Computational Finance and FinTech Numerical and Computational Foundations

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2 Numerical and Computational Foundations

• Further reading: Py4Fi, Chapters 4 and 5

2.1 Arrays with Python lists

Introduction to Python arrays

- Before introducing more sophisticated objects for data storage, let's take a look at the built-in Python list object.
- A list object is a one-dimensional array:

[1]:
$$\mathbf{v} = [0.5, 0.75, 1.0, 1.5, 2.0]$$

- list objects can contain arbitrary objects.
- In particular, a list can contain other list objects, creating two- or higher-dimensional arrays:

```
[2]: m = [v, v, v]
m
```

```
[2]: [[0.5, 0.75, 1.0, 1.5, 2.0], [0.5, 0.75, 1.0, 1.5, 2.0], [0.5, 0.75, 1.0, 1.5, 2.0]]
```

list objects

```
[3]: m[1]
```

```
[4]: m[1][0]
```

[4]: 0.5

• Feel free to push this to higher dimensions...

```
[5]: v1 = [0.5, 1.5]

v2 = [1, 2]
```

```
m = [v1, v2]
c = [m, m]
c
```

```
[5]: [[[0.5, 1.5], [1, 2]], [[0.5, 1.5], [1, 2]]]
```

```
[6]: c[1][1][0]
```

[6]: 1

Reference pointers

- Important: list's work with reference pointers.
- Internally, when creating new objects out of existing objects, only pointers to the objects are copied, not the data!

```
[7]: v = [0.5, 0.75, 1.0, 1.5, 2.0]

m = [v, v, v]

m
```

```
[7]: [[0.5, 0.75, 1.0, 1.5, 2.0], [0.5, 0.75, 1.0, 1.5, 2.0], [0.5, 0.75, 1.0, 1.5, 2.0]]
```

```
[8]: v[0] = 'Python'
m
```

Python array class

- Python also has an array module
- See Documentation

2.2 NumPy arrays

NumPy arrays

- \bullet NumPy is a library for richer array data structures.
- The basic object is ndarray, which comes in two flavours:

Object type	Meaning	Used for
ndarray (regular)	<i>n</i> -dimensional array object	Large arrays of numerical data
ndarray (record)	2-dimensional array object	Tabular data organized in columns

ndarray

Source: Python for Finance, 2nd ed.

- The ndarray object is more specialised than the list object, but comes with more functionality.
- An array object represents a multidimensional, homogeneous array of fixed-size items.
- Here is a useful tutorial

Regular NumPy arrays

• Creating an array:

[16]: 0.7071067811865476

```
[9]: import numpy as np # import numpy
      a = np.array([0, 0.5, 1, 1.5, 2]) # array(...) is the constructor for ndarray's
[10]: type(a)
[10]: numpy.ndarray
        • ndarray assumes objects of the same type and will modify types accordingly:
[11]: b = np.array([0, 'test'])
[11]: array(['0', 'test'], dtype='<U21')</pre>
[12]: type(b[0])
[12]: numpy.str_
     Constructing arrays by specifying a range
        • np.arange() creates an array spanning a range of numbers (= a sequence).
        • Basic syntax: np.arange(start, stop, steps)
        • It is possible to specify the data type (e.g. float)
        • To invoke an explanation of np.arange (or any other object or method), type np.arange?
[13]: np.arange?
[14]: np.arange(0, 2.5, 0.5)
[14]: array([0., 0.5, 1., 1.5, 2.])
     NOTE: The interval specification refers to a half-open interval: [start, stop).
     ndarray methods
        • The ndarray object has a multitude of useful built-in methods, e.g.
             - sum() (the sum),
             - std() (the standard deviation),
             - cumsum() (the cumulative sum).
        • Type a. and hit TAB to obtain a list of the available functions.
        • More documentation is found here.
[15]: a.sum()
[15]: 5.0
[16]: a.std()
```

```
[17]: a.cumsum()
```

Slicing 1d-Arrays

• With one-dimensional ndarray objects, indexing works as usual.

```
[18]: a[1]
```

[18]: 0.5

[19]: array([0., 0.5])

Mathematical operations

- Mathematical operations are applied in a **vectorised** way on an **ndarray** object.
- Note that these operations work differently on list objects.

