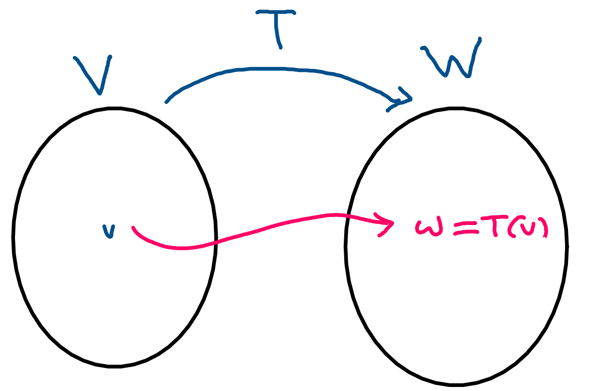
# 1.3 Representación matricial de una Transformación lineal




$$\begin{array}{ccc} V & \xrightarrow{\quad T \quad} & W \\ v & & \omega = T(v) \\ \vdots & & \vdots \\ x = v_{\beta_1} & \xrightarrow{\quad A \quad} & \omega_{\beta_2} = Y = TX \end{array}$$

$$Y = AX$$

$$TX = AX$$

$\dim V = n$ $\qquad\qquad$ $\dim W = m$

$\beta_1 = \{v_1, v_2, \ldots, v_n\}$ $\qquad\qquad$ $\beta_2 = \{\omega_1, \omega_2, \cdots, \omega_m\}$

$$A = ?$$

para $v_1$ $\qquad$ $T(v_1) = \alpha_{1i}w_1 + \alpha_{2i}w_2 + \cdots + \alpha_{mi}w_m$

$$(T(v_i))_{\beta_2} = \begin{bmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \vdots \\ \alpha_{mi} \end{bmatrix}$$

sea $v \in V$

$$T(v) = a_1 w_1 + a_2 w_2 + \cdots + a_m w_m$$

$$v = c_1 v_1 + c_2 v_2 + \cdots + c_n v_n$$

$$T(v) = c_1 T(v_1) + c_2 T(v_2) + \cdots + c_n T(v_n)$$

$$(T(v))_{\beta_2} = c_1 \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{bmatrix} + c_2 \begin{bmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{bmatrix} + \cdots + c_n \begin{bmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \vdots \\ \alpha_{mn} \end{bmatrix}$$

$$\begin{array}{cccc} (T(v_1))_{\beta_2} & (T(v_2))_{\beta_2} & & (T(v_n))_{\beta_2} \\ \downarrow & \downarrow & & \downarrow \end{array}$$

$$(T(v))_{\beta_2} = \underbrace{\begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & & \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}}_{v_{\beta_1}}$$

$$(T(v))_{\beta_2} = A v_{\beta_1}$$

**Ejemplo:** Obtenga la representación matricial de la transformación lineal indicada, donde $B$ es la base en el espacio de origen y $B'$ la base en el espacio imagen

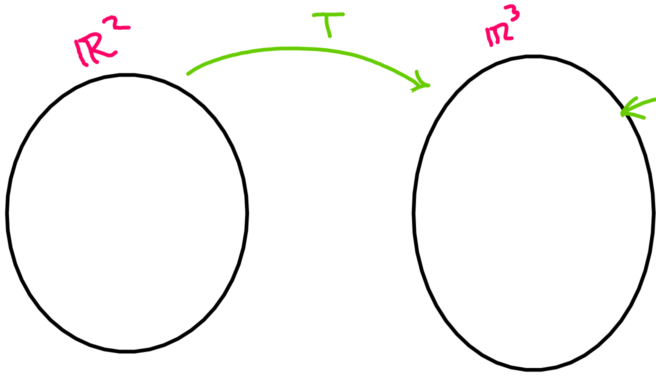
$T: \mathbb{R}^2 \to \mathbb{R}^3; T(x,y) = (2x, x, x+y)$ donde $B = \{(1,4),(3,2)\}$ $B' = \{(1,0,0),(2,-3,5),(3,1,-2)\}$

$$[T] = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad [T]_B^{B'} = ?$$

$\beta = \{e_1; e_2\}$

$e_1 = (1,0)$

$e_2 = (0,1)$




$\beta_1 = \{e_1; e_2; e_3\}$

$e_1 = (1,0,0)$

$e_2 = (0,1,0)$

$e_3 = (0,0,1)$

para $e_1$ $\quad T(e_1) = (2,1,1) = 2\underbrace{(1,0,0)}_{e_1} + 1\underbrace{(0,1,0)}_{e_2} + 1\underbrace{(0,0,1)}_{e_3}$

