Quantum Computing and Post-quantum Cryptography PART 2

André Schrottenloher

Inria Rennes
Team CAPSULE

Summary of Part 1

- Quantum computing (QC) is an enhanced **model of computation**
- ...in which **(only)** some problems admit significant **speedups**

Discrete Logarithms and factoring are solved in **polynomial time** with a large-scale QC.

$\Longrightarrow$ cannot be used as crypto "hard problems" anymore

- Unfortunately, this is (almost) **all** the currently deployed public-key crypto

- Large-scale QCs **do not yet exist**
- The commercial impact of QC is overhyped, but the crypto threat is real

Outline

Preliminaries

Some history

- In 1976, Diffie and Hellman invent the principle of **public-key cryptography** and the **DH key-exchange mechanism**
- In 1978, Rivest, Shamir and Adleman invent the **RSA public-key cryptosystem**

  (British military cryptographers already knew about that, but all of their work remained classified until 1997)

But there's more to the story ...

Diffie, Hellman, "New directions in cryptography", IEEE transactions on information theory, 1976

Rivest, Shamir, Adleman, "A method for obtaining digital signatures and public-key cryptosystems", Commun. ACM 21.2, 1978

The McEliece cryptosystem (1978)

A Public-Key Cryptosystem Based On Algebraic Coding Theory

R. J. McEliece
Communications Systems Research Section

Using the fact that a fast decoding algorithm exists for a general Goppa code, while no such exists for a general linear code, we construct a public-key cryptosystem which appears quite secure while at the same time allowing extremely rapid data rates. This kind of cryptosystem is ideal for use in multi-user communication networks, such as those envisioned by NASA for the distribution of space-acquired data

- The scheme is based on error-correcting codes
- The main drawback is the **public key size**: Megabits vs. Kilobits for RSA
- But it **resists quantum attacks!**

Post-quantum cryptography predates the theory of quantum computing!

McEliece, "A public-key system based on algebraic coding theory", DSN Progress Report 44, Jet Propulsion Lab, 1978

What we need

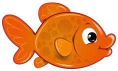
To secure the internet, we need:

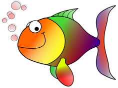
- Public-key encryption $\Longrightarrow$ **more urgent**
- Digital signatures $\Longrightarrow$ **less urgent**
- Secret-key (authenticated) encryption

  $\Longrightarrow$ should be (mostly) OK

Public-key encryption (PKE)

$$\begin{cases} \mathsf{KeyGen}: & 1^\lambda & \mapsto & \mathsf{sk}, \mathsf{pk} \\ \mathsf{Encrypt}: & \mathsf{m}, \mathsf{pk} & \mapsto & \mathsf{c} \\ \mathsf{Decrypt}: & \mathsf{c}, \mathsf{sk} & \mapsto & \mathsf{m} \end{cases}$$





$\mathsf{sk}, \mathsf{pk} = \mathsf{KeyGen}(1^\lambda)$

$$\xrightarrow{\quad \mathsf{pk} \quad}$$

$\mathsf{c} = \mathsf{Encrypt}(\mathsf{m}, \mathsf{pk})$

$$\xleftarrow{\quad \mathsf{c} \quad}$$

$\mathsf{m} = \mathsf{Decrypt}(\mathsf{c}, \mathsf{sk})$

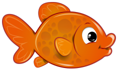
Typical attacks:

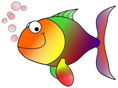
- key security: recover sk from pk
- message security: recover m from c

Key encapsulation mechanism (KEM)

$$\begin{cases} \text{KeyGen}: & 1^\lambda & \mapsto & \text{sk}, \text{pk} \\ \text{Encaps}: & \text{pk} & \mapsto & \text{c}, \text{k} \\ \text{Decaps}: & \text{sk}, \text{c} & \mapsto & \text{k} \end{cases}$$

$\text{k} \in \mathcal{K}$ is to be used as a symmetric secret key.





$\text{sk}, \text{pk} = \text{KeyGen}(1^\lambda)$

$$\xrightarrow{\quad\text{pk}\quad}$$

$\text{c}, \text{k} = \text{Encaps}(\text{pk})$

$$\xleftarrow{\quad\text{c}\quad}$$

$\text{k} = \text{Decaps}(\text{c}, \text{sk})$

Typical attacks:

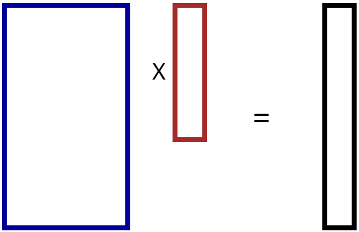
- key security
- session key security: recover k from c

Lattice-based Cryptosystems

Lattice-based crypto: summary

- At the moment, three lattice-based schemes are standardized (or on the way) by NIST: **Kyber** (ML-KEM), **Dilithium** (ML-DSA), **Falcon** (FN-DSA)
- Lattice-based schemes are solid and reach small parameter sizes (PK, SK, ciphertext)
- Compared to RSA, still **doubling** or **quadrupling** the PK size for equivalent security
- Also, implementing these schemes is harder than RSA, **but** the runtime is typically faster

A bad cryptosystem (do not use it!)



