2.988	0	1	2	-0.0049;
136	3.010	0	1	2	-0.0049;
137	3.033	0	2	2	-0.0531;
138	3.055	0	6	2	-0.0672;
139	3.077	0	7	2	-0.0726;
140	3.099	0	7	2	-0.0726;
141	3.121	0	7	2	-0.0726;
142	3.143	0	5	2	-0.0379;
143	3.165	0	4	2	-0.0324;
144	3.188	0	4	2	-0.0342;
145	3.210	0	5	2	-0.0824;
146	3.232	0	6	2	-0.0653;
147	3.254	0	6	2	-0.0672;
148	3.276	0	6	2	-0.0629;
149	3.298	0	5	2	-0.0488;
150	3.320	0	5	2	-0.0488;
151	3.342	0	5	2	-0.0488;
152	3.365	0	5	2	-0.0488;
153	3.387	0	5	2	-0.0488;
154	3.409	0	5	2	-0.0488;
155	3.431	0	5	2	-0.0488;
156	3.453	0	5	2	-0.0488;
157	3.475	0	5	2	-0.0488;
158	3.497	0	5	2	-0.0488;
159	3.520	0	5	2	-0.0488;
160	3.542	0	5	2	-0.0488;
161	3.564	0	5	2	-0.0488;
162	3.586	0	5	2	-0.0488;
163	3.608	0	5	2	-0.0488;
164	3.630	0	5	2	-0.0488;
165	3.652	0	5	2	-0.0488;
166	3.674	0	5	2	-0.0488;
167	3.697	0	5	2	-0.0488;
168	3.719	0	5	2	-0.0488;
169	3.741	0	5	2	-0.0488;
170	3.763	0	5	2	-0.0488;
171	3.785	0	5	2	-0.0488;
172	3.807	0	5	2	-0.0488;
173	3.829	0	5	2	-0.0488;
174	3.852	0	5	2	-0.0488;
175	3.874	0	5	2	-0.0488;
176	3.896	0	5	2	-0.0488;
177	3.918	0	5	2	-0.0488;
178	3.940	0	5	2	-0.0488;
179	3.962	0	5	2	-0.0488]