Berlin School of Economics and Law
Prof. Dr. Natalie Packham

Summer term 2020
LV-Nr. 425093

Computational Finance and FinTech
Option Pricing

Contents

6 Option Pricing

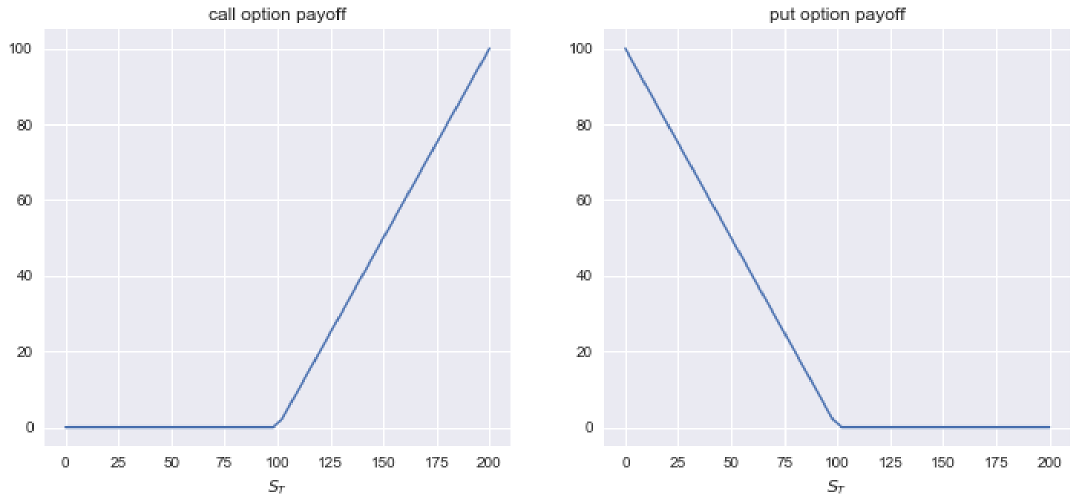
Option Pricing

- This session is *not* directly related to a chapter in **Py4Fi**.
- An **option**, also called **contingent claim**, is a financial derivative whose payoff is a (non-linear) function of an underlying asset's value.
- The most important and liquidly traded contingent claims are **call** and **put options**.
- Option prices are determined by assuming that markets are free of **arbitrage**.
- We cover standard option pricing techniques: the **binomial tree model** and the **Black-Scholes(-Merton) model**.

6.1 Call and put options

- In the following, $(S_t)_{t\geq 0}$ denotes the price process of the underlying asset.
- A **Euopean call option** with **exercise / strike price** K and maturity T has payoff $X = \max(S_T - K, 0) = (S_T - K)^+$.
- The holder of the option has the right, but not the obligation, to buy the stock at time T at price K.
- A **European put option** with **exercise / strike price** K and maturity T has payoff $X = \max(K - S_T, 0) = (K - S_T)^+$.

[3]:
```
x = np.linspace(0,200)
k=100; # strike price
plt.figure(figsize=(12, 5))
plt.subplot(121)
plt.plot(x,(x>k)*(x-k), lw=1.5)
plt.xlabel('$S_T$')
plt.title('call option payoff')
plt.subplot(122)
plt.plot(x, (x<k)*(k-x), lw=1.5)
plt.xlabel('$S_T$')
plt.title('put option payoff');
```



Option valuation

- At first sight, it might be seem impossible to determine a price for an option.
- It turns out that under some general conditions and assumptions, unique prices for contingent claims can be determined!
- The main assumption is that markets are **free of arbitrage**.
- A powerful result, under some further assumptions, then shows that the payoff of a contingent claim can be **replicated** with a dynamic trading strategy in a bond and the underlying asset.
- A rigorous derivation of this result is beyond the scope of the course, but we will sketch the main ingredients.

Discuss: Is the assumption of **absence of arbitrage** plausible in financial markets?

Option valuation - Roadmap

- We begin by setting up a model for a financial market consisting of a **riskless asset**, such as a **bond** or a **money market account** and a **risky asset** e.g. a share of **stock**.
- The bond / money market account accrues at a constant deterministic interest rate, the **risk-free rate**.
- The stock price process is modelled as a **stochastic process**, where, at every point in time the current price and the price history are known, but future prices are unknown.

Option valuation - Roadmap

- First, we consider a **discrete-time model**, where the stock price evolves in a **binomial tree**.
- It turns out that the payoffs of contingent claims can be **replicated** by a dynamic trading strategy in the stock and the bond.
- By a no-arbitrage argument, the price of the contingent claim must equal the cost setting up the replicating strategy.

- Taking the time step length Δt to the limit of 0, yields a **continuous-time model**, the **Black-Scholes(-Merton) model**.
- This gives a closed-form solution for call and put option prices, the famous **Black-Scholes formula**.

6.2 The one-period model

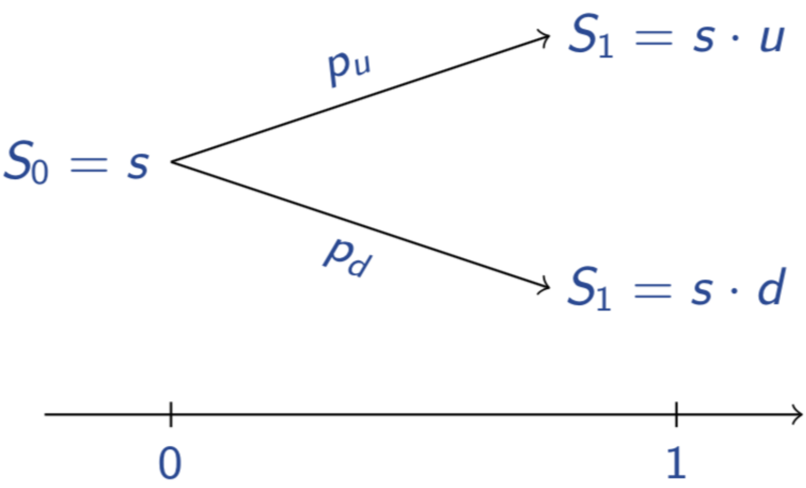
- Example:
  - Suppose you want to buy a **call option** on a stock.

