Ejercicio 6 de la sección 4.6.6

A:matrix([1,3,−1],[3,4,−2],[−1,−2,2]);

$$\begin{pmatrix} 1 & 3 & -1 \\ 3 & 4 & -2 \\ -1 & -2 & 2 \end{pmatrix}$$

I:ident(3);

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Ma:A−x·I;

$$\begin{pmatrix} 1-x & 3 & -1 \\ 3 & 4-x & -2 \\ -1 & -2 & 2-x \end{pmatrix}$$

factor(determinant(Ma));

$$-(x-1)\left(x^2-6\,x-6\right)$$

solve(%=0);

$$[x=3-\sqrt{15}, x=\sqrt{15}+3, x=1]$$

Ma1:Ma,x=1;

$$\begin{pmatrix} 0 & 3 & -1 \\ 3 & 3 & -2 \\ -1 & -2 & 1 \end{pmatrix}$$

Ma2:Ma,x = 3+sqrt(15);

$$\begin{pmatrix} -\sqrt{15}-2 & 3 & -1 \\ 3 & 1-\sqrt{15} & -2 \\ -1 & -2 & -\sqrt{15}-1 \end{pmatrix}$$

Ma3:Ma,x=3−sqrt(15);

$$\begin{pmatrix} \sqrt{15}-2 & 3 & -1 \\ 3 & \sqrt{15}+1 & -2 \\ -1 & -2 & \sqrt{15}-1 \end{pmatrix}$$

Ma1:Ma1.[x,y,z];

$$\begin{pmatrix} 3\,y-z \\ -2\,z+3\,y+3\,x \\ z-2\,y-x \end{pmatrix}$$

ec1:Ma1[1,1]=0; ec2:Ma1[2,1]=0; ec3:Ma1[3,1]=0;

$$3\,y-z=0$$
$$-2\,z+3\,y+3\,x=0$$
$$z-2\,y-x=0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (3)

$$\left[\left[x=\frac{\%r1}{3}, y=\frac{\%r1}{3}, z=\%r1\right]\right]$$

Ma2:Ma2.[x,y,z];

$$\begin{pmatrix} -z+3\,y+(-\sqrt{15}-2)\,x \\ -2\,z+(1-\sqrt{15})\,y+3\,x \\ (-\sqrt{15}-1)\,z-2\,y-x \end{pmatrix}$$

ec1:Ma2[1,1]=0; ec2:Ma2[2,1]=0; ec3:Ma2[3,1]=0;

$$-z+3\,y+(-\sqrt{15}-2)\,x=0$$
$$-2\,z+(1-\sqrt{15})\,y+3\,x=0$$
$$(-\sqrt{15}-1)\,z-2\,y-x=0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (3)

$$[[x=\sqrt{15}\,\%r2-5\,\%r2, y=2\,\%r2-\sqrt{15}\,\%r2, z=\%r2]$$
$$]$$

Ma3:Ma3.[x,y,z];

$$\begin{pmatrix} -z+3\,y+(\sqrt{15}-2)\,x \\ -2\,z+(\sqrt{15}+1)\,y+3\,x \\ (\sqrt{15}-1)\,z-2\,y-x \end{pmatrix}$$

ec1:Ma3[1,1]=0; ec2:Ma3[2,1]=0; ec3:Ma3[3,1]=0;

$$-z+3\,y+\left(\sqrt{15}-2\right)x=0$$

$$-2\,z+\left(\sqrt{15}+1\right)y+3\,x=0$$

$$\left(\sqrt{15}-1\right)z-2\,y-x=0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (3)

$$[[x=-\sqrt{15}\,\%r3-5\,\%r3,y=\sqrt{15}\,\%r3+2\,\%r3,z=\%r3]]$$

```
***********************************
```

```
*****************************
```

B:matrix([1,0,0],[−3,1,0],[4,−7,1]);

$$\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -7 & 1 \end{pmatrix}$$

I:ident(3);

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Mb:B−x·I;

$$\begin{pmatrix} 1-x & 0 & 0 \\ -3 & 1-x & 0 \\ 4 & -7 & 1-x \end{pmatrix}$$

factor(determinant(Mb));

$$-\left(x-1\right)^3$$

solve(%=0);

$$[x=1]$$

Mb1:Mb,x=1;

$$\begin{pmatrix} 0 & 0 & 0 \\ -3 & 0 & 0 \\ 4 & -7 & 0 \end{pmatrix}$$

Mb1:Mb1.[x,y,z];

$$\begin{pmatrix} 0 \\ -3\,x \\ 4\,x-7\,y \end{pmatrix}$$

ec1:Mb1[1,1]=0; ec2:Mb1[2,1]=0; ec3:Mb1[3,1]=0;

$$0 = 0$$
$$-3\,x = 0$$
$$4\,x-7\,y = 0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (1)

$$[[x=0,y=0,z=\%r4]]$$

****

C:matrix([0,0,1,0],[0,0,0,1],[1,0,0,0],[0,1,0,0]);

