Mathematical Implementation of a Gradient Descent Algorithm in a VHDL implemented MLP

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1 Introduction to the Algorithm

The gradient descent algorithm is a simple and efficient method to achieve the minimum value of a given function. Given a function $f(x)$, the gradient descent algorithm starts at a point x_0 and iteratively proceeds in the direction of the negative gradient of f at x_0, $-\nabla f(x_0)$, until it reaches a target point where the calculated gradient is zero. The gradient descent algorithm can be written as follows:

$$x_{n+1} = x_n - \alpha \nabla f(x_n)$$
$$\text{where } \alpha > 0 \text{ is a step size.}$$

The gradient descent algorithm is guaranteed to converge to a local minimum of f, provided that f is a convex function.

2 Example Function: $f(x) = x^2$

Consider the function $f(x) = x^2$. The gradient of the given is $\nabla f(x) = 2x$, and the gradient descent algorithm is derived as the following:

$$\begin{aligned} x_{n+1} &= x_n - \alpha \nabla f(x_n) \\ &= x_n - \alpha(2x_n) \\ &= (1 - 2\alpha)x_n. \end{aligned}$$

This algorithm will converge to the point $x = 0$ regardless of the starting point x_0, given that the following condition is satisfied; $\alpha < \frac{1}{2}$.

3 Application and Objective

The presented gradient descent algorithm will be applied to an MLP (multi layer perceptron) designed in VHDL. The objective is to achieve a functional learning step for the designed MLP.