- The option matures in one year and has a strike price of 105.
- The current stock price is 100, and it is known that the stock price either increases or decreases by 20% within one year.
- The risk-free interest rate is 5% (discrete compounding).

- Questions:
  - What is the **value** of this option? (Pricing)
  - How can you **hedge** this option? (Hedging)

The one-period model

- There are two points in time: $t \in \{0, 1\}$
- Market consists of bond and stock
- Bond price is deterministic: $B_0 = 1$ and $B_1 = 1 + r$
- Stock price S_t is a stochastic process with $S_0 = s$ and $S_1 = s \cdot u$ with probability p_u and $S_1 = s \cdot d$ with probability p_d.



Binomial tree

- We also write $S_1 = S_0 \cdot Z$ where Z is a random variable taking values in $\{u, d\}$.

The one-period model

- Assumptions:
  - s, u, d, p_u, p_d are known
  - $d < u$
  - $p_u + p_d = 1$
- A **portfolio** is a vector (x, y), where
  - x denotes the number bonds held and
  - y denotes the number of stocks held.

The one-period model

- Market assumptions:
  - Short positions and arbitrary holdings are allowed ($(x, y) \in \mathbb{R}^2$).
  - There are no bid-ask spreads.
  - There are no transaction costs.

- The market is completely liquid, i.e., it is always possible to buy and sell unlimited quantities in the market.

Portfolios

- Value process: $V_t = xB_t + yS_t$, $t \in 0, 1$.
- Equivalently:

$$V_0 = x + ys$$
$$V_1 = x(1+r) + yS_0 \cdot Z$$

Arbitrage

- An **arbitrage portfolio** or **arbitrage strategy** is a portfolio (x, y) with

$$V_0 = 0$$
$$\mathbb{P}(V_1 \geq 0) = 1$$
$$\mathbb{P}(V_1 > 0) > 0$$

- In words: An arbitrage opportunity is the possibility of generating at zero cost today a payoff at some time point in the future that is nonnegative with certainty and positive with positive probability.

Arbitrage

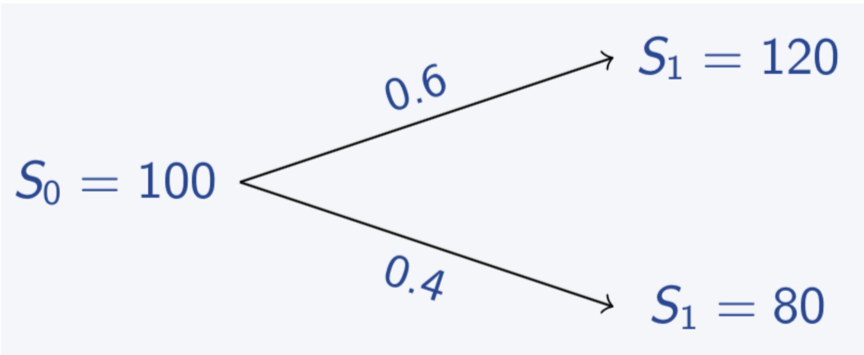
- The one-period model is free of arbitrage if and only if

$$d < 1 + r < u.$$

- Interpretation: return on stock is not allowed to dominate bond, and vice versa.

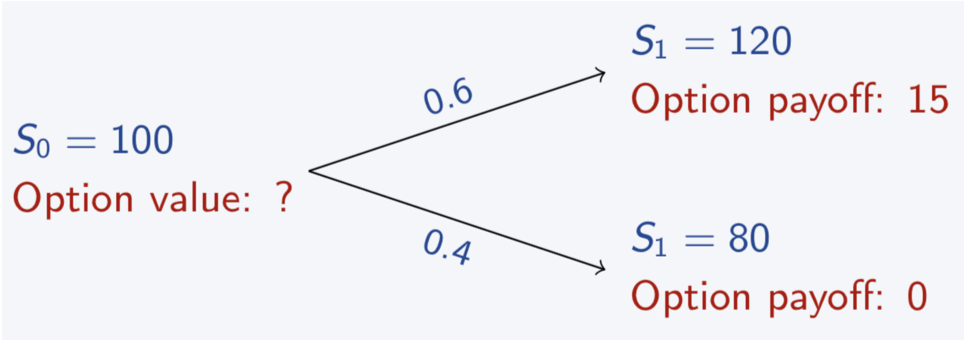
Example

- Consider a market with
  - risk-free interest rate $r = 5\%$,
  - $S_0 = 100$, $u = 1.2$, $d = 0.8$,
  - $p_u = 0.6$, $p_d = 0.4$.



Binomial tree

- The market is free of arbitrage since $0.8 < 1.05 < 1.2$.



Binomial tree

Example - Replicating a call option

- Now consider a call option with strike price $K = 105$:

- In other words, we would like to build a portfolio satisfying:

$$\begin{aligned} x(1+r) + yS_0 \cdot u &= 15 \\ x(1+r) + yS_0 \cdot d &= 0 \end{aligned}$$

Example - replicating a call option

- Plugging in the numbers for $B_t, S_0 \cdot u$ and $S_0 \cdot d$ gives

$$\begin{aligned} x \cdot 1.05 + 120y &= 15 \\ x \cdot 1.05 + \ \ 80y &= 0. \end{aligned}$$

- The solution to this system of linear equations is

$$y = \frac{15}{40} = \frac{3}{8} = 0.375 \quad \text{and} \quad x = -\frac{30}{1.05} = -\frac{30}{21/20} = -\frac{200}{7} = -28.57.$$

- We have found that the portfolio $(-28.57, 0.375)$ **replicates** the option payoff, that is, V_1 is idential to the option payoff.