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

I:ident(4);

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Mc:C−x·I;

$$\begin{pmatrix} -x & 0 & 1 & 0 \\ 0 & -x & 0 & 1 \\ 1 & 0 & -x & 0 \\ 0 & 1 & 0 & -x \end{pmatrix}$$

factor(determinant(Mc));

$$(x-1)^2\,(x+1)^2$$

solve(%=0);

$$[x=-1,x=1]$$

Mc1:Mc,x=1;

$$\begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

Mc1:Mc1.[x,y,z,w];

$$\begin{pmatrix} z-x \\ w-y \\ x-z \\ y-w \end{pmatrix}$$

ec1:Mc1[1,1]=0; ec2:Mc1[2,1]=0; ec3:Mc1[3,1]=0;

$$z-x=0$$
$$w-y=0$$
$$x-z=0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (3)

$$[[x=\%r5, y=w, z=\%r5]]$$

Mc2:Mc,x=−1;

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Mc2:Mc2.[x,y,z,w];

$$\begin{pmatrix} z+x \\ y+w \\ z+x \\ y+w \end{pmatrix}$$

ec1:Mc2[1,1]=0; ec2:Mc2[2,1]=0; ec3:Mc2[3,1]=0;

$$z+x=0$$
$$y+w=0$$
$$z+x=0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (3)

$$[[x=-\%r6,y=-w,z=\%r6]]$$

****

D:matrix([0,1,0,0],[1,0,0,0],[0,0,0,1],[0,0,1,0]);

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

I:ident(4);

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Md:D−x·I;

$$\begin{pmatrix} -x & 1 & 0 & 0 \\ 1 & -x & 0 & 0 \\ 0 & 0 & -x & 1 \\ 0 & 0 & 1 & -x \end{pmatrix}$$

factor(determinant(Md));

$$(x-1)^2 (x+1)^2$$

solve(%=0);

$$[x=-1,x=1]$$

Md1:Md,x=1;

Md1:Md1.[x,y,z,w];

$$\begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} y - x \\ x - y \\ w - z \\ z - w \end{pmatrix}$$

ec1:Md1[1,1]=0; ec2:Md1[2,1]=0; ec3:Md1[3,1]=0;

$$y - x = 0$$
$$x - y = 0$$
$$w - z = 0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (2)

$$[[x = \%r7, y = \%r7, z = w]]$$

Md2:Md,x=-1;

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Md2:Md2.[x,y,z,w];

$$\begin{pmatrix} y + x \\ y + x \\ z + w \\ z + w \end{pmatrix}$$

ec1:Md2[1,1]=0; ec2:Md2[2,1]=0; ec3:Md2[3,1]=0;

$$y + x = 0$$
$$y + x = 0$$
$$z + w = 0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (2)

$$[[x = -\%r8, y = \%r8, z = -w]]$$

****

E:matrix([1,0,0,0],[0,1,0,0],[0,0,-1,0],[0,0,0,-1]);

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

I:ident(4);

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Me:E−x·I;

$$\begin{pmatrix} 1-x & 0 & 0 & 0 \\ 0 & 1-x & 0 & 0 \\ 0 & 0 & -x-1 & 0 \\ 0 & 0 & 0 & -x-1 \end{pmatrix}$$

factor(determinant(Me));

$$(x-1)^2\,(x+1)^2$$

solve(%=0);

$$[x=-1,x=1]$$

Me1:Me,x=1;
Me1:Me1.[x,y,z,w];

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ -2\,z \\ -2\,w \end{pmatrix}$$

ec1:Me1[1,1]=0; ec2:Me1[2,1]=0; ec3:Me1[3,1]=0;

$$0=0$$

$$0 = 0$$
$$-2\,z = 0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (1 2)

$$[[x = \%r9, y = \%r10, z = 0]]$$

Me2:Me,x=-1;

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Me2:Me2.[x,y,z,w];

$$\begin{pmatrix} 2\,x \\ 2\,y \\ 0 \\ 0 \end{pmatrix}$$

ec1:Me2[1,1]=0; ec2:Me2[2,1]=0; ec3:Me2[3,1]=0;

$$2\,x = 0$$
$$2\,y = 0$$
$$0 = 0$$

solve([ec1,ec2,ec3],[x,y,z]);

solve: dependent equations eliminated: (3)

$$[[x = 0, y = 0, z = \%r11]]$$

****

****

Ejercicio 8 de la sección 4.6.6