[21]: [0, 0.5, 1, 1.5, 2]

• ndarray:

[23]: array([0, 1, 2, 3, 4, 5, 6])

Mathematical operations (cont'd)

[26]: array([0, 1, 4, 9, 16, 25, 36])

```
[27]: 2 ** a

[27]: array([ 1,  2,  4,  8,  16,  32,  64])

[28]: a ** a

[28]: array([  1,   1,   4,  27,  256,  3125,  46656])
```

Universal functions in NumPy

• A number of universal functions in NumPy are applied element-wise to arrays:

Multiple dimensions

- All features introduced so far carry over to multiple dimensions.
- An array with two rows:

```
[31]: b = np.array([a, 2 * a])
b
```

```
[31]: array([[ 0, 1, 2, 3, 4, 5, 6], [ 0, 2, 4, 6, 8, 10, 12]])
```

• Selecting the first row, a particular element, a column:

```
[32]: b[0]

[32]: array([0, 1, 2, 3, 4, 5, 6])

[33]: b[1,1]

[33]: 2

[34]: b[:,1]

[34]: array([1, 2])
```

Multiple dimensions

• Calculating the sum of all elements, column-wise and row-wise:

```
[35]: b.sum()
[35]: 63
```

```
[36]: b.sum(axis = 0)
[36]: array([0, 3, 6, 9, 12, 15, 18])
[37]: b.sum(axis = 1)
[37]: array([21, 42])
        Note: axis = 0 refers to column-wise and axis = 1 to row-wise.
     Further methods for creating arrays
        • Often, we want to create an array and populate it later.
        • Here are some methods for this:
[38]: np.zeros((2,3), dtype = 'i') # array with two rows and three columns
[38]: array([[0, 0, 0],
             [0, 0, 0]], dtype=int32)
[39]: np.ones((2,3,4), dtype = 'i') # array dimensions: 2 x 3 x 4
[39]: array([[[1, 1, 1, 1],
              [1, 1, 1, 1],
              [1, 1, 1, 1]],
             [[1, 1, 1, 1],
              [1, 1, 1, 1],
              [1, 1, 1, 1]]], dtype=int32)
[40]: np.empty((2,3))
[40]: array([[1.
                         , 1.41421356, 1.73205081],
                         , 2.23606798, 2.44948974]])
             [2.
     Further methods for creating arrays
[41]: np.eye(3)
[41]: array([[1., 0., 0.],
             [0., 1., 0.],
             [0., 0., 1.]])
[42]: np.diag(np.array([1,2,3,4]))
[42]: array([[1, 0, 0, 0],
             [0, 2, 0, 0],
             [0, 0, 3, 0],
             [0, 0, 0, 4]])
```

NumPy dtype objects

Source: Python for Finance, 2nd ed.

Logical operations

• NumPy Arrays can be compared, just like lists.

dtype	Description	Example
?	Boolean	? (True or False)
i	Signed integer	i8 (64-bit)
u	Unsigned integer	u8 (64-bit)
f	Floating point	f8 (64-bit)
С	Complex floating point	c32 (256-bit)
m	timedelta	m (64-bit)
М	datetime	M (64-bit)
0	Object	O (pointer to object)
U	Unicode	U24 (24 Unicode characters)
V	Raw data (void)	V12 (12-byte data block)

dtype object

```
[43]: first = np.array([0, 1, 2, 3, 3, 6,])
    second = np.array([0, 1, 2, 3, 4, 5,])

[44]: first > second

[44]: array([False, False, False, False, True])

[45]: first.sum() == second.sum()

[46]: True

[46]: np.any([a == 4])

[47]: np.all([a == 4])
```