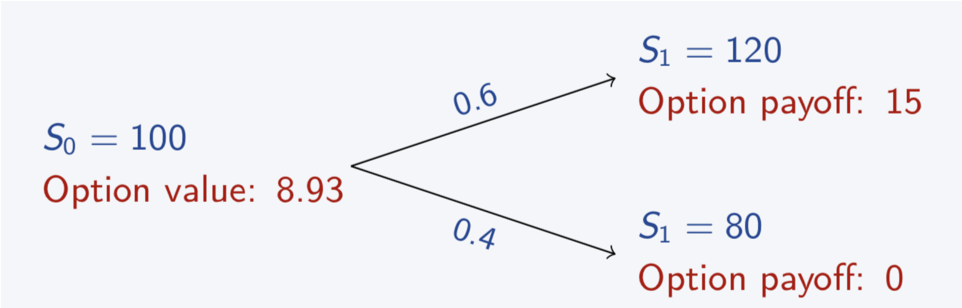
- The value of the portfolio today is $V_0 = -28.57 + 0.375 \cdot 100 = 8.93$.

Example - replicating a call option

- 8.93 is the **only** price for the option that is consistent with the no-arbitrage principle:
- If the option could be traded at a $\begin{Bmatrix} \text{higher} \\ \text{lower} \end{Bmatrix}$ price, then one could create a riskless profit by $\begin{Bmatrix} \text{selling} \\ \text{buying} \end{Bmatrix}$ the option and $\begin{Bmatrix} \text{buying} \\ \text{selling} \end{Bmatrix}$ the replicating portfolio.

Remarks

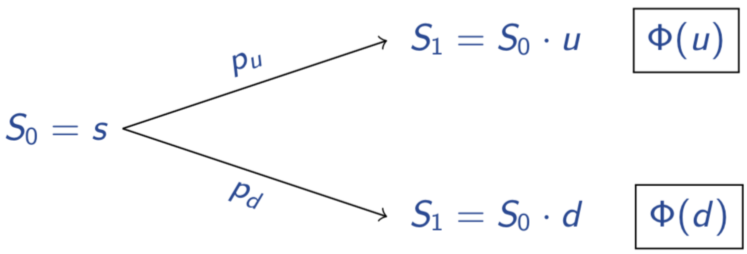
- The portfolio replicating the option payoff is called the **replicating portfolio**.
- The option price is called the **no-arbitrage price** of the option.
- Adding an option to the market that trades at its no-arbitrage price implies that the market remains free of arbitrage.
- A buyer of the option can **hedge** the exposure by entering into **minus** the replicating portfolio.
- The probabilities p_u, p_d did not enter into the calculation of the price.



Binomial tree

Arbitrary contingent claims

- Definition: A **contingent claim (financial derivative)** is any random variable X of the form $X = \Phi(Z)$, where Z is the random variable driving the stock price process.
- Φ is called the **contract function**.



Binomial tree

Arbitrary contingent claims

- **Proposition**: Suppose that a claim X can be replicated with a portfolio (x, y). Then any price of X at $t = 0$ other than $V_0^{(x,y)}$ leads to an arbitrage opportunity.
- **Proposition**: Assume that the binomial model is free of arbitrage. Then the market is **complete**, i.e., all contingent claims can be replicated.

Arbitrary contingent claims

- Proof:
  - Fixing an arbitrary claim X with contract function Φ, we require a solution (x, y) to the system of linear equations,

$$(1+r)x + S_0\, uy = \Phi(u),$$
$$(1+r)x + S_0\, dy = \Phi(d).$$

  - Since $u > d$, the linear system has a unique solution, given by

$$x = \frac{u\Phi(d) - d\Phi(u)}{(1+r)(u-d)} \qquad y = \frac{1}{S_0} \cdot \frac{\Phi(u) - \Phi(d)}{u-d}. \tag{1}$$

Risk-neutral pricing

- Recall that the market is free of arbitrage if and only if $d < 1 + r < u$.
- This is equivalent to saying that: there exist $q_u, q_d > 0$, with $q_u + q_d = 1$, such that

$$1 + r = q_u \cdot u + q_d \cdot d.$$

* q_u, q_d can be interpreted as a **probability measure** $\mathbb{Q}$ with

```
\begin{equation*}
  \mathbb Q(Z=u)=q_u, \qquad \mathbb Q(Z=d)=q_d.
\end{equation*}
```

Risk-neutral pricing

- By definition, we find

$$\frac{1}{1+r}\mathbb{E}^{\mathbb{Q}}[S_1] = \frac{1}{1+r}\left[q_u S_0\, u + q_d S_0\, d\right] = \frac{1}{1+r} \cdot S_0(1+r) = S_0.$$

- The probabilities q_u, q_d are called **risk-neutral probabilities** if the following condition holds:

$$S_0 = \frac{1}{1+r}\mathbb{E}^{\mathbb{Q}}[S_1].$$

Example

- The risk-neutral probabilities in the example are given by

$$\begin{aligned} q_u &= \frac{1.05 - 0.8}{1.2 - 0.8} = \frac{0.25}{0.4} = \frac{5}{8} = 0.625 \\ q_d &= 1 - q_u = 0.375 \end{aligned}$$

- The no-arbitrage price is calculated as

$$\text{Price}(\max(S_1 - 105, 0)) := \frac{1}{1.05}\left(0.625 \cdot 15 + 0.375 \cdot 0\right) = 8.93.$$

Risk-neutral pricing

- We re-state the "no-arbitrage" proposition in a way that is not specific to the one-period model.
- **Proposition (No-arbitrage, First Fundamental Theorem)**: The market model is arbitrage free if and only if there exists a risk-neutral measure $\mathbb{Q}$.