- Choose a public matrix $A \in \mathbb{Z}_q^{\ell \times n}$ at random
- Choose $s \in \mathbb{Z}_q^n$ at random: our private key
- Let (A, As) be our public key

Still a bad cryptosystem (do not use it!)

KeyGen:

- Private key: random $s \in \mathbb{Z}_q^n$
- Public key: random matrix A, $b := As$

Encrypt $m \in \{0,1\}$:

- Pick a random vector $r \in \{0,1\}^\ell$
- Return $c_1, c_2 := rA, (m + r \cdot b)$

Decrypt $(c_1, c_2) \in \mathbb{Z}_q^{n+1}$:

- Return $m = c_2 - c_1 \cdot s$

$$
\begin{aligned}
c_2 - c_1 \cdot s &= (m + r \cdot b) - (rA)s \\
&= m + (r \cdot b) - r(\underbrace{(As)}_{=b}) = m
\end{aligned}
$$

Why is this broken?

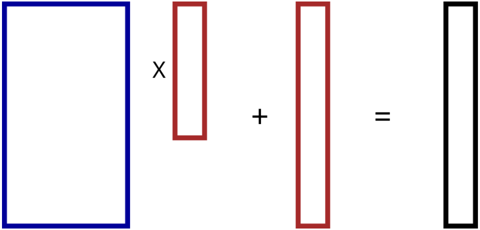
Let's do a Chosen-plaintext attack and always encrypt 0. We observe **samples:**

$$\mathrm{r}\mathrm{A}, (\mathrm{r}\mathrm{A}) \cdot \mathrm{s} \tag{1}$$

for unknown r and s.
After enough samples we have $\boldsymbol{R}, \boldsymbol{R}\mathrm{s}$: invert $\boldsymbol{R}$ to find s.

Linear algebra is not enough for crypto. We need another ingredient.

Learning with errors (LWE)



- Choose a public matrix $A \in \mathbb{Z}_q^{\ell \times n}$ at random
- Choose $s \in \mathbb{Z}_q^n$ at random **and** a "small" error $e \in \mathbb{Z}_q^\ell$
- Public key: $A, b := (As + e)$

Search: find s. **Decision:** distinguish the output from uniform.

We have good reasons to believe that LWE is hard*.

* *Quantum* reduction from average-case LWE to worst-case lattice problems.

Regev, "On lattices, learning with errors, random linear codes, and cryptography", STOC 2005

LWE: encryption scheme

Define:

- **Compress**: decodes an integer mod q into 0 if it's closer to 0 or 1 if it's closer to $q/2$
- **Decompress**: encodes 0 to 0 and 1 to $q/2$

KeyGen:

- Private key: random $s \in \mathbb{Z}_q^n$
- Public key: random matrix A, $b := As + e$ with e "small"

LWE: encryption scheme

Encrypt $m \in \{0,1\}$:

- Pick a random vector $r \in \{0,1\}^{\ell}$
- Return $c_1, c_2 := rA, (\mathrm{Decompress}(m) + r \cdot b)$

Decrypt $(c_1, c_2) \in \mathbb{Z}_q^{n+1}$:

- $m = \mathrm{Compress}(c_2 - c_1 \cdot s)$

Why this works:

$$\begin{aligned} c_2 - c_1 \cdot s &= (\mathrm{Decompress}(m) + r \cdot b) - (rA) \cdot s \\ &= \mathrm{Decompress}(m) + r(As + e) - rAs \\ &= \mathrm{Decompress}(m) + \underbrace{r \cdot e}_{\text{Small}} \end{aligned}$$

The decryption may fail with some insignificant probability (e.g., 2^{-160} in Kyber according to the designers)

Limits of “basic” LWE

- The ciphertext is big: $\mathcal{O}(n)$ bits for only one bit encrypted
- The public key is big (around $\mathcal{O}(n^2)$ bits for n-bit security)
- The computation time is big (around $\mathcal{O}(n^2)$ for a matrix-vector product)

More algebra fixes it: we replace **matrices** over $\mathbb{Z}_q$ by **polynomials**.