Reshape and resize

• ndarray objects are immutable, but they can be reshaped (changes the view on the object) and resized (creates a new object):

```
[48]: ar = np.arange(15)
ar

[48]: array([ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14])

[49]: ar.reshape((3,5))
```

```
[49]: array([[ 0, 1, 2, 3, 4],
             [5, 6, 7, 8, 9],
             [10, 11, 12, 13, 14]])
[50]: ar
[50]: array([ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14])
     Reshape and resize
[51]: ar.resize((5,3))
[52]: ar
[52]: array([[ 0, 1,
                       2],
             [3, 4,
                       5],
             [6, 7, 8],
             [ 9, 10, 11],
             [12, 13, 14]])
        Note: reshape() did not change the original array. ()resize did change the array's shape perma-
     nently.
     Reshape and resize
        • reshape() does not alter the total number of elements in the array.
        • resize() can decrease (down-size) or increase (up-size) the total number of elements.
[53]: ar
[53]: array([[ 0, 1,
                       2],
             [3, 4,
                       5],
             [6, 7, 8],
             [ 9, 10, 11],
             [12, 13, 14]])
[54]: np.resize(ar, (3,3))
[54]: array([[0, 1, 2],
             [3, 4, 5],
             [6, 7, 8]])
     Reshape and resize
[55]: np.resize(ar, (5,5))
[55]: array([[ 0, 1,
                       2,
                           3,
                               4],
             [5, 6, 7, 8,
                              9],
             [10, 11, 12, 13, 14],
                         3,
                      2,
             [ 0, 1,
                               4],
             [5, 6, 7, 8,
                              9]])
[56]: a.shape # returns the array's dimensions
```

[56]: (7,)

Further operations

• Transpose:

Further operations

• Stacking: hstack or vstack can used to connect two arrays horizontally or vertically.

NOTE: The size of the to-be connected dimensions must be equal.

2.3 Structured NumPy arrays

Structured NumPy arrays

- The specialisation of ndarray may be to narrow.
- However, one can instantiate ndarray with a dedicated dtype.
- This allows to build database-like data sets where each row corresponds to an "entry".

Structured NumPy arrays

• Creating a data type:

```
[62]: dt = np.dtype([('Name', 'S10'), ('Age', 'i4'), ('Height', 'f'), ('Children/Pets', 'i4', 2)])
dt
```

```
[62]: dtype([('Name', 'S10'), ('Age', '<i4'), ('Height', '<f4'), ('Children/Pets', '<i4', (2,))])
```

• Equivalently:

```
[63]: dtype([('Name', '0'), ('Age', '<i8'), ('Height', '<f8'), ('Children/Pets', [('f0', '<i8'), ('f1', '<i8')])])
```

Structured NumPy arrays

• Now create the ndarray with the new data type:

```
[64]: s = np.array([('Smith', 45, 1.83, (0, 1)), ('Jones', 53, 1.72, (2, 2))], dtype=dt)
s
```

```
[65]: type(s)
```

[65]: numpy.ndarray

Structured NumPy arrays

• The columns can be accessed through their names:

```
[66]: s['Name']
[66]: array(['Smith', 'Jones'], dtype=object)
[67]: s['Height'].mean()
[67]: 1.775
[68]: s[0]
[68]: ('Smith', 45, 1.83, (0, 1))
[69]: s[1]['Age']
```

2.4 Data Analysis with pandas: DataFrame

Data analysis with pandas

- pandas is a powerful Python library for data manipulation and analysis. Its name is derived from panel data.
- We cover the following data structures:

Source: Python for Finance, 2nd ed.

Object type	Meaning	Used for
DataFrame	2-dimensional data object with index	Tabular data organized in columns
Series	1-dimensional data object with index	Single (time) series of data

Pandas datatypes

DataFrame Class

- DataFrame is a class that handles tabular data, organised in columns.
- Each row corresponds to an entry or a data record.
- It is thus similar to a table in a relational database or an Excel spreadsheet.

```
[70]: import pandas as pd
      df = pd.DataFrame([10,20,30,40], # data as a list
                       columns=['numbers'], # column label
                       index=['a', 'b', 'c', 'd']) # index values for entries
[71]: df
[71]:
         numbers
```

a 10 b 20 С 30 d 40

DataFrame Class

- The columns can be named (but don't need to be).
- The index can take different forms such as numbers or strings.
- The input data for the DataFrame Class can come in different types, such as list, tuple, ndarray and dict objects.