Risk-neutral pricing

- **Proposition**: For the one-period model, the risk-neutral probabilities are given by

$$q_u = \frac{(1+r) - d}{u - d} \tag{2}$$

$$q_d = 1 - q_u = \frac{u - (1+r)}{u - d}. \tag{3}$$

- **Proposition**: If the binomial model is free of arbitrage, then the arbitrage-free price of a contingent claim X is given by

$$\text{Price}(X) = \frac{1}{1+r}\mathbb{E}^{\mathbb{Q}}[X].$$

Risk-neutral probabilities

- Objective probabilities determine which events are possible and which are impossible.
- We compute the arbitrage free price of a financial derivative **as if** we were living in a risk-neutral world.
- This does **not** mean that we believe that we live in a risk-neutral world.
- Rather, investors **do not receive a risk premium** for holding contingent claims that can be entirely risk-managed by replication.
- The valuation formula holds for all investors, regardless of their attitude towards risk.

6.3 The multi-period model

- Discrete time, with time running from $t = 0$ to $t = T$.
- Market consists of two assets, a **bond** and a **stock**
- The **bond price process** denoted by B_t, is given by

$$B_{t+1} = (1 + r)\, B_t$$
$$B_0 = 1,$$

with r the interest rate (a simple period rate).

The multi-period model

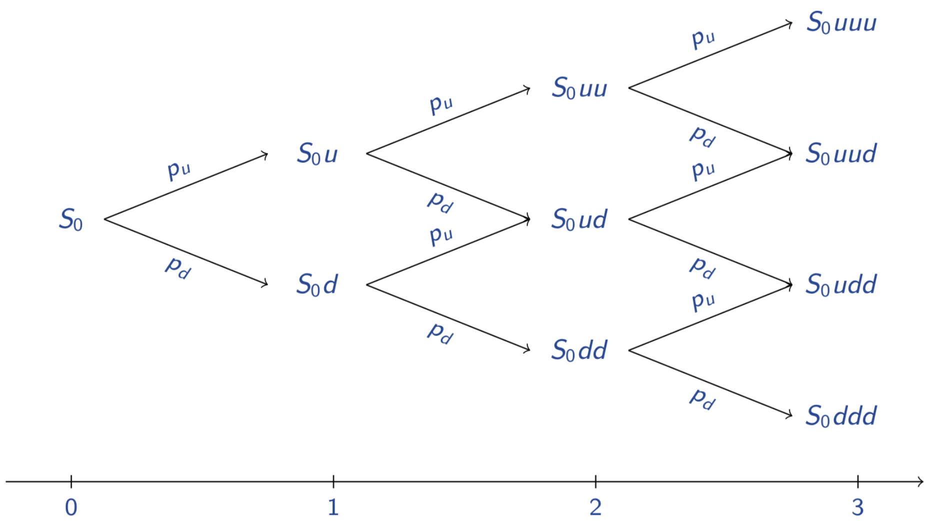
- The dynamics of the **stock process**, denoted by S_t, are

$$S_{t+1} = S_t \cdot Z_t,$$
$$S_0 = s,$$

where $Z_0, \ldots, Z_{T-1}$ are i.i.d. random variables with

$$\mathbb{P}(Z_t = u) = p_u, \qquad \mathbb{P}(Z_t = d) = p_d.$$

A three-period binomial tree:



Binomial tree

Portfolios

- **Definition**: A **portfolio strategy** is a stochastic process

$$\{(x_t, y_t), \quad t = 1, \ldots, T\},$$

  such that h_t is a function of $S_0, S_1, \ldots, S_{t-1}$.

- The **value process** corresponding to portfolio (x, y) is defined by

$$V_t^{(x,y)} = x_t(1 + r) + y_t S_t.$$

- x_t is the amount of money invested at time $t - 1$ and kept until time t.
- y_t is the number of shares bought at time $t - 1$ and kept until time t.
- At any point in time, the portfolio strategy can depend on all information available at that time...
- ... but of course, one cannot look into the future.

Portfolios

- **Definition**: A portfolio strategy (x, y) is said to be **self-financing** if the following condition holds for all $t = 1, \ldots, T - 1$,

$$x_t(1 + R) + y_t S_t = x_{t+1} + y_{t+1} S_t.$$

- This expresses that, at each time t, the market value of the portfolio created at $t - 1$ equals the market value of the portfolio created at t.
- In other words: no funds are injected or withdrawn from the portfolio strategy at times $t = 1, \ldots, T - 1$.
- The portfolio is merely **rebalanced** at every time point, that is, the holdings in the risky asset and risk-free asset are changed subject to keeping the portfolio value constant.

Multi-period model

- Everything else - arbitrage, condition for absence of arbitrage, risk-neutral measure - stays the same.
- One-period risk-neutral pricing formula is a conditional expectation:

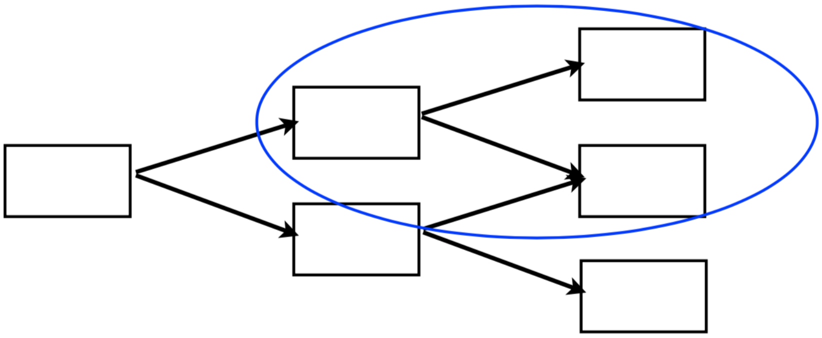
$$S_t = \frac{1}{1 + r} \mathbb{E}^{\mathbb{Q}}[S_{t+1} | S_t].$$

- **Proposition**: The multiperiod binomial model is complete, that is, every claim can be replicated by a self-financing portfolio.

Multi-period model

- Break down multi-period model into one-period models:

- Price and replicate the claim backwards, step-by-step.



