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Introduction to Numerical Methods

Exercise no. 8

Hand in before the beginning of the exercise class on 08.12.2022

Algorithm Cholesky decomposition (for a symmetric and positiv definite matrix $A = (a_{ij})_{1\le i,j\le n}$)

1: **for** $j = 1 : n$ **do**
2: $\quad l_{jj} := \sqrt{a_{jj} - \sum_{m=1}^{j-1} l_{jm}^2} \quad (\sum_{m=1}^{0} l_{jm}^2 = 0$ for $j = 1)$
3: $\quad$ **for** $i = (j+1) : n$ **do**
4: $\quad\quad l_{ij} := (a_{ij} - \sum_{m=1}^{j-1} l_{im} l_{jm})/l_{jj}$
5: $\quad$ **end for**
6: **end for**
7: Set $L = (l_{ij})_{1\le i,j\le n}$

Exercise 8.1 (3 points) We consider a matrix A and a vector b given by

$$A = \begin{pmatrix} 4 & 2 & -10 & 2 \\ 2 & 10 & -14 & -2 \\ -10 & -14 & 50 & -6 \\ 2 & -2 & -6 & 7 \end{pmatrix}, \quad b = \begin{pmatrix} 4 \\ -10 \\ -2 \\ 11 \end{pmatrix}.$$

a) Compute the Cholesky decomposition of A.

b) Use the Cholesky decomposition of A to solve $Ax = b$.

Exercise 8.2 (2 points) Let A have a Cholesky decomposition, i.e., $A = LL^T$ for a regular lower triangle matrix L. Show that A is symmetric and positive definite.

Exercise 8.3 (3 points) A Hilbert matrix of order $n \in \mathbb{N}$ is a matrix $\mathrm{H}_n \in \mathbb{R}^{n\times n}$, $H = (h_{i,j})_{1\le i,j\le n}$ such that

$$h_{i,j} = \frac{1}{i+j-1}.$$

Verify if H_3 admits a Cholesky decomposition, $\mathrm{H}_3 = LL^T$. In case it admits, find the matrix L, and if it doesn't explain what requirement isn't satisfied.