Simple operations

[74]: numbers

30 Name: c, dtype: int64

• Some simple operations applied to a DataFrame object:

```
[72]: df.index
[72]: Index(['a', 'b', 'c', 'd'], dtype='object')
[73]: df.columns
[73]: Index(['numbers'], dtype='object')
     Simple operations
[74]: df.loc['c'] # selects value corresponding to index c
```

```
[75]: df.loc[['a', 'd']] # selects values correponding t indices a and d
         numbers
[75]:
              10
      a
              40
      d
[76]: df.iloc[1:3] # select second and third rows
[76]:
         numbers
              20
      С
              30
     Simple operations
[77]: df.sum()
[77]: numbers
                  100
      dtype: int64
        • Vectorised operations as with ndarray:
[78]: df ** 2
[78]:
         numbers
             100
      b
             400
             900
      С
            1600
      d
     Extending DataFrame objects
[79]: df['floats'] = (1.5, 2.5, 3.5, 4.5) # adds a new column
[80]: df
[80]:
         numbers
                  floats
              10
                      1.5
              20
                      2.5
      b
              30
                      3.5
      С
              40
                      4.5
[81]: df['floats']
[81]: a
           1.5
      b
           2.5
           3.5
      С
      d
           4.5
      Name: floats, dtype: float64
```

Extending DataFrame objects

• A DataFrame object can be taken to define a new column:

```
[82]: df['names'] = pd.DataFrame(['Yves', 'Sandra', 'Lilli', 'Henry'],
                                  index = ['d', 'a', 'b', 'c'])
[83]: df
[83]:
         numbers
                  floats
                            names
              10
                      1.5
                           Sandra
              20
                      2.5
                            Lilli
      b
                      3.5
              30
                            Henry
      С
              40
                      4.5
      d
                             Yves
```

Extending DataFrame objects

• Appending data:

```
[84]: df = df.append(pd.DataFrame({'numbers': 100, 'floats': 5.75, 'names': 'Jill'},
                                   index = ['y',])
[85]: df
[85]:
         numbers
                   floats
                            names
                     1.50
                           Sandra
      a
               10
              20
                     2.50
                            Lilli
      b
              30
                     3.50
                            Henry
      С
      d
              40
                     4.50
                             Yves
             100
                     5.75
                              Jill
      У
```

Extending DataFrame objects

• Be careful when appending without providing an index – the index gets replaced by a simple range index:

```
[86]: df.append({'numbers': 100, 'floats': 5.75, 'names': 'Jill'}, ignore_index=True)
[86]:
         numbers
                   floats
                             names
      0
               10
                     1.50
                            Sandra
      1
               20
                     2.50
                             Lilli
      2
               30
                     3.50
                             Henry
      3
               40
                     4.50
                              Yves
      4
              100
                     5.75
                              Jill
      5
              100
                     5.75
                              Jill
```

Extending DataFrame objects

• Appending with missing data:

```
[87]: df = df.append(pd.DataFrame({'names': 'Liz'},
                                   index = ['z']),
                                   sort = False)
[88]: df
[88]:
         numbers
                  floats
                            names
                     1.50
      a
            10.0
                           Sandra
      b
            20.0
                     2.50
                            Lilli
```

```
С
      30.0
               3.50
                       Henry
d
      40.0
                4.50
                        Yves
     100.0
у
                5.75
                         Jill
       NaN
                NaN
                          Liz
z
```

Mathematical operations on Data Frames

• A lot of mathematical methods are implemented for DataFrame objects:

Time series with Data Frame

- In this section we show how a DataFrame can be used to manage time series data.
- First, we create a DataFrame object using random numbers in an ndarray object.

```
[92]: import numpy as np
      import pandas as pd
      np.random.seed(100)
      a = np.random.standard_normal((9,4))
[92]: array([[-1.74976547,
                            0.3426804 ,
                                         1.1530358 , -0.25243604],
             [ 0.98132079,
                            0.51421884,
                                        0.22117967, -1.07004333],
                            0.25500144, -0.45802699, 0.43516349],
             [-0.18949583,
                            0.81684707, 0.67272081, -0.10441114],
             [-0.58359505,
             [-0.53128038,
                            1.02973269, -0.43813562, -1.11831825],
                            1.54160517, -0.25187914, -0.84243574],
             [ 1.61898166,
                            0.9370822 ,
                                        0.73100034, 1.36155613],
             [ 0.18451869,
                            0.05567601, 0.22239961, -1.443217 ],
             [-0.32623806,
                            0.81645401, 0.75044476, -0.45594693]])
             [-0.75635231,
[93]: df = pd.DataFrame(a)
```

Note: To learn more about Python's built-in pseudo-random number generator (PRNG), see here.