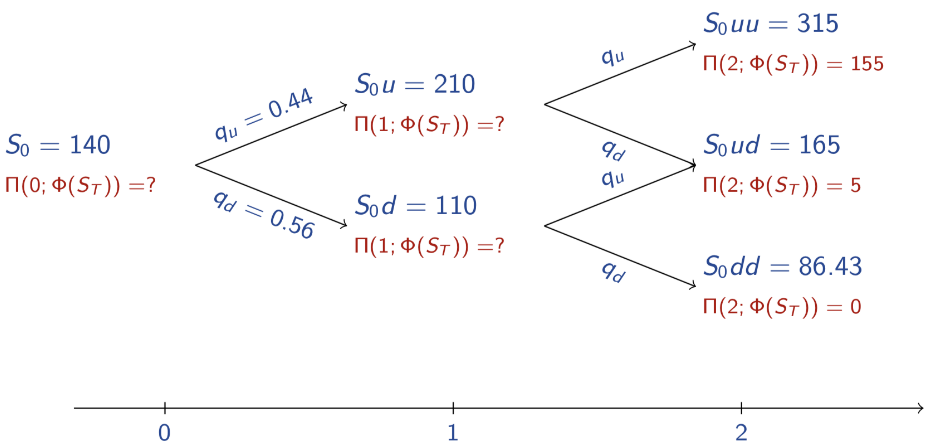
Binomial tree

Example

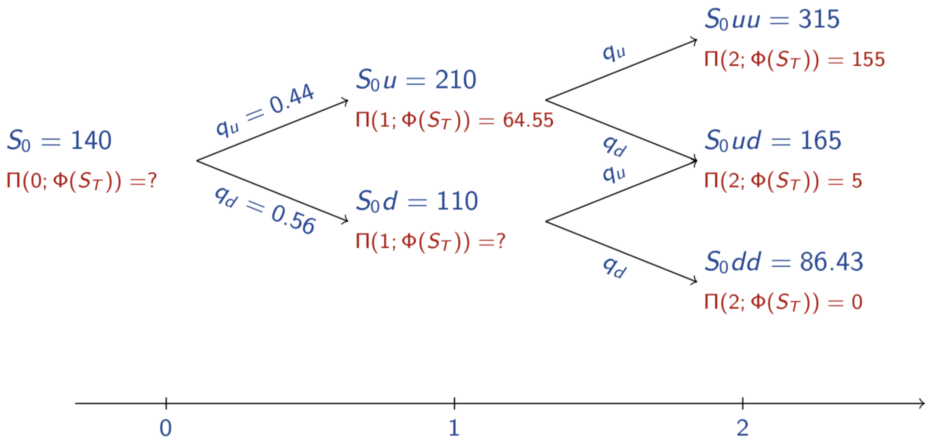
- Spot price $S_0 = 140$
- up and down move factors: $u = 1.5$, $d = 0.78571$
- interest rate $r = 0.1$
- strike $K = 160$
- maturity $T = 2$
- First determine:
  - Risk-neutral probabilities: $q_u = 0.44$, $q_d = 0.56$
  - Contract function:

$$\Phi(S_2) = \max(S_2 - K, 0)$$

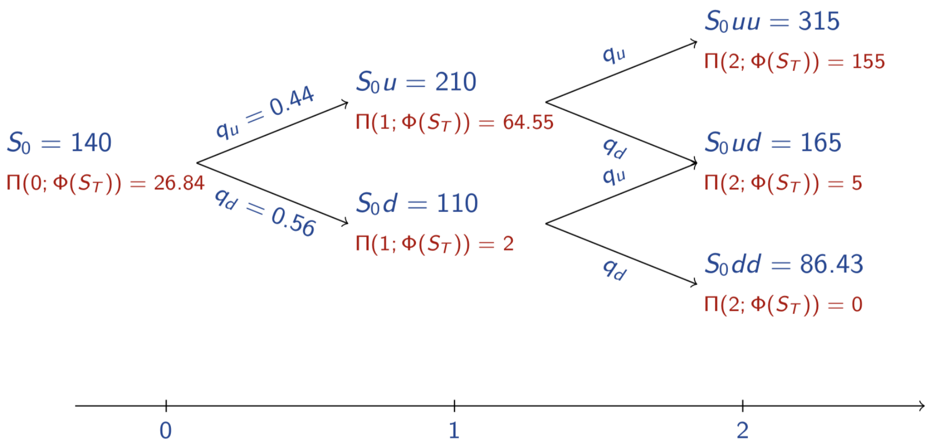
Multi-period model



Binomial tree



Binomial tree



Binomial tree

Multi-period model

Multi-period model

Multi-period model

- **Risk-neutral pricing**:

$$\begin{aligned}
\text{Price}(X) &= \frac{1}{(1+r)^2}\mathbb{E}^{\mathbb{Q}}[\Phi(S_T)] \\
&= \frac{1}{(1+r)^2}\mathbb{E}^{\mathbb{Q}}[q_u^2\Phi(S_0 \cdot u^2) + 2q_u q_d \Phi(S_0 \cdot ud) + q_d^2\Phi(S_0 \cdot d^2)] \\
&= 26.84.
\end{aligned}$$

Multi-period model

- The time-T stock price outcomes follow a **binomial distribution**.
- The price of an option X with contract function Φ is thus:

$$\text{Price}(X) = \frac{1}{(1+r)^T} \cdot \sum_{k=0}^{T} \binom{T}{k} q_u^k \, q_d^{T-k} \Phi(su^k \, d^{T-k}).$$

- Note: This formula applies only to European non-path dependent claims.

Multi-period model

- Let's implement the binomial tree in Python and calculate the price from the example.