Polynomial rings

Fix an integer n, and let: $\mathcal{R} = \mathbb{Z}[X]/(X^{\mathsf{n}} + 1)$, i.e., polynomials where "$X^{\mathsf{n}} = -1$".

$$\boldsymbol{a} = (a_0, \ldots, a_{\mathsf{n}-1}) \leftrightarrow \boldsymbol{a}(X) = a_0 + a_1 X + \ldots + a_{\mathsf{n}-1} X^{\mathsf{n}-1}$$

$\implies$ after all operations (addition, product ...) simply divide the polynomials by $X^{\mathsf{n}} + 1$.

- We can also define such polynomials mod q: $\mathcal{R}_{\mathsf{q}} := \mathbb{Z}_{\mathsf{q}}[X]/(X^{\mathsf{n}} + 1)$

Ring-LWE

- Let's agree on a **distribution of polynomials** $D_{\mathcal{R}}$ with **small coefficients**

- Public $\mathsf{a} \in \mathcal{R}_\mathsf{q}$ chosen u.a.r.
- Secret $\mathsf{s}, \mathsf{e} \in \mathcal{R}_\mathsf{q}$ chosen with $D_{\mathcal{R}}$
- Given $\mathsf{b} = \mathsf{a} \cdot \mathsf{s} + \mathsf{e} \pmod{\mathsf{q}}$, find s (search) or distinguish from random (decision)

With this:

- We have n coefficients (vector a, b) instead of n^2 for a problem of size n!
- We will be able to encrypt n bits instead of 1
- We accelerate the encryption / decryption to $\mathcal{O}(\mathsf{n} \log \mathsf{n})$ operations

Kyber actually uses Module-LWE, which is another variant.

Ring-LWE as in Kyber

KeyGen:

- Private key: $s \in \mathcal{R}_q, e \in \mathcal{R}_q$ sampled using $D_{\mathcal{R}}$ (i.e., small)
- Public key: random $a \in \mathcal{R}_q$, and $b := a \cdot s + e$

Encrypt $m \in \{0,1\}^n$ (as an element of $\mathcal{R}_q$)

- $r \xleftarrow{D_{\mathcal{R}}} \mathcal{R}_q$, $e_1 \xleftarrow{D_{\mathcal{R}}} \mathcal{R}_q$, $e_2 \xleftarrow{D_{\mathcal{R}}} \mathcal{R}_q$
- $c_1 := a \cdot r + e_1$
- $c_2 := b \cdot r + e_2 + \mathrm{Decompress}(m)$
- Return (c_1, c_2)

Decrypt (c_1, c_2)

- $m = \mathrm{Compress}(c_2 - s \cdot c_1)$

$$
\begin{aligned}
c_2 - s \cdot c_1 &= b \cdot r + e_2 + \mathrm{Decompress}(m) - s \cdot (a \cdot r + e_1) \\
&= s \cdot a \cdot r + e \cdot r + e_2 + \mathrm{Decompress}(m) - s \cdot a \cdot r - s \cdot e_1 \\
&= \underbrace{e \cdot r + e_2 - s \cdot e_1}_{\text{Small}} + \mathrm{Decompress}(m) \ .
\end{aligned}
$$

Other Families

Summary

The post-quantum schemes are classified depending on (the objects underlying) their **hardness assumptions**.

Lattices

- SVP/SIS in random or structured lattices
- Mature, upcoming standards

Error-correcting codes

- Decoding random-looking codes
- Larger public key sizes
- Another KEM expected for standardization

Summary

Multivariate systems

- Solving random-looking mulivariate equation systems
- Give quite good signatures (you can expect one standard in the coming years)

Others

- Elliptic curve isogenies: small parameters & heavy computations, not mature
- Hash-based signatures: already standardized (SPHINCS+)
- ...

Code-based crypto

An **error-correcting code** is a way to encode information with redundancy, ex.:

Charlie, **O**scar, **D**elta, **E**cho

Formally we encode words of dimension k over $\mathbb{F}_2$ into codewords of dimension $n > k$ using a vector space defined by an $k \times n$ matrix.

Decoding problems:

- Given a noisy codeword $c = mG + e$ ("small" $e = t$ bit flips), find the original word m
- Given an error syndrome $s = He$, find the weight-t vector e

Decoding random codes is a hard problem

The McEliece scheme

Use a family of codes $\mathcal{F}$ that admit an efficient decoding algorithm, and a procedure to generate such codes:

$$\begin{cases} \mathcal{S} \to \mathcal{F} \\ s \mapsto \mathcal{C}(s) \end{cases}$$

KeyGen:

- Alice picks a secret key $\mathsf{s} \in \mathcal{S}$
- Reveal a parity-check matrix H of the code $\mathcal{C}(\mathsf{s})$

Encrypt: e

- Send $\mathsf{c} = \mathsf{He}$

Decrypt:

- Recover e using a fast decoding algorithm

Bonus: Secret-key Crypto

Secret-key cryptography

Secret-key cryptography seems mostly* secure against QCs.

