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Introduction to Numerical Methods

Exercise no. 9

Hand in before the beginning of the exercise class on 15.12.2022

Exercise 9.1 (4 Points) We consider a matrix A and a vector b given by

$$A = \begin{pmatrix} 3 & 1 & 6 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$$

a) Compute the LU decomposition of A with partial pivoting.

b) Use the decomposition to solve $Ax = b$.

Exercise 9.2 (4 Points) We consider a matrix A and a vector b given by

$$A = \begin{pmatrix} 2 & \alpha & -1 \\ \alpha & 2 & 1 \\ -1 & 1 & 4 \end{pmatrix} \quad b = \begin{pmatrix} 3 \\ 9 \\ 10 \end{pmatrix}$$

a) Find all α so that the matrix A is positive definite.

b) Take the highest integer value possible for α and compute the Cholesky decomposition of A.

c) Use the decomposition to solve $Ax = b$.

(Hint: You can use Sylvester's Criterion which says: A symmetric matrix $A \in \mathbb{R}^{n \times n}$ is positive definite if its principal minors satisfy $\det A[k] > 0, \forall\, k = 1, \ldots, n$.)