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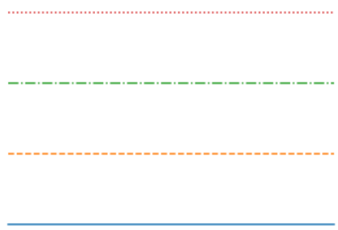
Summer term 2019
LV-Nr. 425093

Computational Finance and FinTech – Problem Set 3

Exercise 1. Write a program that plots lines of different line styles. The output should look something like this:



Exercise 2. Write a function that calculates the Black-Scholes price of a European call option on a stock with price process $(S_t)_{t\geq 0}$.
"European" in this context means that the option can be exercised only at maturity. A call option with strike K gives the owner the right, but not the obligation to purchase a share of stock for a price of K. Hence, the payoff of the call option at maturity T is

$$C_{K,T} = \max(S_T - K, 0),$$

where S_T is the stock price at maturity.
The Black-Scholes model assumes that a stock's log-returns are normally distributed. In this model the price of a call option at time $t = 0$ with current stock price S_0 is given by the *Black-Scholes formula*:

$$C(0, S_0) = S_0\,\mathrm{N}(d_+) - e^{-rT} K\,\mathrm{N}(d_-),$$

with

$$d_\pm = \frac{\ln(S_0/K) + (r \pm \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

where $\mathrm{N}(x)$ is the cumulative distribution function of the standard normal distribution.
The parameters are K: strike price, T: maturity, r: risk-free interest rate and σ: volatility.
(Hint: Use `from scipy.stats import norm` to access the normal distribution function.)

Exercise 3. Load the data from `tr_eikon_eod_data.csv` (see Chapter 5 on Financial Time Series). Create a data frame with log-returns and calculate the relevant summary statistics. Volatility corresponds to the annualised standard deviation of log-returns. To calculate volatilities, scale the standard deviation of daily log-returns by $\sqrt{252}$ and multiply by 100 to express it in percentages. Compare the volatilities of single stocks with the volatility of the S&P 500 index (`.SPX`). Can you find a reasonable explanation for the difference?

Exercise 4. Load the data from `tr_eikon_eod_data.csv` (see Chapter 5 on Financial Time Series). Create a data frame with log-returns and a data frame with discrete returns. Calculate the correlations of the log-returns with the discrete returns. Plot the respective returns in scatter plots. What can you conclude about daily returns?

Exercise 5. Load the data from the file `hprice.xls` into a data frame. The file contains data on $N = 546$ houses sold in Windsor, Canada. The dependent variable Y is the sales price of the house in Canadian dollars. The explanatory variables included in this data set are:

- the lot size of the property (in square feet)
- the number of bedrooms
- the number of bathrooms
- the number of storeys (excluding the basement)
- A dummy variable = 1 if house has a driveway (= 0 otherwise)
- A dummy variable = 1 if house has a recreation room
- A dummy variable = 1 if house has a basement
- A dummy variable = 1 if house has gas central heating
- A dummy variable = 1 if house has air conditioning
- The size of garage (number of cars it will hold)
- A dummy variable = 1 if house is in a desirable neighbourhood

Conduct a simple regression of `saleprice` on `#bedroom` and conduct a multiple regression of `saleprice` on `lot size`, `# bedroom`, `# bath` and `# stories`.

(a) What are the coefficients associated with the number of bedrooms in each regression? Can you explain why they are different?

(b) What is the R^2 in the multiple regression? What is the interpretation?

(c) Are all coefficients statistically significant? Explain!

Exercise 6 (optional)**.** Write an abstract base class `FinancialInstrument` with a `getPrice()` method. Then write concrete classes `Bond`, `Stock` and `EuropeanCallOption`.

- The bond should take a coupon, a maturity and an interest rate as arguments when instantiated.
- The stock should take a dividend, growth rate and discount rate. Implement `getPrice()` via the Gordon Growth model, which assumes that the current stock price is equal to the present value of all future cash flows.
- The call option price at time $t = 0$ and current stock price S_0 is given by the *Black-Scholes formula.*

Determine the prices of the following financial instruments:

- Bond with coupon 5, maturity of five years and a discount rate of 10%.
- Share or stock with dividend of 5, growth rate of 5.2% and a discount rate of 10%.
- European call option with initial stock price 100, strike price 100, maturity of 0.5 years, volatility of 20% and risk-free rate of 5%.