More precisely:

- in theory, we **could** have Shor-like speedups on classically secure ciphers / hash functions
- but these constructions are only theoretical

* Up to the cryptanalysis we tried so far

Yamakawa, Zhandry, "Verifiable Quantum Advantage without Structure", FOCS 2022

Example

The AES-256 block cipher: encrypts 128-bit “blocks” using a 256-bit secret key.

Security of an “ideal” block cipher:

- **classical:** try all the keys (number of keys $= 2^{256}$)
- **quantum:** use Grover’s algorithm

$\Longrightarrow$ generic reduction in security, but a manageable one: $2^{256/2} = 2^{128}$ remains infeasible

Is that all we can do? Nobody knows. We must try to cryptanalyze.

Grover, “A fast quantum mechanical algorithm for database search”, STOC 1996

At the moment

Block cipher key-recovery:

- generic: $T \to T^{1/2}$
- attacks have speedups between T^1 and $T^{1/2}$
- on **specific** (but realistic) designs: between $T^{1/2}$ and $T^{2/5}$

Hash function collision:

- generic: $T \to T^{2/3}$
- attacks have speedups between T^1 and $T^{1/2}$

Hash function preimage:

- generic: $T \to T^{1/2}$
- attacks have speedups between T^1 and $T^{1/2}$

Only in the "store now, decrypt later" attacker model. (There are weirder models.)

The NIST Process

The NIST process

Follows a long tradition of competitions: AES, SHA-3, CAESAR...
It's not only an American thing: NIST standards are *de facto* world standards.

- Bring together all researchers in the field
- Many teams propose their designs
- Let the best win!

https://csrc.nist.gov/projects/post-quantum-cryptography

The NIST process (ctd.)

- No one is safe from cryptanalysis
- Many (most?) of the candidates will be terribly broken
- Including some of your favorite

The NIST process (ctd.)

82 submissions for **KEMs** and **digital signatures**: $\simeq 64$ in the first round.

KEMs	Lattice	Codes	Multivariate	Other
Round 1 (2017)	21	17	2	5
Round 2 (2019)	9	7	0	0
Round 3 (2020) (finalists)	3	1	0	0
Round 3 (2020) (alternate)	2	2	0	1[a]
First standards (2022)	1[b]	0	0	0
Round 4 (2022)	0	3	0	0

a SIKE was broken

b CRYSTALS-Kyber based on Module-LWE

The NIST process (ctd.)

Signatures	Lattices	Codes	Multivariate	Other
Round 1	5	3	9	4
Round 2	3	0	4	2
Round 3 (2020) (finalists)	2	0	1[a]	0
Round 3 (2020) (alternate)	0	0	1[b]	2[c]
First standards (2022)	2[d]	0	0	1

a Rainbow was broken

b GeMSS was broken

c SPHINCS+ and Picnic (only SPHINCS+ survived)

d CRYSTALS-Dilithium and Falcon

There is no round 4 because no one else survived ☹

Thoughts on the NIST process

We learned a lot.*

- Lots of research is still ongoing on the **construction** side: many constructions are **not mature**
- NIST has made rather **conservative** choices, which were **justified**
- Even for the selected standards, lots of (applied) research remains on **secure implementations** and **integration**

* Euphemism: many hopes were broken, many tears were shed.

Bonus track: the NIST PQC-DS process

In Sept. 2023 NIST launched an additional call for signatures.

	Codes	Isogenies	Lattices	MPCitH	Multivar	Symm.	Other
Round 1	6	1	7	7	10	4	5
Round 2	2	1	1	5	4	1	0

With no less than **10** candidates broken during round 1.

Conclusion

On post-quantum crypto

- Even mature solutions need work on implementations and hardware
- Many alternatives, but many of them are not mature (i.e., should not be used in production)

Don't roll your own crypto, post-quantum edition:

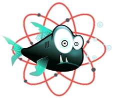
- many designs were broken in the NIST process
- even those of teams with high expertise

$$\text{Security} = \int_0^t \text{cryptanalysis effort } dt$$

Recommendations

- National agencies like ANSSI (France) or BSI (Germany) have made recommendations for post-quantum crypto & transition timelines
- Some of them may recommend schemes which are not NIST standards, but were put to the test (e.g., FrodoKEM)
- Most of them recommend **hybrid encryption** (pre + post-quantum) in the near future

Thank you!



BSI, "Quantum-safe cryptography - fundamentals, current developments and recommendations",

"ANSSI Views on the Post-Quantum Cryptography transition" March 25, 2022

TNO, CWI, AIVD, "The PQC migration handbook", 2023