[4]:
```
import math

S0=140.
T=2
r=0.1
u=1.5
d=0.78571
```

[5]:
```
def calculate_tree():
    S=np.zeros((T+1, T+1))
    S[0,0]=S0
    z=1
    for t in range(1, T+1):
        for k in range(z): # number of up moves
            S[k+1,t] = S[k, t-1] * u
            S[k,t] = S[k,t-1] * d
        z = z + 1
    return np.flipud(S) # flip the result so that up moves are actually "up"
```

Multi-period model

[6]:
```
np.set_printoptions(formatter = {'float':lambda  x: ' %6.2f ' % x})
```

[7]:
```
calculate_tree()
```

[7]:
```
array([[   0.00 ,    0.00 ,  315.00 ],
       [   0.00 ,  210.00 ,  165.00 ],
       [ 140.00 ,  110.00 ,   86.43 ]])
```

6.4 The Cox-Ross-Rubinstein model

- The **Cox-Ross-Rubinstein model (CRR model)**, is a special case of the multiperiod binomial tree model.
- Fix a time interval $[0, T]$ and set $\Delta t = T/N$.
- Trading takes place at times

$$0, \Delta t, 2\Delta t, \dots, (N-1)\Delta t, N\Delta t = T.$$

- The bond price accrues continuously at rate r, so that $B_t = B_0\,\mathrm{e}^{rt}$ and $B_{n\Delta t} = B_{(n-1)\Delta t}\,\mathrm{e}^{r\Delta t}$, where $n = 1, \dots, N$.
- The risk of the stock price is expressed by the **volatility** σ, the standard deviation of the stock price's log-return.
- The volatility for the time period Δt is $\sigma\sqrt{\Delta t}$, yielding

$$u = \mathrm{e}^{\sigma\sqrt{\Delta t}}, \qquad d = 1/u = \mathrm{e}^{-\sigma\sqrt{\Delta t}}.$$

6.5 Towards a continuous-time model: Brownian motion

- Taking $\Delta t \to 0$ gives the **Black-Scholes model** in the continuous-time limit.
- Note that T stays fixed, but the discrete-time grid is made finer and finer.
- The binomial distribution associated with time-T stock prices converges to a normal distribution (this is a consequence of the so-called Central Limit Theorem).

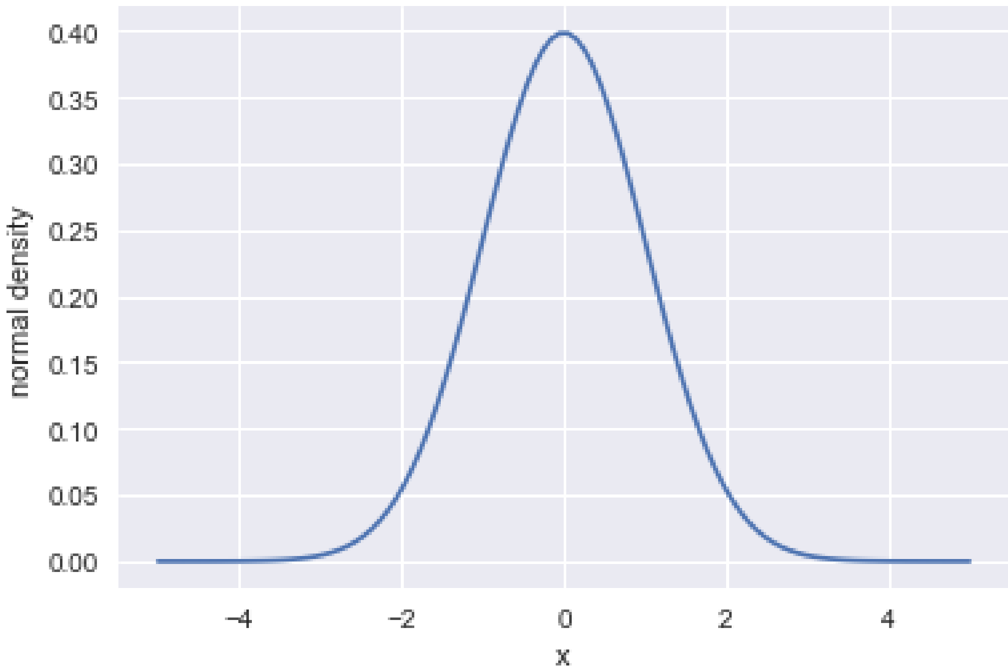
- The up-down moves of the stock price path through the binomial tree converge to a continuous-time stochastic process called a **Brownian motion**.
- This is a process with independent normally distributed increments.

Brownian motion

- A stochastic process $W = (W_t)_{t\geq 0}$ is a **Brownian motion** if
  - $W_0 = 0$ and W has continuous paths,
  - W has independent increments, that is, for $r < s \leq t < u$, the random variables $W_u - W_t$ and $W_s - W_r$ are independent,
  - For $s < t$, $W_t - W_s \sim \mathrm{N}(0, t-s)$, that is, the random variable $W_t - W_s$ is normally distributed with mean 0 and variance $t - s$.

Normal distribution

[8]:
```
import scipy.stats as scs
x=np.linspace(-5,5,100)
plt.plot(x,scs.norm.pdf(x));
plt.xlabel('x')
plt.ylabel('normal density');
```