Practical example using DataFrame class

```
[94]: df

[94]: 0 1 2 3

0 -1.749765 0.342680 1.153036 -0.252436
1 0.981321 0.514219 0.221180 -1.070043
2 -0.189496 0.255001 -0.458027 0.435163
```

Practical example using DataFrame class

• Arguments to the DataFrame() function for instantiating a DataFrame object:

Parameter	Format	Description
data	ndarray/dict/DataFrame	Data for DataFrame; dict can contain Series, ndarray, list
index	Index/array-like	Index to use; defaults to range(n)
columns	Index/array-like	Column headers to use; defaults to range(n)
dtype	dtype, default None	Data type to use/force; otherwise, it is inferred
сору	bool, default None	Copy data from inputs

DataFrame object

Source: Python for Finance, 2nd ed.

Practical example using DataFrame class

• In the next steps, we set column names and add a time dimension for the rows.

```
[95]: df.columns = ['No1', 'No2', 'No3', 'No4']
[96]: df
[96]:
              No1
                        No<sub>2</sub>
                                  No3
                                             No4
      0 -1.749765
                   0.342680
                             1.153036 -0.252436
      1 0.981321
                   0.514219
                             0.221180 -1.070043
      2 -0.189496
                   0.255001 -0.458027 0.435163
      3 -0.583595
                   0.816847
                             0.672721 -0.104411
                   1.029733 -0.438136 -1.118318
      4 -0.531280
      5 1.618982
                   1.541605 -0.251879 -0.842436
        0.184519
                   0.937082
                             0.731000 1.361556
      7 -0.326238
                   0.055676
                            0.222400 -1.443217
      8 -0.756352
                   0.816454
                            0.750445 -0.455947
[97]: df['No3'].values.flatten()
[97]: array([ 1.1530358 , 0.22117967, -0.45802699, 0.67272081, -0.43813562,
             -0.25187914, 0.73100034, 0.22239961, 0.75044476)
```

Practical example using DataFrame class

- pandas is especially strong at handling times series data efficiently.
- Assume that the data rows in the DataFrame consist of monthtly observations starting in January 2019
- The method date_range() generates a DateTimeIndex object that can be used as the row index.

${\bf Practical\ example\ using\ DataFrame\ class}$

• Parameters of the date_range() function:

Parameter	Format	Description
start	string/datetime	Left bound for generating dates
end	string/datetime	Right bound for generating dates
periods	integer/None	Number of periods (if start or end is None)
freq	string/DateOffset	Frequency string, e.g., 5D for 5 days
tz	string/None	Time zone name for localized index
normalize	bool, default None	Normalizes start and end to midnight
name	string, default None	Name of resulting index

Date range parameters

Source: Python for Finance, 2nd ed.

Practical example using DataFrame class

• Frequency parameter of date_range() function:

		Alias	Description
		MS	Month start frequency
		BMS	Business month start frequency
		Q	Quarter end frequency
		BQ	Business quarter end frequency
		QS	Quarter start frequency
		BQS	Business quarter start frequency
		Α	Year end frequency
		ВА	Business year end frequency
Alias	Description	AS	Year start frequency
В	Business day frequency	BAS	Business year start frequency
C	Custom business day frequency (experimental)	Н	Hourly frequency
D	Calendar day frequency	Т	Minutely frequency
W	Weekly frequency	S	Secondly frequency
М	Month end frequency	L	Milliseconds
ВМ	Business month end frequency	U	Microseconds

Source: Python for Finance, 2nd ed.

