Maurice Hensel
André Souza
Winter term 22/23

Introduction to Numerical Methods

Exercise no. 11

Hand in before the beginning of the exercise class on 12.01.2023

Exercise 11.1 (2 points) Let the matrix A be given by

$$A = \begin{pmatrix} -1 & 2 \\ 0 & 2 \\ 1 & 2 \end{pmatrix}.$$

Compute the maximum absolute column sum norm $||A||_1$, the maximum absolute row sum norm $||A||_\infty$ and the spectral norm $||A||_2$.

Exercise 11.2 (2 points) Let the matrix A be given by

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}.$$

Compute the condition numbers $\kappa_1(A), \kappa_2(A)$ and $\kappa_\infty(A)$.

Exercise 11.3 (4 points) We consider the matrix A and the vector b, given by

$$A = \begin{pmatrix} 0.5 & 0.25 \\ 0.25 & 0.5 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

a) Give the iteration rule for the Jacobi method.

b) Consider the initial value

$$x^{(0)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

and calculate two iteration steps of the Jacobi method.

Exercise 11.4 (Bonus) Consider the matrices A_1, A_2 and the vectors b_1, b_2 given by

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 4 & 5 & 1 \\ 2 & 8 & -1 \end{pmatrix}, A_2 = \begin{pmatrix} 4 & 6 & 2 \\ 6 & 25 & 23 \\ 2 & 23 & 62 \end{pmatrix}, b_1 = \begin{pmatrix} 2 \\ 10 \\ 9 \end{pmatrix}, b_2 = \begin{pmatrix} 12 \\ 54 \\ 87 \end{pmatrix}.$$

a) (2 points) Implement the LU decomposition algorithm.

b) (2 points) Change the algorithm of item a) in such a way that it returns the solution to $A_1 x = b_1$.

c) (2 points) Implement the Cholesky decomposition algorithm.

d) (2 points) Change the algorithm of item c) in such a way that it returns the solution to $A_2 x = b_2$. (*Obs: you must submit a written version of your algorithm(s) and submit via email the created files.*)