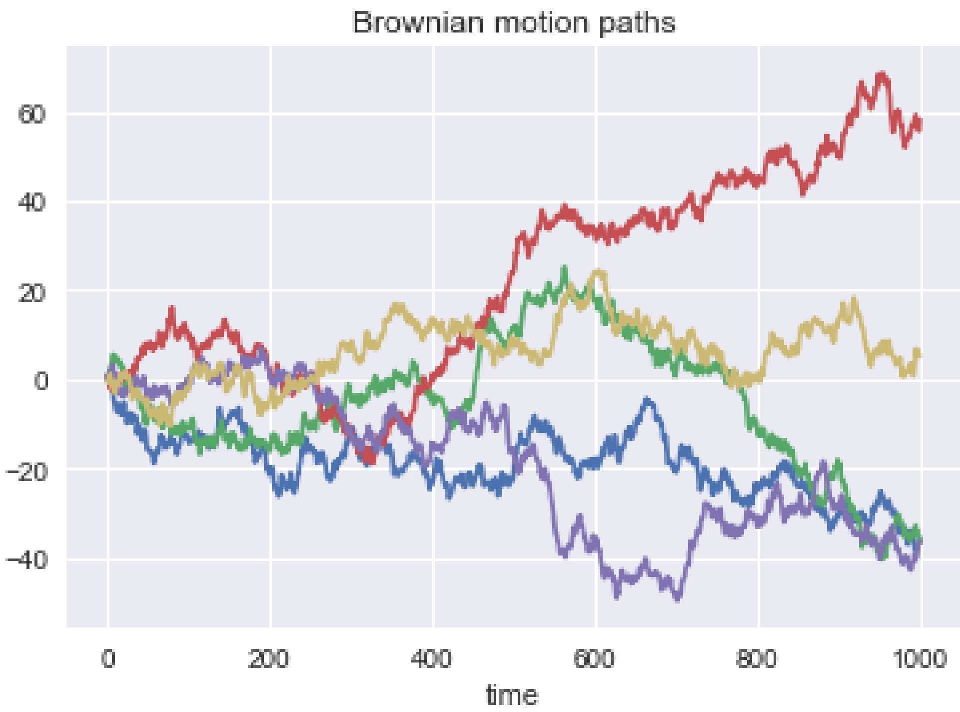


Brownian motion

- Some example paths:

[9]:
```
import numpy.random as npr

npr.seed(1235)
n=1000
z = npr.normal(size=(n,5))
w = np.cumsum(z,axis=0)
plt.plot(w);
plt.xlabel('time')
plt.title('Brownian motion paths');
```



Bond price dynamics

- Continuous compounding at interest rate $r > 0$ of a **bond** or **money market account** is governed by the **differential equation**:

$$dB_t = B_t r\, dt, \quad t \geq 0.$$

- The solution to the differential equation is given by

$$B_t = B_0 \exp(rt), \quad t \geq 0.$$

Stock price dynamics: Geometric Brownian motion

- Similarly, the **Black-Scholes model** specifies the dynamics of a stock price as

$$dS_t = S_t\, \mu\, dt + S_t\, \sigma\, dW_t.$$

- The solution of the differential equation turns out to be

$$S_t = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma W_t\right).$$

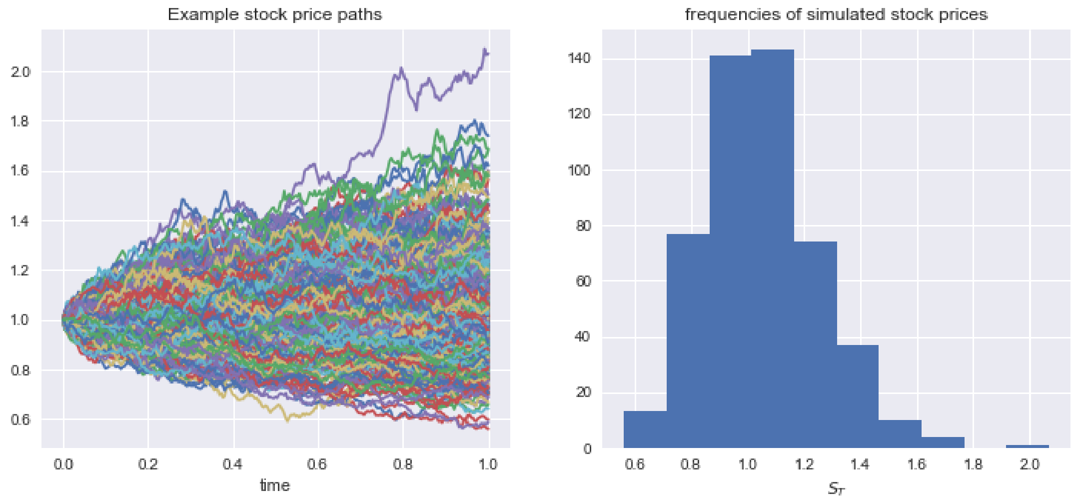
Geometric Brownian motion

[10]:
```
import numpy.random as npr

npr.seed(1235)
n=250
m=500;
dt=1/n;
z2 = npr.normal(size=(n,m))
z = np.zeros((n+1,m))
z[1:]=z2
w = np.cumsum(z,axis=0)
s = np.exp(0.2 * np.sqrt(dt) * w)
det_part = np.exp((0.05-0.5*0.2**2)* dt *np.arange(0,n+1));
for i in range(0,m):
    s[:,i] = s[:,i] * det_part
```

Geometric Brownian motion

[11]:
```
plt.figure(figsize=(12, 5))
plt.subplot(121)
plt.plot(np.arange(0,n+1)*dt,s);
plt.xlabel('time')
plt.title('Example stock price paths');
plt.subplot(122)
plt.hist(s[-1]);
plt.xlabel('$S_T$')
plt.title('frequencies of simulated stock prices');
```



6.6 The Black-Scholes(-Merton) model

- Assume a frictionless financial market in which a bond and a stock are traded.
  - (Assets are liquidly traded, there are no short-selling constraints, there are no transaction costs and there are no bid-ask-spreads.)
- In the **Black-Scholes(-Merton) model** the dynamics of the bond price and the stock price are given by

$$
\begin{aligned}
dB_t &= rB_t\,dt \\
dS_t &= \mu S_t\,dt + \sigma S_t\,dW_t,
\end{aligned}
$$

where * r, μ, σ are deterministic constants, * $W = (W_t)_{t\geq 0}$ is a Brownian motion, * r is the **risk free rate** * μ is the **drift**, * σ is the **volatility**.

Risk-neutral measure

- Because the Black-Scholes model is the result of taking the CRR model to the continuous-time limit, all properties such as the existence of replicating strategies, market completeness, existence of a risk-neutral probability measure carry over.
- The stock price process, under the real-world probability measure $\mathbb{P}$, evolves as

$$dS_t = \mu S_t\,dt + \sigma S_t\,dW_t, \quad t \geq 0.$$

* Under the **risk-neutral probability measure** $\mathbb{Q}$, the stock price process is

$$dS_t = rS_t\,dt + \sigma S_t\,d\overline{W}_t, \quad t \geq 0,$$

where $(\overline{W}_t)_{t\geq 0}$ is a Brownian motion under $\mathbb{Q}$. * Likewise, the bond price process evolves as

$$dB_t = rB_t\,dt, \quad t \geq 0.$$

Risk-neutral pricing

- **Proposition**: The **no arbitrage price** of a contingent claim $Y = f(S_T)$ is given by $\mathrm{e}^{-rT}\mathbb{E}Y$, where expectation is taken under the risk-neutral measure.
- Option pricing boils down to taking expectations under the risk-neutral measure!

