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Winter term 22/23

Introduction to Numerical Methods

Exercise no. 10

Hand in before the beginning of the exercise class on 22.12.2022

Exercise 10.1 (2 Points) How many operations are necessary to perform the LU decomposition, the LU decomposition with partial pivoting and the Cholesky decomposition, respectively.*(Hint: We regard as operations any comparisons, additions, subtractions, multiplications, divisions and root determination .)*

Exercise 10.2 (4 Points) Let the following matrices be given

$$A_1 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 2 & 4 & 17 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 2 & 0 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 1 & 0 & -1 \end{pmatrix}.$$

Which of the above mentioned decomposition methods is the most suitable for these matrices, respectively. Explain your decisions.

Exercise 10.3 (2 Points) Let $p \geq 1$ and $n \in \mathbb{N}$. Then, the p-*Norm* is defined by

$$\|x\|_p = \sqrt[p]{|x_1|^p + |x_2|^p + \ldots + |x_n|^p} \quad \forall x \in \mathbb{R}^n$$

and the *maximum norm* is defined by

$$\|x\|_\infty = \max_{k=1,\ldots n} \{|x_k|\} \quad \forall x \in \mathbb{R}^n.$$

Show that the p-*Norm* converges pointwise towards the maximum norm for $p \to \infty$, i.e.

$$\lim_{p \to \infty} \|x\|_p = \|x\|_\infty \quad \forall x \in \mathbb{R}^n.$$

(Hint: $\sqrt[p]{y} = \exp(1/p \log(y))$ for every $y > 0$.)