1 Portfolio Optimization

Suppose we have a market consisting of two stocks whose *price vector* is represented as $P = (A \quad B)$. Our portfolio vector can be represented as a vector $\Lambda = \begin{pmatrix} a \\ b \end{pmatrix}$ with $a + b = 1$. Then the return on our portfolio is simply $\Pi = P \cdot \Lambda$. Unfortunately their is some amount of *risk*, r, () associated with the portfolio. The risk has two components: *diversifiable risk* and *undiversifiable risk*. The diversifiable risk can be minimized by optimizing Λ. Undiversifiable risk is the risk due to the market. For example the 2008 housing market crisis is an example of undiversifiable risk.

For investors, problem of finding the correct portfolio vector requires the computation of the risk associated with A and B. We now treat A and B, as random variables. One issue of computing the risk comes from probability theory

$$\mathbb{E}(A + B) = \mathbb{E}(A) + \mathbb{E}(B)$$

where $\mathbb{E}$ denotes the expected value. However for variances

$$\text{var}(A + B) = \text{var}(A) + \text{var}(B) + 2 * \text{cov}(A, B)$$

Therefore if we want to compute the variance of our portfolio, P, we will need to compute the covariance matrix. In the language of finance, the covariance of two stocks is called the *beta coefficient*.

We have two main equations. The factor model equation

$$R = \mu + LF + \varepsilon \tag{1}$$

and the forecasting equation

$$K : F_n \to R_{n+1} \tag{2}$$

Let us set $t = 0$ corresponding to Day 756 and count from there.

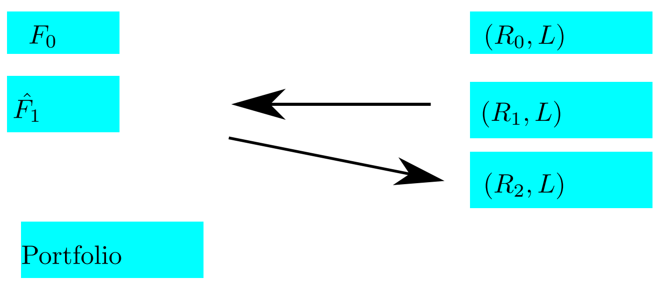
1. Train coefficients L_{ij} from the F and R coming from training data. F are the features we are given. R is the asset price. (Method - *Linear Regression*)

2. Use the given price, R_1, on Day 757 and the trained coefficients L_{ij} to compute the $\tilde{F_1}(R_1, L)$ vector on Day 757. (Method - Invert (1). μ, the "mean of all stocks", is easily computed. $\mathbb{E}(\varepsilon) = 0$ and then check condition number for inverting L)

3. Having computed $\tilde{F_1}(R_1, L)$, we use equation (2) to forecast the price vector $\tilde{R_2}$ on Day 758. (The Barra Model assumes that $\tilde{F_1} \implies \tilde{R_2}$).

4. Dynamically allocate portfolio and risk and shit

5. Dynamically update the matrix L at some point given F and R values.



1: $R_n = \mu + LF_n + \varepsilon$

2: $K : F_n \to R_{n+1}$

Figure 1: Flow of logic in the ol' factor model