Practical example using DataFrame class

• Now set the row index to the dates:

```
[99]: df.index = dates
    df
```

```
[99]:
                       No1
                                 No2
                                           No3
      2019-01-31 -1.749765
                           0.342680
                                     1.153036 -0.252436
      2019-02-28 0.981321
                           0.514219
                                     0.221180 -1.070043
      2019-03-31 -0.189496
                           0.255001 -0.458027 0.435163
      2019-04-30 -0.583595
                           0.816847
                                     0.672721 -0.104411
      2019-05-31 -0.531280
                           1.029733 -0.438136 -1.118318
      2019-06-30 1.618982
                           1.541605 -0.251879 -0.842436
      2019-07-31 0.184519
                                     0.731000 1.361556
                           0.937082
      2019-08-31 -0.326238
                           0.055676
                                     0.222400 -1.443217
      2019-09-30 -0.756352  0.816454  0.750445 -0.455947
```

Practical example using DataFrame class

• Next, we visualise the data:

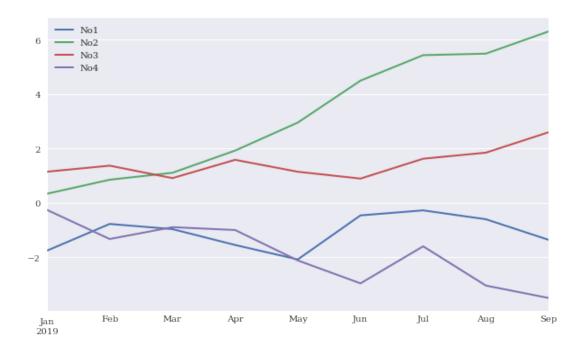
```
[100]: from pylab import plt, mpl # imports for visualisation plt.style.use('seaborn') # This and the following lines customise the plot style mpl.rcParams['font.family'] = 'serif' %matplotlib inline
```

• More about customising the plot style: here.

Practical example using DataFrame class

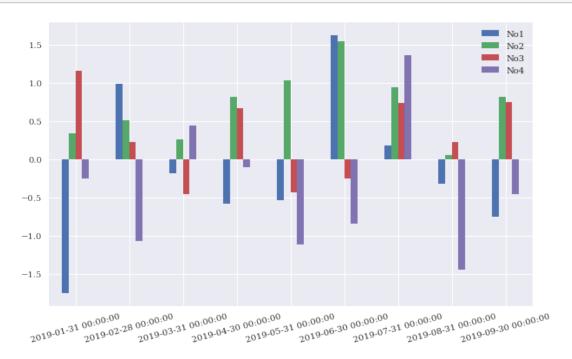
• Plot the cumulative sum for each column of df:

```
[101]: df.cumsum().plot(lw = 2.0, figsize = (10,6));
```



Practical example using DataFrame class

• A bar chart:



Practical example using DataFrame class

• Parameters of plot() method:

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Practical example using DataFrame class

• Parameters of plot() method:

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${\bf Practical\ example\ using\ DataFrame\ class}$

• Useful functions:

```
[103]: df.info() # provide basic information
```

<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 9 entries, 2019-01-31 to 2019-09-30
Freq: M
Data columns (total 4 columns):
 # Column Non-Null Count Dtype

_____ 0 9 non-null float64 9 non-null float64 1 No2 2 No3 9 non-null float64 float64 3 No4 9 non-null

dtypes: float64(4)

memory usage: 360.0 bytes

Parameter	Format	Description
×	label/position, default None	Only used when column values are x-ticks
у	label/position, default None	Only used when column values are y-ticks
subplots	boolean, default False	Plot columns in subplots
sharex	boolean, default True	Share the x-axis
sharey	boolean, default False	Share the y-axis
use_index	boolean, default True	Use DataFrame.index as x-ticks
stacked	boolean, default False	Stack (only for bar plots)
sort_columns	boolean, default False	Sort columns alphabetically before plotting
title	string, default None	Title for the plot
grid	boolean, default False	Show horizontal and vertical grid lines
legend	boolean, default True	Show legend of labels
ax	matplotlib axis object	matplotlib axis object to use for plotting
style	string or list/dictionary	Line plotting style (for each column)
kind	string (e.g., "line", "bar", "barh", "kde", "density")	Type of plot
logx	boolean, default False	Use logarithmic scaling of x-axis
logy	boolean, default False	Use logarithmic scaling of y-axis
xticks	sequence, default Index	X-ticks for the plot