$$(T(e_1))_{\beta_1} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

para $e_2$ $\quad T(e_2) = (0,0,1) = 0\underbrace{(1,0,0)}_{e_1} + 0\underbrace{(0,1,0)}_{e_2} + 1\underbrace{(0,0,1)}_{e_3}$

$$(T(e_2))_{\beta_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{array}{cc} \checkmark & \checkmark \end{array}$$

$$[T] = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$T: \mathbb{R}^2 \to \mathbb{R}^3; T(x,y) = (2x, x, x+y)$ donde $B = \{\overset{v_1}{(1,4)}, \overset{v_2}{(3,2)}\}$ $\quad B' = \{\underset{w_1}{(1,0,0)}, \underset{w_2}{(2,-3,5)}, \underset{w_3}{(3,1,-2)}\}$

Para $v_1 = (1,4)$ $\quad T(v_1) = T(1,4) = (2,1,5) = aw_1 + bw_2 + cw_3 = a(1,0,0) + b(2,-3,5) + c(3,1,-2)$

$$(2,1,5) = (a+2b+3c, -3b+c, 5b-2c)$$

$$(T(v_1))_{\beta'} = (T(1,4))_{\beta'} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{aligned} a+2b+3c &= 2 \\ -3b+c &= 1 \\ 5b-2c &= 5 \end{aligned} \qquad \begin{aligned} a &= 76 \\ b &= -7 \\ c &= -20 \end{aligned}$$

$$(T(1,4))_{\beta'} = \begin{bmatrix} 76 \\ -7 \\ -20 \end{bmatrix}$$

Para $v_2 = (3,2)$ $\quad T(v_2) = T(3,2) = (6,3,5) = rw_1 + sw_2 + tw_3 = r(1,0,0) + s(2,-3,5) + t(3,1,-2)$

$$(6,3,5) = (r+2s+3t, -3s+t, 5s-2t)$$

$$(T(v_2))_{\beta'} = (T(3,2))_{\beta'} = \begin{bmatrix} r \\ s \\ t \end{bmatrix}$$

$$\begin{aligned} r+2s+3t &= 6 \\ -3s+t &= 3 \\ 5s-2t &= 5 \end{aligned} \qquad \begin{aligned} r &= 118 \\ s &= -11 \\ t &= -30 \end{aligned}$$

$$(T(3,2))_{\beta'} = \begin{bmatrix} 118 \\ -11 \\ -30 \end{bmatrix}$$

Por tanto

$$A = [T]_{\beta}^{\beta'} = \begin{bmatrix} 76 & 118 \\ -7 & -11 \\ -20 & -30 \end{bmatrix}$$

$$T(x,y) = (2x, x, x+y)$$

$X=(x,y)$

$Y=(T(v))_{\beta'}$ $\quad X=v_\beta$

$Tx=Ax$

$(T(v))_{\beta'}=A\,v_\beta$

$Tx \qquad Y \quad =Ax$

$Tx=Ax$

$$T(x,y)=(2x,x,x+y)$$

Para $v=(1,1)$

$T(v)=T(1,1)=(2,\ 1,\ 2)$

Usando la representación matricial

$T(1,1)=?$

$(T(v))_{\beta'}=Av_\beta$

$v_\beta=?$ $\qquad v=av_1+bv_2 \longrightarrow (1,1)=a(1,4)+b(3,2)$

$v_\beta=\begin{bmatrix}a\\b\end{bmatrix}$ $\qquad (1,1)=(a+3b,4a+2b)$

$(T(v))_{\beta'}=?$

$a+3b=1 \qquad a=0.1$

$4a+2b=1 \qquad b=0.3$

$v_\beta=\begin{bmatrix}1/10\\3/10\end{bmatrix}$

$$(T(v))_{\beta'} = Av_{\beta} = \begin{bmatrix} 76 & 118 \\ -7 & -11 \\ -20 & -30 \end{bmatrix} \begin{bmatrix} 1/10 \\ 3/10 \end{bmatrix} = \begin{bmatrix} 43 \\ -4 \\ 11 \end{bmatrix}$$

$$T(v) = 43w_1 - 4w_2 - 11w_3$$

$$T(1,1) = 43(1,0,0) - 4(2,-3,5) - 11(3,1,-2)$$

$$T(1,1) = (2,1,2)$$