ECE 316 – Lecture Notes Chapters 1-3

# Chapter 1: Combinatorial Analysis

## The Basic Principle of Counting

There are ways to order objects.

Example: A small community has 10 women, who each have 3 children. If one woman and one of her children are to be chosen as mother and child of the year, how many different choices are there?

What if any woman and any child could be chosen?

Example: How many 7-place license plates are possible if the first three places are letters and the rest are numbers?

What if no letters or numbers could be repeated?

## Permutations

There are permutations of objects where objects are alike with respective repetitions.

Example: How many different ordered arrangements of , , and are possible?

Example: How many ways can you put 10 books on the bookshelf, where 4 are math, 3 are chemistry, 2 are history, and 1 is language, and books of the same subject must be put together?

Example: A class consists of 6 men and 4 women. An exam is given and no students obtain the same score.

1. How many different class rankings are possible?
2. If men and women are ranked separately, how many rankings are possible?

Example: 6 men and 4 women are to form a committee of 4 people, 2 men and 2 women. Man A doesn’t want to be in the same committee as Woman B. How many different ways can the committee be formed?

Example: How many letter arrangements can be formed from the word ?

Example: How many different ways can 9 flags be hung in a line, with 4 white, 3 red, and 2 blue?

## Combinations

* Either choose item A and choose items from remaining items, or don’t choose item A and choose items from items

Example: How many different groups of 3 can be selected from the items A, B, C, D, E?

Example: antennas with defective, what is the probability that the system is functional, if it is functional as long as there are no two consecutive defective antennas?

Example: How many subsets are there of a set containing elements, ?

## Binomial Theorem

* For each of the terms, choose either or
* Each term has total ’s and ’s – choose ’s and remaining are ’s

## Multinomial Coefficients

Example: distinct items are to be divided into distinct groups of sizes respectively, where . How many divisions are possible?

Example: 10 children are to be divided into 2 basketball teams, each 5 players, to play against each other. How many divisions are possible?

Example: 10 children are to be divided into 2 basketball teams, each 5 players; one team will play at the provincial level, the other at the national level. How many divisions are possible?

## Multinomial Theorem

To divide items into distinct groups of :

# Chapter 2: Axioms of Probability

## Sample Space and Events

* – **sample space**: the set of all possible outcomes of an experiment
* – **event**: any subset of a sample space
  + – **complement**: all elements not in
  + – **union** of two events
  + – **intersection** of two events
    - If then the events are **disjoint**

**Commutative Laws:**

**Associative Laws:**

**Distributive Laws:**

## DeMorgan’s Laws

## Axioms of Probability

1. , for any event
2. , for any sample space
3. For any sequence of disjoint events

## Some Simple Propositions

Proof:

1. If , then

Proof:

## Inclusion-Exclusion Identity

Example: Mike is taking courses and . With probability 0.8, he will like . With probability 0.7, he will like . With probability 0.6, he will like both. What is the probability that he will like neither?

## Sample Space with Equally Likely Outcomes

If a sample space has equally likely outcomes, then the probability of each outcome is

Then for an event , the probability is:

Example: If 3 balls are drawn randomly from a bowl of 6 white and 5 black, what is the probability that one is white and the two others are black?

Example: In a room of people, what is the probability that no two people celebrate their birthday on the same day of the year?

Example: 52 playing cards are randomly divided into 4 piles of 13 cards each. What’s the probability that each pile has exactly 1 ace?

# Chapter 3: Conditional Probability and Independence

## Conditional Probability

The conditional probability that an event happens, given that an event happened, is:

Example: A student is taking an exam with an one-hour time limit. Suppose the probability that the student will finish the exam in less than hours is , for all . Given that the student is still working after 0.75 hours, what is the probability that the full hour is used?

## Chain Rule

## Baye’s Formula

For any two events and :

Example: People are either accident-prone or not accident-prone. Accident-prone people have probability 0.4 of having an accident in the first year, while not accident-prone people have probability 0.2.

1. Assume 30% of the population is accident-prone. What is the probability that a new policy-holder will have an accident within the first year?
2. Suppose that the new policy-holder gets in an accident in the first year. What is the probability that he is accident-prone?

## Independent Events

If , then this implies:

If and are independent events, then so are and .

Proof:

Example: An infinite sequence of independent trials is to be performed. Each trial results in a success with probability and a failure with probability . What is the probability that:

1. At least one success occurs in the first trials

1. Exactly successes occur in the first trials
2. All [infinite] trials result in success
3. At least successes occur in the first trials