Parameters of plot method

Parameter	Format	Description
yticks	sequence, default Values	Y-ticks for the plot
xlim	2-tuple, list	Boundaries for x-axis
ylim	2-tuple, list	Boundaries for y-axis
rot	integer, default None	Rotation of x-ticks
secondary_y	boolean/sequence, default False	Plot on secondary y-axis
mark_right	boolean, default True	Automatic labeling of secondary axis
colormap	string/colormap object, default None	Color map to use for plotting
kwds	keywords	Options to pass to matplotlib

${\bf Plot_parameters}$

${\bf Practical\ example\ using\ DataFrame\ class}$

```
[104]: df.sum()
[104]: No1
             -1.351906
       No2
              6.309298
       No3
              2.602739
       No4
             -3.490089
       dtype: float64
[105]: df.mean(axis=0) # column-wise mean
[105]: No1
             -0.150212
       No2
              0.701033
       No3
              0.289193
       No4
             -0.387788
       dtype: float64
```

```
[106]: df.mean(axis=1) # row-wise mean
[106]: 2019-01-31
                   -0.126621
       2019-02-28
                    0.161669
       2019-03-31
                    0.010661
       2019-04-30
                    0.200390
       2019-05-31
                   -0.264500
       2019-06-30
                    0.516568
       2019-07-31
                    0.803539
       2019-08-31
                   -0.372845
       2019-09-30
                    0.088650
       Freq: M, dtype: float64
      Useful functions: groupby()
[107]: df['Quarter'] = ['Q1', 'Q1', 'Q1', 'Q2', 'Q2', 'Q2', 'Q3', 'Q3', 'Q3',]
[108]: df
[108]:
                       No1
                                 No2
                                           No3
                                                     No4 Quarter
       2019-01-31 -1.749765
                            0.342680
                                     1.153036 -0.252436
                                                              Q1
       2019-02-28 0.981321
                            0.514219 0.221180 -1.070043
                                                              Q1
       2019-03-31 -0.189496
                            0.255001 -0.458027 0.435163
                                                              Q1
       2019-04-30 -0.583595
                            0.816847
                                     0.672721 -0.104411
                                                              02
       2019-05-31 -0.531280
                            1.029733 -0.438136 -1.118318
                                                              Q2
       2019-06-30 1.618982
                            1.541605 -0.251879 -0.842436
                                                              Q2
       2019-07-31 0.184519
                            0.937082
                                     0.731000 1.361556
                                                              QЗ
       2019-08-31 -0.326238
                            0.055676
                                     0.222400 -1.443217
                                                              QЗ
       QЗ
      Useful functions: groupby()
[109]: groups = df.groupby('Quarter')
[110]: groups.mean()
[110]:
                     No<sub>1</sub>
                              No2
                                        No3
                                                  No4
       Quarter
                         0.370634 0.305396 -0.295772
       Q1
               -0.319314
                         1.129395 -0.005765 -0.688388
       Q2
               0.168035
               -0.299357
                         0.603071 0.567948 -0.179203
[111]: groups.max()
[111]:
                              No2
                                        No3
                                                  No4
                    No1
       Quarter
       Q1
               0.981321
                         0.514219 1.153036 0.435163
                1.618982
       Q2
                         1.541605 0.672721 -0.104411
       QЗ
                0.184519
                         0.937082 0.750445 1.361556
      Useful functions: groupby()
[112]: groups.aggregate([min, max]).round(3)
```

```
[112]:
                 No1
                              No2
                                            No3
                                                          No4
                 \min
                              min
                                            min
                                                          min
                        max
                                     max
                                                   max
      Quarter
              -1.750 0.981 0.255 0.514 -0.458 1.153 -1.070 0.435
      Q1
              -0.584 1.619 0.817 1.542 -0.438 0.673 -1.118 -0.104
      Q2
      QЗ
              -0.756  0.185  0.056  0.937  0.222  0.750 -1.443  1.362
```

