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Computational Finance and FinTech – Problem Set 8

Exercise 1. The data in `dax.csv` contains daily DAX index levels and returns from 1990 to 2016. Value-at-risk (VaR) quantifies the loss boundary that will not be exceeded with a given probability α.
Using the last 300 days as historical data, calculate one-day value-at-risk (VaR) at the level $\alpha = 0.99$ using the delta-normal, historical simulation and GARCH approaches.
The delta-normal VaR at the level α is given as:

$$\mathrm{VaR}_\alpha = -V_0 \cdot \mathrm{N}_{1-\alpha} \cdot \sigma,$$

where V_0 is the current DAX index value, $\mathrm{N}_{1-\alpha}$ is the $1-\alpha$-quantile of the standard normal distribution and σ is the volatility of the historical DAX returns. A typical level is $\alpha = 0.99$.
For historical simulation, VaR is given as

$$\mathrm{VaR}_\alpha = -V_0 \cdot \hat{Q}_{1-\alpha},$$

where $\hat{Q}_{1-\alpha}$ is the empirical $1-\alpha$-quantile
For a GARCH model, observing that conditional on the volatility forecast, returns are normally distributed, VaR is just the delta-normal VaR with the volatility replaced by the vol forecast:

$$\mathrm{VaR}_\alpha = -V_0 \cdot \mathrm{N}_{1-\alpha} \cdot \sigma_{\mathrm{GARCH}}.$$