6.7 The Black-Scholes formula

- The famous Black-Scholes formula can be proved by calculating the expected value of a call option payoff.

- **Theorem (Black-Scholes formula)**: In the Black-Scholes model, the price of a European call option with parameters K, T, r, σ and S_0 is given by

$$S_0\,\mathrm{N}(d_+) - \mathrm{e}^{-rT}K\,\mathrm{N}(d_-),$$

with

$$d_\pm = \frac{\ln(S_0/K) + (r \pm \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

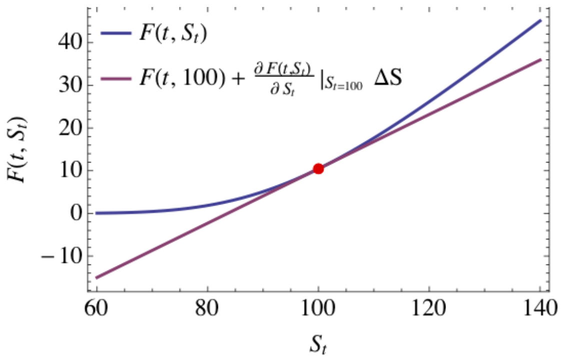
where $\mathrm{N}(x)$ is the cumulative distribution function of the standard normal distribution.

Hedging in the Black-Scholes model

- At any point in time $t \leq T$, the **replicating portfolio** of a European call option maturing at T with time-t price $F(t, S_t)$ consists of
  - $\dfrac{\partial F(t,S_t)}{\partial S} = \mathrm{N}(d_{t,+})$ units of stock and
  - $F(t,S_t) - \dfrac{\partial F(t,S_t)}{\partial S}\,S_t = -\mathrm{e}^{-r(T-t)}K\mathrm{N}(d_{t,-})$ units of the bond.

Hedging in the Black-Scholes model

- The partial derivative is the **sensitivity** of the call option with respect to the underlying asset.
- Small movements ΔS_t in either direction in the asset price change the option price by approximately $\dfrac{\partial F(t,S_t)}{\partial S}\,\Delta S_t$:



Hedginng

Greeks

- The **sensitivities** of an option position are called **"Greeks"**.
- Greeks: The sensitivities of a contingent claim's price process $F(t, S_t)$ with respect to the input parameters are:

$$
\begin{aligned}
\text{Delta:} \quad & \Delta_t = \frac{\partial F(t, S_t)}{\partial S} \\
\text{Gamma:} \quad & \Gamma_t = \frac{\partial^2 F(t, S_t)}{\partial S^2} && \text{(sensitivity of Delta)} \\
\text{Theta:} \quad & \Theta_t = \frac{\partial F(t, S_t)}{\partial t} \\
\text{Vega:} \quad & \nu_t = \frac{\partial F(t, S_t)}{\partial \sigma} && \text{("model risk")} \\
\text{Rho:} \quad & \rho_t = \frac{\partial F(t, S_t)}{\partial r}
\end{aligned}
$$

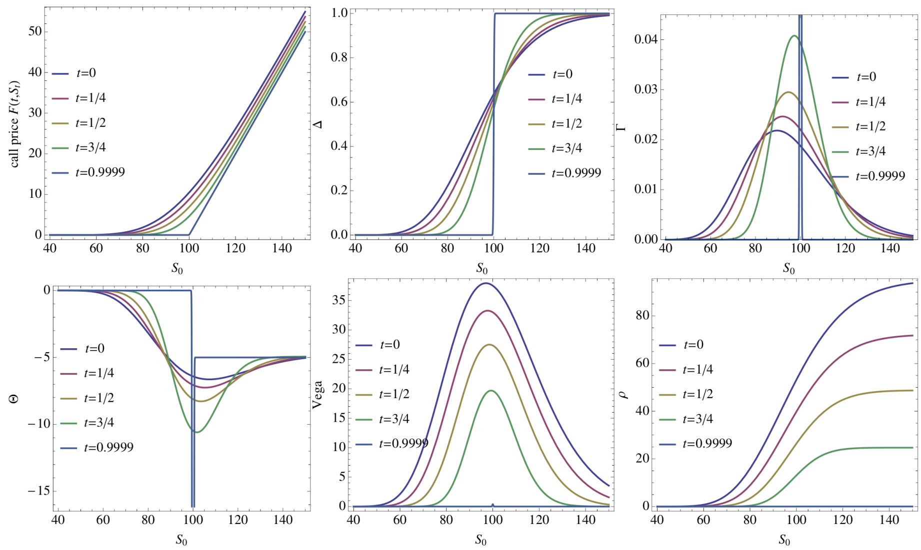
Greeks

- In the case of a call option, the Greeks are given by:

$$
\begin{aligned}
\text{Delta:} \quad & \Delta_t = \mathrm{N}(d_{t,+}) \\
\text{Gamma:} \quad & \Gamma_t = \frac{\mathrm{N}'(d_{t,+})}{S_0 \sigma \sqrt{T}} \\
\text{Theta:} \quad & \Theta_t = -\frac{S_0 \mathrm{N}'(d_{t,+})\sigma}{2\sqrt{T}} - rKe^{-rT}\mathrm{N}(d_{t,-}) \\
\text{Vega:} \quad & \nu_t = S_0 \sqrt{T}\mathrm{N}'(d_{t,+}) \\
\text{Rho:} \quad & \rho_t = Ke^{-rT}\mathrm{N}(d_{t,-})T
\end{aligned}
$$

Greeks

- Call option prices and greeks:


call price $F(t,S_t)$
Δ
Γ
Θ
Vega
ρ
S_0
t=0
t=1/4
t=1/2
t=3/4
t=0.9999

Greeks