Selecting and filtering data

```
• Logical operators can be used to filter data.
         • First, construct a DataFrame filled with random numbers to work with.
[113]: data = np.random.standard_normal((10,2))
[114]: df = pd.DataFrame(data, columns = ['x', 'y'])
[115]: df.head(2) # the first two rows
[115]:
                 х
       0 1.189622 -1.690617
       1 -1.356399 -1.232435
[116]: df.tail(2) # the last two rows
[116]:
                 Х
       8 -0.940046 -0.827932
       9 0.108863 0.507810
      Selecting and filtering data
[117]: (df['x'] > 1) & (df['y'] < 1) # check if value in x-column is greater than 1 and
        \rightarrow value in y-column is smaller than 1
[117]: 0
             True
            False
       2
            False
       3
            False
       4
            True
       5
            False
       6
            False
       7
            False
       8
            False
            False
       dtype: bool
[118]: df[df['x'] > 1]
[118]:
       0 1.189622 -1.690617
       4 1.299748 -1.733096
[119]: df.query('x > 1') # query()-method takes string as parameter
[119]:
       0 1.189622 -1.690617
       4 1.299748 -1.733096
```

Selecting and filtering data

```
[120]: (df > 1).head(3) # Find values greater than 1

[120]: x y
0 True False
1 False False
2 False False
[121]: df [df > 1].head(3) # Select values greater than 1 and put NaN (not-a-number) in the

→ other entries

[121]: x y
0 1.189622 NaN
1 NaN NaN
2 NaN NaN

Concatenation

• Adding rows from one data frame to another data frame can be done with append() or concat():
```

Concatenation

Concatenation

Joining

- In Python, join() refers to joining DataFrame objects according to their index values.
- There are four different types of joining:
 - 1. left join
 - 2. right join
 - 3. inner join
 - 4. outer join

Joining

```
[125]: df1.join(df2, how = 'left') # default join, based on indices of first dataset
[125]:
            Α
                  В
       a
          100
                NaN
       b
          200
                150
          300
               NaN
       С
          400
                50
[126]: df1.join(df2, how = 'right') # based on indices of second dataset
[126]:
            Α
                  В
       f
          {\tt NaN}
                200
          200
               150
       d 400
                50
      Joining
[127]: df1.join(df2, how = 'inner') # preserves those index values that are found in both
        \rightarrow datasets
[127]:
            Α
                  В
       b
          200
                150
       d 400
                50
[128]: df1.join(df2, how = 'outer') # preserves indices found in both datasets
[128]:
            Α
                  В
          100
               NaN
       a
          200
       b
                150
       С
          300
                NaN
       d
          400
                50
       f
          NaN
               200
      Merging
```

- Join operations on DataFrame objects are based on the datasets indices.
- \bullet $\mathbf{Merging}$ operates on a shared column of two <code>DataFrame</code> objects.
- To demonstrate the usage we add a new column C to df1 and df2.

```
[129]: c = pd.Series([250, 150, 50], index = ['b', 'd', 'c'])
    df1['C'] = c
    df2['C'] = c
```

Merging

250.0

150.0

50.0

NaN

200

NaN

 ${\tt NaN}$

150

NaN

NaN

NaN

250.0

200

300

NaN

1

2

3 400

```
[130]: df1
[130]:
             Α
                    С
          100
                  NaN
       a
       b
          200
                250.0
          300
                 50.0
       С
          400
               150.0
[131]: df2
[131]:
             В
                    C
       f
           200
                  NaN
          150
                250.0
       b
           50
                150.0
      Merging
          • By default, a merge takes place on a shared column, preserving only the shared data rows:
[132]: pd.merge(df1, df2)
[132]:
                    С
                          В
             Α
       0
          100
                        200
                  {\tt NaN}
          200
                250.0
                        150
                150.0
          400
                         50
          • An outer merge preserves all data rows:
[133]: pd.merge(df1, df2, how = 'outer')
[133]:
             Α
                    С
                          В
       0
           100
                  NaN
                        200
          200
       1
                250.0
                        150
       2
          300
                 50.0
                        NaN
          400
                150.0
                         50
      Merging
          • There are numerous other ways to merge DataFrame objects.
          • To learn more about merging in Python, see the pandas document on DataFrame merging.
[134]: pd.merge(df1, df2, left_on = 'A', right_on = 'B')
[134]:
                  C_x
                          В
                            С_у
             Α
       0 200 250.0
                        200
                             NaN
[135]: pd.merge(df1, df2, left_on = 'A', right_on = 'B', how = 'outer')
[135]:
                  C_x
                          В
                               С_у
             A
           100
                  {\tt NaN}
                        NaN
                               {\tt NaN}
```